Universitat Autònoma de Barcelona
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Macroeconomic Policy
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Problem Set # 3, (due November 11th)

1. Consider the following version of the Diamond model (an OLG economy with production). Agents live for two periods and the growth rate of the population is $n \geq 0$. In the first period agents work (they inelastically supply one unit of time in exchange of the market wage rate, w_t), and choose how much to consume and how much to save for the second period (in the second period agents cannot work). Saving is materialized in capital, which can be rented to firms in the second period. The problem of one of those agents can be described as:

$$\max_{\{c_t^t, c_{t+1}^t, s_t^t\}} \frac{{c_t^t}^{1-\sigma} - 1}{1-\sigma} + \beta \frac{{c_{t+1}^t}^{1-\sigma} - 1}{1-\sigma},$$

with $\beta \in (0,1)$, $\sigma > 0$ and $\sigma \neq 1$ (if $\sigma = 1$ then we have log utility), and subject to:

$$\begin{aligned} c_t^t + s_t^t &= w_t, \\ c_{t+1}^t &= (1 + r_{t+1} - \delta) s_t^t, \\ c_t^t, s_t^t, c_{t+1}^t &\geq 0, \end{aligned}$$

where r_{t+1} is the return to saving, and where δ is the depreciation rate of capital. In the production side of the economy there is a competitive firm endowed with a Cobb-Douglas technology, such that output is given by $Y_t = K_t^\theta L_t^{1-\theta}$, (with $\theta \in (0,1)$.

1.1 State a notion of sequential markets equilibrium for the previous economy.

1.2 Determine the function describing the evolution of capital over time: $k_{t+1} = s(k_t)$. Are transitional dynamics monotone in k (i.e., $k_t^1 \geq k_t^2$ implies that $k_{t+1}^1 = s(k_t^1) \geq s(k_t^2) = k_{t+1}^2$)? State conditions for the current economy to display monotone dynamics.

1.3 Introduce a notion of stationary equilibrium and determine the corresponding steady state. Is the steady state stable under your previous conditions?

1.4 Consider the introduction of a pay-as-you-go (PAYG) system of social security similar to the one we introduced in class for the exchange economy:

$$b_t = (1+n)d_t, \tag{1}$$

Remember that in this system young agents pay lump-taxes (here we call them d) which are distributed amongst the current old, and in the following period the current old collect the benefits b (out of the taxes d payed by the new young). Analyze the effects of the introduction of such a system at the steady state. In particular, can the introduction of a PAYG social security system promote gains in efficiency? If your answer is yes, provide conditions under which it would be so. If your answer is no, explain why not.

2. Consider the following version of the Diamond model in which preferences are given by:

$$U(c_t^t, c_{t+1}^t) = \log c_t^t + \beta c_{t+1}^t, \text{ with } \beta \in (0,1).$$

The technology to produce goods is

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} - \delta K_t, \text{ with } \alpha, \delta \in (0,1).$$

Notice that we have incorporated the depreciation rate directly in the technology. We keep the same assumptions we introduced in class: agents can only work when they are young, population grows at a rate $n > 0$, and $m = 0$.

2.1 Define the competitive equilibrium with SM.

2.2 Find the dynamic equilibrium.

2.3 Discuss the stability of the equilibrium.

2.4 Repeat 2.2 and 2.3 under the assumption that the technology is given by

$$Y_t = \log(1 + K_t/L_t)^{L_t}.$$