Universitat Autònoma de Barcelona
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Macroeconomic Policy
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Problem Set # 1, (due October 28th)

Consider the following version of an OLG exchange economy similar to the one introduced in the classes. Agents live for two periods, and are endowed with w_1 units of consumption goods when young (which are non storable), and with w_2 when old. Preferences over consumption goods of an agent born in a period t are of the form

$$u(c_t^t, c_{t+1}^t) = \frac{((c_t^t)^\gamma (c_{t+1}^t)^{1-\gamma})^{1-\theta} - 1}{1-\theta},$$

with $\gamma \in (0,1)$ and $-1 < \theta \neq 1$. We assume that the population stays constant over time, so for every old agent in period t there is one and only one young agent in that same period. There is an initial generation of old agents, with preferences given by

$$u(c^0) = \log c_1^0.$$

The endowment of the initial old consists of w_2, and of m units of "money". Hence in principle young agents may decide to save by accepting units of money from old agents. Saving entails giving up some consumption units when young, in exchange of a (potentially) increased consumption when old.

1. Write down the utility maximization problem of one of the agents as if she was living in an Arrow-Debreu economy, and compute FOC characterizing her optimal choices.

2. Using your previous findings, find expressions for excess demand when young, $y(p_t, p_{t+1}) = c_t^t - w_1$, and excess demand when old, $z(p_t, p_{t+1}) = c_{t+1}^t - w_2$ (notice that by construction, these expressions correspond to optimal excess demands given (p_t, p_{t+1}).

3. Using your definitions of $y(p_t, p_{t+1}) = c_t^t - w_1$ and of $z(p_t, p_{t+1}) = c_t^t - w_1$, state a notion of AD equilibrium corresponding to the previous economy.

4. Produce a graph of the Offer Curve in the $(y, z(y))$ space. Plot in the same space the market clearing condition for goods you stated in the previous exercise.

5. Introduce a notion of stationary equilibrium corresponding to the previous economy.

6. Assume $m = 0$. Argue that only the stationary autarky equilibrium (agents simply consume their own endowments) is possible.

7. State conditions (in terms of w_1, w_2, σ, γ) under which the autarky equilibrium is *efficient*.

8. Assume now $m > 0$. Is the autarky equilibrium still possible? Is there any other stationary equilibrium? That is, can you state conditions (in terms of w_1, w_2, σ, γ) under which there is another equilibrium? Explain the links (if any!) between your current answer and the one to the previous exercise.

9. Construct a non stationary equilibrium. What happens to the equilibrium sequence of equilibrium prices? Find out the $\lim_{t\to+\infty} p_t$. Argue that within ϵ distance from your equilibrium, there is another non stationary equilibrium.

10. State a notion of stationary SM equilibrium, and show that it is equivalent to the stationary AD equilibrium (i.e., that there is a mapping between the prices in each equilibrium and that the allocation corresponding to each set of prices is the same).