Universitat Autònoma de Barcelona
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Macroeconomic Policy
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Problem Set # 2, (due November 4th)

1. Consider the following version of an OLG exchange economy similar to the one introduced in the classes. Agents live for two periods, and are endowed with w_1 units of consumption goods when young, and with w_2 when old (these endowments are non storable). Preferences over consumption goods of an agent born in a period t are of the form

$$u(c_t^t, c_{t+1}^t) = \log(c_t^t) + \gamma \log(c_{t+1}^t),$$

with $\gamma > 0$. We assume that the population stays constant over time, so for every old agent in period t there is one and only one young

1.1 Write down the utility maximization problem of one of the agents under the assumption there are sequential markets, and determine the corresponding demand functions of the agent.

1.2 Determine the autarky interest rate.

1.3 State conditions in terms of γ under which the economy will be type Classical and type Samuelson.

1.4 Take $\gamma = 1$. State conditions in terms of w_1, w_2 under which the economy will be type Classical and type Samuelson.

2. Keep the same assumptions as before, but suppose now that preferences are given by

$$u(c_t^t, c_{t+1}^t) = \frac{1}{\sigma}(c_t^t)^\sigma + \frac{\alpha}{\sigma}(c_{t+1}^t)^\sigma,$$

with $\alpha > 0$ and $\sigma \in (0, 1)$.

2.1 Determine the interest rate under autarky.

2.2 State the utility maximization problem of one of the agents corresponding to the Arrow-Debreu market arrangement, and compute FOC for optimality. Normalize $p_1 = 1$, and fond the sequence of equilibrium princes.

2.3 State conditions under which the equilibrium will be efficient.

3. Consider an OLG economy in which goods are storable. In particular, assume that the endowments are given by $(w_1, w_2) = (w, 0)$, and that goods that are not consumed in the first period are still available for consumption in the second period, but only a fraction $(1-\delta)$ (hence, $\delta \in (0,1)$ is the depreciation rate). Determine the autarky equilibrium, and discuss under what conditions it is efficient (assume $U(c_t^t, c_{t+1}^t) = u(c_t^t) + \beta u(c_{t+1}^t)$, and that it satisfies standard assumptions).

4. Consider again an OLG economy in which agents live for two periods, and are endowed with w_1 units of consumption goods when young (which are non storable), and with w_2 when old. Preferences over consumption goods of an agent born in a period t are of the form

$$u(c_t^t, c_{t+1}^t) = \frac{(c_t^t)^{1-\gamma} - 1}{1-\gamma} + \beta \frac{(c_{t+1}^t)^{1-\gamma} - 1}{1-\gamma}$$

with $-1 < \gamma \neq 1$ and $\beta \in (0,1)$. We assume that the population stays constant over time, so for every old agent in period t there is one and only one young agent in that same period. There is an initial generation of old agents, with preferences given by

$$u(c^0) = \frac{(c_1^0)^{1-\gamma} - 1}{1-\gamma}.$$

The endowment of the initial old consists of w_2, and of m units of "money". Hence in principle young agents may decide to save by accepting units of money from old agents. Saving entails giving up some consumption units when young, in exchange of a (potentially) increased consumption when old.

4.1 Write down the utility maximization problem of one of the agents as if she was living in an Arrow-Debreu economy, and compute FOC characterizing her optimal choices.

4.2 Using your previous findings, find expressions for excess demand when young, $y(p_t, p_{t+1}) = c_t^t - w_1$, and excess demand when old, $z(p_t, p_{t+1}) = c_{t+1}^t - w_2$ (notice that by construction, these expressions correspond to optimal excess demands given (p_t, p_{t+1}).

4.3 Produce a graph of the Offer Curve in the $(y, z(y))$ space. Is it possible for this economy (for some combination of β, σ, w_1, w_2) to display cycles?

4.4 State conditions (in terms of w_1, w_2, β, σ) under which the autarky equilibrium is *efficient*.