Universitat Autònoma de Barcelona
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Macroeconomic Policy
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Problem Set # 4, (due November 25th)

1. Consider an OLG economy of exchange in which the endowments of young and of old are, respectively, w^y, w^o. Assume that preferences of agents born in $t \geq 1$ are given by $\log(c_t^t) + \beta \log(c_{t+1}^t)$, with $\beta \in (0,1)$. In this economy there is a government that follows a rule with respect to money creation: $M_{t+1} = M_t(1+\sigma)$. Finally, population is constant and equal to N.

1.1 Assume $\sigma = 0$, and that the initial old hold the initial M. State conditions for the existence of a stationary monetary equilibrium. How do these conditions relate to the efficiency of the non monetary equilibrium?

1.2 Assume now that $\sigma > 0$, and that money is introduced in the economy as a transfer proportional to money holdings. This means that the budget constraints (in the SM equilibrium we are considering) are given by

$$p_t c_t^t = p_t w^y - s_t^t, \text{ and } p_{t+1} c_{t+1}^t = p_{t+1} w^o + s_t^t (1+\sigma).$$

Investigate whether money is super-neutral in a stationary monetary equilibrium of the economy.

1.3 Continue assuming that $\sigma > 0$, but now the government uses money creation to buy goods from the private sector and throws them into the sea (this can be seen as a way to finance useless public expenditure which reduces the available resources for private consumption). Who are the traders with the government? Introduce a notion of stationary monetary equilibrium in a SM arrangement. Is money super-neutral in this economy? Find an expression for the revenue raised by the government as a function of σ, and explain your findings.

1.4 After the result introduced in class and your findings in the previous exercises, is there any general result about the super-neutrality of money in OLG economies? Explain.

2. Consider an OLG economy of exchange in which population stays constant. There is an initial amount M of money in the hands of the initial old. Also, in this economy there is available a storage technology such that converts z units of the endowment of the young (w^y) into $z(1+x)$ units of consumption in the following period (for completeness, the endowment of the old is w^o). Preferences are given by $\log(c_t^t) + \beta \log(c_{t+1}^t)$. We consider a SM arrangement in which a government introduces new money at a rate σ by means of lump-sum transfers to the old (this is the arrangement we developed in class).

2.1 Introduce a notion of stationary SM equilibrium corresponding to the previous economy.

2.2 Characterize non monetary equilibria. Find conditions under which there will be "active" storage (i.e., agents decide to use the storage technology).

2.3 Find conditions for the existence of monetary equilibrium.

2.4 Describe the optimality/nonoptimality of the equilibria above as $x > 0$ or $x \leq 0$ (in this second case, the good depreciates).

3. Consider an economy in which a continuum of agents (consumer-workers) choosing consumption, and saving, over an infinite horizon. There are two types of agents, in that a fraction π of them are able to supply h_1 units of labor, and the remaining fraction $(1-\pi)$ supply h_2 units of labor, with $h_1 > h_2$. In addition to heterogeneity in their labor productivity, agents differ in their initial endowment of assets, denoted a^j (hence, $j \in [0,1]$ stands for the "name" of the agents). The generic problem of a type-i agent (for $i = 1, 2$) is given by:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t^j + \bar{c}),\ \beta \in (0,1),$$

(so that since leisure is not an argument in the utility function labor supply is inelastic). The corresponding budget constraint reads:

$$c_t^j + a_{t+1}^j = w_t h_i + R_t a_t^j,$$

where $R_t = r_t + 1 - \delta$ represents the return factor on assets, the same for all agents, and w_t is the wage rate, also the same for all agents. In this model, therefore, we encompass heterogeneity in the two endowments, initial assets and efficient units of labor.

To close the model, we assume there is a representative firm which operates in competitive markets in order to maximize profits. The associated FONC read:

$$r_t = F_1(K_t, H_t), \quad \text{and } w_t = F_2(K_t, H_t),$$

where $F(K_t, H_t)$ represents a constant returns to scale technology in capital and labor. Notice in particular that the firm simply buys units of labor (an aggregate amount), irrespectively of its type.

3.1 Define the competitive equilibrium for this economy. In particular, state the market clearing condition for each good (pay attention to the labor market).

3.2 Define "life-time" wealth ω_t^i, and show that consumption for a type i agent satisfies

$$c_t^j B_t^1 + \bar{c} B_t^2 = \omega_t^i.$$

Do B_t^1 and B_t^2 depend on i, or on j?

3.3 Show that the amount of assets for each agent satisfies the following recursion:

$$a_{t+1}^j = \beta R_t a_t^j + D_t^i, \forall t \geq 0.$$

Write down an expression for the coefficient of variation in assets in $t+1$ as a function of the coefficient in t. Use that expression to describe the dynamics of heterogeneity over a transition to the steady state.

3.4 Given your previous results, state and demonstrate an "aggregation theorem" (i.e., can we perfectly aggregate the decisions of all agents? Can we construct a representative agent who's decisions mimic the aggregate decisions of the heterogeneous agents?). If in your opinion such a theorem does not exist, then explain in detail why not. If in your opinion the answer is yes, then state precisely the problem of the "representative agent" (preferences, constraints, etc.).