Notes
on
the
Overlapping
Generations
Model

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1. Social security revisited

There is a large literature considering the benefits of switching from a Pay-As-You-Go (PAYG) system to a Fully Funded (FF) system (see for instance Kotlikoff, St. Louis Fed, 1998, and Birkeland and Prescott, Minneapolis Fed, 2007, and the references therein among others). A motivation for this interest comes from the fact that due to changes in demographics, the number of available "young" agents to sustain the existing "old" is declining very rapidly, and so the current PAYG systems currently in place in may countries will be unsustainable in the near future.

One possibility to overcome this situation is to increase the contributions to the system, or reduce the benefits at retirement, or a combination of both.

Furthermore, there is evidence suggesting that the competitive equilibrium is dynamically efficient: in the equilibrium without the social security system it would be impossible to increase welfare of at least a generation without decreasing it for any other generation. This means that in principle there are welfare gains to be materialized from switching to a FF system.

If the given economy could jump from the initial steady state with the PAYG system, to the new steady state corresponding to the FF system, then there is no question about the gains of adopting the new regime. All in all, there are two important considerations that need to be dealt with:

1. In general there is no reason to expect to jump in one period to the new steady state after a policy change. Hence a transition toward the new steady state will take place, and we need a proper assessment of gains and losses over such a transition.

2. It is obvious that such a transition involves defaulting on the debts implicit in the PAYG system. Is this fact making impossible a Pareto improvement in the OLG economy?

We look at these issues in a simple model.

2. Steady states with social security

From the perspective of an individual, a social security system is a technology that allows to transfer resources from the present to the future. The cost of operating this technology is the contributions the individual needs to finance in the present. The following budget constraints correspond to an individual born in t:

$$c_t^t + a_{t+1}^t \leq w_t l_t - w_t l_t \tau_t,$$

$$c_{t+1}^t \leq a_{t+1}^t (1 + r_{t+1}) + p_{t+1},$$

where τ_t represent the contributions to the social security system in period t, and where p_{t+1} represent the payments received in period $t+1$. The inter temporal budget constraint corresponding to the agent reads

$$c_t^t + \frac{c_{t+1}^t}{1+r_{t+1}} \leq w_t l_t - w_t l_t \tau_t + \frac{p_{t+1}}{1+r_{t+1}}.$$

2.1 Budget balance in the PAYG system

The PAYG system is an unfunded system in which the young generation of workers pays the pensions of the current old generation of retirees. If the population grows at a rate $n \geq 0$, then we have that the budget constraint of the government in period $t+1$ satisfies

$$N_t p_{t+1} = N_{t+1} w_{t+1} l_{t+1} \tau_{t+1},$$

hence

$$p_{t+1} = (1+n) w_{t+1} l_{t+1} \tau_{t+1}.$$

2.2 Budget balance in the FF system

In the FF system the contributions of young workers are saved at the market interest rate. In the following period the agents retire and get back their savings. Hence, every worker pays her/his own pension through the market mechanism. We therefore have that the pension of the old generation in period $t+1$ (that was born in period t) satisfies:

$$p_{t+1} = w_t l_t \tau_t (1 + r_{t+1}).$$

Assume that the economy is stationary: $l_t = l_{t+j}$, $\tau_t = \tau_{t+j}$, $r_t = r_{t+j}$, for all $j \geq 0$, and that $w_{t+1} = (1+g)w_t$, with $g \geq 0$.

We have that the pension in the FF system is larger than under the PAYG system provided that

$$\begin{aligned}\frac{p_{t+1}^{FF}}{(1+r_{t+1})} &= w_t l_t \tau_t > \frac{(1+n)(1+g)w_t l_t \tau_t}{(1+r_{t+1})} \\ &= (1+n)w_{t+1} l_{t+1} \tau_{t+1} \\ &= \frac{p_{t+1}^{PAYG}}{(1+r_{t+1})}\end{aligned}$$

holds whenever $(1+r) > (1+n)(1+g)$.

The last condition is equivalent to the condition in the Diamond model to assert that the competitive equilibrium is *dynamically efficient*.

It seems, therefore, that when the condition for efficiency holds there is room for improvement in going from the PAYG system to the FF system.

Notice, however, that switching from PAYG to FF means that the initial generation of young pays no taxes, and thus, the initial generation of old receive no pension benefits. Unless some brute force arrangement is in place, such a reform would hardly be passed in current democracies. Hence, the government would need to engineer some strategy to implement the FF.

In what follows we keep the analysis at its simples terms: the change in regime does not alter nor wages nor returns.

3. Default on the old

The simplest possibility to consider when implementing the FF system is to default on the promises to the current old generation. In this scenario we assume that the government in period T unexpectedly decides to suspend the PAYG system and switches to the FF system.

- The old generation in period T (that was born in period $T-1$) is clearly worse off: they contributed to the system when they were young (in period $T-1$), and receive nothing in exchange in period T.

- The young generation in period T, however, is better off: they can save the same as before (assets plus private "retirement plan"), and the return of their saving is larger than what they would have got in the PAYG system.

- All future generations benefit form being in the FF system.

4. Default on the young

Assume that the government keeps the promises to the old in period T. To pay for these pensions, the government issues debt that will have to be repayed in period $T+1$. Budget balance in period T requires that $N_{t-1}p_T = N_t b_{T+1}$, or that

$$p_T = (1+n)b_{T+1}.$$

Since the old in T are payed the expected pension, they are indifferent between the old and the new system.

The young generation in period T buys the debt of the government, they know that they will get no pension from the system (which is abolished in T), and thus, they save accordingly. The budget constraints of this generation read

$$c_T^T + a_{T+1}^T + b_{T+1}^T \leq w_T l_T$$

and

$$c_{T+1}^T \leq (a_{T+1}^T + b_{T+1}^T)(1 + r_{T+1}),$$

which reduce to

$$c_T^T + \frac{c_{T+1}^T}{1 + r_{T+1}} \leq w_T l_T.$$

This generation is better off because the economy is dynamically efficient, and thus the return in the FF system is larger than under the PAYG system.

In period $T+1$, however, the government needs to redeem its debt and chooses some combination of debt and taxes on the young. The budget constraint of the government in period $T+1$ is therefore given by

$$N_T b_{T+1}(1 + r_{T+1}) = N_{T+1}(b_{T+2} + t_{T+1}),$$

which we write as

$$b_{T+1}(1 + r_{T+1}) = (1 + n)(b_{T+2} + t_{T+1}).$$

Notice that unless some taxes are introduced, over time the amount of debt would explode.

Suppose the government keeps aggregate debt constant:

$$b_{T+1} = (1+n)b_{T+2},$$

so that taxes are just used to pay the interest generated by the outstanding debt:

$$b_{T+1}r_{T+1} = (1+n)t_{T+1}.$$

Substituting the bonds from the budget constraint in period T we finally get that

$$t_{T+1} = \frac{r_{T+1}p_T}{(1+n)^2}.$$

Under the new regime, the generation born in $T+1$ faces the following budget constraints:

$$c_T^{T+1} + a_{T+2}^{T+1} + b_{T+2}^{T+1} \leq w_{T+1} l_{T+1} - t_{T+1}$$

and

$$c_{T+2}^{T+1} \leq (a_{T+2}^{T+1} + b_{T+2}^{T+1})(1 + r_{T+2}),$$

which produce the following inter temporal budget constraint:

$$c_{T+1}^{T+1} + \frac{c_{T+2}^{T+1}}{1 + r_{T+2}} \leq w_{T+1} l_{T+1} - t_{T+1}.$$

Had the system remained unchanged, the inter temporal budget constraint of these agents would have been

$$c_{T+1}^{T+1} + \frac{c_{T+2}^{T+1}}{1 + r_{T+2}} \leq w_{T+1} l_{T+1} - w_{T+1} l_{T+1} \tau_{T+1} + \frac{p_{T+2}}{1 + r_{T+2}}.$$

We will simply compare net taxes under each regime, and show that under the PAYG system these net taxes are smaller.

In the old PAYG system net taxes are given by

$$NT_{\mathsf{PAYG}} = w_{T+1} l_{T+1} \tau_{T+1} - \frac{p_{T+2}}{1 + r_{T+2}}.$$

Under our assumptions we have that $w_{T+1} l_{T+1} \tau_{T+1} = (1+g)p_T/(1+n)$, and that $p_{T+2} = (1+g)^2 P_T$. Substituting these expressions in the equation above we get that

$$NT_{\mathsf{PAYG}} = (1+g)p_T \left[\frac{1}{(1+n)} - \frac{(1+g)}{(1+r_{T+2})} \right].$$

Suppose, toward a contradiction, that $t_{T+1} < NT_{\mathsf{PAYG}}$. Then we would have that

$$\frac{r_{T+1}}{(1+n)^2} < (1+g)\left[\frac{1}{(1+n)} - \frac{(1+g)}{(1+r_{T+2})}\right].$$

Rearranging, we get that

$$\frac{r(1+r)}{(1+n)} < (1+g)\left[(1+r) - (1+g)(1+n)\right],$$

where we are assuming that the interest rate is also constant. Since $(1+r) > (1+n)(1+g)$, then we have that

$$\frac{r(1+n)(1+g)}{(1+n)} < (1+g)\left[(1+r) - (1+g)(1+n)\right],$$

simplifying and rearranging then we finally have that

$$(1+g)(1+n) < 1,$$

which is the desired contradiction.

We conclude that defaulting on the young also creates a welfare loss, even if the economy transits to a new system in which the return is larger than under PAYG.

5. A neutral transition

It is possible to implement a transition from the PAYG to the FF system that leaves indifferent all generations. This transition involves making explicit the debt the system has with every cohort.

In period T the government collects taxes form the current young and gives them a monetary transfer. The proceeds from taxation are used to finance the pensions of the current old, as in the PAYG system. This means that the old generation in T is indifferent between the two systems.

The transfer to the young is equal to the present value of their pension:

$$tr_T = \frac{p_{T+1}}{(1 + r_{T+1})}$$

In order to finance this monetary transfer, the government issues debt (that will have to be repayed in period $T + 1$). Budget balance implies that

$$N_T b_{T+1} = N_T tr_T, \text{ hence } b_{T+1} = \frac{p_{T+1}}{(1 + r_{T+1})}.$$

The budget constraints in every period for the young generation in T are:

$$c_T^T + a_{T+1}^T + b_{T+1}^T \leq w_T l_T - w_T l_T \tau_T + tr_T$$

and

$$c_{T+1}^{T} \leq (a_{T+1}^{T} + b_{T+1}^{T})(1 + r_{T+1}),$$

which reduce to

$$\begin{aligned} c_T^T + \frac{c_{T+1}^T}{1+r_{T+1}} &\leq w_T l_T - w_T l_T \tau_T + tr_T \\ &= w_T l_T - w_T l_T \tau_T + b_{T+1} \\ &= w_T l_T - w_T l_T \tau_T + \frac{p_{T+1}}{(1+r_{T+1})}. \end{aligned}$$

Notice that the expression above is exactly the inter temporal budget constraint under the old PAYG system. This means that the young in period T are indifferent between the to systems.

In period $T+1$ the government repeats the same strategy: collects taxes form the young to pay back the debt plus interest to the currently old. At the same time, makes a monetary transfer to the young which is equal to the present value of their pensions, and issues debt to finance the transfer..., and so on.

- The preceding analysis suggests that once that transition from the inefficient PAYG system to the efficient FF system is properly taken into account, there are no welfare gains from such a change in regime.

- When the economy is dynamically inefficient, the social security system transfers less resources than it would be possible by simply using the market. The problem, however, is that if there is no default in changing the system, then the government will have to issue debt, which is also subject to a higher cost. This increase in the cost implies higher taxes for the future generations, which end up offsetting the positive effects of the reform.

- The conclusion, therefore, is that a reform of the social security system may be desirable only if it helps to reduce distortions or it increases the consumption possibility set. That is, it needs additional reforms.

6. Social security in production economies

In the two period OLG model with production we showed that saving from the young is a function of their wage rate, w_t, and of the interest rate in the following period, r_{t+1}:

$$s_t = s(w_t, r_{t+1}).$$

We also stated that in equilibrium, the available amount of capital was no more and no less than what the young saved in the previous period:

$$K_{t+1} = N_t s(w_t, r_{t+1}), \quad \text{or } k_{t+1} = \frac{1}{1+n} s(w_t, r_{t+1}).$$

We showed that a sufficient condition for stability and monotonic dynamics around the steady state was:

$$0 < \frac{-s_w(w(k^*), r(k^*)) f''(k^*) k^*}{1 + n - s_r(w(k^*), r(k^*)) f''(k^*)} < 1,$$

which would imply that

$$0 < \frac{dk_{t+1}}{dk_t} < 1.$$

If the competitive equilibrium is dynamically inefficient, then there is "too much saving", and policies that reduce the incentives to save would promote welfare improvements. Notice that under standard assumptions on technology, reducing the amount of capital (by reducing saving) would lead to a reduction of the wage rate, and to an increase of the interest rate. Hence the relevant policies will harm agents when young in exchange of increased benefits when old (through a higher return to their saving).

A PAYG social security system is a way to reduce capital accumulation. Assume the government introduces taxes (τ) on young workers, and that it returns them when they are old in the form of pensions (p), but in the usual schedule such that current young pay for the current old:

$$N_t p_{t+1} = N_{t+1}\tau, \text{ or } p_{t+1} = (1+n)\tau.$$

The budget constraints corresponding to an agent born in period t read:

$$c_t^t + s_t = w_t - \tau,$$

and

$$c_{t+1}^t = (1 + r_{t+1} - \delta)s_t + (1+n)\tau$$

The FOC for assets takes the usual form:

$$u'(w_t - \tau - s_t) = \beta(1 + r_{t+1} - \delta)u'((1 + r_{t+1} - \delta)s_t + (1+n)\tau).$$

It is straightforward to use the implicit function theorem to get that

$$\frac{ds}{d\tau} = s_\tau = \frac{-u''(c_t^t) - \beta(1 + r_{t+1} - \delta)(1+n)u''(c_{t+1}^t)}{u''(c_t^t) + \beta(1 + r_{t+1} - \delta)^2 u''(c_{t+1}^t)} < 0$$

This means that increasing taxes effectively reduces saving (when prices are taken as given). This result is useful when we try to evaluate the effect of taxes on the available capital labor ratio in $t+1$. In this case we have

$$\begin{aligned} k_{t+1} &= \tfrac{s(w_t, r_{t+1}, \tau)}{1+n} \\ &= \tfrac{s(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), \tau)}{1+n} \end{aligned}$$

An application of the implicit function theorem to the equation above reveals that

$$\frac{dk_{t+1}}{d\tau} = \frac{s_\tau}{1 + n - s_r f''(k_{t+1})}.$$

We found before that $s_\tau < 0$, and the condition for local monotonicity implies that $1 + n - s_r f''(k_{t+1}) > 0$. Hence, we conclude that

$$\frac{dk_{t+1}}{d\tau} < 0.$$

The implication is that if the economy is dynamically inefficient, then the planner can improve welfare by implementing (unexpectedly) a marginal PAYG social security system. Notice that over the transition toward the steady state, the stock of capital will be declining to its new level (i.e., that wages will be falling and interest rates rising).

References

These notes are based on Aubuchon, Conesa, and Garriga (2010, still working on it), and on Conesa and Garriga (IER, 2008). This last paper contains many interesting references, and it is an excellent example of what “quantitative macro” does. See also Breyer (JITE, 1989). Feldstein and Liebman (Handbook of Public Economics, 2002), and Diamond (2004) review much of the literure on social security reforms.

The results for production economies come from the the classical paper by Diamond (AER, 1965). Diamond considered an economy with inelastic labor supply. Fanti and Spataro (JM, 2006) look at the case of endogenous labor supply, and show that the conclusions obtained by Diamond may not be robust.