Univeristat Autònoma de Barcelona
Macroeconomic Policy, 2012-2013
IDEA

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Problem Set # 2 (due April 5th)

1. Precautionary savings and not having access to insurance markets. Consider a two periods economy in which there is a continuum, mass one, of agents. In the first period, all agents receive a certain labor productivity endowment, normalized to one. In the second period the endowment of labor productivity is random: a fraction π of agents will receive e_h and the remaining $1 - \pi$ will receive e_l (with $e_h > e_l$). Preferences are such that agents only care about consumption in the first and in the second period: $U(c_1, c_2) = \log c_1 + \beta \log c_2$, with $\beta \in (0, 1)$. We take the rental price of labor w as exogenously given, and we assume that it is the same in both periods. Finally, we assume there is a single asset to transfer wealth over time: agents can save in a risk-less asset, which pays a return $(1 + r)$ in the second period (again, r is exogenously given).

1.1 Assume that agents can buy insurance against the low productivity endowment in the beginning of the second period (before the uncertainty in that period is realized). In particular, suppose that this insurance market is perfectly competitive and that there is free entry (which simply means that profits are zero, and this condition will tell you what are insurance prices). Solve the utility maximization problem of one of these agents and in particular, characterize saving of each of the agents. By the way, are markets complete (i.e., would having access to a full set of Arrow securities increase the possibilities of trade/insurance compared to the current arrangement with a single insurance?

1.2 Compare your previous answers to the one you obtain when all agents are alike: in the second period each agent receives a labor productivity endowment equal to $\bar{e} = e_h\pi + e_l(1 - \pi)$.

1.3 Compare your previous answers to the one you obtain when the only available asset is the one paying $(1+r)$ units in the second period (i.e., when there are no insurance markets available). Explain in terms of precautionary savings, i.e., state the analogy between precautionary savings and having (or not) access to insurance markets.

Notes:
(1) In the 70's, Leland and Sandmo showed that in two periods economies, a convex marginal utility of consumption ($u''' > 0$) is enough to have precautionary savings. Since this condition is satisfied in the exercise, you should obtain the same kind of result. It is also important to realize that in the Aiyagari-Huggett model we developed in class, precautionary savings will always obtain, irrespectively of the sign of u'''. Hence, there are important differences between two periods (finite horizon) models and infinite horizon models.
(2) The initial notion of "precautionary saving" was meant to reflect the effect of increased uncertainty (for instance, about the effect of moving from zero variance to positive variance in the second period environment). The exercise above suggests that this notion of precautionary saving is intimately linked to the availablem market arrangement.

2. Binding constraints in a deterministic setting. Consider the problem of a consumer who chooses sequences of consumption goods and assets $\{c_t, a_{t+1}\}$ so as to maximize $\sum_{t=0}^{\infty} \beta^t u(c_t)$, with u strictly increasing, strictly concave, and differentiable. We write the budget constraint of the consumer as $c_t + a_{t+1} = y_t + (1 + r)a_t$, in which $\{y_t\}$ is an exogenously given, deterministic, sequence of labor incomes. The consumer performs this maximization subject to $c_t \geq 0$ and $a_{t+1} \geq B$. Finally, assume that $a_0 = 0$, $\beta(1 + r) = 1$, and that $B = 0$ (hence *no borrowing* is permitted).

2.1 Determine the sequence $\{c_t, a_{t+1}\}$ that solves the consumer's problem under the assumption that $\{y_t\} = \bar{y}, \underline{y}, \bar{y}, \underline{y}, ...$ with $0 < \underline{y} < \bar{y}$. Show, in particular, that the borrowing constraint is never binding.
2.2 Show that if $\{y_t\} = \underline{y}, \bar{y}, \underline{y}, \bar{y}, ...$, then the borrowing constraint binds in the first period.

3. Is "more trade" unambiguously desirable? In this exercise we try to asses whether alleviating a friction by increasing trade is desirable or not from a welfare perspective (it can be seen as a version of Athreya 2002, and Li and Sarte, 2006). Hence, we try to look at the effects of alleviating constraints in incomplete market economies.

Consider an exchange economy in which a continuum, mass one, of agents live for two periods. We assume that a fraction $1/2$ of the population is endowed with $0 < y_l$ units of consumption goods in the first period, and that the endowment of the remaining $1/2$ fraction is $y_h > y_l$. In the second period all agents receive the same endowment, $\bar{y} > 0$. Agents can smooth out their consumption over time by trading a safe asset (uncontingent), a bond with market price q (in terms of the consumption good of the first period). Borrowing is indicated by a negative amount of assets, and it is allowed up to an exogenous limit $-B$ (of course, debt issues in the first period convey the obligation to deliver consumption goods in the second, i.e., no default is possible). The objective of a consumer type i can be written as

$$\begin{aligned} \max_b v_i(B) = \quad & \log c_1^i + \beta \log c_2^i \\ \text{s. to} \quad & c_1^i = y^i + q b^i, \\ & c_2^i = \bar{y} - b^i, \\ & b^i \geq -B. \end{aligned}$$

3.1 Define the competitive equilibrium corresponding to the previous economy, and determine the equilibrium under the assumption that the borrowing limit is not binding (i.e., B is so large that the borrowing of the borrower satisfies $b^l > -B$). Show, in particular, that if $B > \bar{y}(y_h - y_l)/((1+\beta)(y_h + y_l))$, then the borrowing limit does not bind. Pay special attention to the market clearing condition for bonds: in equilibrium the amount sold must be equal to the amount bought.

3.2 Suppose now that the borrowing limit is smaller than before so it is binding for the borrower, hence $b^l = -B$. Show that equilibrium prices satisfy $q(B) = \beta y_1^h/(\bar{y} + B(1+\beta))$. Given this, evaluate the change in equilibrium welfare of the savers that arises when the borrowing limit is reduced.

3.3 Suppose now that the borrowing limit is given by $\tilde{B} = \bar{y}(y_h - y_l)/((1+\beta)(y_h + 2y_l))$. What is the effect of a slight increase in the borrowing limit on the welfare of borrowers? And on the welfare of lenders (savers)? What do you conclude about the effect of the borrowing limit on welfare?

4. Use the method of interpolations in the decision rules for assets we explained in class to solve a version of Huggett 1994 incomplete markets model. To this end, assume $u(c) = \log c$, $\beta = .99$, $e_h = 1.1$, $e_l = .9$, $\pi_{e_h|e_h} = .9$, and $\pi_{e_l|e_l} = .8$. Given this information, determine the *natural debt limit.* In your numerical approximation you cannot assume that the borrowing limit is equal to the borrowing limit (why?), so let fix a more stringent limit than the natural limit. Remember that in this model agents are only allowed to trade a one period risk-less bond at a price q. Solving for the equilibrium entails finding a price q^*, such that the asset markets clears. A simple strategy to check market clearing is to simulate a long time series of the actual decision rule, and compute the average over time. A more sophisticated approach is to determine the actual distribution of wealth, and then integrate the decision rule for assets using that distribution as requested in our equilibrium definition. The paper by Huggett contains all the details about this procedure.

Suggested readings about heterogeneous agents due to incomplete markets

A useful reference on the general theme is:
Ríos-Rull, J.V. (1995): *"Models with heterogeneous agents"*, Ch. 4 in *Frontiers of Business Cycles Research*, T. Cooley ed.

See also Chapter 14 in the volume by Ljungqvist and Sargent.

The literature about incomplete markets is large an expanding fast. The basic references are:

- Aiyagari, R. (1994): *"Uninsured idiosyncratic risk and aggregate saving"*, Quarterly Journal of Economics.
- Huggett, M. (1993): *"The Risk-Free Rate in Heterogeneous-Agent Incomplete Insurance Economies*, Journal of Economic Dynamics and Control.
- Huggett, M. (1997): *"The one-sector growth model with idiosyncratic shocks: Steady states and dynamics.* Journal of Monetary Economics.
- Krusell, P., and A. Smith (1998): *"Income and wealth heterogeneity in the macroeconomy"*, Journal of Political Economy.

On the general theme of the (sometimes surprising) effects of alleviating constraints in IM economies (here we look at borrowing constraints):

- Athreya, K.B. (2002): *"Welfare implications of the Bankruptcy Reform Act of 1999".* Journal of Monetary Economics.
- Li, W. and P.D. Sarte (2006): *"U.S. consumer bankruptcy choice: The importance of general equilibrium effects"*, Journal of Monetary Economics.
- Livshits, I., MacGee, J. and M. Tertilt (2007): *"Consumer Bankruptcy: A Fresh Start"*, American Economic Review.