# Inflation Targeting under Collective Wage Bargaining*

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**Abstract**

The purpose of the paper is to design optimal monetary policy rules in a New-Keynesian model featuring the presence of large unions. It is shown that concentrated labor markets call for more aggressive inflation stabilization. This is because the central bank is able to induce wage restraint and to push output towards Pareto efficiency by implementing tougher stabilization policies. Moreover, the welfare cost of deviation from the optimal policy is increasing in wage setting centralization. The analysis is performed in the context of a linear-quadratic approach where the welfare measure is derived resorting to a second order approximation to households' lifetime utility.

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# 1 Introduction

Wages are set through collective agreements in most of the OECD countries. Also, wage bargaining systems vary across countries, ranging from completely decentralized, such as the United States and the United Kingdom, to highly centralized, such as Norway[1]. Under collective wage bargaining, negotiations are delegated to few large unions, whose decisions affect the aggregate level of wages, hence the real cost of labor and inflation. In such an environment strategic interaction is an issue: wage rises trigger a policy response on the part of the central bank, as long as the objective of price stability is at risk. In this paper we study optimal monetary policy in an otherwise standard New-Keynesian model (NK, henceforth) featuring the presence of large unions.

Early contributions have shown that if wage setters are large, the systematic behavior of the central bank has an impact on labor supply decisions and, as a consequence, on the long-run equilibrium level of employment and production. In fact, unions anticipate that wage pressures will lead to a surge in inflation and to labor demand reductions through monetary policy tightening. Therefore, tougher inflation stabilization policies raise steady state employment by restraining wage demands. In this context, the monetary policy rule is non-neutral as a change of the rule has permanent real effects[2]. A representative though not exhaustive sample of the earlier literature on the topic includes Bratsiotis and Martin (1999), Iversen and Soskice (2000) and Lippi (2002, 2003)[3]. Gnocchi (2009) integrates the non-neutrality of the monetary policy rule emphasized by previous contributions into the baseline NK model and shows that inflation stabilization yields steady state welfare gains that are increasing with the centralization of the wage bargaining process. In addition, those gains have the same order of magnitude of welfare losses associated to suboptimal stabilization policies. It follows that models abstracting from the strategic interaction between large wage setters and the central bank may overlook some of the benefits brought about by price stability, depending on wage setting institutions. Optimal monetary policy in this framework is an open question.

Here, we allow for an inflation-output stabilization trade-off generated by wage mark-up shocks and we study how the optimal policy varies with the labor market structure. The design of the optimal policy rule is performed by using the methodology introduced by Rotemberg and Woodford (1997) and further de-

[1]The OECD Employment Outlook 2004 provides a survey about labor market institutions in OECD countries and 2000 figures on union density, coverage and wage-setting centralization. Typically, centralized countries tend to have high union coverage despite of a low union density, the reason being that the wage bargained through the collective agreement extend to all workers in a given sector, independently of whether they are affiliated to a union or not.

[2]Note however that the classical neutrality result is not challenged: a shock to the policy instrument dies off in the long-run.

[3]See also Cukierman and Lippi (1999) and Coricelli, Cukierman and Dalmazzo (2006). Holden (2005) took this literature a step forward by considering the effects of the monetary regime on wage setters' incentives to coordinate their decisions.

veloped by Benigno and Woodford (2005). The method resorts to a second order approximation to households' lifetime utility as an approximate welfare measure. As it well known since Benigno and Woodford (2005), the methodology requires the second order approximation of some of the equilibrium conditions in order to eliminate the first order terms due to steady state distortions and then obtain a purely quadratic welfare measure. Because of the long-run non-neutrality of the rule, the objective function of the monetary authority is decomposed in such a way to disentangle the steady state and the stabilization effects of policy.

The paper shows that the presence of large wage setters creates an additional dimension of the policy trade-off, with respect to the one traditionally considered by the literature. This is because in a model with unions, being more aggressive in stabilizing inflation allows to reduce steady state distortion by inducing wage restraint. But, as tougher inflation stabilization policies amplify output gap volatility, the policy maker has to trade-off steady state gains against stabilization losses. Two are the forces underlying the policy dilemma: wage setting centralization and the volatility of the cost push shock. Highly centralized labor markets are associated to high gains of being aggressive in exchange for less average distortion. In fact, larger unions internalize to a greater extent the impact of their wage policy on inflation, making more effective the strategic interaction channel of monetary policy. On the other hand, the more volatile is the cost push shock, the more costly is price stability in terms of gap fluctuations. This implies a high stabilization cost of reducing average distortion.

The two forces interact resolving the policy trade-off and determining the following optimal policy results.

If the volatility of the cost push shock is sufficiently low and the concentration of the labor market is high enough, strict inflation targeting is optimal, even in the presence of wage mark-up shocks. A high volatility of the cost push shock induces the policy maker to accept some volatility of inflation, optimal aggressiveness however is still increasing in labor market concentration. Finally, a decomposition of the approximate welfare measure allows to compute the cost of deviating from the optimal policy and to decompose the total effect in a steady state cost and a stabilization cost. It is showed that the steady state cost, as a fraction of the total, decreases with the standard deviation of the cost push shock and increases with wage setting centralization.

The paper is organized as follows. Section 2 describes the model economy, Section 3 derives and gives an economic interpretation to the welfare criterion, Section 4 computes the optimal simple interest rate rule. Section 5 concludes.

# 2 The Model

The model economy consists of a continuum of households and firms and a finite number of unions. Households and firms are modelled as in the baseline NK model with goods prices staggered à la Calvo (1983)[4]. The main differences with respect to the standard framework are in the structure of the labor market. Households indeed delegate wage setting decisions to unions and, for given wage, they are willing to supply whatever quantity of labor is required to clear the markets. This section summarizes the private sector equilibrium for an economy with large unions, generalizing the model by Iversen and Soskice (2000) and Lippi (2003) to the baseline NK set-up. Gnocchi (2009) is closely followed.

The central bank sets the nominal interest rate, reacting to endogenous variations in inflation according to the following policy rule[5]

$$i_t = \rho + \gamma_\pi \pi_t \tag{1}$$

where $i_t$ is the log of the nominal interest rate factor, $\rho$ is the steady state level of $i_t$, inflation is defined as $\pi_t = logP_t - logP_{t-1}$ and $\gamma_\pi > 1$.

It is assumed that the fiscal policy is responsible for offsetting the static distortions arising because of imperfectly competitive goods markets, while, differently from the baseline model, the inefficiency arising in labor markets is not corrected for[6]. Lump-sum transfers and taxes are available and they are free to adjust in order to balance the government budget constraint at all times.

## 2.1 Households

The economy is populated by a continuum of infinitely lived households indexed by $i$ on the unit interval [0,1], each of them consumes a continuum of differentiated goods and supplies a differentiated labor type. Households have preferences defined over consumption and hours worked described by the utility function[7]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ logC_{t,i} - \frac{L_{t,i}^{1+\phi}}{1+\phi} \right] \tag{2}$$

where $C$ is aggregate consumption, obtained aggregating in the Dixit-Stiglitz form the quantities consumed of each variety

$$C_{t,i} = \left[ \int_0^1 C_{t,i}(f)^{\frac{\theta_p - 1}{\theta_p}} df \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{3}$$

[4]For derivations of the baseline model I refer to Calvo (1983), Clarida, Galí and Gertler (1999), Galí (2008), Walsh (2003) and Woodford (2003)

[5]A thorough discussion about the implication of choosing such a simple rule is postponed to Section 3.

[6]Maintaining a fiscal subsidy to correct goods market inefficiency is inconsequential for all the results. In fact, because of the lack of any strategic interaction between firms and the monetary authority, the subsidy is simply re-scaling the steady state independently of monetary policy.

[7]The analysis is restricted to the case of log utility. In this case not only the model is more tractable, but the policy analysis is particularly intuitive and transparent. An additional appendix, which is available upon request, shows that all results derived here continue to hold in the more general case of a CRRA utility function.

and the parameter $\theta_p > 1$ is representing the elasticity of substitution among varieties. Defining the aggregate price index[8] as

$$P_t = \left[ \int_0^1 P_t(f)^{1-\theta_p} df \right]^{\frac{1}{1-\theta_p}} \tag{4}$$

the budget constraint faced by households in each period is

$$C_{t,i} + \delta_{t,t+1} B_{t,i} \leq B_{t-1,i} + \frac{W_{t,i}}{P_t} L_{t,i} + T_{t,i} + Div_{t,i} \tag{5}$$

$\delta_{t,t+1}$ is the price vector of a state contingent asset paying one unit of consumption in a particular state of nature in period t+1, $B_t$ is the vector of the corresponding state contingent claims purchased by the household and $B_{t-1}$ the value of the claims for the current realization of the state of nature. $\frac{W_{t,i}}{P_t} L_{t,i}$ represents real labor income. Finally, each consumer receives a share $Div_{t,i}$ of the aggregate profits and lump-sum government transfers $T_{t,i}$. Households maximize their lifetime utility (2) subject to the budget constraint (5) choosing state contingent paths of consumption and assets. Optimal allocation of consumption over time implies a standard Euler equation that, combined with the monetary policy rule and the clearing of all goods markets, reads as

$$Y_t = \Pi_t^{-\gamma_\pi} \left[ E_t \left\{ \Pi_{t+1}^{-1} Y_{t+1}^{-1} \right\} \right]^{-1} \tag{6}$$

where $\Pi_t$ is the gross inflation rate, defined as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \tag{7}$$

## 2.2 Firms

Infinitely many monopolistically competitive firms lie on the unit interval and are indexed by $f$. Each one produces a differentiated good using a continuum of labor types, according to the following constant return to scale technology

$$Y_t(f) = A_t L_{t,f} \tag{8}$$

Productivity (TFP), denoted by $A_t$, follows an autoregressive process represented by

$$logA_{t+1} = \rho_a logA_t + \varepsilon_{t+1,a} \tag{9}$$

where $\varepsilon_t$ is white noise with standard deviation $\sigma_{\varepsilon,a}$. The effective labor input is obtained aggregating in the Dixit-Stiglitz form the quantities hired of each differentiated labor type

$$L_{t,f} = \left[ \int_0^1 L_{t,f}(i)^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}$$

[8] The price index has the property that the minimum cost of a consumption bundle $C_t$ is $P_t C_t$

The parameter $\theta_w > 1$ is representing the elasticity of substitution among labor types. Firms do not have market power in the labor market, then they take wages as given. Defining the aggregate wage[9] as

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}} \tag{10}$$

cost minimization implies

$$L_{t,f}^*(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\theta_w} L_{t,f} \tag{11}$$

Firms set the price in order to maximize profits, subject to the constraint that demand must be satisfied at the posted price. Prices are set in staggered contracts with random duration as in Calvo (1983): in any period each firm faces a constant probability $1-\alpha$ to re-optimize and charge a new price. A subsidy is used by the fiscal authority to undo the steady state distortion induced by firms' market power in the goods markets. The definition of the price index and profit maximization imply

$$\left[\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right]^{\frac{1}{1-\theta_p}} = \frac{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j MC_{t+j}\Pi_{t,t+j}^{\theta_p}}{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j \Pi_{t,t+j}^{\theta_p-1}} \tag{12}$$

where $\Pi_{t,t+j} \equiv \frac{P_{t+j}}{P_t}$ and the real marginal cost is identical across firms and equal to

$$MC_t = \frac{W_t}{P_t A_t} \tag{13}$$

Integrating (11) across firms yields total demand of labor faced by household $i$

$$L_t^*(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\theta_w} L_t; \qquad L_t = \int_0^1 L_{t,f} df \tag{14}$$

Finally, it can be easily shown that the aggregate production function is given by

$$Y_t \Delta_t = A_t L_t \tag{15}$$

where $\Delta_t$[10] is defined as

$$\Delta_t = \int_0^1 \frac{Y_t(f)}{Y_t} df \tag{16}$$

and represents a measure of relative price dispersion.

[9]As for the price index, aggregate wage has the property that the minimum cost of a unit of composite labor input $L_t$ is $W_t L_t$

[10]It can be proved that $log(\Delta)$ is a function of the cross sectional variance of relative prices and it is of second order.

## 2.3 Unions

The economy is populated by a finite number of unions indexed by $j$, where $j \in \{1, ..., n\}$, $n \geq 2$. All workers are unionized and they split equally among unions so that each union has mass $n^{-1}$. The mass can be interpreted as the degree of wage setting centralization (CWS) as well as unions' ability to internalize the consequences of their actions.

It is assumed that wages are fully flexible and any possibility of pre-commitment to future wage policies is ruled out. Unions set real wages simultaneously and each of them takes other unions' real wages as given[11]. Each union $j$ sets the real wage on behalf of her members to maximize their lifetime utility function (2) subject to the budget constraint[12] (5) and labor demand (14) for all members $i \in j$. Since unions are non-atomistic, aggregate labor and aggregate wages cannot be taken as given. Hence, union $j$ also internalizes the aggregate demand (6), the aggregate wage index (10), the short run aggregate supply (12) and the production function (15).

The solution to unions' problem implies the following relation

$$\frac{W_t}{P_t} = \frac{\eta}{\eta - 1} L_t^{\phi} C_t \tag{17}$$

where $\eta$ denotes the labor demand elasticity to the real wage. The first order condition for unions has the same form as in the standard case with atomistic wage setters: the real wage is set at a mark-up over the marginal rate of substitution. However, differently from the standard set-up, the elasticity of labor demand

$$\eta = \theta_w (1 - \frac{1}{n}) + \frac{1}{n} \Sigma_L \tag{18}$$

is a weighted average of the elasticity of substitution among labor types and the

[11]Note that as long as wages are flexible no additional real effects stem from assuming nominal rather than real wage bargaining. The result differs from Lippi (2003). The monetary policy commitment is the assumption responsible for the difference. Nominal wage inflation in the wage of union $j$ generates price inflation, which is reducing the real wage of the others. This additional effect alters labor demand elasticity. However, under commitment to a policy rule, the magnitude of the effect does not depend on the inflation coefficient of the rule. This can be easily proved by looking at the inflation equation *conditional on the wage set by unions* which is the equation that unions would actually internalize. The intuition is that the existence of an inflation bias, as in Lippi (2003), is key for the channel to work. In fact, large unions understand that they can manipulate the bias through their wage decision. This channel is absent here.

[12]Fiscal policy and dividends are taken as given, as it is usually assumed in the literature. See Lippi (2002, 2003)

elasticity of aggregate labor demand to the real wage, $\Sigma_L$[13]

$$\Sigma_L = \gamma_\pi \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{19}$$

which is in turn an increasing function of $\gamma_\pi$: tough inflation stabilization policies discourage wage pressures by punishing a wage increase with a contraction of aggregate demand. It follows that wage mark-ups negatively depend both on the elasticity of substitution between two any labor types *and* the Taylor rule inflation coefficient. This has been labelled as the strategic interaction channel of transmission of monetary policy. The strength of the channel increases in CWS, because larger unions anticipate to a greater extent the impact of their wage policy on aggregate variables. As a consequence, the marginal impact of a change in the monetary policy stance is magnified by wage setting centralization. In the limiting case of infinitely many unions, the wage mark-up only depends on the elasticity of substitution and the model converges to the standard NK.

An unpleasant feature of the baseline NK model with nominal price rigidity is the lack of a non-trivial policy trade-off, which is perceived to be as an empirically relevant problem by any central banker: it is needed to create a tension between inflation and output gap stabilization. Therefore, it is assumed from now on that the wage mark-up is fluctuating exogenously around its mean value[14]. The first order condition is modified accordingly to include a random shock

$$\frac{W_t}{P_t} = exp\{\mu_t^w\}\frac{\eta}{\eta-1}L_t^\phi C_t \tag{20}$$

$\mu_t^w$ follows an autoregressive process represented by

$$\mu_{t+1}^w = \rho_u \mu_t^w + \varepsilon_{t+1,u} \tag{21}$$

where $\varepsilon_{t,u}$ is white noise with standard deviation denoted by $\sigma_{\varepsilon,u}$.

## 2.4 The Steady State and the Pareto Optimum

The non-stochastic steady state of the model is derived by setting the shocks to their mean value. It is straightforward to prove that the steady state level of the gross inflation rate is equal to one. Moreover, output, employment and consumption read as

[13] For the derivation of $\Sigma_L$ see Appendix B. Note that $\Sigma_L$ is not constant over time. However, it is possible to show that, for empirically relevant values of the parameters and for the calibrations considered below, elasticity fluctuations do not generate quantitatively significant variation out of the steady state at a second-order accuracy. To this purpose the model has been approximated to second order and simulated using the method developed by Schmitt-Grohé and Uribe (2004). Then it is assumed in the rest of the paper that elasticity is constantly equal to its steady state value. This is inconsequential also for the results obtained in the welfare analysis.

[14] This can be seen as a shortcut to include other forms of nominal rigidities, such as wage stickiness. See also Clarida et al. (1999), Galí (2003) and Woodford (2003)

$$Y = L = C = \left[1 - \frac{1}{\eta}\right]^{\frac{1}{1+\phi}} \tag{22}$$

In contrast, the Pareto efficient levels of output, employment and consumption at the steady state

$$Y^* = C^* = L^* = 1 \tag{23}$$

are obtained equalizing the marginal rate of substitution between consumption and leisure to the corresponding marginal rate of transformation.

Two are the main features of the steady state. On the one hand it is not efficient, since the presence of unions drive a wedge between the marginal rate of substitution and marginal productivity of labor. On the other hand, the steady state is not independent of the monetary policy rule: the central bank can push output towards Pareto efficiency. In fact, the monetary authority can induce wage restraint by making labor demand more elastic through a more aggressive inflation stabilization policy. In particular, in the case of strict inflation targeting, $\gamma_\pi \to \infty$, steady state efficiency is restored

$$\lim_{\gamma_\pi \to \infty} L = \lim_{\gamma_\pi \to \infty} \left[1 - \frac{1}{\eta}\right]^{\frac{1}{1+\phi}} = 1 \tag{24}$$

The outcome of the model does not challenge the conventional neutrality result: a transitory shock to the nominal interest rate dies off in the long-run and leaves the steady state unaffected. The way in which the central bank systematically behaves, however, has an impact on real economic activity. The non-neutrality result of the monetary policy rules emphasized by Bratsiotis and Martin (1999) and Iversen and Soskice (2000) still holds in the baseline NK model and extends to a simple Taylor rule.

Before introducing the policy problem, it is convenient to define a measure of average distortion. A reasonable candidate is the wedge between marginal productivity and the marginal rate of substitution. While the efficient steady state implies the following marginal rate of substitution

$$mrs^* = (L^*)^\phi C = 1$$

at the actual steady state

$$mrs = L^\phi C = 1 - \eta^{-1}$$

so that $\Phi \equiv \eta^{-1}$ can be defined as a measure of steady state inefficiency.

# 3 The Policy Problem

The previous section analyzes the behavior of private agents and unions when the central bank credibly commits to a monetary policy rule. The policy problem faced by the central bank can then be described as the choice of the coefficients entering the rule, taking into account the reaction of the agents to the policy commitment.

I wish to find the optimal monetary policy rule within a class of simple and implementable rules of the kind described by equation (1). A rule is said to be implementable if it brings about a locally unique rational expectation equilibrium in a neighborhood of the non-stochastic steady state, under the assumption of sufficiently tightly bounded exogenous processes. An implementable rule is optimal, within the particular family of policies taken into consideration, if it yields the highest value for a suitably defined welfare criterion.

The definition of such a criterion and the analysis of its implications for the monetary policy problem are the objects of the section. The issue is addressed using the linear-quadratic approach introduced by Rotemberg and Woodford (1997) and further developed by Benigno and Woodford (2005). Because of the long-run non-neutrality of the rule, the welfare measure is decomposed in such a way to disentangle the steady state and the stabilization effects of policy.

Optimality is judged from a timeless perspective. For a policy to be optimal in this sense, it is sufficient to limit central bank's ability to exploit the expectations already in place at the time the commitment is chosen.

## 3.1 The Welfare Criterion

The conditional expectation of lifetime utility as of time zero is

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ logC_t - \frac{L_t^{1+\phi}}{1+\phi} \right] \tag{25}$$

It might seem natural to define the optimal policy rule at time zero as the one that maximizes (25) subject to the constraints imposed by the behavior of the private sector. However, the use of (25) leads to a time inconsistent selection of the rule. This is because the optimal choice correctly takes into account the effects of policy on future expectations, but not on the expectations formed prior to time zero. Past expectations about current outcomes are in fact given at the time of policy selection. As a consequence, should the policy be reconsidered at a later period, the new commitment would not be a continuation of the original plan: the policy maker has the incentive to fool the agents whenever she has the possibility of revising her commitments. I closely follow Benigno and Woodford (2005) who propose to penalize the rules exploiting the expectations already in place at the time the commitment is chosen. According to their method the welfare criterion can be defined in three steps. The intuition of the procedure is described below while I refer to the appendix for the technical details.

First, one needs to characterize the unconstrained timelessly optimal policy. The term unconstrained here refers to the fact that the optimal policy does not necessarily need to be implemented by a simple policy rule of the kind described by equation (1). Note also that, differently from the case studied by Benigno and Woodford (2005), average distortion is controlled by the monetary authority.

Second, it is computed the gain of fooling the agents, that is the value of choosing a policy that does not validate past expectations about current equilibrium outcomes. This is equivalent to compute the gain of deviating from the timelessly optimal plan.

Finally, the welfare criterion is constructed by subtracting from $U_0$ the gain of fooling the agents, $\Psi(\mu^{w,0})$, associated to the policy under scrutiny

$$\hat{U}_0 = U_0 - \Psi(\mu^{w,0})$$

Since $\Psi(\mu^{w,0})$ is a function of the whole history of cost push disturbances up to time zero, it is computed the unconditional expected value of the modified welfare criterion, integrating over all possible histories of shocks. A second order approximation to $\hat{U}_0$ yields the purely quadratic approximate welfare measure

$$\begin{aligned}\hat{W}_0 &= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu_t^w)^2 + \\ &\quad -\frac{1}{2}\frac{\theta_p}{\lambda}E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2\right] - E\Psi(\mu^{w,0}) \qquad (26)\end{aligned}$$

where $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ and $\bar{U}$ is the steady state level of utility. All variables are expressed in log deviations from the non-stochastic steady state and the welfare relevant output gap

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^*$$

is defined as the output deviation from a properly defined target

$$\hat{y}_t^* \equiv a_t - \frac{\Phi}{1+\phi}\mu_t^w$$

The welfare criterion can be used not only to determine the rule that is optimal within a given class, but also to compute the cost of deviating from the optimized rule. Consider two policy regimes, R (reference) and A (alternative), respectively characterized by the induced allocations $(\{C_t^R, L_t^R\}_{t=0}^{\infty})$ and $(\{C_t^A, L_t^A\}_{t=0}^{\infty})$. Then the associated welfare is

$$U^R = U(\{C_t^R, L^R\}_{t=0}^{\infty}) \text{ and } U^A = U(\{C_t^A, L^A\}_{t=0}^{\infty})$$

Let the cost of regime A be denoted by $\gamma$. I measure $\gamma$ as the fraction of regime R's consumption that households would be willing to give up in order to be as well off as under regime A. Formally it is implicitly defined by

$$U(\{(1-\gamma)C_t^R, L^R\}_{t=0}^{\infty}) = U(\{C_t^A, L^A\}_{t=0}^{\infty})$$

It can be easily shown that, given the functional form of the utility function

$$\gamma = 1 - exp\{(1-\beta)(U^A - U^R)\} \qquad (27)$$

### 3.2 Average Distortion, Inflation Stabilization and Welfare

A well defined approximate welfare measure allows to analyze what are the objectives of a benevolent central bank willing to choose the state-contingent path of the economic variables preferred by the private sector. It turns out that, differently from a standard NK framework, the evaluation of alternative policies cannot disregard possible effects stemming from the policy rule non-neutrality due to the presence of unionized labor markets.

In fact, the welfare function can be decomposed into two parts: a stabilization component measuring the welfare effects of fluctuations around the non-stochastic steady state

$$W_0^{Stab} = -\frac{1}{2}\frac{\theta_p}{\lambda}E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2\right] - \Psi(\mu^{w,0}) \tag{28}$$

and a steady state component measuring the welfare effects due to a change in the average distortion of the economy

$$W_0^{StSt} = \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu_t^w)^2 \tag{29}$$

The stabilization component provides a rationale for minimizing inflation and output gap deviations from properly defined targets. Inflation fluctuations are penalized in that they create unnecessary variability in the relative price dispersion. The target level of inflation is zero, because only complete price stability would remove any dispersion in relative prices. Fluctuations in the output gap are also costly. This is because price stickiness implies inefficient changes in the average mark-up charged by firms. As in the case of atomistic agents studied by Benigno and Woodford (2005), the output target is a linear combination of the natural and the efficient output

$$\hat{y}_t^* = \Phi\hat{y}_t^n + (1-\Phi)\hat{y}_t^{FB} \tag{30}$$

where $\hat{y}_t^n$

$$\hat{y}_t^n \equiv a_t - \frac{1}{1+\phi}\mu_t^w \tag{31}$$

is the natural output and the efficient output is

$$\hat{y}_t^{FB} = a_t \tag{32}$$

The case of non-atomistic agents exhibits however an interesting additional feature. For the policy rule has permanent real effects, steady state distortion, which is commonly disregarded as independent of policy, cannot be taken as given. In particular, one cannot abstract from the contribution of the steady state component $W_0^{StSt}$. Looking at (29), two are the channels through which average distortion affects welfare. The first one is represented by the term

$$\frac{\bar{U}(\Phi)}{1-\beta}$$

This is the discounted steady state level of utility, which is a decreasing function of $\Phi$. Recall that $\Phi$ is the wedge between the marginal rate of substitution and the marginal rate of transformation. As long as $\Phi$ is positive, the agents are willing to give up leisure in exchange for consumption at a rate that is on average higher than the one implied by the technological constraints. Hence, they would be better off consuming less leisure and more goods. Tougher stabilization policies induce unions to restrain wages, increasing the steady state level of employment and then enhancing efficiency and welfare. The second component

$$-\frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu_t^w)^2$$

isolates the negative effect of inefficient wage mark-up fluctuations. When the steady state is non distorted, this term disappears and wage mark-up fluctuations do not matter per se but only to the extent they create output gap variability. Only when the steady state is distorted, changes in the mark-up directly and negatively affect welfare. The result is quite intuitive: though transitory, inefficient fluctuations add on top of a positive and permanent level of average distortion, then it would be welfare improving to smooth them over the cycle. It can be proved that the steady state component is strictly decreasing in average distortion.

The analytical expression of the welfare criterion allows to get the intuition of how the policy problem is affected by the strategic interaction channel of monetary policy. Big players in the labor markets internalize the consequences of their actions on aggregate variables. This gives the monetary authority a chance of controlling average distortion that in turn reduces welfare through the two channels described above. As a consequence, the central bank has an additional reason to stabilize inflation other than the usual concern about relative price dispersion: the policy maker has to face an additional dilemma.

## 3.3 The Trade-Off: an Additional Dimension

Being the welfare criterion purely quadratic, it is sufficient to approximate the structural equations to first order, to obtain an approximation to the optimal policy at a first order accuracy. Hence, the policy problem consists in selecting the inflation coefficient entering the policy rule in order to maximize $\hat{W}_0$ subject to the following log-linear constraints

$$\hat{x}_t = E_t \hat{x}_{t+1} - (i_t - E_t \pi_{t+1} - r_t^*) \tag{33}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + (1 - \Phi)\lambda \mu_t^w \tag{34}$$

where (33) is the IS equation and (34) is the New-Keynesian Phillips curve (NKPC). $r_t^*$ is a composite disturbance defined as follows

$$r_t^* = -(1 - \rho_a) a_t + (1 - \rho_u) \frac{\Phi}{1 + \phi} \mu_t^w + \rho$$

Looking at the policy problem, it is possible to isolate an additional dimension of the trade-off with respect to the one traditionally studied in the literature.

Because of the cost push disturbance, it is not feasible to fully stabilize inflation and output gap simultaneously: it is possible to reduce inflation volatility only at the cost of increasing gap volatility. This is the classical trade-off between inflation and output gap stabilization. In an economy populated by atomistic agents, its solution determines optimal fluctuations and provides a complete description of optimal monetary policy. In a model with unions, however, it may be optimal to deviate from those optimal fluctuations in exchange for less average distortion by being more aggressive in stabilizing inflation. Therefore, static efficiency can be enhanced only at the cost of more volatility in the output gap: this is the additional dilemma faced by the policy maker.

The economic intuition suggests that the key forces underlying the new policy trade-off are the standard deviation of the cost push shock relatively to the TFP shock, as in the baseline NK model, and wage setting centralization. The higher the relative standard deviation of the cost push shock ($RS$), the higher the cost of price stability relatively to gap stability. Then, also the cost of reducing average distortion has to be higher in terms of dynamic efficiency. On the other hand, the more the labor market is concentrated, the bigger are unions and then the stronger is the strategic interaction channel of monetary policy. This implies that being tough in stabilizing inflation pays more in terms of average distortion, so that the additional dimension of the trade-off gains importance relatively to the traditional stabilization concerns.

## 3.4 The Trade-Off and the Monetary Policy Rule: Some Discussion

The analysis focuses on a simple monetary policy rule targeting inflation. In fact, the main goal of the paper is to reassess optimal inflation stabilization and its benefits in a model featuring large wage-setters. Assuming a simple rule as the one considered here is a way of quantifying the benefits from inflation stabilization that are overlooked by the literature when abstracting from unionization.

A more general rule may improve upon the optimal policy outcome and eventually a sufficiently flexible rule may also allow to resolve the inefficient level of employment, without necessarily creating a stabilization policy dilemma. For instance, targeting the real marginal cost *and* the shocks directly would be one of the possible ways to threaten the unions against wage pressures, without inducing inefficient stabilization. However, such a rule is non-simple, in that it requires not only the knowledge of the model and the parameter values on the part of the central bank. It also requires observability of all the shocks in real time. Hence, this rule may be seen as hardly implementable, though optimal. This is the very reason why simple rules have been given widespread attention in the literature. Given that it is needed the fully optimal policy so as to evaluate suboptimal simple rules, we assume that the central bank can implement a sufficiently flexible instrument rule so as to derive the fully optimal policy. Still, we keep this part in the Appendix, while we restrict the attention to simple Taylor rules in the rest of the paper. This is also because we do not see the fully optimal policy in this case as an interesting issue. Under commitment indeed the central bank is endowed by assumption with enough credibility to undo the trade-off between optimal stabilization and steady state efficiency via a trigger strategy. In fact, a threat to punish wage pressures would not need to be implemented at equilibrium if credible and, because of commitment, it is *by assumption* a credible out-of-equilibrium outcome. Therefore, we keep the discussion in terms of optimal simple rules targeting inflation.

# 4 Optimal Simple Policy Rules

I turn now to the design of the optimal simple rule which is subsequently used as a benchmark to evaluate the performance of alternative suboptimal rules. The welfare criterion is computed analytically. However, welfare maximization is performed numerically over a grid since first order conditions do not have a closed form solution. Before stating the optimal monetary policy results, it is useful to study the behavior of the welfare function.

It has been established so far that, under a timeless perspective, a benevolent policy maker is choosing the rule in order to maximize

$$\begin{aligned}\hat{W}_0 &= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu_t^w)^2 + \\ &\quad -\frac{1}{2}\frac{\theta_p}{\lambda}E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2\right] - E\Psi(\mu^{w,0})\end{aligned}$$

subject to the constraints imposed by private agents' behavior

$$\hat{x}_t = E_t\hat{x}_{t+1} - (i_t - E_t\pi_{t+1} - r_t^*)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\hat{x}_t + (1-\Phi)\lambda\mu_t^w$$

Using the IS equation, the Phillips curve and the policy rule the equilibrium dynamics can be represented by a system of stochastic difference equations

$$\begin{bmatrix} \hat{x}_t \\ \pi_t \end{bmatrix} = AE_t \begin{bmatrix} \hat{x}_{t+1} \\ \pi_{t+1} \end{bmatrix} + B(r_t^* - \rho) + C\lambda(1-\Phi)\mu_t^w \tag{35}$$

where

$$\Omega = \frac{1}{1+\kappa\gamma_\pi}$$

$$A = \Omega \begin{bmatrix} 1 & 1-\beta\gamma_\pi \\ \kappa & \kappa+\beta \end{bmatrix}$$

$$B = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

$$C = \Omega \begin{bmatrix} -\gamma_\pi \\ 1 \end{bmatrix}$$

The system has a unique solution and the state-contingent evolution of inflation and output gap is

$$\pi_t = f_{\pi,a}a_t + f_{\pi,u}\mu_t^w \tag{36}$$

$$\hat{x}_t = f_{x,a}a_t + f_{x,u}\mu_t^w \tag{37}$$

where $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$ are a function of structural parameters and of the coefficients entering the policy rule. The solution of inflation and output gap are used in the welfare criterion to solve for expectations. Finally, (26) can be related to the monetary policy stance.

$$\begin{aligned} E\hat{W}_0 \quad = \quad & \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}\frac{\sigma_u^2}{1-\beta} - \frac{1}{2}\frac{\sigma_a^2}{1-\beta}\frac{\theta_p}{\lambda}(f_{\pi,a}^2 + \tilde{\lambda}f_{x,a}^2) + \\ & -\frac{1}{2}\frac{\sigma_u^2}{1-\beta}\frac{\theta_p}{\lambda}(f_{\pi,u}^2 + \tilde{\lambda}f_{x,u}^2) + f_{\pi,u}\lambda\Gamma \end{aligned} \tag{38}$$

Appendix D and E show how to recover coefficients $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$ and function (38). $\Gamma$ and $\tilde{\lambda}$ are convolutions of parameters defined in the Appendix.

Before computing the optimal monetary policy, it is instructive to look at the shape of the welfare criterion and to study how it changes when CWS and *RS* vary. In order to plot the welfare function it is considered a range of values for the monetary policy stance, chosen from an equally spaced grid on the interval [1.25,125]. The length of each subinterval is fixed to 0.25. Given the very high value of the upper bound of the grid, a policy setting $\gamma_\pi = 125$ is referred to as strict inflation targeting. Parameters are calibrated as it is reported in Table 3. These values are conventionally used in the NK literature. It has been checked that results are robust to alternative plausible calibrations. Concerning the cost push shock, autocorrelation is set to zero while alternative calibrations of $\sigma_{\varepsilon,u}$ are considered in order to match different values of the relative standard deviation, as it is displayed in Table 4. It is labelled as high, medium or low a cost push shock standard deviation that is respectively twenty, ten or five times TFP standard deviation. These are the three representative cases commented below. Note that in general the values considered for the standard deviation of the cost push shock are quite high. Hence the calibration is relatively conservative in the sense that results are biased against the argument that unionized labor markets matter for optimal monetary policy.

Two are the main results suggested by the numerical analysis.

First, given wage setting centralization and the chosen bounds for aggressiveness in inflation stabilization, you can find a value of the relative standard deviation, $RS^*$, such that if $RS < RS^*$ strict inflation targeting performs better than any other policy considered within the bounds. If relative standard deviation is higher, the welfare function has a maximum within the bounds[15]. This is because a high relative standard deviation implies high marginal costs of over-stabilizing inflation relatively to marginal gains in terms of average distortion: the stabilization dimension of the trade-off dominates the second one. The intuition is confirmed looking at the graphs.

The left hand panel of Figure 1 displays the welfare criterion for an economy with three unions and low $RS$. The function is strictly increasing in the inflation coefficient, hence strict inflation targeting is the optimal policy. The right hand panel shows the welfare cost of deviating from the optimized value. To grasp some insight, total welfare is decomposed in steady state and stabilization component in Figures 2 and 3 respectively. In both charts the solid line represents actual welfare while the dotted line is the value that corresponds to the inflation coefficient maximizing total welfare. Looking at Figures 2 and 3, it is immediate to see that in the optimal policy the steady state component is maximized while the stabilization component is not. Hence, given the degree of concentration in the labor markets a low $RS$ resolves the trade-off between stabilization and average distortion in favor of the latter. The opposite is observed in the case of a high $RS$. Figure 4 again displays total welfare for an economy with three unions. Now the function has a maximum. If the effect of policy is decomposed, as in Figure 5 and 6, it is evident that the stabilization part is maximized while steady state welfare is not. The additional dilemma is dominated by the traditional concerns about stabilization.

Then, it can be inferred that the higher is the relative standard deviation of the cost push shock the less labor market unionization matters in terms of optimal monetary policy.

[15] The apparent discontinuity is induced only by the fact that the welfare function is evaluated numerically over a grid. The most plausible conjecture, however, is that it can always be found a maximum if the upper bound of the grid is sufficiently high. Moreover, the results considered altogether do not suggest any discontinuity: high CWS always calls for higher $\gamma_\pi$ and high $RS$ always requires lower $\gamma_\pi$.

The second result is that $RS^*$ is increasing with the centralization of wage setting: it is more likely to prefer strict inflation targeting when labor markets are concentrated. The intuition is that high CWS implies high steady state marginal gains from inflation stabilization. Once again it is insightful to have a look at the plots.

Consider the case of three unions and low, high or medium $RS$ as depicted in Figures 1, 4 and 7 respectively. It can be easily seen that $RS^*$=MEDIUM, i.e. if the relative standard deviation of the cost push shock is higher than or equal to the medium value, then strict inflation targeting is not optimal. However, if you consider the case with two unions as in figures 8 and 9, it is clear that $RS^*$=HIGH. This means that when the degree of CWS increases it is needed a higher volatility of the cost push shock to rule out strict inflation targeting as the optimal policy.

With a clear intuition of how the welfare criterion is affected by the key forces underlying the policy trade-off, it is straightforward to interpret the optimal monetary policy results.

Optimal monetary policy is defined by the inflation coefficient entering the Taylor rule that maximizes the welfare criterion over the grid. Table 5 shows the value of $\gamma_\pi$ as a function of the degree of centralization of wage setting and of the relative standard deviation of the cost push shock. The main result is that the optimal stance is always increasing in the centralization of the wage bargaining process. Interestingly, if the volatility of the cost push shock is sufficiently low and the concentration of the labor market is high enough, then strict inflation targeting is optimal even in the presence of inefficient fluctuations of output. This is the case of low $RS$ and 2, 3 or 5 unions. On the other hand, for high values of the volatility of the cost push shock, the policy maker accepts some volatility of inflation as in the standard NK model. However, the more the labor market is concentrated, the higher is the optimal aggressiveness.

Then, it can be concluded that the optimal policy is significantly affected by the labor market structure.

Welfare analysis allows to assess more closely the relevance of the changes induced in the policy prescriptions by the presence of a unionized labor force. Tables 6 and 7 display the welfare cost of adopting an ad-hoc Taylor rule with a coefficient $\gamma_\pi = 1.5$ instead of the optimal one. The two extreme cases of high and low $RS$ are considered for an economy characterized by 2, 3 or 15 unions.

If $RS$ is high, welfare costs are almost entirely accounted for by the stabilization component that is however implausibly high (always more than three percentage points). In the case of $N = 2$ the steady state cost is not negligible (0.2473 percentage points of consumption) while it is not significant for $N = 3$ and $N = 15$ (less than a hundredth of a percentage point). On the other hand, if $RS$ is low and the labor market is highly concentrated (as in the case of $N = 2$ or $N = 3$), not only the steady state component is not negligible, it is also the most important part of the welfare cost. Finally, if the wage bargaining process is sufficiently decentralized, as for $N = 15$, the steady state component is again negligible as in the case of high $RS$.

Hence, welfare analysis suggests that both the total and the steady state cost of deviating from the optimal policy are increasing in the centralization of wage setting. In particular, the steady state cost as a fraction of the total increases with CWS and decreases with the relative standard deviation of the cost push shock.

We can conclude that, unless implausibly high values for the standard deviation of the cost push shock are assumed, it is costly to disregard the labor market structure as a determinant of the optimal monetary policy. This is because the central bank can induce wage restraint and then reduce average distortion through aggressive inflation stabilization. The gains stemming from aggressiveness are greater than the costs associated to a higher variability of the output gap. The fact that that most of the cost is coming from the steady state component is in line with the economic intuition.

# 5 Conclusion

The paper studies whether and how the labor market structure affects the monetary policy problem in a model with nominal rigidities and non-atomistic unions. In particular, it is computed the optimal simple interest rate rule as a function of the degree of wage setting centralization.

The main finding is that the optimal aggressiveness in stabilizing inflation is increasing in wage setting centralization. Moreover, the relevance of policy prescriptions is assessed resorting to welfare analysis. It turns out that it is significantly costly to disregard possible inefficiencies stemming from high degrees of centralization of the bargaining process.

# A Appendix: The Private Sector Equilibrium

It is convenient to collect the equations defining the private sector equilibrium. Given $\Delta_{-1}$, exogenous stochastic processes $A_t$ and $\mu_t^w$ and given a value for the policy parameter $\gamma_\pi$, the rational expectation equilibrium for the sticky price economy is a process $\{Y_t, \Pi_t, \Delta_t, F_t, K_t\}_{t=0}^{\infty}$ that satisfies the following system of equations

$$Y_t^{-1} = \Pi_t^{\gamma_\pi} E_t\{\Pi_{t+1}^{-1} Y_{t+1}^{-1}\} \tag{39}$$

$$\frac{1-\alpha\Pi_t^{\theta_p - 1}}{1-\alpha} = \left(\frac{K_t}{F_t}\right)^{1-\theta_p} \tag{40}$$

$$K_t = \frac{\eta}{\eta - 1} exp\{\mu_t^w\} \left(\frac{Y_t}{A_t}\right)^{1+\phi} \Delta_t^{\phi} + \alpha\beta E_t \left\{(\Pi_{t+1})^{\theta_p} K_{t+1}\right\} \tag{41}$$

$$F_t = 1 + \alpha\beta E_t \left\{(\Pi_{t+1})^{\theta_p - 1} F_{t+1}\right\} \tag{42}$$

$$\Delta_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right)^{\frac{\theta_p}{\theta_p-1}} + \alpha\Pi_t^{\theta_p}\Delta_{t-1} \tag{43}$$

$$\eta = \theta_w(1-\frac{1}{n}) + \frac{1}{n}\gamma_\pi \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{44}$$

# B Appendix: The Impact of Union's $j$ Wage on The Aggregate Wage Index

Let the real wage be

$$w_t = \frac{W_t}{P_t} \tag{45}$$

hence

$$w_t = \left[\int_0^1 w_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}} \tag{46}$$

Considering that the representative union takes as given the wage of the workers of other unions and that the wage is the same for the workers of union $j$

$$\begin{aligned}
\frac{\partial w_t}{\partial w_{t,j}} &= \frac{\partial}{\partial w_{t,j}}\left[\int_0^1 w_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}} \\
&= \frac{\partial}{\partial w_{t,j}}\left[\int_{i\in j} w_t(i)^{1-\theta_w} di + \int_{i\notin j} w_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}} \\
&= \frac{1}{n}\left[\frac{w_{t,j}}{w_t}\right]^{-\theta_w} = \frac{1}{n}
\end{aligned} \tag{47}$$

the result follows immediately from the definition of the real aggregate wage index. The last equality holds because of symmetry at equilibrium. Note that, because of symmetry, it is also true that

$$\frac{\partial w_t}{\partial w_{t,j}} \frac{w_{t,j}}{w_t} \quad = \quad \frac{\partial w_t}{\partial w_{t,j}} = \frac{1}{n} \tag{48}$$

# C Appendix: Labor Demand Elasticity

The elasticity of labor demand perceived by the j-th union can be derived in three steps

**Step 1: The elasticity of inflation to the aggregate real wage**
From equations (12), (41) and (42) the elasticity of inflation to the aggregate real wage is

$$\Sigma_{\Pi,t} \equiv \frac{\partial log\Pi}{\partial log w} = \Pi_t^{1-\theta_p} \left(\frac{K_t}{F_t}\right)^{1-\theta_p} \frac{1-\alpha}{\alpha} \frac{MC_t}{K_t} \tag{49}$$

At the zero inflation steady state

$$\Sigma_{\Pi} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{50}$$

**Step 2: The elasticity of aggregate labor demand to the aggregate wage index**
Aggregate labor demand is a function of aggregate demand faced by firms. The elasticity of aggregate labor to aggregate demand is constant and equal to 1. It follows from aggregate demand (6) and the elasticity of inflation to the aggregate real wage index (49) that

$$\Sigma_{L,t} \equiv \ -\frac{\partial log L}{\partial log w} = -\frac{\partial log L}{\partial log \Pi}\Sigma_{\Pi,t} = \gamma_\pi \Sigma_{\Pi,t} \tag{51}$$

At the zero inflation steady state

$$\Sigma_L = \gamma_\pi \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{52}$$

**Step 3: The elasticity of type $j$ labor demand to union's $j$ real wage**
From firms' optimization problem

$$L_t^*(j) = \left[\frac{w_t(j)}{w_t}\right]^{-\theta_w} L_t \tag{53}$$

Equation (53) allows the j-th wage setter to compute the perceived elasticity of its own labor demand with respect to the real wage charged (**differently from the standard case**, aggregate labor is **NOT** taken as given, but it is perceived to be a function of the real wage through the strategic interaction with the central bank as it is showed in steps 1 and 2). Hence,

$$\begin{aligned}\eta_t &\equiv -\frac{\partial log L_{t,j}}{\partial log w_{t,j}} \\ &= \theta_w - \frac{1}{n}\theta_w + \frac{1}{n}\Sigma_{L,t} \\ &= \theta_w - \frac{1}{n}\theta_w + \frac{1}{n}\gamma_\pi \Pi_t^{1-\theta_p}\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\frac{1-\alpha}{\alpha}\frac{MC_t}{K_t} \qquad (54)\end{aligned}$$

$\theta_w$ is assumed to be such that labor demand is elastic, that is $\eta > 1$. It is immediate to see from (54) that labor elasticity is not constant over time. This implies that the wage mark-up fluctuates over time. At the zero inflation steady state

$$\eta = \theta_w - \frac{1}{n}\theta_w + \frac{1}{n}\gamma_\pi \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \qquad (55)$$

# D Appendix: Derivation of equation (26)

The welfare criterion (26) is derived using the method proposed by Benigno and Woodford (2005) for the evaluation of suboptimal policy rules. First, it is characterized the timelessly optimal policy, i.e. an optimal policy that validates private sector's expectations at time zero. Then it is computed an approximation to the value of deviating from the timelessly optimal policy. That value is finally subtracted from the second order approximation of households' lifetime utility.

In the case of non-atomistic wage setters the procedure differs with respect to the one treated in Benigno and Woodford (2005) in that average distortion is not independent of policy. However it can be shown that the timelessly optimal problem can be suitably redefined and solved in two steps: the determination of the timelessly optimal allocation as a function of average distortion and then the choice of the average distortion that maximizes households' utility subject to the constraint of implementing a timelessly optimal allocation.

This further complication makes convenient to introduce the notion of timelessly optimal fluctuations (or timelessly optimal stabilization policy). Recall that $x_t = (Y_t, \Pi_t, \Delta_t)$ and $X_t = (F_t, K_t)$.

**Definition 1**: *Let $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^{\infty}$ be the solution to the following problem*

Max $U_0$ s.t.

$$\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha} = \left(\frac{K_t}{F_t}\right)^{1-\theta_p} \qquad (56)$$

$$K_t = [1-\Phi]^{-1} exp\{\mu_t^w\}\left(\frac{Y_t}{A_t}\right)^{1+\phi}\Delta_t^{\phi} + \alpha\beta E_t\left\{(\Pi_{t+1})^{\theta_p} K_{t+1}\right\} \qquad (57)$$

$$F_t = 1 + \alpha\beta E_t \left\{ (\Pi_{t+1})^{\theta_p - 1} F_{t+1} \right\} \quad (58)$$

$$\Delta_t = (1-\alpha)\left( \frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha} \right)^{\frac{\theta_p}{\theta_p-1}} + \alpha\Pi_t^{\theta_p}\Delta_{t-1} \quad (59)$$

$$X_0 = X_0^* \quad (60)$$

*given* $\Delta_{-1}$, $\{A_t, \mu_t^w\}_{t=0}^{\infty}$, $X_0^*$ *and a value for average distortion* $\Phi$. *If* $X_0^*$ *is chosen in such a way that* $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^{\infty}$ *is a time invariant function of exogenous states*[16], *then* $\{x_t^*(\Phi), X_t^*(\Phi)\}_{t=0}^{\infty}$ *is defined to be the timelessly optimal stabilization policy.*

Note that the timelessly optimal stabilization policy is **conditional** on $\Phi$, it is in other terms the best response to shocks, given a certain degree of average distortion. In line with the timeless perspective, the initial value of forward looking variables is constrained in the stabilization policy problem. Technically, these constraints allow to make recursive a problem that naturally is not. Economically, imposing those constraints is equivalent to ask the policy maker not to take advantage of expectations already in place at the time of choosing the commitment.

If the central bank were not constrained by a simple rule, she could choose whatever degree of average distortion she liked, $\Phi^*$, and then implement the timelessly optimal stabilization policy consistent with that degree of average distortion by selecting an appropriate policy rule. $\{x_t^*(\Phi^*), X_t^*(\Phi^*), \Phi^*\}_{t=0}^{\infty}$ would then be a full characterization of the timelessly optimal policy. Formally the follow definition applies.

**Definition 2**: *Let* $\{x_t^*, X_t^*, \Phi^*\}_{t=0}^{\infty}$ *be the solution to the following problem*

$$\text{Max } U_0 \text{ s.t.}$$

$$\{x_t^*\}_{t=0}^{\infty} = \{x_t^*(\Phi)\}_{t=0}^{\infty}$$

$$\{X_t^*\}_{t=0}^{\infty} = \{X_t^*(\Phi)\}_{t=0}^{\infty}$$

*given* $\Delta_{-1}$ *and* $\{A_t, \mu_t^w\}_{t=0}^{\infty}$. *Then* $\{x_t^*, X_t^*, \Phi^*\}_{t=0}^{\infty}$ *is defined to be the timelessly optimal policy.*

[16] see Woodford (2003) and Giannoni and Woodford (2002)

Hence, the timelessly optimal policy problem can be broken in two steps: first the choice of optimal fluctuations compatible with any degree of average distortion and then the choice of average distortion or, equivalently, the choice of the non-stochastic steady state.

Concerning the second step, it is assumed that whenever the bank has the chance to choose monetary policy without restricting to a simple rule, the best average distortion is zero. This amounts to assume that the marginal benefits of reducing average distortion are greater than the marginal costs. It has been checked numerically that this is always the case for all calibrations considered here.

The rest of the section develops as follows: section 1 characterizes and approximates to first order the timelessly optimal stabilization policy; section 2 derives the welfare criterion for the evaluation of simple policy rules.

## D.1 Timelessly Optimal Fluctuations

The problem associated to Definition 1 has no closed form solution. However, using a linear-quadratic approach allows to obtain an approximate characterization of the timelessly optimal stabilization policy at a first order accuracy. Before resorting to local approximation techniques it is shown the existence of a non-stochastic steady state.

The constraints implied by the initial commitments $X_0 = X_0^*$ can be equivalently rewritten as

$$\Pi_0^{\theta_p - 1} F_0 = \Pi_0^{*\theta_p - 1} F_0^* \tag{61}$$

$$\Pi_0^{\theta_p} K_0 = \Pi_0^{*\theta_p} K_0^* \tag{62}$$

where $\Pi_0^*$ is the inflation rate consistent with $X_0^*$ according to equation (56). Let $\psi_{1,t}$ through $\psi_{4,t}$ denote the Lagrange multipliers corresponding to constraints (56) through (59) and let $-\alpha\psi_{2,-1}^*$ $-\alpha\psi_{3,-1}^*$ denote the Lagrange multipliers corresponding to constraints (61) and (62). Hence, the problem associated to Definition 1 can be restated using the following Lagrangian function

$$\begin{aligned}
\Lambda_t &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ h(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) \right\} \\
&- \psi_{2,-1}^* \alpha \left[ \Pi_0^{\theta_p - 1} F_0 - \Pi_0^{*\theta_p - 1} F_0^* \right] \\
&- \psi_{3,-1}^* \alpha \left[ \Pi_0^{\theta_p} K_0 - \Pi_0^{*\theta_p} K_0^* \right]
\end{aligned}$$

where $\psi$ is the vector of Lagrange multipliers and $h(\cdot)$ is defined as

$$
\begin{aligned}
h(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) = u_t \quad &+ \quad \psi_{1,t}\left[K_t\left(\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right)^{\frac{1}{\theta_p-1}} - F_t\right] \\
&+ \quad \psi_{2,t}\left[F_t - 1 - \alpha\beta(\Pi_{t+1})^{\theta_p-1}F_{t+1}\right] \\
&+ \quad \psi_{3,t}\left[K_t - MC_t - \alpha\beta(\Pi_{t+1})^{\theta_p}K_{t+1}\right] \\
&+ \quad \psi_{4,t}\left[\Delta_t - (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right)^{\frac{\theta_p}{\theta_p-1}} - \alpha\Pi_t^{\theta_p}\Delta_{t-1}\right]
\end{aligned}
$$

For convenience the following definitions have been used

$$
MC_t = [1-\Phi]^{-1}\, exp\{\mu_t^w\}\left(\frac{Y_t}{A_t}\right)^{1+\phi}\Delta_t^{\phi}
$$

$$
u_t = logY_t - \frac{\left(\frac{Y_t\Delta_t}{A_t}\right)^{(1+\phi)}}{1+\phi}
$$

The marginal benefit of relaxing constraints (61) and (62) is equal to the value of the corresponding Lagrange multipliers and it can be interpreted as the marginal gain of fooling agents at time zero.

Rearranging terms, the Lagrangian can be rewritten (up to a constant) in the following discounted stationary form so that a time invariant system of first order conditions can be trivially obtained

$$
\Lambda_t \quad = \quad E_0\sum_{t=0}^{\infty}\beta^t g(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) \tag{63}
$$

where $g(\cdot)$ is now defined as

$$
\begin{aligned}
g(\psi_t, \psi_{t-1}; x_t, x_{t-1}, X_t) = u_t \quad &+ \quad \psi_{1,t}\left[K_t\left(\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right)^{\frac{1}{\theta_p-1}} - F_t\right] \\
&+ \quad \psi_{2,t}[F_t - 1] - \alpha\psi_{2,t-1}\left[(\Pi_t)^{\theta_p-1}F_t\right] \\
&+ \quad \psi_{3,t}[K_t - MC_t] - \alpha\psi_{3,t-1}\left[(\Pi_t)^{\theta_p}K_t\right] \\
&+ \quad \psi_{4,t}\left[\Delta_t - (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_p-1}}{1-\alpha}\right)^{\frac{\theta_p}{\theta_p-1}} - \alpha\Pi_t^{\theta_p}\Delta_{t-1}\right]
\end{aligned}
$$

(63) has the same form as the one used by Benigno and Woodford (2005)[17] and it can be immediately seen that their results apply to the case with non-atomistic wage setters. Hence I refer to their paper in stating the following results.

**Proposition 1**: *The non-stochastic steady state of the problem associated to Definition 1 exists and is such that*

$$K = F = (1 - \alpha\beta)^{-1}$$

$$\Pi = \Delta = 1$$

$$Y = (1 - \Phi)^{\frac{1}{1+\phi}}$$

**Proposition 2**: *A second order approximation to lifetime utility (25) yields*

$$\frac{\bar{U}}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \left[ \Phi \hat{y}_t - \frac{1}{2} u_{yy} \hat{y}_t^2 - \frac{1}{2} u_\pi \hat{\pi}_t^2 + u_{ya} \hat{y}_t a_t - \frac{1}{2} u_{aa} a_t^2 + u_a a_t \right] \quad (64)$$

where $\hat{y}_t$ measures deviations of aggregate output from its steady state level and the coefficients entering equation (64) are

$$\begin{aligned} u_{yy} = u_{ya} = u_{aa} &= (1-\Phi)(1+\phi) \\ u_\pi &= (1-\Phi)\frac{\theta_p}{\lambda} \\ u_a &= (1-\Phi) \\ \lambda &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \end{aligned}$$

Note that when the steady state is distorted, a non-zero linear term appears in (64), implying that you cannot evaluate utility to the second order using an approximate solution for output that is accurate to first order only. However, the linear term can be substituted out using a second order approximation to the aggregate supply (56)

**Proposition 3**: *The second order approximation to lifetime utility (64) can be rewritten in the following purely quadratic form*

$$\begin{aligned} W_0 &= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\theta_p}{\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p} \hat{x}_t^2 \right] + \\ &\quad - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi} E_0 \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + E_0 \sum_{t=0}^{\infty} \beta^t \hat{y}_t^* + T_0 \end{aligned} \quad (65)$$

[17] see their Appendix B1

**Proof**: *A second order approximation to the aggregate supply (56) yields*

$$V_0 = \lambda E_0 \sum_{t=0}^{\infty} \beta^t [v_y \hat{y}_t + \frac{1}{2} v_{yy} \hat{y}_t^2 + \frac{1}{2} v_\pi \hat{\pi}_t^2 - v_{ya} \hat{y}_t a_t + \frac{1}{2} v_{aa} a_t^2 - v_a a_t + \\ + \mu_t^w + \frac{1}{2} (\mu_t^w)^2 + (1+\phi) \mu_t^w (\hat{y}_t - a_t)] \quad (66)$$

*where*

$$\begin{aligned} v_{yy} = v_{ya} = v_{aa} &= (1+\phi)^2 \\ v_\pi &= (1+\phi)\frac{\theta_p}{\lambda} \\ v_a &= (1+\phi) \\ v_y &= (1+\phi) \\ \hat{x}_t &= \hat{y}_t - \hat{y}_t^* \\ \hat{y}_t^* &= a_t - \frac{\Phi}{1+\phi}\mu_t^w \end{aligned}$$

*Subtracting* $\frac{\Phi}{\lambda(1+\phi)} V_0$ *from* $U_0$ *one can obtain (65) where* $T_0$

$$T_0 = \frac{\Phi}{\lambda(1+\phi)} V_0$$

*is a deterministic component that depends only on the initial commitments on the forward looking variables and that is predetermined at the time of the policy choice.*

These results can be used to derive a first order approximation to the timelessly optimal stabilization policy. Within a linear-quadratic framework the problem associated to Definition 1 can be reformulated as follows

$$\begin{gathered} \text{Min } \tfrac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{\lambda} \hat{x}_t^2\right] \text{ s.t.} \\ \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + (1-\Phi)\lambda \mu_t^w \\ \pi_0 = \pi_0^* \end{gathered}$$

where

$$\begin{aligned} \kappa &= \lambda(1+\phi) \\ \tilde{\lambda} &= \kappa/\theta_p \end{aligned}$$

Defining $\varphi$ as the Lagrange multiplier associated to the log-linear version of the aggregate supply, the first order conditions are

$$\begin{aligned} \pi_t + \varphi_t - \varphi_{t-1} &= 0 \\ \tilde{\lambda}\hat{x}_t - \kappa\varphi_t &= 0 \end{aligned}$$

These conditions can be rearranged in order to have either a targeting rule

$$\pi_t + \frac{\tilde{\lambda}}{\kappa}(\hat{x}_t - \hat{x}_{t-1}) = 0 \tag{67}$$

or an equation describing the evolution of the Lagrange multiplier

$$E_t\varphi_{t+1} - \frac{1}{\beta}(1 + \beta + \frac{\kappa^2}{\tilde{\lambda}})\varphi_t + \frac{1}{\beta}\varphi_{t-1} = \frac{1}{\beta}(1 - \Phi)\lambda\mu_t^w \tag{68}$$

It can be shown that the characteristic equation

$$\mu^2 - \frac{1}{\beta}(1 + \beta + \frac{\kappa^2}{\tilde{\lambda}})\mu + \frac{1}{\beta} = 0$$

has two roots $\mu_1$ and $\mu_2$ such that $0 < \mu_1 < 1 < \mu_2$. Hence, equation (68) has a unique bounded solution and

$$\varphi_t = \mu\varphi_{t-1} - \mu(1 - \Phi)\lambda E_0 \sum_{t=0}^{\infty}(\beta\mu)^t\mu_t^w \tag{69}$$

where $\mu \equiv \mu_1$. If a process for the mark-up shock of the form (21) is assumed, (69) becomes

$$\varphi_t = \mu\varphi_{t-1} - \frac{\mu(1 - \Phi)\lambda}{1 - \beta\rho_u\mu}\mu_t^w \tag{70}$$

The Lagrange multiplier can be solved as a function of the history of wage mark-up shocks

$$\varphi_t = -\frac{\mu(1 - \Phi)\lambda}{1 - \beta\rho_u\mu}\sum_{j=0}^{\infty}\mu^j\mu_{t-j}^w \tag{71}$$

Finally (71), together with the log-linear version of the Phillips curve and the first order conditions, determines inflation and output gap as a function of the history of shocks and average distortion. In the timelessly optimal policy average distortion is zero, hence it follows that

$$\varphi_t^* = -\frac{\mu\lambda}{1 - \beta\rho_u\mu}\sum_{j=t}^{\infty}\mu^j\mu_{t-j}^w \tag{72}$$

(72) can be interpreted as a first order approximation to the marginal value of deviating from the timelessly optimal policy.

## D.2 Evaluation of suboptimal rules

Although expected lifetime utility as of time zero has been used in determining the timelessly optimal policy, $W_0$ cannot serve the purpose of evaluating policy rules. This is because of the time inconsistency issue.

In a timeless perspective, initial commitments guarantee that policy confirms past expectations about current outcomes. However, it may be the case that the optimal initial commitments are not feasible within the class of rules under consideration. In turn the violation of initial commitments may give an advantage to those rules, because of the usual time inconsistency that naturally arises in any Ramsey problem.

Notwithstanding, it is undesirable to prefer rules that are improving the stabilization trade-off by fooling the agents at the time of policy selection. Therefore, Benigno and Woodford (2005) propose to use a welfare criterion that is still based on expected lifetime utility but that penalizes deviations from the timelessly optimal commitments. In particular the criterion is modified in such a way that if the class is flexible enough to contain the timelessly optimal policy, then the rule implementing the timelessly optimal policy is selected as the best one. Hence the new criterion becomes

$$\hat{U_0} = U_0 - \psi^*_{2,-1}\alpha \left[\Pi_0^{\theta_p - 1} F_0 - \Pi_0^{*\theta_p - 1} F_0^*\right] - \psi^*_{3,-1}\alpha \left[\Pi_0^{\theta_p} K_0 - \Pi_0^{*\theta_p} K_0^*\right] \quad (73)$$

Note that any rational expectation equilibrium that is maximizing (73) and is satisfying the timelessly optimal commitments is by definition the timelessly optimal allocation. It is in fact the solution to the problems associated to Definition 1 and Definition 2. In addition the following result holds

**Proposition 4**: *A second order approximation to the modified welfare criterion (73) can be written in the following purely quadratic form*

$$\begin{aligned} W_0 - \varphi^*_{-1}(\pi_0 - \pi_0^*) = {} & \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi} E_0 \sum_{t=0}^{\infty} \beta^t (\mu_t^w)^2 + E_0 \sum_{t=0}^{\infty} \beta^t \hat{y}_t^* \\ & - \frac{1}{2}\frac{\theta_p}{\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2\right] - \varphi^*_{-1}(\pi_0 - \pi_0^*) \end{aligned} \quad (74)$$

*where $\varphi^*_{-1}$ is the Lagrange multiplier associated to the timelessly optimal policy problem in its linear-quadratic version and $\pi_0^*$ is a first order approximation to the timelessly optimal initial commitment*

Since the Lagrange multiplier depends on the history of shocks prior to the policy choice, in the spirit of the timeless it is computed the unconditional expectation of (74) integrating over all possible histories of the shocks

$$
\begin{aligned}
\hat{W}_0 &= E\left\{W_0 - \varphi^*_{-1}(\pi_0 - \pi^*_0)\right\} \\
&= \frac{\bar{U}(\Phi)}{1-\beta} - \frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu^w_t)^2 + \\
&\quad -\frac{1}{2}\frac{\theta_p}{\lambda}E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + (1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2\right] - E\left\{\varphi^*_{-1}(\pi_0 - \pi^*_0)\right\} \qquad (75)
\end{aligned}
$$

Defining $E\Psi(\mu^{w,0})$ as $E\left\{\varphi^*_{-1}(\pi_0 - \pi^*_0)\right\}$, (75) becomes (26).

# E Appendix: Derivation of coefficients $f_{\pi,a}$, $f_{\pi,u}$, $f_{x,a}$ and $f_{x,u}$

The system of stochastic difference equation (35) has a unique solution of the form

$$
\begin{aligned}
\begin{bmatrix} \hat{x}_t \\ \pi_t \end{bmatrix} &= -(1-\rho_a)\left[I - \rho_a A\right]^{-1} B a_t + \\
&\quad \left[I - \rho_u A\right]^{-1}\left[\frac{(1-\rho_u)\Phi B}{1+\phi} + \lambda(1-\Phi)C\right]\mu^w_t
\end{aligned}
$$

Defining

$$TFP = -(1-\rho_a)\left[I - \rho_a A\right]^{-1} B$$

and

$$CP = \left[I - \rho_u A\right]^{-1}\left[\tfrac{(1-\rho_u)\Phi B}{1+\phi} + \lambda(1-\Phi)C\right]$$

it follows that $f_{\pi,a} = TFP(2,1)$, $f_{x,a} = TFP(1,1)$, $f_{\pi,u} = CP(2,1)$ and $f_{x,u} = CP(1,1)$.

# F Appendix: Derivation of equation (38)

Define

$$\sigma_u^2 = \frac{\sigma^2_{\varepsilon,u}}{1-\rho_u^2}$$

$$\sigma_a^2 = \frac{\sigma^2_{\varepsilon,a}}{1-\rho_a^2}$$

Using (36) and (37), the third term of (75) becomes

$$E\left\{\sum_{t=0}^{\infty}\beta^t[\pi_t^2+(1+\phi)\frac{\lambda}{\theta_p}\hat{x}_t^2]\right\}=\frac{\sigma_a^2}{1-\beta}(f_{\pi,a}^2+\tilde{\lambda}f_{x,a}^2)+\frac{\sigma_u^2}{1-\beta}(f_{\pi,u}^2+\tilde{\lambda}f_{x,u}^2) \quad (76)$$

From the solution of the Lagrange multiplier (71)

$$E\{\varphi_{t-1}\mu_t^w\}=-\frac{\mu\lambda\rho_u\sigma_u^2}{(1-\beta\rho_u\mu)(1-\rho_u\mu)}$$

Substituting the previous equation and (36) in the fourth term of (75) yields

$$\begin{aligned} E\left\{\varphi_{-1}^*(\pi_0-\pi_0^*)\right\} &= f_{\pi,u}E\{\varphi_{-1}^*\pi_0\}+t.i.p. \\ &= -f_{\pi,u}\frac{\mu\lambda\rho_u\sigma_u^2}{(1-\beta\rho_u\mu)(1-\rho_u\mu)}+t.i.p. \\ &= -f_{\pi,u}\lambda\Gamma+t.i.p. \end{aligned} \quad (77)$$

where

$$\Gamma=\frac{\mu\rho_u\sigma_u^2}{(1-\beta\rho_u\mu)(1-\rho_u\mu)}$$

Finally, given the stochastic properties of the wage mark-up shock,

$$\frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}E\sum_{t=0}^{\infty}\beta^t(\mu_t^w)^2=\frac{1}{2}\frac{\Phi(1-\Phi)}{1+\phi}\frac{\sigma_u^2}{1-\beta} \quad (78)$$

Using (76), (77) and (78) in (75), (38) can be immediately obtained.

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Figure 1: Low $RS$, 3 Unions

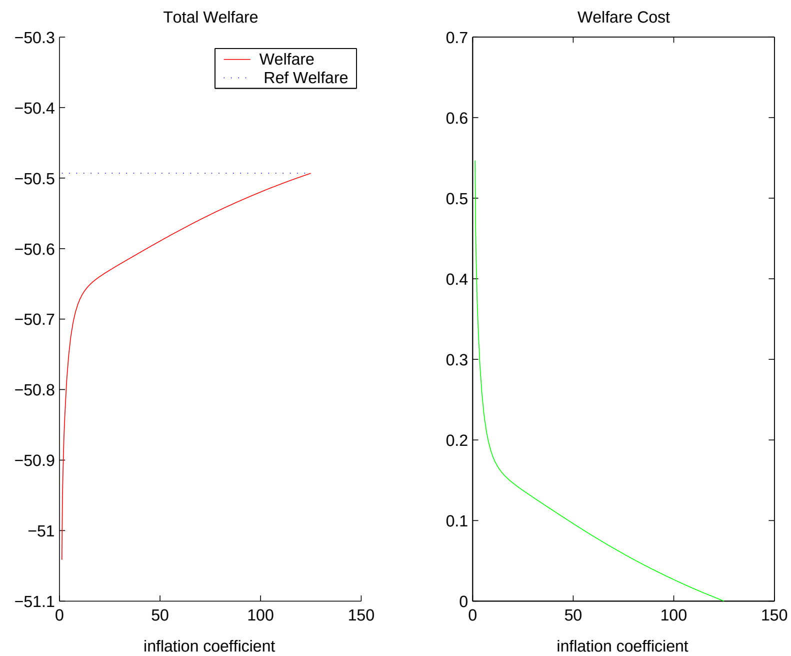


Figure 2: Low $RS$, 3 Unions

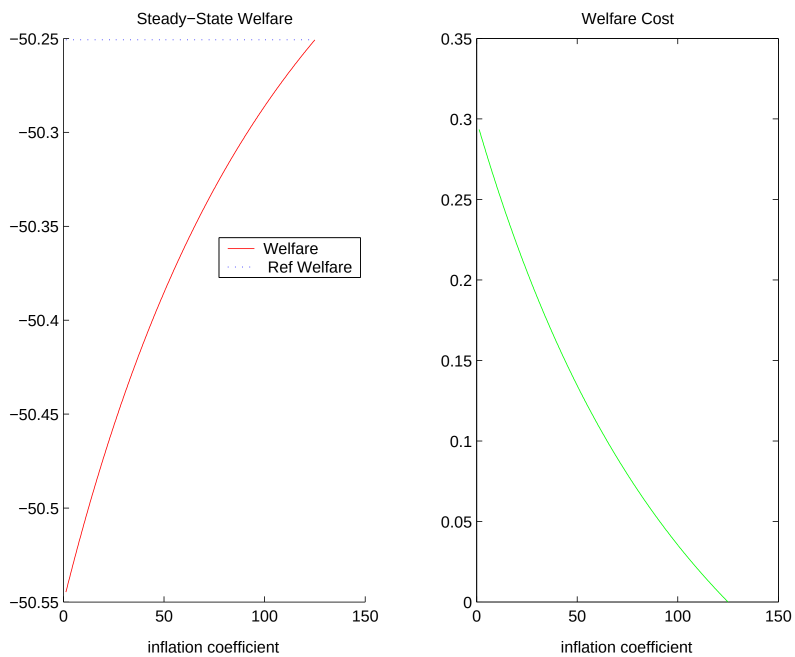

Figure 3: Low $RS$, 3 Unions

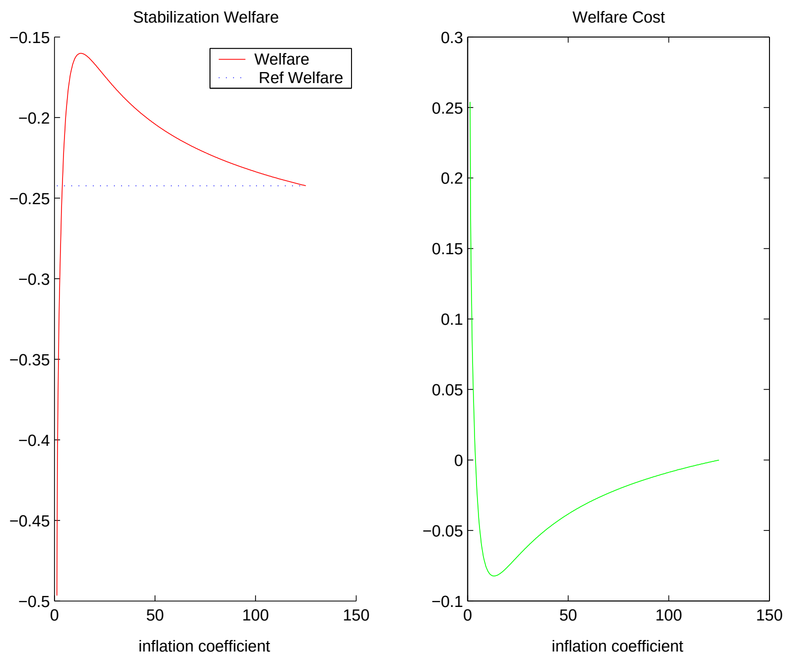


Figure 4: High $RS$, 3 Unions

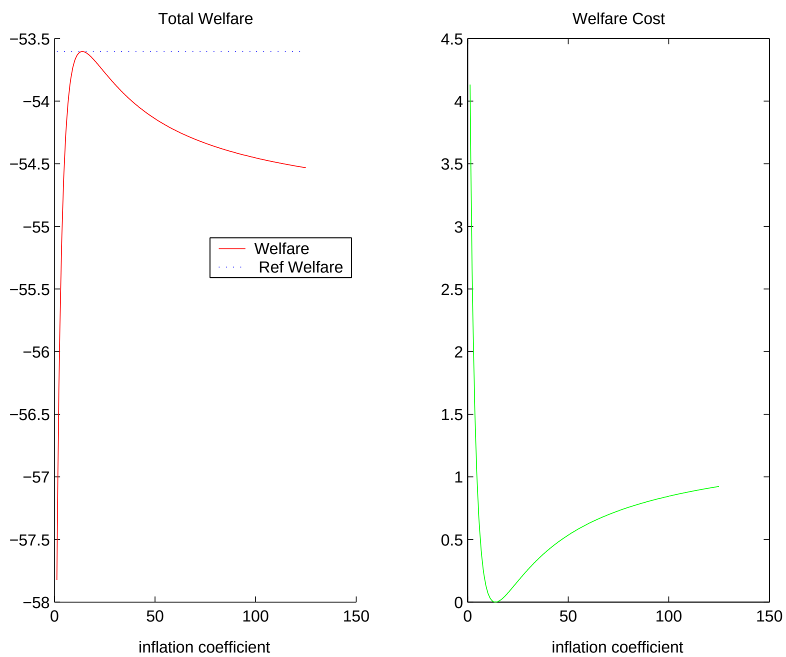

Figure 5: High $RS$, 3 Unions

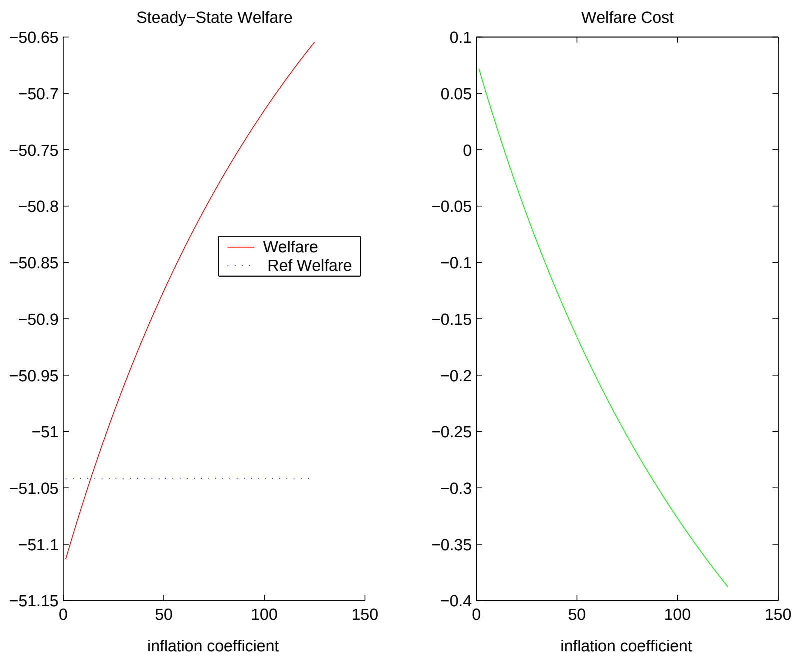


Figure 6: High $RS$, 3 Unions

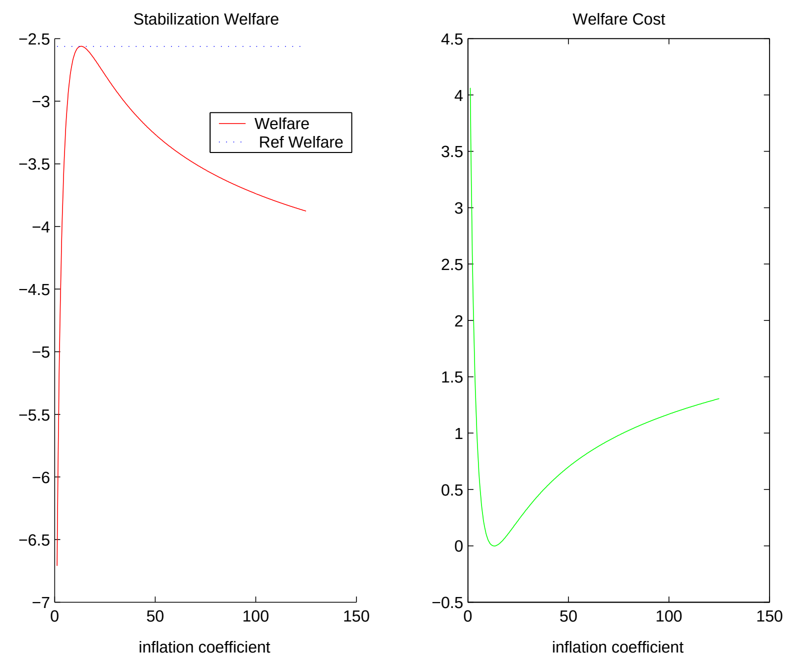

Figure 7: Medium $RS$, 3 Unions

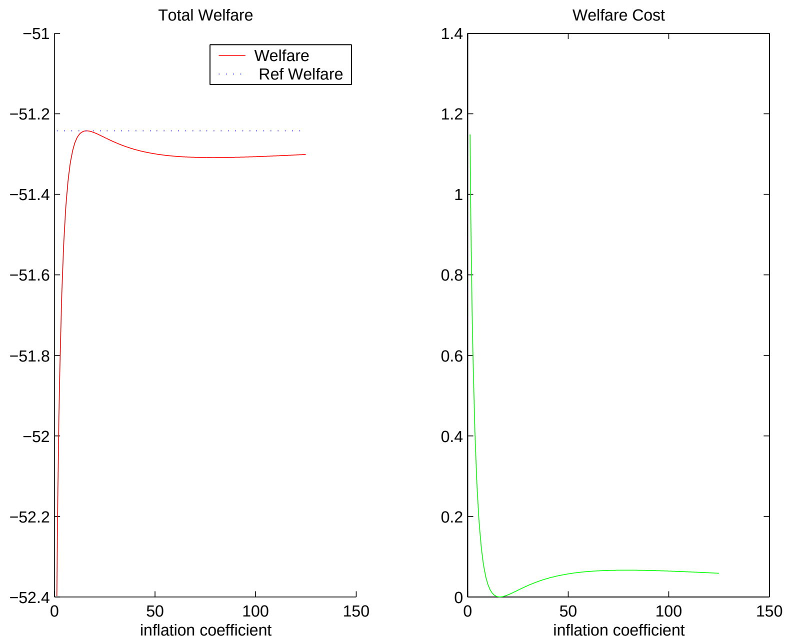


Figure 8: Medium $RS$, 2 Unions

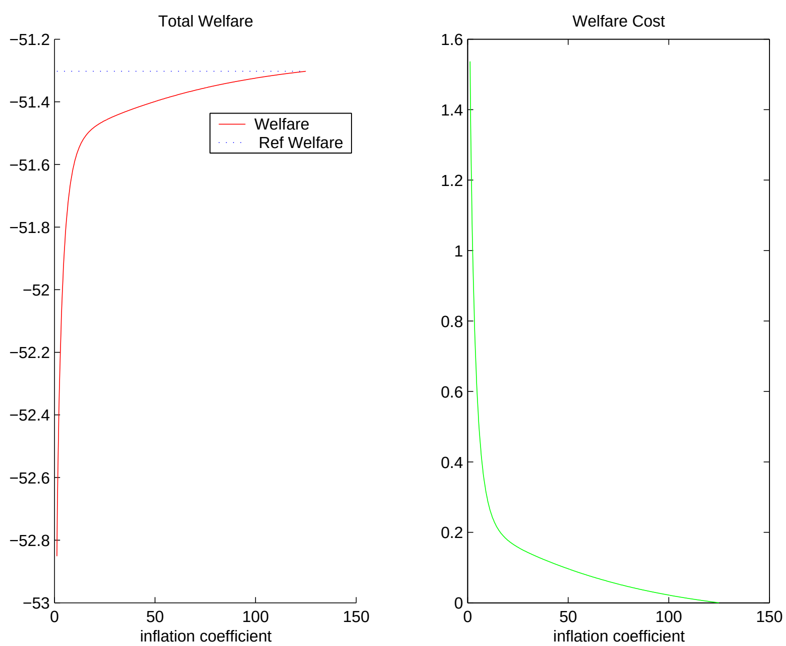

Figure 9: High $RS$, 2 Unions

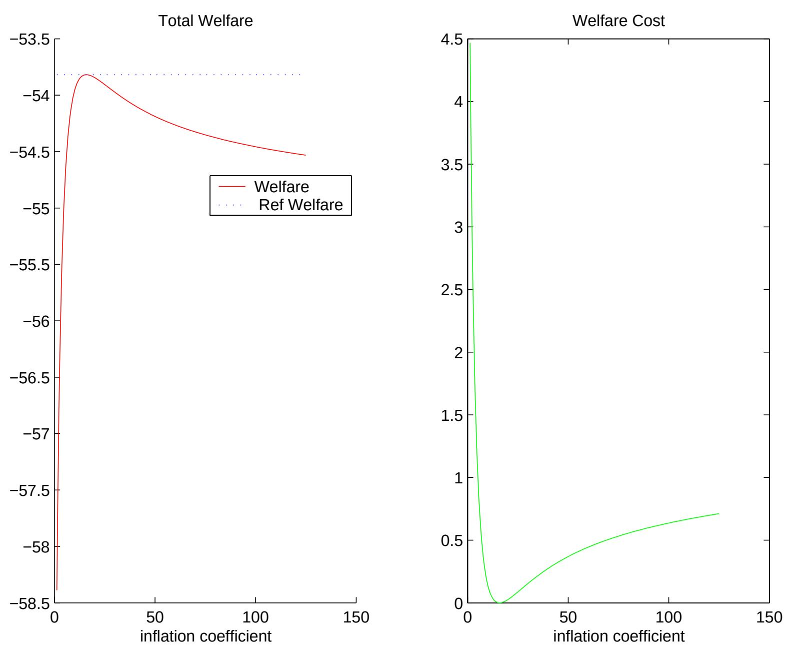

Table 1: **Baseline Calibration**

| **Parameter** | **Value** |
|---|---|
| Price Stickiness | 0.75 |
| Discount Factor | 0.99 |
| Elast. Subst. Goods | 11 |
| Elast. Subst. Labor Types | 11 |
| Elast. Marginal Disutility Labor | 1 |
| TFP autocorrelation | 0.95 |
| TFP Std. Dev. Innovation | 0.0071 |

Table 2: **Cost Push Shock Calibration**

| **Std. Dev. Innovation** | **Relative Std. Dev.** |
|---|---|
| 0.0227 | 1 |
| 0.0455 | 2 |
| **0.1137** | 5 (**LOW**) |
| **0.2274** | 10 (**MEDIUM**) |
| **0.4548** | 20 (**HIGH**) |

Table 3: **Optimal Monetary Policy**

| Std. Dev. CP | Number of Unions | Optimal Stance |
|---|---|---|
| High | 2 | 16 |
| High | 3 | 13.75 |
| High | 5 | 13.25 |
| High | 10 | 13.00 |
| High | 15 | 12.75 |
| Low | 2-3-5 | Strict Inflation Targeting |
| Low | 10 | 14.00 |
| Low | 15 | 13.50 |

Table 4: **Welfare Costs of Deviation from Optimal Policy: High Cost Push Shock Standard Deviation.** The cost is measured relatively to the optimized rule. It is expressed as the percentage decrease in the output process associated to the optimal policy necessary to make welfare under the ad-hoc rule as high as under the optimized rule.

| **N** | $\frac{\textbf{Optimal Stance}}{\textbf{Ad-hoc Rule}}$ | **Total Cost** | **Steady-State Cost** | **Stabilization Cost** |
|---|---|---|---|---|
| 2 | $\frac{16}{1.5}$ | 3.9858 | 0.2473 | 3.7477 |
| 3 | $\frac{13.75}{1.5}$ | 3.6646 | 0.0703 | 3.5968 |
| 15 | $\frac{12.75}{1.5}$ | 3.4470 | 0.0058 | 3.4415 |

Table 5: **Welfare Costs of Deviation from Optimal Policy: Low Cost Push Shock Standard Deviation.** The cost is measured relatively to the optimized rule. It is expressed as the percentage decrease in the output process associated to the optimal policy necessary to make welfare under the ad-hoc rule as high as under the optimized rule.

| N | $\frac{\textbf{Optimal Stance}}{\textbf{Ad-hoc Rule}}$ | **Total Cost** | **Steady-State Cost** | **Stabilization Cost** |
|---|---|---|---|---|
| 2 | $\frac{125}{1.5}$ | 0.8802 | 0.7118 | 0.1696 |
| 3 | $\frac{125}{1.5}$ | 0.4627 | 0.2925 | 0.1708 |
| 15 | $\frac{13.5}{1.5}$ | 0.2462 | 0.0036 | 0.2426 |