Univeristat Autònoma de Barcelona
Macroeconomic Policy, 2012-2013
IDEA

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Problem Set # 1 (due March 22th)

1. Consider an economy in which a large number of agents choose labor supply, consumption, and saving, over an infinite horizon. There are two types of agents, denoted $i = 1, 2$, and the corresponding fractions are given by π and $(1 - \pi)$. The objective of an agent type i is given by:

$$\sum_{t=0}^{\infty} \beta^t \{\log(c_t^i + \bar{c}^i) + A \log(T^i - h_t^i)\},\ A \geq 0,\ \beta \in (0, 1).$$

The corresponding budget constraint reads:

$$c_t^i + a_{t+1}^i = w_t h_t^i + R_t a_t^i,$$

where $R_t = r_t + 1 - \delta$ represents the return factor on assets (think of a as of capital k), the same for all agents, and w_t is the wage rate, also the same for all agents. In the previous formulation $\bar{c}^i$ is not necessarily equal to zero, and T^i is the endowment of time for type i agents. This model, therefore, is able to encompass heterogeneity in the two endowments (assets and time), and in the term $\bar{c}$.

There is a representative firm which operates in competitive markets in order to maximize profits. The associated FONC read:

$$r_t = F_1(K_t, H_t), \quad \text{and } w_t = F_2(K_t, H_t),$$

where $F(K_t, H_t)$ represents a constant returns to scale technology in capital and labor.

1.1 Define the competitive equilibrium for this economy. In particular, state the market clearing condition for assets.

1.2 Assume from now on that $\bar{c}^1 = \bar{c}^2$, $A = 0$ and that $T^1 = T^2 = 1$. Define formally the steady state. What can you say about the distribution of assets at the steady state? Is it unique? Why?

1.3 Define "life-time" wealth ω_t^i, and show that consumption for a type i agent satisfies

$$c_t^i B_t^1 + \bar{c}^i B_t^2 = \omega_t^i.$$

Do B_t^1 and B_t^2 depend on i?

1.4 Given your previous results, state and demonstrate an aggregation theorem. If in your opinion such a theorem does not exist, then explain in detail why not. Propose an example where perfect aggregation does not hold. For instance, what happens if we assume $A \neq 0$ and $T^i = 1$ for all i? What happens if $\bar{c}^1 \neq \bar{c}^2$?

2. Assume again that A=0 and $T^i = 1$ for all i. Assume also that $\bar{c}^i = \bar{c}$ for all i. Finally, assume $\beta = 0.99$, $F(K, L) = K^\alpha L^{1-\alpha}$, with $\alpha = 0.36$, and that $\delta = 0.025$. Under the assumptions we introduced in class (that the economies in this Problem Set satisfy), the competitive equilibrium is efficient.

2.1 Determine the steady state level of capital, k^*. Notice that for this calculation you do not need to know $\bar{c}$.

2.2 Imagine that one of the (many!) agents in the previous economy has $k^i = \theta k^*$, with $\theta \in (0, 1)$. Since the economy is at the steady state, then prices are constant. Fix a small number for $\bar{c}$ (like $\bar{c} = 0.0001$), fix a value for θ, say $\theta = 1/2$, and compute the present value of the utility of this agent:

$$v(k^i) = \sum_{t=0}^{\infty} \beta^t u(c(k^i)),$$

where $c(k^i) = w(k^*) + (r(k^*) - \delta)k^i$ (i.e., consumption satisfies the stationary budget constraint.

2.3 Assume that the single agent in the previous problem considers to abandon the stationary economy, and settle down in an isolated island. In the island the available technology, depreciation rate, discount factor, etc. is the same as in the large economy. The only difference is that the available stock of capital is $k^i = \theta k^*$, which is smaller that k^*. Hence the autarky economy will grow, and it will converge to k^*. What is the present value of welfare along this transition? Does it pay off to move to the island (even if the move was "free")? Does you result change when you vary $\bar{c}$ or θ?

To do this exercise you need do two basic operations. First, solve the planner's problem of "Robinson Crusoe". You can do this by value function iterations, and the result of this is essentially policy functions for capital, and for consumption (as labor supply is inelastic). In the second step you use the optimal policies to simulate welfare toward the steady state. In general this means that you know $k' = g(k)$ only if k is a point of your grid. Nothing guarantees the k' as given above is also in your grid... The solution is to do some sort of interpolation. Along this transition you also find the corresponding consumption and welfare, and compute the associated present value, which is what you need to compare with your result in 2.2.