

# Monetary Policy Credibility and the Term Structure of Interest Rates

Preliminary\*

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## Abstract

We investigate the impact of monetary policy credibility on the term structure of interest rates. By credibility we mean the ability of the monetary authority to commit to future policy and fulfill her promises. We find that credibility steepens the yield curve by increasing term premia. This is because it allows to optimally smooth shocks over time, by shifting the effect of current shocks to future consumption and so increasing autocorrelation of consumption. Because of habits, higher autocorrelation of consumption makes marginal utility less volatile at long, relatively to short forecasting horizons. It follows that agents ask a higher compensation for buying long term debt, rather than rolling over short-run investments.

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# 1 Introduction

In this paper we investigate the implications for the term structure of interest rates of monetary policy credibility.

On the one hand, the term structure of interest rates plays a prominent role in macroeconomics. Not only the slope of the yield curve has proved to be an informative business cycle leading indicator: a flattening of the curve predicts recessions, a steepening predicts expansions. Also, monetary policy effectiveness relies on the ability to affect the path of future expected interest rates, and then the whole term structure, rather than their current level only.

On the other hand, credibility plays a prominent role in optimal monetary policy literature. The optimal policy prescribes to smooth the impact of shocks over time. Such a smoothing allows to contain the volatility of both output gap and inflation and can be achieved only by credibly committing to the optimal interest rate path. If the central bank lacks the credibility to effectively use expectations as a policy instrument, she also has to accept higher volatility of inflation and output gap. This does not mean that expected rates and yields do not react to shocks, simply they are not under the direct control of the monetary authority. Clarida, Galí and Gertler (1999) and Woodford (2003) have forcefully emphasized the stabilization gains from commitment, as opposed to a discretionary regime.

It is worth stressing that, though subsuming the importance of establishing sound and clear policy objectives, this view advocates something more than adopting an inflation target. It also requires that the central bank effectively signals future policy intentions, fulfill her promises and the private sector believes them. This is what we mean by credibility. For a thorough discussion of the link between credibility and stabilization gains from commitment, see Woodford (2005).

Given that credibility allows to achieve better stabilization outcomes by controlling interest rate expectations and yields *and* by smoothing risk over time, we think it is interesting to know whether and how a credibility improvement affects the term structure. In fact, it has been showed by Den Hann (1995) and Hordal, Tristani and Vestin (2008) among others that term premia crucially depend on the expected volatility of marginal utility at long relative to short forecasting horizon. The intuition of that result is straightforward and is connected to precautionary savings: agents would like to save by going short so as to hedge consumption risk. As an implication, a credibility improvement may well have an impact on the slope of the yield curve by smoothing the shocks over the cycle and affecting how they propagate at different horizons. Monetary policy is commonly thought to move the yield curve more at the short rather than at the long-end and hence its slope. Evans and Marshall (1998) make this point. Our argument, though related, is different. We do not focus indeed on monetary policy shocks, but we investigate the effects of the policy regime, i.e. the way the central bank systematically reacts to shocks and affects aggregate risk.

We answer this question by computing the yield curve implied by the optimal

monetary policy, under both regimes of discretion and commitment. We choose and parameterize the simplest possible model delivering monetary non neutrality, sizeable term premia and time inconsistency of the solution to the Ramsey planner. To this purpose, we assume nominal price stickiness, habits in consumption<sup>1</sup> and wage mark-up shocks. We interpret as a credibility improvement a switch from discretion to commitment.

We find that the unconditional mean of the term spread, that is the difference between short and long maturity yields, is higher under commitment than under discretion. Therefore, a credibility improvement makes the term structure steeper. Commitment *and* habits are responsible for the result. In fact, committed monetary policy smoothes the impact of shocks over time. As a consequence, though the economy is less prone to aggregate risk, part of the impact of current shocks is transferred to the future, destabilizing expected consumption far ahead. Still, the variance of marginal utility is dampened, because higher consumption persistence dampens consumption changes too. Consumption changes indeed are exactly what matters to households, when they form habits and only care about current relatively to past consumption. It follows that agents ask higher compensation for holding long term maturities rather than rolling over short run investments.

Our result has some interesting implications. First, improved monetary policy credibility and communication cannot account for the flattening of the yield curve observed over recent years. As pointed out by Backus and Wright (2007), a downward trend in (real) term premia seems to be responsible for the decrease in the term spreads. The explanation of such a change cannot rely on a switch in the monetary policy regime. Second, a flattening of the yield curve *ceteris paribus* signals worse, rather than better, monetary policy. Obviously, this is not a claim about observed monetary policy, since we don't identify whether some structural change has occurred. And looking at the lively debate about the Great Moderation, this would not certainly be an easy task. However, we believe that our analysis sheds some light on the yield curve implications of the *optimal* policy, as advocated by the currently widespread theoretical literature.

It may well be the case that establishing an inflation target accounts for a flattening of the yield curve, as convincingly argued by Gurkaynak, Sack and Swanson (2005). But this argument is complementary, not alternative to ours. Our point, rather than positive, is normative and tightly linked to an evaluation of monetary policy, based on welfare theoretic grounds. Importantly, such an evaluation assigns to policy the crucial role of shaping expectations so as to improve stabilization trade-offs. *Without* necessarily requiring full stabilization of inflation around the target. Whether observed monetary policy is reaping the benefits of credibility or not would require an empirical investigation that is

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<sup>1</sup>Alternative explanations to habits have been offered to account for sizeable term and risk premia in DSGE models. One possibility is departure from rational expectations, as pointed out by Cogley and Sargent (2008). Another is the "rare disaster" explanation put forward by Barro (forthcoming). Stochastic volatility and regime shifts in the law of motion of shocks have been considered. See for instance Amisano and Tristani (2008).

beyond the scope of this paper.

Our contribution is strictly related to and borrows heavily from a growing part of the monetary policy literature, showing that business cycle models augmented with real and nominal frictions are able to match the yield curve and asset pricing evidence. A representative, though not exhaustive, sample is Hordal et al. (2008), Ravenna and Seppala (2007), Giannitsarou and Challe (2009), De Paoli and Zabczyk (2009). Differently from these authors, rather than assuming a monetary policy rule, we compute optimal monetary policy.

It is well known that while helping business cycle models to match the evidence on term and risk premia, habits generate counterfactually high volatility of interest rates. In addition, a recent paper by Rudebusch and Swanson (2008) highlights that habits also boost volatility of all other endogenous variables, such as consumption and hours worked. We show that our optimal policy model is not subject to this critique, though interest rate volatility is over-predicted. This is because we face an additional hurdle with respect to models with habits and ad-hoc policy rules: the zero lower bound of the nominal interest rate is binding under the optimal policy. Hence, a proper solution method should explicitly take care of the bound and limit the volatility of the policy instrument. We follow most of the contributions in the field by disregarding the problem. We can defend ourselves only by arguing that we have a reason more to do it, as for our purposes we need to solve the model non-linearly via a standard collocation method. The reason is purely technical. Not only we want to accurately compute first moments, *but more importantly* we need to apply the method by Klein, Krusell and Rios-Rull (2008) to solve for the Markov-perfect equilibrium of the policy problem. This method involves two fixed points, rather than one as under commitment, since the first order conditions contain the unknown derivatives of the policy functions. Collocation methods are computationally more convenient than perturbation to attack the problem, at least at an approximation order higher than one.

## 2 The Model

We consider an infinite horizon closed production economy with imperfectly competitive product and labor markets and sticky prices. Exogenous stochastic variation in wage mark-ups is the only source of aggregate uncertainty and financial markets are complete. The interest rate term structure is derived from equilibrium prices of one period state contingent bonds. It is assumed that the fiscal policy is responsible for offsetting the static distortions arising because of imperfectly competitive markets. Lump-sum transfers and taxes are available and they are free to adjust in order to balance the government budget constraint at all times. Finally, monetary policy is in charge of setting the nominal interest rate.

## 2.1 Households

The economy is populated by a continuum of infinitely lived households indexed by  $i$  on the unit interval  $[0,1]$ , each of them consumes a continuum of differentiated goods and supplies a differentiated labor type. Preferences are defined over consumption and hours worked and they are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{t,i} - \kappa C_{t-1,i})^{1-\gamma}}{1-\gamma} - \delta \frac{N_{t,i}^{1+\phi}}{1+\phi} \right] \quad (2.1)$$

where  $C$  is aggregate consumption, obtained aggregating in the Dixit-Stiglitz form the quantities consumed of each variety

$$C_{t,i} = \left[ \int_0^1 C_{t,i}(f)^{\frac{\theta_p-1}{\theta_p}} df \right]^{\frac{\theta_p}{\theta_p-1}} \quad (2.2)$$

the parameter  $\theta_p > 1$  is representing the elasticity of substitution among varieties and  $\kappa C_{t-1}$ , is the habit consumption stock, where  $\kappa \in [0,1]$ . Defining the aggregate price index<sup>2</sup> as

$$P_t = \left[ \int_0^1 P_t(f)^{1-\theta_p} df \right]^{\frac{1}{1-\theta_p}} \quad (2.3)$$

optimal allocation of expenditure among varieties implies

$$C_{t,i}^*(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\theta_p} C_{t,i} \quad (2.4)$$

The parameter  $\delta$ , entering the utility function, is pinning down steady state marginal utility of leisure, relatively to consumption.  $\delta$  is calibrated to normalize steady state hours worked to 1.

Given optimal allocation of expenditure, the period budget constraint is

$$P_t C_{t,i} + E_t Q_{t,t+1} B_{t+1,i} \leq B_{t,i} + (1 + \tau^w) W_{t,i} N_{t,i} + T_t + Div_t \quad (2.5)$$

$Q_{t,t+1}$  is the nominal one period ahead stochastic discount factor at time  $t$  and  $B_{t+1}$  is the nominal payoff of a riskless portfolio of state contingent bonds in period  $t+1$ .  $W_{t,i} N_{t,i}$  is nominal labor income and  $\tau_w^i$  a proportional subsidy. Finally, each consumer receives a share  $Div_t$  of the aggregate profits and lump-sum government transfers  $T_t$ .

Labor services offered by households are regarded by firms as imperfect substitutes, where the elasticity of substitution is equal to  $\theta_w > 1$ . Total labor demand faced by each household is given by

$$N_{t,i} = \left[ \frac{W_{t,i}}{W_t} \right]^{-\theta_w} N_t \quad (2.6)$$

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<sup>2</sup>The price index has the property that the minimum cost of a consumption bundle  $C_t$  is  $P_t C_t$ .

where

$$N_t = \left[ \int_0^1 N_{t,i}^{\frac{\theta_w-1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w-1}} \quad (2.7)$$

is the aggregate labor index combining in the Dixit-Stiglitz from the total quantity sold of each variety and

$$W_t = \left[ \int_0^1 (W_t(i))^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}} \quad (2.8)$$

can be interpreted as the aggregate wage, defined so that the minimum cost of the aggregate labor index  $\int_0^1 N_{t,i} W_t di$  is  $W_t N_t$ .

## 2.2 Firms

Consider a continuum of monopolistically competitive firms, each producing a differentiated good according to a linear technology

$$Y_{t,f} = N_{t,f} \quad (2.9)$$

We follow Rotemberg (1982) and we model nominal price stickiness by assuming that firms face quadratic adjustment costs when changing prices

$$\frac{\eta}{2} \left( \frac{P_t(f)}{P_{t-1}(f)} - 1 \right)^2 \quad (2.10)$$

so that nominal profits read as

$$\sum_{j=0}^{\infty} \left\{ Q_{t,t+j} \left[ P_t(f) Y_{t,f} - (1 - \tau^p) W_t N_{t,f} - P_t \frac{\eta}{2} \left( \frac{P_t(f)}{P_{t-1}(f)} - 1 \right)^2 \right] \right\} \quad (2.11)$$

where  $\tau^p$  is a subsidy the government can use to offset the steady state distortions due to monopolistic competition and  $Q_{t,t+j}$  is the stochastic discount factor in period  $t$  for profits  $j$  periods ahead.

## 2.3 The Private Sector Equilibrium

In a symmetric equilibrium  $P_t(f) = P_t$  and  $W_{t,i} = W_t$  for all  $t$ ,  $f$  and  $i$ , implying that all firms also hire the same amount of labor input so that  $N_{t,f} = N_t$ . After defining marginal utility  $\mu_t$

$$\mu_t = (C_t - \kappa C_{t-1})^{-\gamma} - \beta \kappa (C_{t+1} - \kappa C_t)^{-\gamma} \quad (2.12)$$

the private sector equilibrium is a set of plans  $\{C_t, N_t, Y_t, \Pi_t\}_{t=0}^{\infty}$  satisfying the following equations

$$Y_t = N_t = C_t + \frac{\eta}{2} (\Pi_t - 1)^2 \quad (2.13)$$

$$\Pi_t(\Pi_t - 1) = \beta E_t \frac{\mu_{t+1}}{\mu_t} \Pi_{t+1} (\Pi_{t+1} - 1) - \frac{\theta_p N_t}{\eta} \left( \frac{(1 - \tau^p) W_t}{P_t} - \frac{1 - \theta_p}{\theta_p} \right) \quad (2.14)$$

$$R_{1,t}^B \beta E_t \frac{\mu_{t+1}}{\mu_t} \frac{P_t}{P_{t+1}} = 1 \quad (2.15)$$

$$\frac{W_t}{P_t} = \exp\{\nu_t^w\} \frac{(N_t^i)^\varphi}{\mu_t} \quad (2.16)$$

where  $\Pi_t$  is the gross inflation rate and the nominal interest rate factor,  $R_{1,t}^B$ , is chosen by the monetary authority. Equation (2.13) defines the resource constraint, while (2.14) and (2.15) are the conventional aggregate supply and Euler equations. The wage equation (2.16) already takes into account that the subsidy to labor income is set so as to offset steady state distortion due to market power. In addition, in order to introduce a tension between inflation and output gap stabilization, it is assumed that the wage mark-up fluctuates exogenously around its mean value<sup>3</sup>.  $\nu_t^w$  follows an autoregressive process represented by

$$\nu_{t+1}^w = \rho_\nu \nu_t^w + \varepsilon_{t+1,\nu} \quad (2.17)$$

where  $\varepsilon_{t+1,\nu}$  is white noise with standard deviation denoted by  $\sigma_{\varepsilon,\nu}$  and  $\rho_\nu \in [0, 1)$ .

## 2.4 The Term Structure of Interest Rates

Prices of one period risk-free bonds, delivering one unit of currency or one unit of consumption at date  $t + 1$ , respectively read as

$$p_{1,t}^B = \beta E_t \frac{\mu_{t+1}}{\mu_t} \frac{P_t}{P_{t+1}} \quad (2.18)$$

$$p_{1,t}^b = \beta E_t \frac{\mu_{t+1}}{\mu_t} \quad (2.19)$$

Let the one period ahead nominal and real stochastic discount factors at time  $t$ ,  $Q_{t,t+1}$  and  $q_{t,t+1}$ , be

$$Q_{t,t+1} = p_{1,t}^B \quad (2.20)$$

$$q_{t,t+1} = p_{1,t}^b \quad (2.21)$$

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<sup>3</sup>The assumption could be rationalized by real or nominal frictions in the wage contracting process. See also Clarida et al. (1999), Galí (2003) and Woodford (2003)

then, nominal and real risk-free bonds with maturity  $n$  can be priced at time  $t$  by no arbitrage

$$p_{n,t}^B = E_t Q_{t,t+1} p_{n-1,t+1}^B \quad (2.22)$$

$$p_{n,t}^b = E_t q_{t,t+1} p_{n-1,t+1}^b \quad (2.23)$$

Prices can be rewritten as

$$p_{n,t}^B = p_{n-1,t}^B E_t p_{1,t+n-1}^B + Cov_t\{Q_{t,t+n-1}, p_{1,t+n-1}^B\} \quad (2.24)$$

$$p_{n,t}^b = p_{n-1,t}^b E_t p_{1,t+n-1}^b + Cov_t\{q_{t,t+n-1}, p_{1,t+n-1}^b\} \quad (2.25)$$

Equations (2.24) and (2.25) state that the price of a risk-free bond with maturity  $n$  is equal to the expected value of a one period bond in  $t+n-1$  discounted by the relevant interest rate, if and only if agents are risk neutral or there is no uncertainty. When these latter conditions do not hold, agents may ask a premium for holding a period bond with maturity  $n$ , rather than waiting until  $t+n-1$  and lend short term. Such a term premium is the covariance between the stochastic discount factor and the price of a one period bond at time  $t+n-1$ , with opposite sign. The intuition is the standard one from asset pricing theory: the premium is positive if the future short term interest rate increases when consumption unexpectedly falls. In fact, in that case agents face capital losses from reselling the bond they hold whenever they need to resell it so as to smooth consumption.

As it is standard in the term structure literature, we define nominal and real interest rates as yields to maturity. Hence,

$$r_{n,t}^B = -\frac{1}{n} \log(p_{n,t}^B) \quad (2.26)$$

$$r_{n,t}^b = -\frac{1}{n} \log(p_{n,t}^b) \quad (2.27)$$

(2.26) and (2.27) denote the nominal and real interest rate respectively. It follows from the definition of interest rates that

$$r_{n,t}^B = \frac{1}{n} \sum_{j=0}^{n-1} E_t r_{1,t+j}^B + TP_{n,t}^B \quad (2.28)$$

$$r_{n,t}^b = \frac{1}{n} \sum_{j=0}^{n-1} E_t r_{1,t+j}^b + TP_{n,t}^b \quad (2.29)$$

where  $TP_{n,t}^B$  and  $TP_{n,t}^b$  are defined as residuals. Those residuals are labelled as *term premia*, since, up to an approximation error referred to as convexity bias<sup>4</sup>,

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<sup>4</sup>The convexity bias arises since expected interest rates  $E_t r_{n,t+j}$  differ from  $-n^{-1} \log \left\{ \beta^j E_t \frac{\mu_{t+n}}{\mu_t} \right\}$  because of Jensen's inequality. The bias is conventionally left as a part of the term premium.

they are zero if and only if agents are risk neutral or there is no uncertainty. If this is the case, long term rates are equal to average future short term rates and the *expectation hypothesis is said to hold*. When premia are positive, agents ask a compensation for investing long term, rather than rolling over short term investments. In this case, the expectation hypothesis is violated and the spread between short and long term maturities is not useful to infer market expectations about the future interest rate path.

Yields as a function of maturity define the term structure of interest rates. It is worth stressing that under the assumption of stationarity and ergodicity of yields

$$E\{r_{1,t}\} = E\{r_{1,t+j}\} \quad (2.30)$$

for all  $t$  and  $j$ . This implies that the unconditional expected slope of the term structure, equivalently the spread between short and long term maturities, entirely reflects term premia. For the sake of the interpretation, it is insightful to recall that a second order approximation to the unconditional slope yields<sup>5</sup>

$$E\{r_{n,t}^b - r_{1,t}^b\} = -\frac{1}{2} \left( E \left[ \frac{1}{n} \text{Var}_t \Delta^n \hat{\mu}_{t+n} \right] - E [\text{Var}_t \Delta \hat{\mu}_{t+1}] \right) \quad (2.31)$$

boiling down to

$$E\{r_{n,t}^b - r_{1,t}^b\} = -\frac{\gamma^2}{2} \left( E \left[ \frac{1}{n} \text{Var}_t \Delta^n \hat{c}_{t+n} \right] - E [\text{Var}_t \Delta \hat{c}_{t+1}] \right) \quad (2.32)$$

when there is no habits persistence. Hats denote log deviations from the non-stochastic steady state. Equation (2.31) and (2.32) relates term premia with the precautionary savings motive. Risk-averse agents delay consumption by saving more, the higher is the amount of uncertainty. Since this is not feasible at equilibrium, interest rates fall to discourage savings. The term structure is positively sloped if consumption uncertainty is lower far away in the future relatively to the short run, so that households are more willing to lend short term relatively to long term. It is well known in the literature that this is the case when consumption growth is positively autocorrelated and habits persistence is sufficiently high<sup>6</sup>.

Some further discussion on habits is needed, as they are crucial for both the behavior of the interest rates and our results. Backus, Gregory and Zin (1989) first pointed out that business cycle models are incapable to generate positive term premia without requiring negatively autocorrelated consumption growth. This feature of the model has been labelled as a puzzle, since autocorrelation of consumption growth and term premia are both positive in the data. The puzzle is

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<sup>5</sup>The expression also corresponds to the standard one obtained under the assumption of log-normal shocks in partial equilibrium models. See Hordal et al. (2008) for further discussion.

<sup>6</sup>See Hordal et al. (2008) and Ravenna and Seppala (2007).

due to the fact that consumption, and marginal utility absent habits persistence, is more volatile at short rather than long forecasting horizons when consumption growth is negatively autocorrelated. In this case indeed, the variance is larger at higher frequencies. Den Hann (1995) showed that habits solve the puzzle by disconnecting the volatility of consumption from the volatility of marginal utility. In fact, if households care about changes in consumption rather than consumption per se, marginal utility can be negatively autocorrelated and then more volatile at high frequencies, independently of the process of consumption growth. In more intuitive terms, in the long run the stock of habits has adjusted and then consumption risk is lower relatively to the near future, though the swings in the level of consumption are expected to be more abrupt at lower frequencies, as it is when the autocorrelation of consumption growth is positive.

In our model monetary policy affects the slope of the yield curve via two mechanisms. On the one hand, commitment reduces volatility of consumption and marginal utility with respect to discretion. On the other hand, it is well known in the literature<sup>7</sup> that optimal policy under commitment is inertial, meaning that the central bank makes endogenous variables more persistent so as to improve stabilization trade-offs. Hence, higher persistence of consumption through habits reduces the conditional variance of future marginal utility to a different extent at different forecasting horizons. The way these two forces interact shapes the slope of the yield curve.

### 3 Term Structure and Monetary Policy Credibility

Two are the policy regimes we consider, commitment and discretion. In both cases, the central bank maximizes households' utility function (2.1) subject to the constraints defining the competitive equilibrium, (2.12)-(2.16). Under commitment, a state contingent plan is chosen at time zero. In contrast, under discretion the policy maker is allowed to re-optimize in every period and chooses the current policy instrument, given market expectations about future variables. We follow Klein et al. (2008) to solve for the Markov-perfect equilibrium under discretion. In particular, when evaluating the impact of current policy on future variables, the central bank takes as given the policy functions mapping current states into future values of inflation, consumption and marginal utility. Therefore future variables can only be influenced through current policy, rather than by announcing future plans. All the details, as well as a recursive formulation of the discretionary policy problem, are left to the Appendix.

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<sup>7</sup>For instance, see Galí (2008).

### 3.1 Solution Method

Given that non linearities are crucial to explain term premia, we solve for equilibrium policy functions by using a standard collocation method with Chebychev polynomials as basis functions<sup>8</sup>. Let  $s_t$  be the vector of endogenous state variables and  $x_t$  the vector of controls. The solution is a collection of  $N = \dim(s) + \dim(x)$  functions  $g^i(s_{t-1}, \nu_t^w)$  with  $i = 1, 2, \dots, N$ . Let  $K$  be the number of functions we parameterize, then we approximate  $g^k$  with a function  $z^k$  as follows

$$z^k(s_{t-1}, \nu_t^w) = \sum_{i=0}^n \varphi_i^k f_i(s_{t-1}, \nu_t^w) \quad (3.1)$$

Coefficients  $\varphi_i^k$  are delivered by our solution method and functions  $f_i$  are the  $n$  basis functions we use to parameterize the policy function. Given the allocation, we can recover bond prices and interest rates by computing the conditional expectation of the stochastic discount factor at different maturities. Since quadrature methods are computationally unfeasible for long enough maturities, we resort to Monte-Carlo techniques.<sup>9</sup> Hence, we first run 30.000 simulations of endogenous variables. Then, for each maturity  $j$  we estimate by OLS

$$\beta^j \frac{\mu_{t+j}}{\mu_t} \frac{P_t}{P_{t+j}} = \sum_{i=0}^{\bar{n}} \varphi_i^{j,B} f_i^B(s_{t-1}, \nu_t^w) + \epsilon_t^{j,B} \quad (3.2)$$

$$\beta^j \frac{\mu_{t+j}}{\mu_t} = \sum_{i=0}^{\bar{n}} \varphi_i^{j,b} f_i^b(s_{t-1}, \nu_t^w) + \epsilon_t^{j,b} \quad (3.3)$$

where we use the same basis, but we possibly parameterize prices policy functions at a different, typically lower, order. Finally, we use the fitted values to approximate conditional expectations.

### 3.2 Parametrization: Baseline

In the baseline parametrization, we set deep parameters to the conventional values used in this literature (see Table 1). We follow Ravenna and Seppala (2007) to choose risk-aversion, habits persistence, the Frisch-elasticity of labor supply and the elasticity of substitution among goods. The elasticity of substitution among labor types is also set to 11. The preference parameter  $\delta$  is such that steady state consumption is normalized to 1. Price adjustment costs are calibrated to replicate an average price duration of four quarters. To this purpose, we log-linearize the Phillips curve (2.14) around the deterministic steady state

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<sup>8</sup>High order Taylor approximation may also be used to characterize first and second moments. However, discretion requires to solve additional fixed point problems, as the derivatives of unknown policy functions appear in the first order conditions. For high enough approximation orders, it is computationally more convenient to use projection techniques.

<sup>9</sup>This is the same approach that has been followed by Evans and Marshall (1998) and Ravenna and Seppala (2007).

$$\pi_t = \beta E_t \pi_{t+1} + \frac{1 - \theta_p}{\eta} \hat{m}c_t \quad (3.4)$$

and we determine  $\eta$  to match the slope implied by a Calvo pricing model with four quarter average duration. Lower case letters stand for logs and  $\hat{m}c_t$  for the log-deviation of the real marginal cost from the deterministic steady state.

The serial correlation and the standard deviation of the innovation to the shock are set respectively to 0 and 0.11 in the baseline parametrization. This implies that the standard deviation of wage mark-up shocks amounts to five times the standard deviation of TFP shocks, a reasonable value if compared with the structural estimates of large scale models. We omit TFP shocks from the model as they would leave unaltered our message and unnecessarily burden computation.

Given that we need an accurate non-linear solution to convey our message, we deliberately keep the model as simple as possible. Hence, we give up on a quantitatively reliable replication of business cycle evidence. In addition, the optimal policy obviously requires a less flexible parametrization with respect to ad-hoc monetary policy rules.

Our baseline parametrization delivers sizeable term premia under both policy regimes at the cost of producing excessively high volatility of interest rates. Then, we want to check whether our results survive when we discipline the model not to unreasonably boost volatilities. Further discussion is left to the following sections.

### 3.3 Policy Regime, Term Structure and Term Premia

Figure 1 plots the unconditional mean of real interest rates as a function of maturity in deviation from the short term real interest rate. We omit to report the nominal yield curve, since the optimal policy under both regimes implies low inflation volatility around a zero mean value. As a consequence, inflation risk premia are negligible. This is the case also when the central bank is assumed to follow a monetary policy rule that is consistent with positive inflation and is calibrated to match business cycle evidence as in Ravenna and Seppala (2007).

By stationarity, the unconditional mean of future expected interest rates in deviation from the short rate is zero. This implies that the whole slope is accounted by the presence of term premia. Under both regimes the term premium is positive at all maturities. This is because agents, due to habits, prefer to save short relatively to long term. However, it is evident that commitment implies higher term premia and then a steeper yield curve. The intuition is that to stabilize positive (negative) wage mark-up shocks it is optimal to generate expectations of future deflation (inflation). As a consequence, part of the recessionary (expansionary) impact of the shock is shifted to future periods. Since this policy is not time consistent, it is implementable only when the central bank is credible enough to reap the benefits of commitment. Under commitment, higher consumption persistence reduces through habits the volatility of marginal utility

and it does so to a greater extent far ahead in the future. Then, though switching from commitment to discretion aggregate consumption risk falls, households perceive the short run as riskier than the long run.

### 3.4 Sensitivity analysis

Business cycle models augmented with habits have two peculiar flaws. On the one hand habits magnify the volatility of nominal and real interest rates. On the other hand, the high variance of shocks required to match the observed yields spreads amplifies the standard deviation of all endogenous variables. As a consequence of that, when calibrating the model, there is a trade-off between replicating the standard deviation of endogenous variables and the average slope of the term structure. An extensive discussion has been provided by Rudebusch and Swanson (2008). However, we have to face an additional hurdle. Optimal policy models deliver high volatility of interest rates in exchange for low volatility of output and inflation, likely violating the nominal interest rate zero bound. Requiring policy to satisfy the zero bound on nominal interest rates or introducing transaction frictions would yield on average higher and more volatile inflation and would also help in terms of interest rate volatility. However, adding the zero bound implies additional computational burden we decided to avoid and leave to future research. Table 2, displaying the moments in the model and in the the data, makes the point: habits and optimal policy boost the volatility of interest rates, while keeping low the volatility of consumption. With this section, we want to check whether putting some discipline on the model to fix, at least partially, those flaws invalidates our result.

We choose parameters of the stochastic process governing wage mark-up shocks and habits persistence<sup>10</sup> to match under commitment the standard deviation of consumption in the data<sup>11</sup>. We leave free *all* other moments. Table 3 reports the moments of endogenous variables and parameters.

We choose to target only consumption for several reasons. First, real wages do not contain additional relevant information to identify  $\kappa$ ,  $\rho_\nu$  and  $\sigma_{\epsilon,\nu}$ . Second, consumption output and hours in the model have roughly the same volatility, as we do not have neither capital nor government expenditure and the variance of inflation is extremely low. It follows that it is meaningless to target all of them simultaneously. By choosing consumption, less volatile than output and hours in the data, we bias the model against positive term premia<sup>12</sup>. This also allows

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<sup>10</sup>Some preliminary explorative analysis indeed reveals that the parameters to which second moments are most sensitive are the ones we calibrate.

<sup>11</sup>For choosing parameters we perform a search over a grid of equally spaced intervals where  $\kappa \in [0.66, 0.8]$ ,  $\rho_\nu \in [0.8, 0.9]$  and  $\sigma_{\epsilon,\nu} \in [0.05, 0.12]$ , with the number of intervals for the three parameters being equal respectively to 14, 10 and 24. The endpoints for the grid on  $\kappa$  are the minimum and maximum values used in the literature, while the choice of the grid for the shocks has been restricted to generate sizeable term premia.

<sup>12</sup>and in fact, observed volatility of consumption, which is lower in our baseline parametrization, drives up serial correlation and down the standard deviation, implying lower term structure slope.

the volatility of interest rates to fall, at least to some extent. Finally, we believe it would be unfair to ask the model to replicate interest rate volatility. Given that we do not impose the zero bound nor we have additional frictions calling for rate stabilization, targeting interest rates would force consumption volatility to be artificially high or equivalently, habits and serial correlation to be too low. To disentangle the excess volatility of rates due to habits and to the zero bound problem, we look for parameters replicating the volatility of consumption imposing  $\kappa = 0$ . Table 5 shows that, even without habits, interest rate volatility is still high. We conclude that we prefer not to ask the calibration to fix a problem that would be better solved by enriching the model, was this computationally feasible.

Some implications of this alternative parametrization need to be stressed.

Set aside the interest rates, we do not generate counterfactually high standard deviation of any endogenous variable. This mitigates the criticism that has been put forward by Rudebusch and Swanson (2008). In addition, the volatility of real wages and hours worked are under-predicted. However, the fit of the model in this respect can be easily fixed by introducing TFP shocks. Inflation volatility is under-predicted, but we think that this is strictly connected with the high standard deviation of rates.

Figure 2 again plots the slope of the term structure. Under both regimes the slope, and then term premia, are lower than in the previous parametrization and in the data. However, our result survives. Still, the monetary policy regime accounts for approximately one fourth of the slope.

## 4 Conclusions

We compute the term structure of risk-free bond yields implied by a standard business cycle model, augmented with habits and nominal price rigidities under the assumption that policy is set optimally. We consider both cases of commitment and discretion and we find that requiring time consistency of policy lower the term premium households ask to hold long-term bonds. Our intuition is that discretion prevents policy makers not only from containing inefficient volatility, but also from smoothing consumption over the cycle. It follows that agents perceive consumption far ahead in the future to be relatively riskier than in the short-term, if compared with the time-inconsistent solution. If commitment is interpreted as a credibility improvement with respect to discretion, our result implies that a flattening of the yield curve *ceteris paribus* signals worse, rather than better, monetary policy.

## A Appendix: Commitment

The central bank chooses a state contingent plan  $\{c_t, \mu_t, \lambda_{1,t}, \lambda_{2,t}\}_{t=0}^{\infty}$  so as to maximize (2.12)-(2.16). Let  $u_t$  be the period utility function

$$u_t \equiv \frac{(C_t - \kappa C_{t-1})^{1-\gamma}}{1-\gamma} - \delta \frac{N_t^{1+\phi}}{1+\phi}$$

After defining the vector of endogenous state variables  $s_t \equiv \{c_t, \mu_t\}$ , the vector of control variables  $x_t \equiv \{\pi_t\}$  and the vector of Lagrange multipliers  $\lambda_t \equiv \{\lambda_{1,t}, \lambda_{2,t}\}$ , the Lagrangian function associated to the policy problem is

$$\Lambda_0 = E_0 \sum_{t=0}^{\infty} \beta^t h(\lambda_t, \lambda_{t-1}; s_t, s_{t-1}, x_t, x_{t+1})$$

with the function  $h()$  defined as

$$\begin{aligned} h &= u_t + \\ &+ \lambda_{1,t} \left\{ \Pi_t(\Pi_t - 1) - \beta E_t \frac{\mu_{t+1}}{\mu_t} \Pi_{t+1}(\Pi_{t+1} - 1) + \frac{\theta_p N_t}{\eta} \left( \frac{(1 - \tau^p) W_t}{P_t} - \frac{1 - \theta_p}{\theta_p} \right) \right\} \\ &+ \lambda_{2,t} \left\{ \mu_t - (C_t - \kappa C_{t-1})^{-\gamma} + \beta \kappa (C_{t+1} - \kappa C_t)^{-\gamma} \right\} \end{aligned}$$

where for convenience it has been used the following

$$\frac{W_t}{P_t} = \frac{\nu_t^w}{\mu_t} \left[ C_t + \frac{\eta}{2} (\Pi_t - 1)^2 \right]^\phi \quad (\text{A.1})$$

$$N_t = \left[ C_t + \frac{\eta}{2} (\Pi_t - 1)^2 \right] \quad (\text{A.2})$$

implied by intra-temporal allocation between consumption and leisure (2.16) and the resource constraint (2.13). Since Marcet and Marimon (1998), it is well know that by setting  $\lambda_{-1} = 0$ , the function  $\Lambda_0$  can be easily rewritten recursively

$$\Lambda_0 = E_0 \sum_{t=0}^{\infty} \beta^t g(\lambda_t, \lambda_{t-1}; s_t, s_{t-1}, x_t)$$

with  $g()$  defined as

$$\begin{aligned} g &= u_t + \lambda_{1,t} \left\{ \Pi_t(\Pi_t - 1) + \frac{\theta_p N_t}{\eta} \left( \frac{(1 - \tau^p) W_t}{P_t} - \frac{1 - \theta_p}{\theta_p} \right) \right\} \\ &- \lambda_{1,t-1} \left\{ \frac{\mu_t}{\mu_{t-1}} \Pi_t(\Pi_t - 1) \right\} \\ &+ \lambda_{2,t} \left\{ \mu_t - (C_t - \kappa C_{t-1})^{-\gamma} \right\} + \lambda_{2,t-1} \left\{ \kappa (C_t - \kappa C_{t-1})^{-\gamma} \right\} \end{aligned}$$

Hence, there exist time invariant functions solving the policy problem. As it is conventional in this class of models, it is possible to recover the nominal interest rate implementing the optimal policy allocation by using the Euler equation (2.15).

## B Appendix: Discretion

Under discretion, the central bank chooses in each period current allocations only, given how market expectations about future variables are rationally formed by agents. As it is well known, this is equivalent to allow the policy maker to choose a state contingent plan  $\{c_{t+j}, \mu_{t+j}, \lambda_{1,t+j}, \lambda_{2,t+j}\}_{j=0}^{\infty}$  in each period  $t$  from 0 onwards, but ruling out as an equilibrium whatever plan that is not credible. We follow Klein et al. (2008) in focusing on Markov equilibria featuring differentiable policy functions. This refinement is intended to avoid anything resembling a reputation-like mechanism for sustaining good-equilibria.

### B.1 Equilibrium

The equilibrium can be formally stated as follows. Define the vector of endogenous state variables  $s_t \equiv \{c_t\}$ , the vector of control variables  $x_t \equiv \{\pi_t, \mu_t\}$  and the vector of Lagrange multipliers  $\lambda_t \equiv \{\lambda_{1,t}, \lambda_{2,t}\}$ . An equilibrium consists of a value function  $V$  and policy functions  $S = \mathcal{C}, \mathcal{L}$  and  $X = \{\mathcal{P}, \mathcal{M}\}$  such that for any  $s_{t-1}$ ,  $s_t = S(s_{t-1}, \nu_t^w)$ ,  $\lambda_t = \mathcal{L}(s_{t-1}, \nu_t^w)$  and  $x_t = X(s_{t-1}, \nu_t^w)$  solve

$$\min_{\lambda} \max_{x_t, s_t} \left\{ \mathcal{H}(\lambda_t, x_t, s_t, s_{t-1}, \mathcal{E}_1(s_t, \nu_t^w), \mathcal{E}_2(s_t, \nu_t^w), \nu_t^w) + \beta E_t V(s_t, \nu_{t+1}^w) \right\}$$

where

$$\begin{aligned} \mathcal{H} &= u_t \\ &+ \lambda_{1,t} \{ \Pi_t (\Pi_t - 1) \} \\ &- \lambda_{1,t} \left\{ \mathcal{E}_1 + \frac{\theta_p N_t}{\eta} \left( \frac{(1 - \tau^p) W_t}{P_t} - \frac{1 - \theta_p}{\theta_p} \right) \right\} \\ &+ \lambda_{2,t} \{ \mu_t - (C_t - \kappa C_{t-1})^{-\gamma} + \mathcal{E}_2 \} \\ \mathcal{E}_1 &= \beta E_t \frac{\mathcal{M}(s_t, \nu_{t+1}^w)}{\mu_t} \mathcal{P}(s_t, \nu_{t+1}^w) (\mathcal{P}(s_t, \nu_{t+1}^w) - 1) \\ \mathcal{E}_2 &= \beta E_t \kappa (C(s_t, \nu_{t+1}^w) - \kappa C_t)^{-\gamma} \\ V(s_{t-1}, \nu_t^w) &= \mathcal{H}(\mathcal{L}, X, S, s_{t-1}, \mathcal{E}_1, \mathcal{E}_2, \nu_t^w) \\ &+ \beta E_t V(S, \nu_{t+1}^w) \end{aligned} \tag{B.1}$$

and definitions (A.1)- (A.2) still apply while in the last two lines function arguments have been suppressed. It is immediate to note that the problem is recursive and that the policy maker correctly takes into account that expectations are influenced by future policy through the endogenous state variables. Also, this dynamic program features an additional fixed point, since the unknown derivatives of unknown policy functions appear in the first order conditions. Hence, it is not obvious that perturbation techniques are more appropriate than a collocation method. This is why we resort to the latter.

## B.2 Envelope Condition

It is straightforward to show that the conventional envelope argument holds also in this environment.

We use subindexes for the variable with respect to which we take derivatives, denote states predetermined in the following period  $s_t$  as  $s'$  and drop time indexes and function arguments whenever it is making notation less cumbersome without creating ambiguity. First order conditions require

$$\mathcal{H}_x = 0 \tag{B.2}$$

$$\mathcal{H}_\lambda = 0 \tag{B.3}$$

$$\mathcal{H}_{s'} + \frac{\partial \mathcal{H}}{\partial \mathcal{E}_1} \frac{\partial \mathcal{E}_1}{\partial s'} + \frac{\partial \mathcal{H}}{\partial \mathcal{E}_2} \frac{\partial \mathcal{E}_2}{\partial s'} + \beta V_{s'} = 0 \tag{B.4}$$

Differentiation of the value function with respect to states  $s$  yields

$$\begin{aligned} V_s &= \mathcal{H}_\lambda \mathcal{L}_s + \mathcal{H}_x X_s + \mathcal{H}_{s'} S_s + \mathcal{H}_s \\ &+ \frac{\partial \mathcal{H}}{\partial \mathcal{E}_1} \frac{\partial \mathcal{E}_1}{\partial s'} S_s + \frac{\partial \mathcal{H}}{\partial \mathcal{E}_2} \frac{\partial \mathcal{E}_2}{\partial s'} S_s + \beta V_{s'} S_s \end{aligned} \tag{B.5}$$

It is immediate to see that first order conditions (B.2)-(B.4) and equation (B.5) imply the standard envelope condition

$$V_s = H_s$$

where the unknown derivatives of the unknown policy functions disappear, though not vanishing in the first order conditions. Therefore, standard dynamic programming allows to find the generalized Euler equations. They are named generalized by Klein et al. (2008), as they contain the unknown derivatives.

Table 1: **Baseline Parametrization**

Parameter	Mnemonic	Value
Pref. Discount Factor	$\beta$	0.99
Habits Persistence	$\kappa$	0.8
RRA Coefficient	$\gamma$	2.5
Leisure Weight	$\delta$	11.62
Inv. Frisch Elasticity	$\phi$	0.5
Goods Elast. Subst.	$\theta_p$	11
Labor Type Elast. Subst.	$\theta_w$	11
Price Adj. Cost	$\eta$	116.55
Serial Corr. Shock	$\rho_\nu$	0
Std. Dev. Innovation Shock	$\sigma_{\epsilon,\nu}$	0.11

Table 2: **Percentage standard deviations: baseline.** The following table reports the second moments of selected variables from the data and from the model under the baseline parametrization. In the model, volatility of hours and output equal volatility of consumption. Rates and inflation have been annualized.

Variable	Data	Model
Consumption	1.19	0.29
Inflation	2.52	0.08
Real Wage	0.82	0.25
Short-term Real Interest Rate	2.30	44
Long-term Real Interest Rate	2.71	44

Table 3: **Sensitivity Analysis: matching consumption.** Second moments in the data and in the model under the parametrization matching the standard deviation of consumption. Values of  $\kappa$ ,  $\rho_\nu$  and  $\sigma_{\epsilon,\nu}$  are respectively 0.8, 0.8 and 0.05.

Variable	Data	Model
Consumption	1.19	1.18
Inflation	2.52	0.11
Real Wage	0.82	0.14
Short-term Real Interest Rate	2.30	6.64
Long-term Nominal Interest Rate	2.71	6.68

Table 4: **Sensitivity Analysis: habits versus optimal policy.** Second moments in the data and in the model under the parametrization matching the standard deviation of consumption and imposing  $\kappa = 0$ . Values of  $\rho_\nu$  and  $\sigma_{\epsilon,\nu}$  are respectively 0.5 and 0.04.

Variable	Data	Model
Consumption	1.19	1.18
Inflation	2.52	0.4
Real Wage	0.82	1.34
Short-term Real Interest Rate	2.30	4.03
Long-term Nominal Interest Rate	2.71	4.20

Figure 1: The Real Term Structure of Interest Rates. Baseline

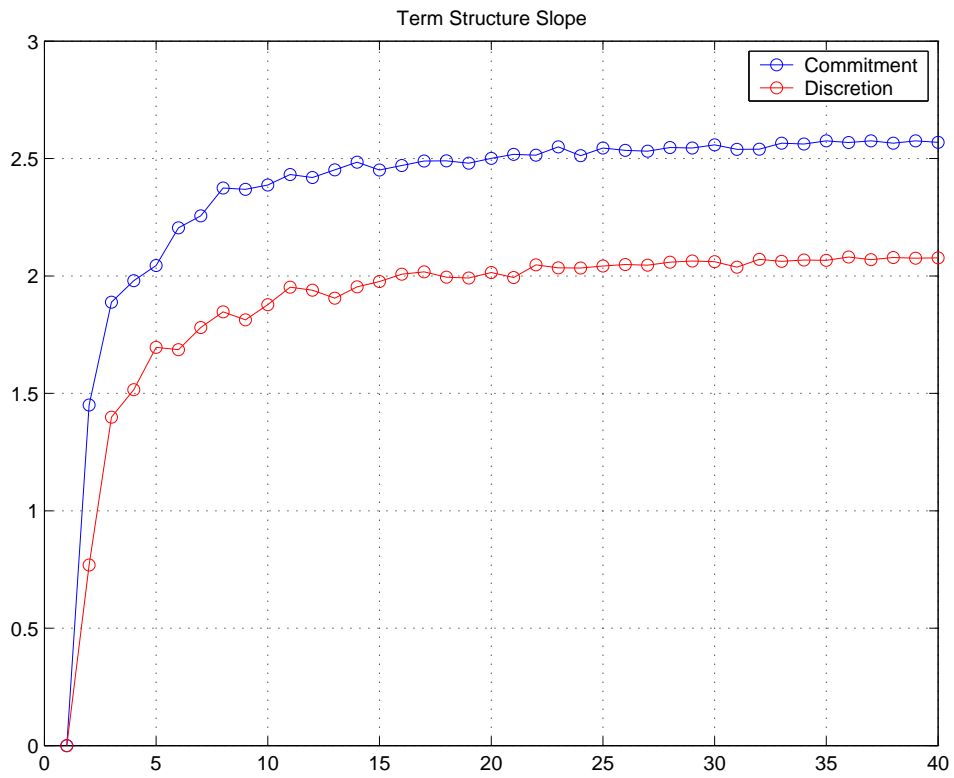
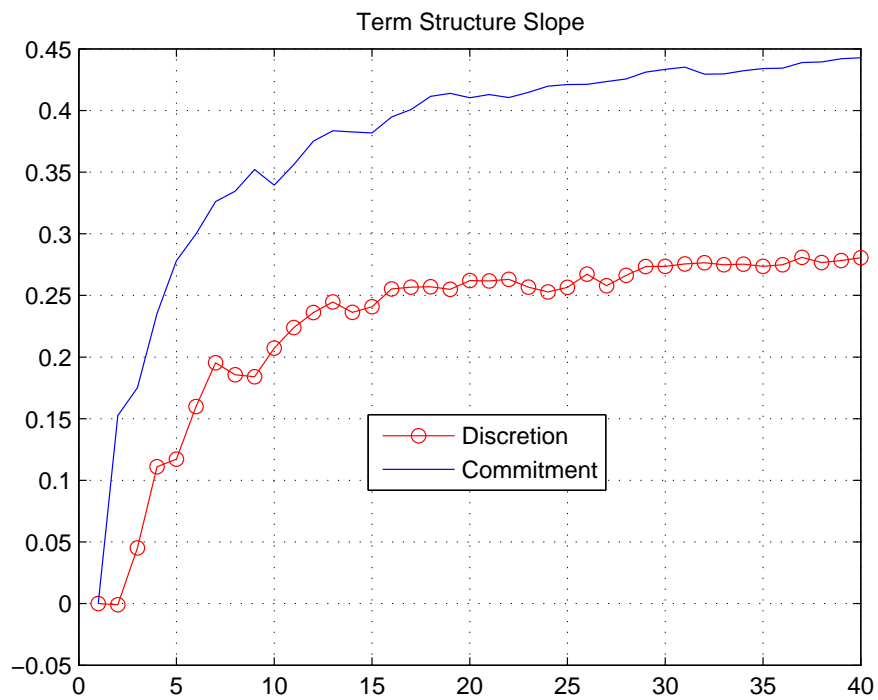


Figure 2: The Real Term Structure of Interest Rates. Sensitivity Analysis



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