Notes on the Overlapping Generations Model

3

1. Income distribution and Macroeconomics

Between the 50's and 70's there was a remarkable interest in understanding equilibrium in the log run using exogenous growth models, as it was impossible to come up with an explanation of endogenous growth.

After the works about endogenous growth by Romer (JPE, 1986, 1990) and Lucas (JME, 1988, Econometrica 1993) in the 80's and early 90's, a new literature appeared trying to understand the role of institutions and policies (such as democracy), and the effect of initial conditions (such as the initial degree of wealth/income inequality). Examples of this line of research in the Neoclassical tradition include Chatterjee

(J. Pub. E. 1994), Caselli and Ventura (AER, 2000), Chetterjee and Ravikumar (MD, 1999), among others).

An interesting result often found in this literature is that heterogeneity/inequality has no effects on equilibrium dynamics, and yet, equilibrium dynamics do have effects on heterogeneity. This is interesting because it suggests that in terms of long run, there is nothing to be lost/gained by having more or less inequality. This conclusion holds in frictionless economies and in environments in which wealth effects are negligible.

Sorger (JET, 2002), is an example of the relevance of wealth effects: the initial distribution matters to determine the long run equilibrium, hence initial conditions do affect aggregate dynamics.

The OLG economies are a natural example of economies with frictions.

2. A simple model

Consider an OLG economy in which agents live for two periods, and in which there are two sectors operating different technologies.

2.1 Technologies

Skilled sector: Firms in this sector have access to a CRS technology in capital and labor, satisfying standard assumptions about differentiability, concavity, etc.

$$Y_t^s = F(K_t, L_t^s)$$

The FOC for profit maximization in this sector dictate that

$$F_K(K_t, L_t^s) = r_t, \ \text{ and } F_L^s(K_t, L_t^s) = w_t.$$

We assume that the economy of interest is a small open economy and that there is perfect mobility of capital and labor.

The first implication of this is that the equilibrium return of capital is given in international markets, r.

The second implication of this is that the FOC with respect to capital determines the capital to output ratio that prevails in the economy: there is a k^* such that $f'(k^*) = r = F_K(K_t, L_t^s)$, with $k = K/L$.

The wage of skilled workers in this economy is also constant and given by $w^s = f(k^*) - f'(k^*)k^*$.

Unskilled sector: Firms in this sector have access to a linear technology

$$Y_t^{ns} = w^{ns} L_t^{ns},$$

where w^{ns} is labor productivity in this sector, and in equilibrium coincides with the wage rate (also constant).

2.2 Preferences and endowments

Households live for two periods, but consume only in the second.

In the first period the must choose between becoming a skilled worker, or remaining unskilled.

If the worker remains unskilled, then she/he works in both periods.

In case the worker decides to become skilled, then in the first period the agent does not work, and instead acquires education/skills. Education (denoted h)is an *indivisible* good, so for every agent:

$$h_t \in \{0, h_0\}.$$

We also denote the cost of education, in terms of consumption goods, by h_0. Once the agent is endowed with skills, then she/he works in the skilled sector during the second period.

We assume there is *altruism*, and specify preferences as

$$u = \alpha \log c + (1 - \alpha) \log b,$$

where c is consumption in the second period, and where b is the bequest for each of the $n \geq 0$ descendants. Households are all identical (no differences in ability or learning capacity), and differ only in the amount of bequest they receive in the first period, and the bequest they leave when they die.

2.3 The credit market

The lending rate is given in the international market: $r > 0$. A household may decide to lend (or save) in case she/he is not interested in obtaining education, or in case the initial bequest is large enough to cover the cost of education, h_0.

In case the initial bequest does not cover the cost of education, then the agent can borrow in the credit market. The credit market is competitive, but there are frictions: borrowers may try to scape and avoid payment of debts. Furthermore, lenders may incur in tracking activities to make more difficult to borrowers to scape. These tracking activities are also costly. We formalize these ideas in what follows

1. An agent with a debt level d needs to return $d(1+i)$, where $i > 0$ is the interest rate for borrowers (lenders -and firms which are less mobile- face the interest rate r).

2. A no arbitrage condition in the credit market states that given d:

$$di = dr + z,$$

where z is the cost for the lender to keep track of the borrower. We rewrite the last equation as $z = d(i - r)$.

3. There is an incentive compatible constraint, such that the higher is z, the higher is the cost of a borrower to scape (or otherwise she/he would do it). The

incentive compatibility constraint makes sure that the borrower has the right incentives to repay the debts:

$$d(1+i) \leq \beta z,$$

where $\beta > 1$. Inserting in the expression above the fact that $z = d(i-r)$, and rearranging, we get that $i = (1+\beta r)/(\beta - 1)$. It is straightforward to show that $r < i$.

2.4 The problem of the household

We need to consider the optimal choice for bequests in three possible situations, as the household may choose to remain unskilled, educated but having to ask for a loan, or educated and saving part of the bequest she/he received in the first period.

In case the household decides to remain unskilled the budget constraint reads (x is the bequest the household receives in the first period)

$$c + b = (x + w^{ns})(1 + r) + w^{ns}.$$

The FOC of the household problem implies (this FOC is the same in all the cases we consider below):

$$b = \frac{1 - \alpha}{\alpha} c,$$

and so we have,

$$b = (1-\alpha)[x(1+r) + w^{ns}(2+r)],$$

and

$$u_{ns}(x) = \log(x(1+r) + w^{ns}(2+r)) + \epsilon,$$

with $\epsilon = \alpha \log \alpha + (1-\alpha)\log(1-\alpha)$.

In case the initial bequest is larger than h_0, then the agent may choose to be educated. The budget constraint reads $c + b = (x - h_0)(1+r) + w^s$, and so we get

$$b = (1-\alpha)[(x-h_0)(1+r) + w^s],$$

and

$$u_s(x) = \log((x-h_0)(1+r) + w^s)) + \epsilon.$$

Finally, if the agent decides to borrow to finance education we have

$$b = (1 - \alpha)[(x - h_0)(1 + i) + w^s],$$

and

$$u_{sb}(x) = \log((x - h_0)(1 + i) + w^s)) + \epsilon.$$

Notice that b (the bequest leaved for the following generation) is linear in x. The slope of this function is either $(1 - \alpha)(1 + r)$, or $(1 - \alpha)(1 + i)$. We assume that $(1 - \alpha)(1 + r) < 1$, which prevents bequest (from rich households) to explode. We also assume $(1 - \alpha)(1 + i) > 1$ which helps to transit from relatively low initial bequests to higher final bequests.

We also impose that obtaining education is better than remaining unskilled: $x(1+r)+w^{ns}(2+r) \leq x(1+r)+w^s - h_0(1+r)$, or that

$$w^{ns}(2+r) \leq w^s - h_0(1+r).$$

Notice also that a borrower with a bequest x_t invests in education only if $x_t(1+r)+w^{ns}(2+r) \leq (x_t - h_0)(1+i)+w^s$, which we write as

$$\widehat{x} = \frac{w^{ns}(2+r) - w^s + h_0(1+i)}{i-r} \leq x_t.$$

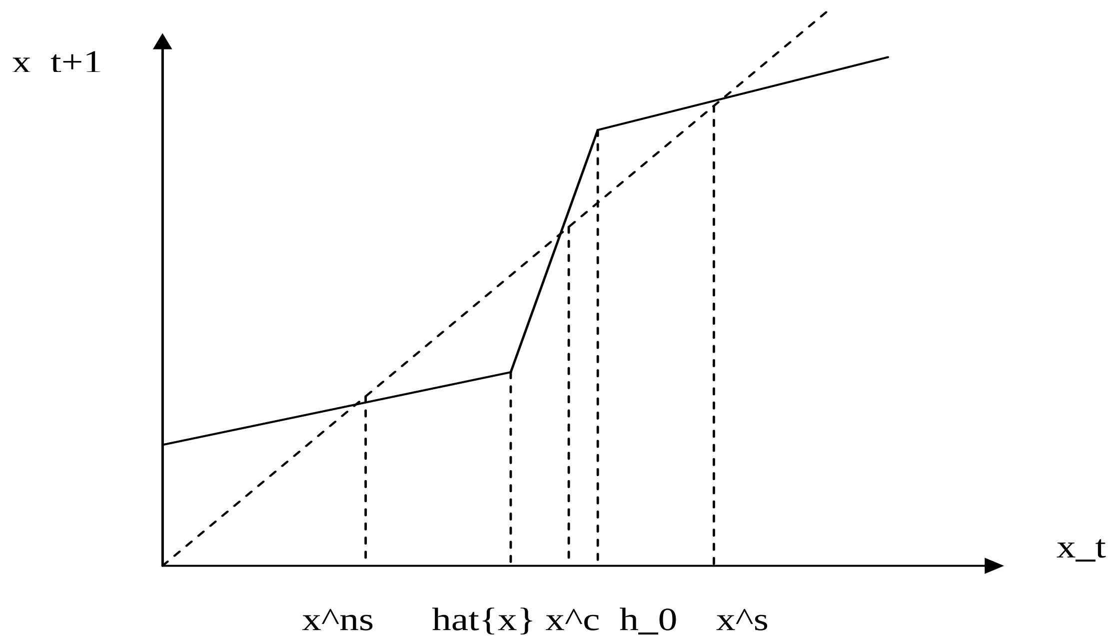
Let $D_t(x_t)$ be the cdf of bequests in period t (i.e., the "mass" od agents receiving every possible level of bequest in period t). Then we have that the mass of agents that remain unskilled is given by

$$L_t^{ns} = \int_0^{\widehat{x}} D_t'(x_t),$$

and $L_t^s = \int_{\widehat{x}}^L D_t'(x_t)$.

We summarize the evolution of bequests in the following equation, and figure

$$x_{t+1} = \begin{cases} (1-\alpha)[x_t(1+r) + w^{ns}(2+r)] \\ (1-\alpha)[(x_t - h_0)(1+i) + w^s] \\ (1-\alpha)[(x_t - h_0)(1+r) + w^s] \end{cases}$$


x t+1
x_t
x^ns
hat{x}
x^c
h_0
x^s

All agents with an initial bequest at least as large as $\hat{x}$ acquire education. However, not all their descendants will continue being educated. The critical value is x^c:

$$x^c = \frac{(1-\alpha)[h_0(1+i) - w^s]}{(1-\alpha)(1+i) - 1}.$$

For agents with $x_t > x^c$, bequests grow over time and converge to $\bar{x}^s$:

$$\bar{x}^s = \frac{1-\alpha}{1-(1-\alpha)(1+r)}[w^s - h_0(1+r)].$$

For agents with $x_t < x^c$, bequests shrink over time and converge to $\bar{x}^{ns}$:

$$\bar{x}^{ns} = \frac{1-\alpha}{1-(1-\alpha)(1+r)} w^{ns}(2+r).$$

- In the long run the distribution of wealth is polarized, with all households receiving and leaving a bequest equal to $\bar{x}^{ns}$ and remaining unskilled, or to $\bar{x}^{s}$ and receiving education.

- In the short run (during the transition) the mass of agents that acquire education is decreasing over time.

- In this model the initial distribution of wealth determines the long run steady state. In particular, if we take aggregate wealth in the economy $X = \bar{x}^{s} L_{\infty}^{s} + \bar{x}^{ns} L_{\infty}^{ns}$, then it straightforward to show that average wealth satisfies:
$$\frac{X}{L} = \bar{x}^{s} - \frac{L_{\infty}^{ns}}{L} \bar{x}^{ns},$$

which is decreasing in the ratio L_{∞}^{ns}/L. Thus, there is a clear link between the mass of unskilled households and average wealth.