Univeristat Autònoma de Barcelona
Macroeconomic Policy, 2012-2013
IDEA

Prof. F. Obiols and D. Matveev

Problem Set # 3 (due April 19th)

During the classes this week we have briefly discussed the result that with lump-sum taxation the timing of taxes does not matter (the so-called Ricardian Proposition), and the result about the fact that "taxing capital income is a bad idea". The plan in this exercise list is to review these issues from a slightly different perspective.

1.1 About the Ricardian proposition. Consider a household that lives for $T \geq 2$ periods. Her labor income is exogenously given $\{w_t\}_{t=0}^T$, and suppose the agent has access to a bonds market, in which the interest rate is $\{r_t\}_{t=0}^T$ (also exogenously given). The household has preferences $\sum_t^T \beta^t u(c_t)$, with u satisfying standard assumptions. In addition, there is a government that needs to finance the sequence of public expenditure $\{g_t\}_{t=0}^T$. The government has access to lump-sum taxes, and to the bonds market, so that the corresponding budget constraint in period t reads:

$$g_t + (1 + r_t)B_t = T_t + B_{t+1}.$$

Let us assume that $g_t < w_t$ for all t (this assumption is stronger than needed, and it makes sure that the sequence of public expenditure is feasible). Prove that the Ricardian proposition holds in this economy. That is, sow that two different sequences of taxes and bonds, say $\{T_t, B_t\}_{t=0}^T$ and $\{\hat{T}_t, \hat{B}_t\}_{t=0}^T$ that satisfy the budget constraint of the government for the same sequence of public expenditure, produce exactly the same *aggregate* consumption and saving.

1.2 Suppose there is a borrowing constraint, and that it is binding in some periods. Argue that in general the Ricardian proposition wont hold.

1.3 The previous results are not due to the fact that prices are exogenous. To determine endogenously the prices, assume there is the usual competitive firm which produces consumption and investment goods by means of capital and labor. Hence, in this extension of the basic model the agent has access to a capital market, to a bonds market, and to a labor market (you can still

treat labor supply as exogenous, but nothing would change if you modify preferences to make it endogenous). Show that if borrowing constraints do not bind, then the Ricardian proposition holds.

1.4 Given the results above, Do you think that the Ricardian Proposition should still hold if there is aggregate uncertainty (as in the RBC model)? Do you think heterogeneity among agents would be able to break the result? Develop the intuition and explain your answer.

2. Consider the optimal taxation problem in an economy in which the technology uses three different factors of production: in addition to capital and labor, it uses a new factor z_t. Hence, we have that output y_t is given by

$$y_t = F(k_t, h_t, z_t),$$

where F is of constant returns to scale in the three inputs. We retain the assumption of competitive markets (let's call p_t the price of factor z_t in period t).

2.1 Write down the problem of the representative (competitive) firm.

We assume that there is a representative household with preferences defined over consumption and leisure $u(c_t, l_t)$. In every period the agent receives an endowment of one unit of time, which can be divided between leisure (l_t), working in the market (h_t^m), and working at home (h_t^h). In particular, if the agent devotes h_t^h units of time to produce at home, she obtains $z_t = g(h_t^h)$ units of inputs, which have a market price p_t. We assume that the home production technology satisfies $g' > 0$ and $g'' < 0$.

In addition, the agent is born with some units of initial capital which evolves over time by means of the usual law of motion

$$k_{t+1} = i_t + (1 - \delta)k_t,$$

where $\delta \in (0, 1)$ represents the depreciation rate.

2.2 Write down the feasibility constraint of the previous economy.

2.3 Write down the Planner's problem associated to the economy (to this end, assume as usual that the representative agent is infinitely lived, and that future utility is discounted at a constant rate $\beta \in (0, 1)$).

Finally, there is a government that finances a given sequence of public expenditure $\{G_t\}$, by means of a combination of taxes on labor income obtained in the market, on capital income, and by issuing debt (one period bonds). Notice in particular that it is not possible to tax the income from z_t.

2.4 Write down the budget constraint of the government (to fix some notation, let the price of bonds be q_t. This is the price the agent pays today to buy a bond, and it delivers as usual one unit of consumption in the following period). The initial amount of bonds is zero.
2.5 Write down the problem of the representative household in the market economy (i.e., taking all prices and taxes as given), and taking into account the fact that the household can also sell/buy one period bonds at the same market price than the government.
2.6 Define the competitive equilibrium.
2.7 Formulate the optimal taxation problem under the assumption of commitment.
2.8 What is the optimal tax rate on capital income in the steady state? (i.e., find the optimal tax rate under the assumption that in the long run G_t converges).

3. This problem should help you to understand the role of (lack of) commitment. Consider a risk sharing problem between two agents (denotes 1 and 2) in an endowment economy. Both agents have preferences of the form $\sum_{t=0}^{\infty} \beta^t \log c_t^i$, with $\beta \in (0,1)$ (for $i = 1, 2$). The endowment process in every period $\omega^i \in \{\underline{\omega}, \bar{\omega}\}$ for $i = 1, 2$ and with $0 < \underline{\omega} < \bar{\omega}$, satisfies: $Prb(\omega^i = \underline{\omega}) = 1/2$, with $Prb(\omega^1 = \omega^2) = 0$. Assume it is impossible to store the endowment.

3.1 Use recursive language to write the utility maximization problem for each agent in isolation (i.e., under autarky). Be precise about what are state and control variables.

3.2 Suppose there was a benevolent planner that wishes to maximize the joint welfare of the two agents: $\sum_{t=0}^{\infty} \beta^t 1/2(\log c_t^1 + \log c_t^2)$ (where $1/2$ is the weight each agent receives in the objective function of the planner. Write down the optimization problem and characterize as much as possible its solution.

3.3 Would it be possible that the two agents *by themselves* implement the optimal allocation you just found in the previous point? Explain way in detail.

3.4 If your answer to the point above was yes, you are wrong... Now, is there anything these two agents can do to improve their welfare? I.e., Is autarky all that is feasible/possible? Write down the utility maximization problem in which agents exchange a contract (explain what is at stake in this contract), that is actually *implementable*, i.e., that no agent would be willing to deviate from.

Suggested readings about optimal income taxation with commitment

- A classical reference is: Chamley, Ch., 1986. *Optimal Taxation of Capital Income in General Equilibrium with infinite lives.* Econometrica, Vol. 54, 607-22.

- See also: Ljungqvist and Sargent (2000): *Recursive Macroeconomic Theory*, MIT press., Ch. 9 and 12.

A few interesting papers:
- Aiyagari, S.R., 1995. *Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints and Constant Discounting.* Journal of Political Economy, Vol. 103, 1158-75.
- Atkeson, A., Chari, V.V. and P.J. Kehoe, 1999. *Taxing Capital Income: A Bad Idea.* Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 23, 3-17.
- Barro, Robert R., 1974. *Are Government Bonds Net Wealth?*, Journal of Political Economy, Vol.82, 1095-1117.
- Barro, Robert R., 1979. *On the determination of Public Debt*, Journal of Political Economy, Vol. 87, 940-971.
- Chari, V.V., and P.J. Kehoe, 1998. *Optimal Fiscal and Monetary Policy.* Staff Report 251, Minneapolis Fed.
- Erosa, A., and M. Gervais, 2001. *Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review*, Federal Reserve Bank of Richmond Economic Quarterly, Vol. 87, 23-44.
- Garcia-Mila, T., A. Marcet and E. Ventura, 2001. *Supply side Interventions and Redistribution* UPF, working paper.
- Jones, L. E., Manuelli, R. E., and P. E. Rossi. 1993. *Optimal Taxation in Models of Endogenous Growth*, Journal of Political Economy, Vol. 101, 485-517.
- Judd, K.L., 1985. *Redistributive Taxation in a Simple Perfect Foresight model.* Journal of Public Economics, Vol. 28, 59-83.

- A curiosity, and a recent advancement:

- Ennis, H., and T. Keister (2005): *Optimal fiscal policy under multiple equilibria*, Journal of Monetary Economics, Vol. 52, 1359-1377
- Conesa, J.C., Kitao, S., and D. Krueger, 2009. *Taxing Capital? Not a Bad Idea After All!*, American Economic Review, 99, 25-48.