Notes on the Overlapping Generations Model

3.2

3. Variable wages

The previous version of the model neglected any action from changes in wages: labor income was fixed in each sector and independent of relative abundance of each type of workers. We extend the previous model to allow for such an interaction, from the distribution of types (skilled/unskilled) to wages, and from wages to the distribution of types.

Assume that in the unskilled sector the technology is now

$$Y_t^{ns} = G(L_t^{ns}, N),$$

where N is land, and where G is such that labor displays decreasing returns to scale. We write the profit maximizing condition as

$$w_t^{ns} = G_L(L_t^{ns}, N) = P(L_t^{ns}).$$

The equation above describes labor demand in the unskilled sector as a function of the market wage rate in period t, w_t^{ns}.

Labor supply in the unskilled sector is given by the amount of agents that decide not to acquire education. Following the intuition from the previous version of the model, all agents with current bequest smaller that $\widehat{x}$ choose not to be educated. Since in the current version of the model the wage rate in the unskilled sector is likely to change over time, we make this dependence explicit and write $\widehat{x}(w_t^{ns})$:

$$\widehat{x}(w_t^{ns}) = \frac{1}{i-r}[w_t^{ns}(1+r) + h_0(1+i) - w^s].$$

Notice that $\widehat{x}(w_t^{ns})$ increases with w_t^{ns}. Given this, labor supply in period t is given by

$$s_t = \int_0^{\widehat{x}(w_t^{ns})} D_t'(x_t).$$

Note that labor supply in the unskilled sector is increasing in w_t^{ns}. To see this, take D_t as given, and remember that $\widehat{x}(w_t^{ns})$ is increasing in w_t^{ns}. This means that the larger is the wage rate in this sector, the larger is the threshold bequest, and so the larger is the fraction of households who's bequest is too small to be profitable to pay for their education and work in the skilled sector. Hence labor supply is increasing in w_t^{ns}.

Finally, we will also assume that if an agent decides to remain unskilled, then she/he will only work in the first period (consumption will still take place in the second).

An economy is called "developed" if $\widehat{x}(w_t^{ns}) > x^c$. An economy is called "less developed" if $\widehat{x}(w_t^{ns}) \leq x^c$.

We then have that $\widehat{x}(w_t^{ns}) > x^c$ iff

$$\frac{1}{i-r}[w_t^{ns}(1+r)+h_0(1+i)-w^s] > \frac{(1-\alpha)[h_0(1+i)-w^s]}{(1-\alpha)(1+i)-1},$$

and rearranging, we get

$$w_t^{ns} > \frac{\alpha+\alpha r-r}{(1+r)(\alpha+\alpha i-i)}[w^s-h_0(1+i)] = w^c.$$

Hence, we have that an economy is *developed* when the wage rate in the unskilled sector is above certain threshold (and constant) level: $w_t^{ns} > w^c$.

3.1 The case of a less developed economy

In this case we have that $\widehat{x}(w_t^{ns}) \leq x^c$: The amount of wealth needed to be able to borrow to obtain education and still be able to keep constant the bequest is larger than the amount of wealth that is needed to be indifferent between studding or not. This means that:

1. Households with $x_t \leq \widehat{x}(w_t^{ns})$ do not acquire eduction and thus, work as unskilled workers.

2. All agents with x_t smaller than x^c leave a bequest to their descendants that is smaller than the one they received. Among those, however, a few of them (for $x_t \in (\widehat{x}(w_t^{ns}), x^c])$ do acquire education and work in the skilled sector in the current period.

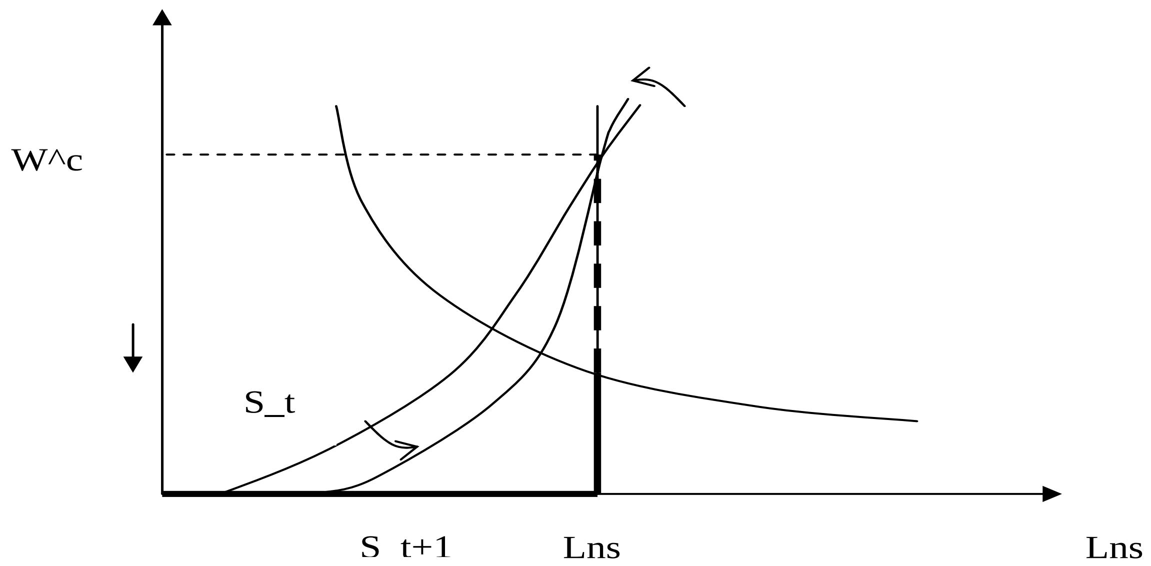
3. Agents with $x_t > x^c$ leave a bequest larger than the one they received: $x_{t+1} > x_t$. All these agents acquire education and work in the skilled sector.

Fact 2 implies that the mass of agents that decides not to acquire education is larger in $t + 1$ than in t. In particular, this means that for any wage rate smaller or equal that w_t^{ns}, the labor supply in the unskilled sector increases. When we look at the current schedule fr labor supply (i.e., at the mass of agents willing to work for any given wage rate in the unskilled sector), notice that labor supply shifts to the right for any $w^{ns} < w^c$, but that for $w^{ns} = w^c$ the mass of agents that are willing to work in the unskilled sector in the current period is the same as the mass of agents that were willing to do so in the previous one (since no agent with $x_t \leq x^c$ left a bequest larger than x^c. Furthermore, for agents with a bequest at least as large as x^c, the bequest they leave is larger than the one they received, so the mass of agents with $x_t \geq x^c$ is shifted to the right.

Hence, the implication is the labor supply rotates around w^c.

Thus, labor supply shifts to the right for $w^{ns} \leq w^c$ (and to the left for $w^{ns} \geq w^c$). The intersection between labor demand ($P(L^{ns})$) and the new labor supply determines the new equilibrium wage rate in $t+1$, w^{ns}_{t+1}, and it follows that $w^{ns}_t > w^{ns}_{t+1}$.

Over a transition of a *less developed economy* the wage rate in the unskilled sector decreases, more mass of agents is “poor” (and poorer over time), and the mass of “rich” agents is constant (but they are richer over time. Hence, during a transition in a *less developed economy* inequality increases.


W^c
S_t
S t+1
Lns
Lns

3.2 The case of a developed economy

In this case $w^{ns} > w^c$, and thus all individuals leave a bequest larger than the one they received. This means that labor supply in the unskilled sector decreases over time, and the wage rate in that sector increases during a transition. In the long run heterogeneity completely disappears.

We summarize these results in the following proposition.

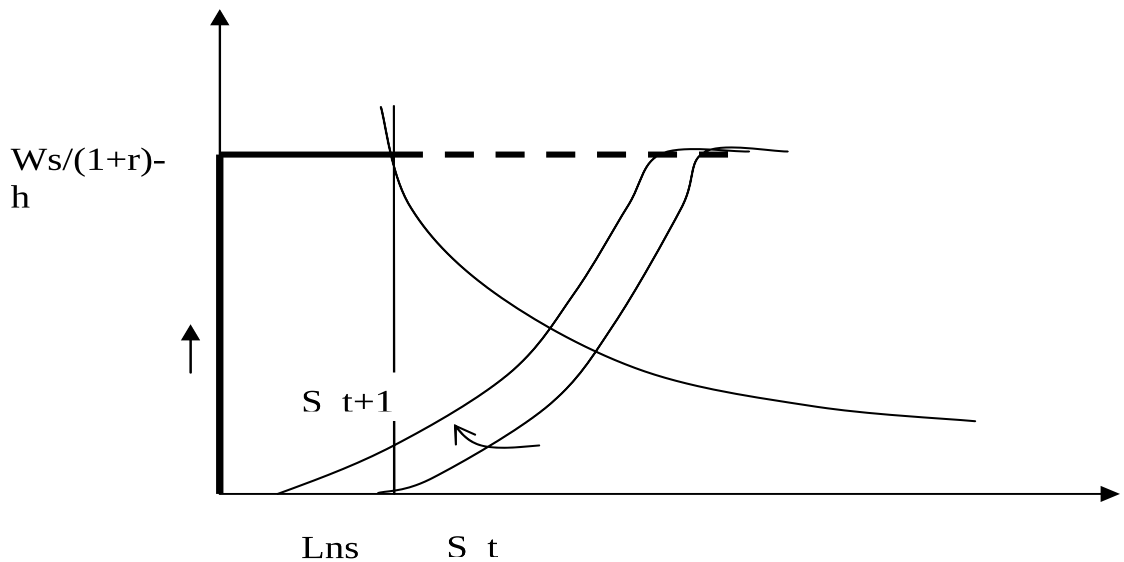
Proposition: If an economy satisfies $0 < x^c < \bar{x}^s$, then its dynamics depend on the number of individuals with inheritances less than x^c in period t, L_t^c.

a) A less developed economy, where $P(L_t^c) \leq w^c$, converges to an unequal distribution of income, with

$$w_\infty^{ns} < \frac{w^s}{1+r} - h_0.$$

b) A rich economy, where $P(L_t^c) > w^c$, converges to an equal distribution of income, with

$$w_\infty^{ns} = \frac{w^s}{1+r} - h_0.$$



Ws/(1+r)-
h
S t+1
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References

The material in this section follows Galor and Zeira, RES 1993. See also the related paper by Banerjee and Newman, RES 1991, and Greenwood and Jovanovic, JPE 1990 for a different approach. Lars Ljungqvist has an interesting paper on the importance of the market for human capital (J. Dev. Econ., 1993). D. Ray has a series of papers on transitional dynamics emphasizing additional issues such as the importance of contracts (his wage page contains many papers along these lines). Finally, see “Taking intermediation seriously”, by Bruce Smith, if you want to know more about intermediation in markets with frictions. Bruce was by far the leader in this literature.