MINIEXAMEN 4

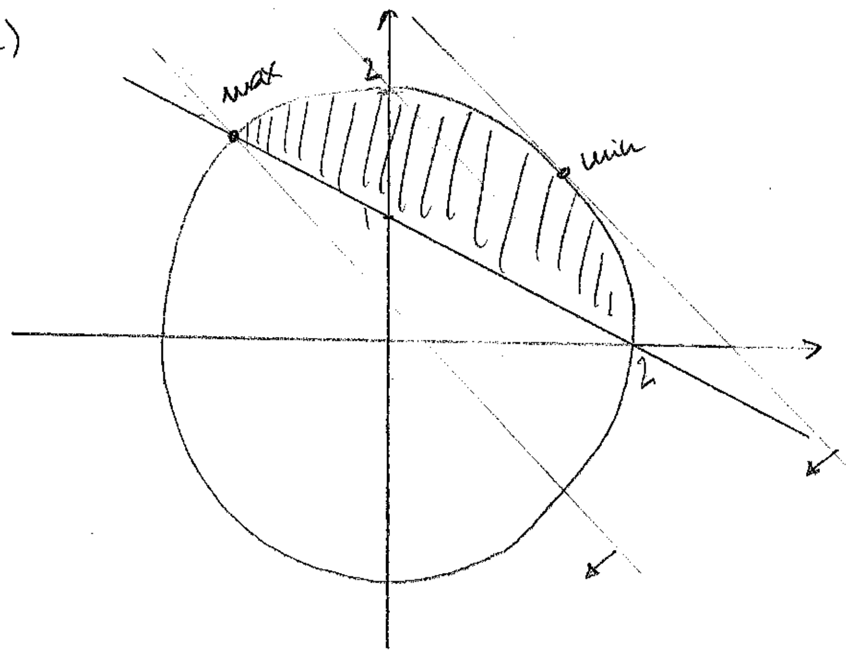
PROBLEMA 1

$$f(x,y) = -x-y$$

s.a. $\quad x+2y \geq 2 \qquad (g_1)$

$\quad x^2+y^2 \leq 4 \qquad (g_2)$

a) b) c)



d) $\mathcal{L} = -x-y - \lambda_1\left[-x-2y+2\right] - \lambda_2\left[x^2+y^2-4\right]$

$\frac{\partial \mathcal{L}}{\partial x} = -1+\lambda_1 - 2x\lambda_2 = 0 \qquad \lambda_1\left[-x-2y+2\right] = 0$

$\frac{\partial \mathcal{L}}{\partial y} = -1+2\lambda_1 - 2y\lambda_2 = 0 \qquad \lambda_2\left[x^2+y^2-4\right] = 0$

En el mínim només es satura $g_2 \Rightarrow g_1 < 0 \quad \lambda_1 = 0$

Sistema a resoldre:

$$\left.\begin{array}{l} -1+\cancel{\lambda_1}^{0} - 2x\lambda_2 = 0 \\ -1+\cancel{2\lambda_1}^{0} - 2y\lambda_2 = 0 \end{array}\right\} \quad \left.\begin{array}{l} \lambda_2 = -\frac{1}{2x} \\ \lambda_2 = \frac{-1}{2y} \end{array}\right\} \quad x = y$$

$x^2+y^2 = 4$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$(-\sqrt{2}, -\sqrt{2}) \Rightarrow$ no factible $g_1 \not< 0 \qquad \left(\frac{-1}{2\sqrt{2}}, \sqrt{2}, \sqrt{2}\right) \Rightarrow$ mínim local màxim: les dues restriccions es saturen:

$-1+\lambda_1-2x\lambda_2=0$ ①

$-1+2\lambda_1-2y\lambda_2=0$ ②

$$\left.\begin{array}{l} x^2+y^2=4 \\ x+2y=2 \end{array}\right\} \Rightarrow y=1-\frac{1}{2}x \; \Big\} \; x^2+\left(1-\frac{1}{2}x\right)^2=4$$

$$x^2+1-x+\frac{1}{4}x^2=4 \Rightarrow \frac{5}{4}x^2-x-3=0$$

$$x=\frac{1\pm\sqrt{1+15}}{\frac{5}{2}}=\frac{1\pm4}{\frac{5}{2}} \left\langle \begin{array}{l} 2 \\ -\frac{6}{5} \end{array}\right.$$

$$y= \left\langle \begin{array}{l} 1-1=0 \\ 1+\frac{3}{5}=\frac{8}{5} \end{array}\right.$$

Punts $(0,2)$ $\left(-\frac{6}{5},\frac{8}{5}\right)$ ⊛

Queda trobar les λ's mitjançant ① i ②

$\left(-\frac{6}{5},\frac{8}{5}\right)$

$$\left.\begin{array}{l} -1+\lambda_1+\frac{12}{5}\lambda_2=0 \Rightarrow \lambda_1=1-\frac{12}{5}\lambda_2 \\ -1+2\lambda_1-\frac{16}{5}\lambda_2=0 \Rightarrow \lambda_1=\frac{1}{2}+\frac{8}{5}\lambda_2 \end{array}\right\} \quad 1-\frac{12}{5}\lambda_2=\frac{1}{2}+\frac{8}{5}\lambda_2$$

$$\frac{1}{2}=\frac{20}{5}\lambda_2$$

$$\lambda_2=2>0$$

$$\lambda_1=\frac{1}{2}+\frac{16}{5}>0 \quad ✓$$

màxim!

f) El conjunt factible és convex i la funció objectiu és còncava i convexa. Per tant el problema és còncau i convex.

La condició de Slater es satisfà perquè en el punt $(1,1)$

$$x^2+y^2 = 2 < 4$$

i

$$x + 2y = 1+2 = 3 > 2$$

✓

PROBLEMA 2

x = unitats de 100g de Pinita

y = " " " " Tulita

min $20x + 30y$

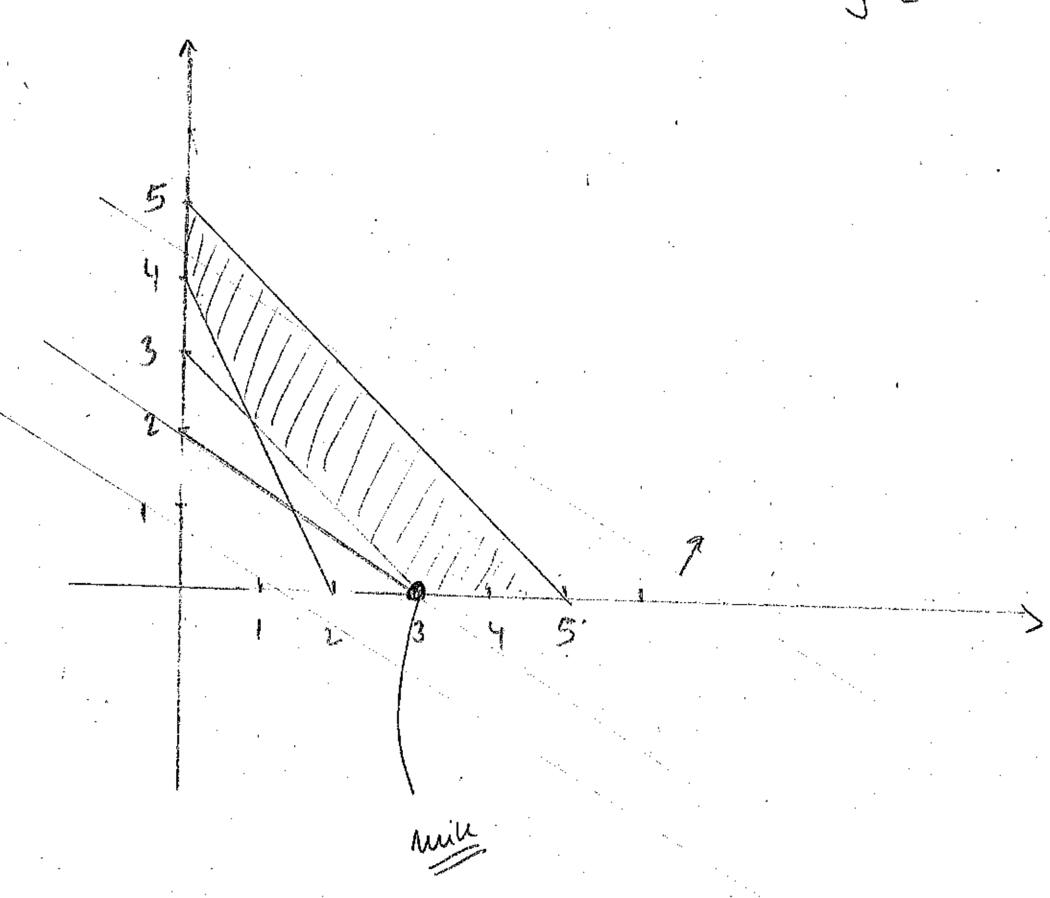
s.a.

$8x + 12y \geq 24$ ⇒ greix superior a 24g.
$y \geq 2 - \frac{2}{3}x$.

$12x + 12y \geq 36$ ⇒ carbohidrats superiors a 36g.
$y \geq 3 - x$

$2x + y \geq 4$ ⇒ proteïna superior a 4g
$y \geq 4 - 2x$

$x + y \leq 5$ ⇒ total de pes inferior a 500g
$y \leq 5 - x$



(3,0) 300g de pinita i gens de tulita.

MINIEXAMEN 4

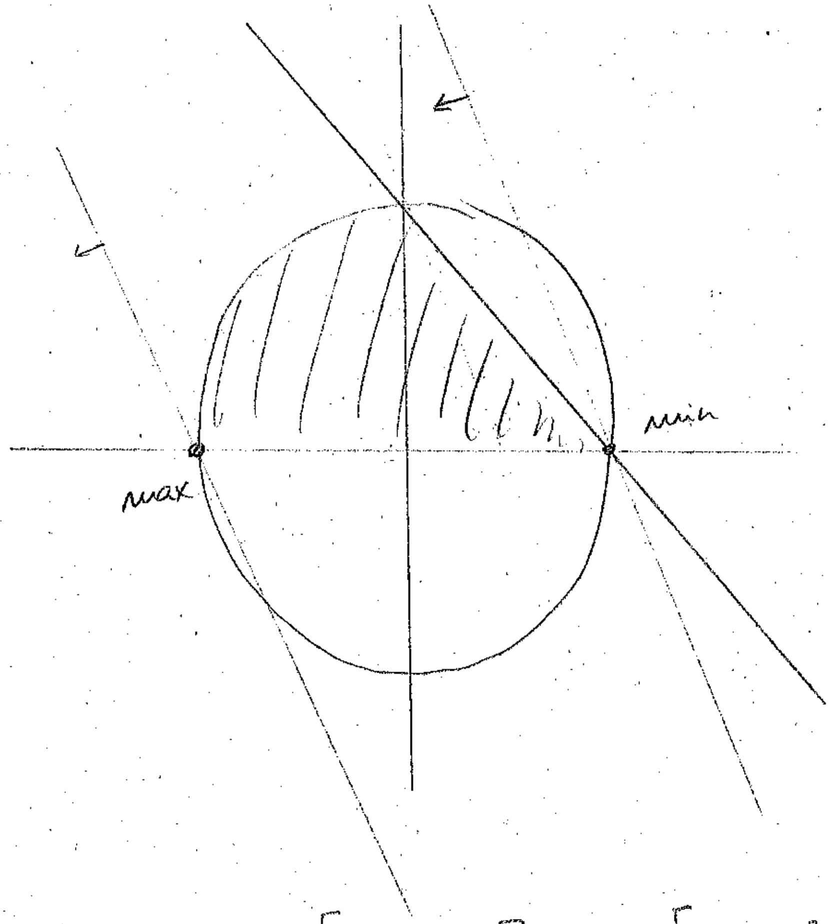
PROBLEMA 1

opt. $-2x-y$

a) b) c) s.a. $x+y \leq 3$

$x^2+y^2 \leq 9$

$y \geq 0$



$$\mathcal{L} = -2x-y - \lambda_1[x+y-3] - \lambda_2[x^2+y^2-9] - \lambda_3[-y]$$

$$\frac{d\mathcal{L}}{dx} = -2 - \lambda_1 - 2x\lambda_2 = 0$$

$$\frac{d\mathcal{L}}{dy} = -1 - \lambda_1 - 2y\lambda_2 + \lambda_3 = 0$$

$$\lambda_1(x+y-3) = 0$$

$$\lambda_2(x^2+y^2-9) = 0$$

$$\lambda_3(-y) = 0$$

En el mínim g_1, g_2, g_3 es saturen

$$\left.\begin{array}{l} x+y=3 \\ x^2+y^2=9 \\ y=0 \end{array}\right\} \boxed{\begin{array}{l} x=3 \\ y=0 \end{array}}$$

$$\left.\begin{array}{l} -2-\lambda_1-6\lambda_2=0 \\ -1-\lambda_1+\lambda_3=0 \end{array}\right\}$$

Mirant el gràfic veig que si em relaxessin g_3 no canviaria l'òptim $\Rightarrow \lambda_3=0$

Llavors $\lambda_3=0 \Rightarrow \lambda_1=-1 \quad <0$

$$-2+1=6\lambda_2 \Rightarrow \lambda_2=\frac{-1}{6} < 0$$

e) En el màxim g_2 i g_3 es saturen $g_1 \neq 0, g_1<0 \Rightarrow \lambda_1=0$.

$$\left.\begin{array}{l} x^2+y^2=9 \\ y=0 \end{array}\right\} \begin{array}{l} y=0 \\ x=\pm 3 \end{array}$$

$(3,0)$ era el mínim

$(-3,0)$

$$-2-\cancel{\lambda_1}^{0}+6\lambda_2=0 \Rightarrow \lambda_2=\frac{1}{3}>0$$

$$-1-\cancel{\lambda_1}^{0}+\lambda_3=0 \Rightarrow \lambda_3=1 \quad >0$$

$(1, \frac{1}{3}, -3, 0) \Rightarrow$ màxim local ✓

x: unitats de 100g de Rumita

PROBLEMA 2 y: " " " " Tunita

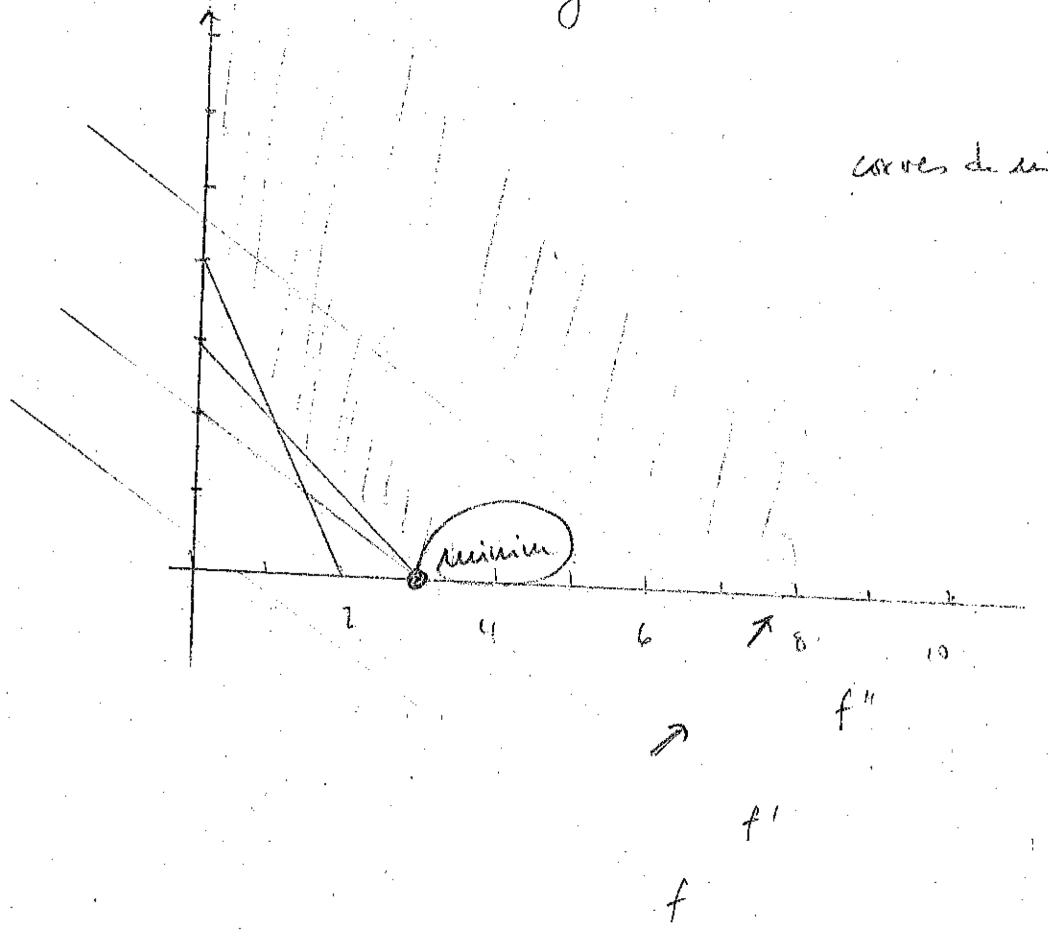
min $10x + 15y$

s.a. $8x + 12y \geq 24 \Rightarrow y \geq 2 - \frac{2}{3}x$

$12x + 12y \geq 36 \Rightarrow y \geq 3 - x$

$2x + y \geq 4 \Rightarrow y \geq 4 - 2x$

$x + y \leq 10$



$\left(\frac{2}{3}\right)$

corbes de nivell f: $y = \frac{K}{15} - \frac{10}{15}x$

3 unitats de Rumita satisfan les necessitats alimentàries i minimitzen la despesa.