Decentralizing Equality of Opportunity

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Abstract

In a global justice problem, equality of opportunity is satisfied if individual well-being is independent of exogenous irrelevant characteristics. Policymakers, however, address questions involving local justice problems. We interpret a collection of local justice problems as the decentralized global justice problem. We show that controlling for effort locally, which is not required by the global justice objective, is sufficient for decentralizing equality of opportunity. Moreover, under some conditions, equalizing rewards to effort is not only sufficient but necessary. This implies in particular that most affirmative action policies may not contribute to providing equality of opportunity.

Keywords: Global Justice, Local Justice, Decentralizing, Equality of Opportunity, Effort
JEL Classification: I00, I20

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1 Introduction

Although global and local distributive justice problems seem to be related, a critical gap exists between them. In the first chapter of his book *Equity*, Peyton Young describes this disconnect: "[t]heories of justice in the large do not tell us how to solve concrete, everyday distributive problems such as who should get into medical school or how much to charge for a subway ride." He argues that the reason for this is that "[i]ssues of local justice tend to be compartmentalized. Societies make no effort to coordinate distributive decisions across different domains." For example, kidney agencies do not give precedence to patients who failed to get into university; or a worker's wage at a given job is not increased because the government located a waste dump near his home. He concludes by saying that "[T]here is no mechanism comparable to the invisible hand of the market for coordinating distributive justice at the micro into just outcomes at the macro level." He then argues that local distributive justice should be derived by induction, that is, formulating general rules from what we observe is being done in different local distributive justice problems.

In this paper we interpret a collection of local distributive justice problems as essentially the decentralized global distributive justice problem. Then, a local justice rule can be defined as the rule that decentralizes a given distributive justice criterion when information is disperse and decision making decentralized. We then ask whether we can solve the global distributive justice problem by solving a series of local distributive justice problems. In particular, we characterize the set of local distributive justice rules that decentralize equality of opportunity.

There is a wide literature dealing with the issue of compensation and responsibility.[1] The results in this literature are mainly axiomatic characterizations clarifying the conflicts and interdependencies of different ethical principles and their different axiomatic representations. But the context is that of a global justice problem. To analyze the point in this paper we focus on a very simple formalization of equality of opportunity, initiated by Rawls (1971) and formalized and discussed in Roemer (1998). Equality of opportu-

[1] For an overview see Fleurbaey and Maniquet (2004) "Compensation and Responsibility" in the Handbook of Social Choice and Welfare for an excellent review of this literature.

nity requires that an individual's success in life be independent of *irrelevant* characteristics, that is, of characteristics that the individual should not be responsible for. Such irrelevant characteristics may include, for example, race, gender or parents' income.[2] Large part of the social debate concerns precisely what the set of irrelevant characteristics should be, but this paper abstracts from this debate and takes the set of irrelevant characteristics as given.

In the literature the difference between global and local problems has been largely ignored. Specifically, local equality of opportunity has been interpreted as local outcomes or local utilities being independent of irrelevant characteristics affecting the local outcome. This is the analogous definition of equality of opportunity in the global problem for the local problem.[3] We show that such notion of local equality of opportunity may not lead to equality of opportunity globally. Specifically, local independence of outcomes from irrelevant characteristics will not aggregate into independence of total welfare from irrelevant characteristics. This paper shows that a collection of local mechanisms will decentralize the provision of equality of opportunity if it provides local equality of rewards to effort, where effort is a productive individual choice variable, for example, time devoted to a given activity.[4] Moreover, it shows that if the mechanisms and the environment are *rich*, notion that will be made precise later in the paper, global equality of opportunity will be decentralized only if the collection of mechanisms provides local equality of rewards to effort. Although equalizing opportunities does not require explicitly controlling for effort in the global problem, it does for the decentralized problem. Effort, an endogenous variable, needs to be controlled for in the local problems in order to achieve equality of opportunity.

When decentralizing equality of opportunity, a problem that arises is that of properly accounting for the cost of local effort, because it generally depends on information that the local policymaker does not have.[5] This is the

[2]Whether welfare or outcomes should be equalized has been part of the debate, but this paper does not directly address this issue, although indirectly it relates to it, since the global objective is welfare but locally the planner focuses on outcomes. See for example Dworkin (1981a,b) or Roemer (1986) for a discussion on this issue.

[3]Some examples are Benabou and Ok (2001), Betts and Roemer (2002), Llavador and Roemer (2001) or Roemer et al (2003).

[4]Roemer (1998) proposes an algorithm that equalizes outcomes for individuals who exert the same *degree* of effort. This differs substantially from equalizing rewards to effort as shown in section 2.

[5]For example, colleges may be interested in equalizing the probability of admission

main reason why providing equality of opportunity in a decentralized manner requires that two individuals differing in irrelevant characteristics receive the same outcome *when* they exert the same effort. This suggests that when designing policies one should analyze the effect of the policy on the rewards to effort and not only on the outcome it will generate. Implementing this may be hard and may require mechanism design techniques because effort is usually unobservable. Therefore some may interpret this as a critique to the equal opportunities enterprise since we present a more difficult problem. Others, however, may interpret this result as a step toward understanding what the provision of global equality of opportunities in society entails and view the local rule as a useful tool for policy design.

This paper does not analyze how this moral objective interacts with other objectives that policymakers may have. It does not address how efficient or profitable it may be for the policymaker to equalize rewards to effort. So clearly future research should embed this criterion in a more general welfare function that includes other desirable properties or objectives that the policymaker may care for. A general approach to this issue would involve an axiomatic analysis similar to that of Fleurbaey and Maniquet (2005, 2006). In more concrete settings it will be important to analyze how equalizing rewards to effort limits the policymaker. This would allow us address important issues like which is the most efficient way of providing equality of rewards to effort or how restrictive equalizing rewards to effort is in other respects. However, in competitive environments like auctions, tournaments or contests it has been shown that asymmetries between the players reduces their incentives to compete and that reducing these asymmetries may be beneficial to the administrator.[6] How these asymmetries are resolved, however, has a big impact for the administrator. In particular, equalizing chances of winning of asymmetric players through a lump sum bonus may decrease the efforts exerted by all individuals. But equalizing rewards to effort enhances

into college for students that only differ in their neighborhood characteristics. But if different individuals have invested a different number of hours of study, colleges want to compensate for the individual's cost of studying the extra hours by admitting them with a higher probability. But this opportunity cost depends on how much effort or time the individual is spending on the other activities available to him, for example, training basketball or working in a paid job.

[6]See Maskin and Riley (1995), Myerson (1981), McAfee and McMillan (1989), Corns and Schotter (1996) for details.

competition and increases the efforts exerted by all individuals.[7] In this case therefore, properly providing local equality of opportunity maximizes the performance of the competition.

The structure of the remainder of the paper is as follows. Section 2 presents an example to illustrate the main ideas underlying the results. Section 3 presents the benchmark model and derives the main result. Section 4 presents some concluding remarks.

2 Education and Basketball: An Example

Paul and Richard are intelligent, highly motivated and hardworking teenagers, denoted by $i \in \{P, R\}$. Both would like to become professional basketball players and get a college degree. Paul and Richard are identical except that Richard's neighborhood has better public schools.[8] Unlike education, Paul and Richard have identical resources to improve their basketball skills, that is, they have the same number of courts and teams of similar qualities in which to train.

Paul and Richard have to decide how much time to spend on homework, denoted by e_S, and on playing basketball, denoted by e_B. Their ultimate objective is to get into college and the NBA.

In this example we assume that individuals are characterized by a vector of relevant characteristics $r = (r_S, r_B)$, which correspond to education and basketball innate ability, and a vector of irrelevant characteristics, $b = (b_S, b_B)$, representing neighborhood school resources and basketball facilities. In particular let $(b_S^P, b_B^P) = (\frac{1}{2}, 1)$ and $(b_S^R, b_B^R) = (\frac{7}{10}, 1)$, and $(r_S^P, r_B^P) = (r_S^R, r_B^R) = (\frac{1}{3}, \frac{1}{5})$. Note that Paul and Richard only differ in b_S.

College admissions decide who gets admitted into college and observe education-specific characteristics, r_S and b_S, and NBA recruiters decide who gets admitted into the NBA and observe basketball-specific characteristics, r_B and b_B.

Out of the characteristics and effort, Paul and Richard get a performance in education and basketball that can be measured by the outcome functions "normalized SAT score" and "normalized NBA test score". The normalized SAT score (percentile in the distribution of SAT scores in the population)

[7] See Franke (2006) and Calsamiglia, Franke and Rey (2007).

[8] Paul (P) comes from the Poor neighborhood and Richard (R) from the Rich neighborhood.

can be represented by:

$$a_S = a_S(b_S, r_S, e_S) = b_S r_S (e_S)^{\frac{1}{2}}$$

College admissions therefore know that $a_S = \frac{1}{3} b_S (e_S)^{\frac{1}{2}}$. The normalized NBA test score (percentile in the distribution of NBA scores in the population) can be represented by:

$$a_B = a_B(b_B, r_B, e_B) = b_B r_B (e_B)^{\frac{1}{2}}$$

NBA recruiters therefore know that $a_B = \frac{1}{5}(e_B)^{\frac{1}{2}}$. The cost of training and studying to Paul and Richard is represented by:

$$c(e_S, e_B) = \frac{1}{500}(e_S + e_B)^2$$

College admissions officers and NBA recruiters, with education-specific and basketball-specific information respectively, decide the probability of individuals being admitted into college and into the NBA, that is $p_S(b_S, r_S, e_S)$ and $p_B(b_B, r_B, e_B)$.[9] Paul and Richard's utility is determined by the probability of them being admitted into college or the NBA and by the cost of devoting time to either of the two activities:

$$U = p_S(b_S, r_S, e_S) + p_B(b_B, r_B, e_B) - c(e_S, e_B)$$

We say that college and NBA recruiters follow a *market policy* if they admit individuals with probability $p_j = a_j$.

The marginal cost in one activity depends on the effort being spent at the other activity. Specifically, the more time one spends in one activity, the higher the marginal cost of effort for the other activity. As seen in Figure 1, when maximizing his utility, Richard devotes relatively more time to school than Paul given his higher productivity of studying. This leaves Richard with a higher marginal cost of playing basketball, leading him to train fewer hours than Paul. In particular, Paul will choose $(e_S^P, e_B^P) = (4.2, 6)$ and get $p_S^P = 0.34$, $p_B^P = 0.5$ and $U^P = 0.62$; and Richard will choose $(e_S^R, e_B^R) = (6.6, 4.8)$ and get $p_S^R = 0.6$, $p_B^R = 0.44$ and $U^R = 0.77$. Notice

[9] In this example we are not modelling an admission policy that is allocating a fixed number of seats, but assuming that the admissions policy for Paul can be changed without Richard's being affected, which is somewhat unrealistic, but greatly simplifies presenting the point of the example.

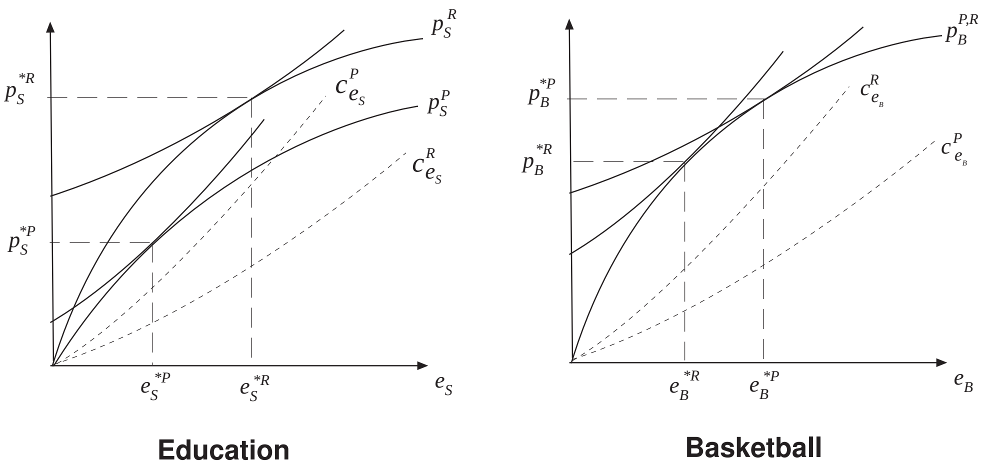

Figure 1: Choices and outcomes when college and NBA recruiters follow the market policy

that equality of opportunity is violated because Richard has a higher welfare than Paul even though they only differ in irrelevant characteristics.

Equality of opportunity is a concern when irrelevant characteristics affecting individual's achievement differ. Since irrelevant characteristics affecting basketball are the same for Paul and Richard, NBA recruiters need not worry about violating equality of opportunity. For them the market policy, for example, will satisfy equality of opportunity.[10]

Suppose that college admissions had as a first priority the contribution to the provision of equality of opportunity through providing local equality of opportunity or equality of educational opportunity. It is not clear, a priori, what equality of educational opportunity would require. One possibility is that a consultant, having read some political philosophy, Roemer's book on Equality of Opportunity, or the literature on compensation and responsibility tells them that since Paul and Richard are identical except for their irrelevant characteristic, the compensation principle suggests equalizing the

[10]In the literature an important principle of justice that authors have imposed is the *principle of natural reward*, introduced by Fleurbaey (1995a), which requires respecting inequalities resulting from aspects other than irrelevant characteristics. In particular then it would suggest that if individuals are identical with respect to irrelevant characteristics there would be no reason to add any modification to the policy. See Fleurbaey and Maniquet (2004) for further discussion.

chances that they get admitted into college. This will lead to college admissions designing an admissions policy that will ensure Paul and Richard being admitted with equal probability. There will generally be a large set of policies satisfying this property. The following examples point out two policies that are often discussed and even applied in the political debate that are in this set but do not lead to global equality of opportunity.

Example 1: equalizing local outcomes through Affirmative Action Colleges set the admissions probability according to a modified SAT score, $p_S = a_S + T(b_S)$, to evaluate the different applicants. The transfers $T(b_S)$ are such that Paul and Richard get the same modified score. In particular it gives a positive transfer of $T(b_S^P) = a_S^R - a_S^P = 0.26$ to Paul and a zero transfer to Richard.[11]

As seen in Figure 2, $p_S'^P$ corresponds to p_S^P shifted up until $p_S'^{*P} = p_S^{*R}$. Given college admissions policy Paul and Richard will not change their hours devoted to education and basketball because of the lump sum nature of the transfer, i.e., the rewards to time into either activity are left unchanged. We observe that Paul is better off than Richard under this policy ($U^P = 0.88$ and $U^R = 0.77$).

College admissions accept both Paul and Richard with equal probabilities. NBA recruiters accept Paul with higher probability than Richard because his performance is better—he chooses to devote more time to basketball.[12]

This intervention gives Paul admission to college at a lower "price" in terms of effort than to Richard. Richard devotes more time to it and is equally rewarded with the same chances of being admitted. Paul can then spend more time training basketball at a lower marginal cost. Equality of opportunity is violated since Paul's utility is higher than Richard's.

Example 2: equalizing local outcomes "paternalistically"

[11]Affirmative Action policies have been of this form in some universities and have been formalized in this manner in the literature. Disadvantaged minorities were given a fixed number of extra points in their evaluation procedure. See for example Schotter and Weigelt (1992).

[12]Notice that we did not consider Paul sophisticated enough to know that the college admissions were equalizing outcomes. If that were the case Paul would have spent NO time in school because he'd know that would not affect his education degree relatively to Richard's.

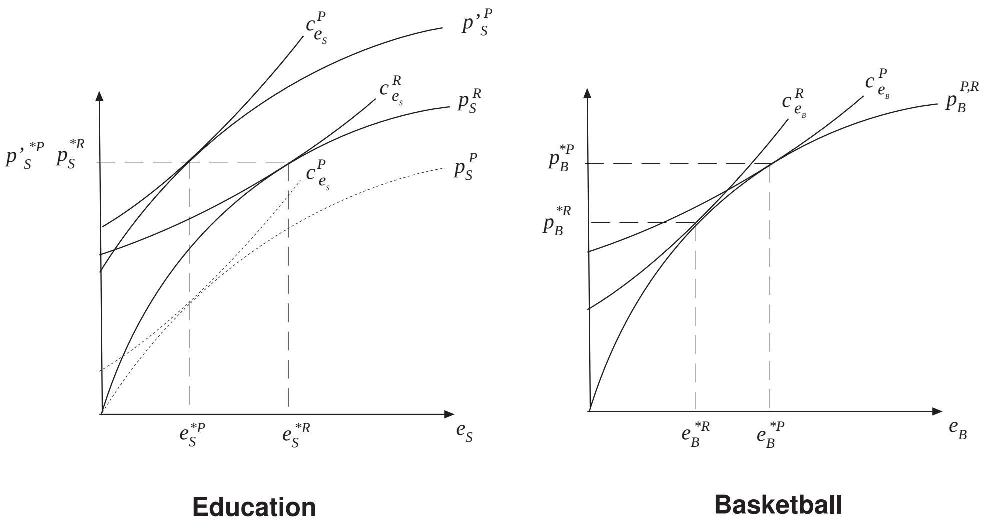

Figure 2: Effect of Affirmative Action consisting of adding a constant number of points to SAT scores

College admissions paternalistically force Paul to study long enough (through having the teacher force him to stay more hours in school) so that he gets the same SAT score as Richard. That is, force Paul to study a certain amount of hours, $e_S^{\prime P}$, so that $a_S(\frac{1}{3}, e_S^{\prime P}, 0.5) = a_S(\frac{1}{3}, e_S^R, 0.7)$. If the market policy is now implemented the probability of being admitted is equalized.

Figure 3 shows that Paul is now forced to study long enough so that he gets the same SAT score as Richard. Paul has to study longer hours, $e_S^{\prime P} = 13$, and therefore faces a larger marginal cost for training basketball that leads him to choose to train less hours, $e_B^{\prime P} = 2.6$. This lowers his utility to $U^P = 0.44$. Paul and Richard are equally likely to get into college, but Richard has higher chances of entering the NBA. Richard gets a utility of $U^R = 0.77$. Paul is now not only worse than Richard, but worse than he was under no equalizing educational opportunity policy!

These two examples illustrate that independence of local outcomes to irrelevant characteristics does not lead to equality of opportunity. In particular a policymaker trying to contribute to the provision of equality of opportunity by making local outcomes independent of irrelevant characteristics may end up generating new and stronger inequalities. In the first example Richard, originally advantaged, ends up with a lower utility than Paul. In the second

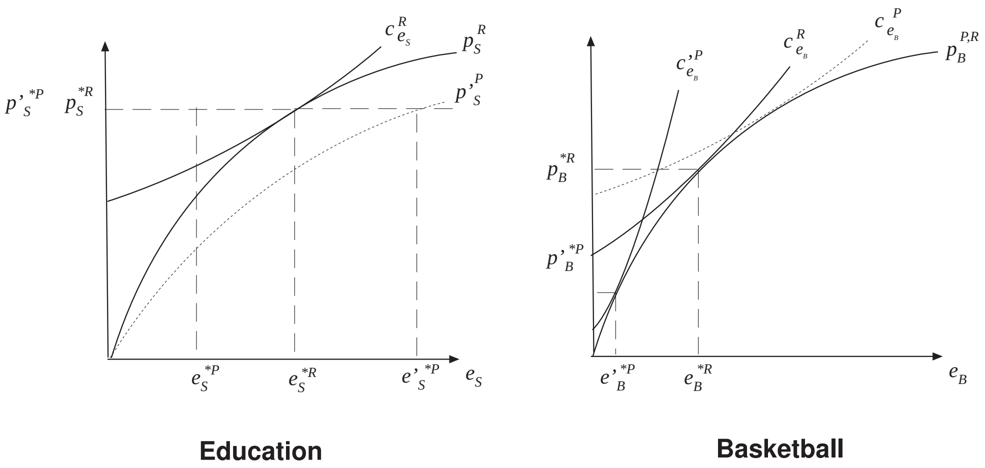

Figure 3: Effect of policy that forces Paul to stay in school long enough to get the same educational outcome as Richard

example, Paul, originally disadvantaged, is made worse off than before the intervention.

This paper shows that the rule that decentralizes equality of opportunity is one that equalizes rewards to effort or time, that is, that it requires the same "price" in terms of effort for each unit of local outcome. In this case, the goal is that for every extra hour of studying, Paul and Richard get the same increase in their probability of being admitted. There will generally be a large set of policies that satisfy this property.[13] The following is just an example of a policy that would accomplish such goal.

Example 3: equalizing rewards to effort
College admissions correct not only for the bias in outcomes but also in rewards to effort and bases his admissions decisions on the following modified SAT score: $a'_S(b_S) = T(b_S) \cdot a_S$ and sets $p_S = a'_S$. This would imply increasing the probability of being admitted proportionally to the SAT achieved. In this case $T^R(b_S^R, b_S^P) = 1$ and $T^P(b_S^R, b_S^P) = \frac{b_S^P}{b_S^R} = \frac{7}{5}$ would make the marginal increase in the probability of being admitted for one more hour

[13] The policymaker will generally have other objectives, for example efficiency, that will help him decide which policy to implement from the set satisfying equality of rewards to effort. We do not address this issue in this example but comment on it in the introduction of the paper.

studied equal for Paul and Richard. As Figure 4 shows, under this policy Paul and Richard's choice sets are equalized, that is, the probabilities of entering college and the NBA for any chosen level of effort are independent of neighborhood education.

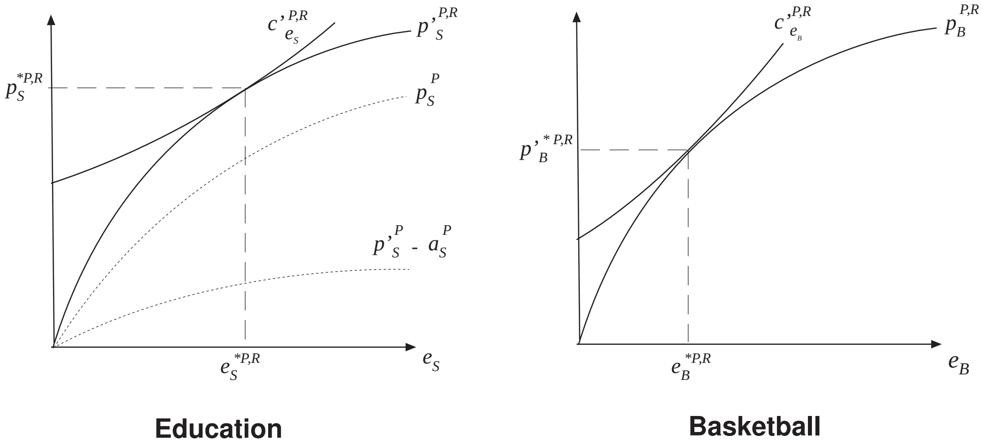

Figure 4: Effect of Affirmative Action consisting of adding a proportion of extra points to SAT scores

The problems Paul and Richard face, the tradeoffs and payoffs from studying and training are exactly the same, and therefore their welfare will be equal. Paul and Richard choose to study the same number of hours, train the same number of hours, get the same chances to enter college and the NBA and and get equal utilities.

3 The Model

This section shows that the results derived from section 2 are no specific to the example, but can be extended to general environments. The model only includes two activities, but the results can be easily extended to n activities.

There is a continuum of individuals in an interval I and two utility-providing activities, $j \in \{1, 2\}$.[14]

[14]Having a continuum of individuals is necessary in the proof of Theorem 2, but is not needed for Theorem 1. See footnote 23 for exposition.

3.1. Individual characteristics and preferences. Each individual $i \in I$ is characterized by two vectors of exogenous characteristics: *irrelevant characteristics*, $b = (b_1, b_2) \in \mathbb{R}^2$, which represent the characteristics that society thinks should not affect individual well-being; and *relevant characteristics*, $r = (r_1, r_2) \in \mathbb{R}^2$, which represent the characteristics that society thinks may affect individual well-being.[15] Individual characteristics are described by the joint distribution $F : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$. Let Ω be the set of conceivable joint distributions F.

Individuals devote *effort* or time to each of the activities, $e = (e_1, e_2) \in \mathbb{R}^2$.[16] An individual with characteristics (b, r) will exert an effort $e = \varphi(b, r)$, where $\varphi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$. Let Δ be the set of conceivable such functions.

Individual utility depends on the outcome consumed for each activity and the effort exerted in each activity:

$$U(x, e) = u(x) - c(e)$$

where u is a strictly increasing function representing the utility out of the outcome vector $x = (x_1, x_2)$ and c is a *non separable* function in e_1 and e_2 that represents the cost of the effort vector, $e = (e_1, e_2)$.[17]

[15] There may be activities for which society believes that no characteristic is irrelevant. This may correspond to spheres in life for which societies believe individuals are fully responsible and therefore no government intervention is required. By having all characteristics in an activity be relevant this model is perfectly compatible with there being a private sphere, which has been defended by political philosophers and economists.

[16] The analysis contains only one dimensional efforts, one for each activity. This may well correspond to similar underlying efforts. For instance, eating healthy food will improve your performance in all activities (some foods will help you more for some activities, but clearly restraining yourself of eating some foods will improve most activities). This could be included in the model by choosing the appropriate cost function. Another aspect of importance would be the efforts being multidimensional. In that case the sufficiency results presented in this section would clearly hold. As for the necessity part some modifications in the modeling would need to be included, which would not incorporate any additional intuitions but would complicate the analysis.

[17] The analysis can be extended to the case where relevant characteristics in the preferences are included, that is, for the case where $U(t, x, e) = u(t, x) - c(t, e)$, where t is a relevant characteristic. The model can also be extended to the case where the asymmetry in preferences is an irrelevant characteristic but linked to a specific activity. For example, if some individuals get less utility out of education because of their parent's background and influence than. The results in this paper apply equivalently to these extended frameworks. More general asymmetries in preferences could be included but in some cases defining what policy maker is responsible for compensating it in the decentralized case will be less natural.

3.2. Production. Outcome produced in each activity depends only on the characteristics of the specific activity. That is, outcome in activity j will depend on the marginal distribution of characteristics (b_j, r_j), F_j, and the efforts exerted, $e_j \in \varphi_j(b_j, r_j)$, where $\varphi_j(\hat{b_j}, \hat{r_j}) = \{\hat{e_j} | \varphi(\hat{b_j}, b_{-j}, \hat{r_j}, r_{-j}) = (\hat{e_j}, e_{-j})\}$. There are no externalities in production between these two activities.

The outcome production function of individuals with characteristics and effort (b, r, e) can be represented by a vector of strictly increasing functions:

$$a(b, r, e; F, \varphi) = (a_1(b_1, r_1, e_1; F_1, \varphi_1), a_2(b_2, r_2, e_2; F_2, \varphi_2)) \in I\!R^2$$

where $a_j : I\!R^3 \times \Omega_j \times \Delta_j \to I\!R$ represents the outcome produced by individual with characteristics and effort (b_j, r_j, e_j) in activity j when living in society with characteristics described by F_j and exerting efforts according to φ_j. Therefore b_j and r_j are relevant and irrelevant characteristics potentially affecting the production in activity j.

Definition 1 (social mechanism): A *social mechanism* μ is a mapping that assigns for every distribution $F \in \Omega$ and effort function $\varphi \in \Delta$ an outcome vector to each individual:

$$\mu : I\!R^3 \times I\!R^3 \times \Omega \times \Delta \to I\!R \times I\!R$$

$$\mu \equiv (\mu_1, \mu_2) : (b, r, e; F, \varphi) \to (x_1, x_2)$$

$$s.t. \int_{\boldsymbol{R}^2 \times \boldsymbol{R}^2} \mu_j(b, r, e; F, \varphi)\, dF \leq \int_{\boldsymbol{R}^2 \times \boldsymbol{R}^2} a_j(b_j, r_j, e_j; F_j, \varphi_j)\, dF_j \text{ for } j = 1, 2$$

That is, x_j is the outcome that the individual with characteristics (b, r) and effort e gets out of activity j when the distribution of individual characteristics is F and the efforts exerted are described by φ. For example, the social mechanism could assign to each individual what he produces, in which case $\mu(b, r, e; F, \varphi) = a(b, r, e; F, \varphi)$.

3.3. Individual choice and equilibrium. Given a social mechanism, an individual's effort vector will maximize his utility, given the efforts of others:

$$e^*(b, r; F, \varphi) \equiv \arg\max_{e \in \boldsymbol{R}^2} u(\mu(b, r, e; F, \varphi)) - c(e)$$

φ^* is an equilibrium if for all individuals $(b, r) \in \text{supp}F$, $\varphi^*(b, r) = e^*(b, r; F, \varphi^*)$. Every individual is behaving optimally given what society is doing.

Individual utility in equilibrium is denoted by:

$$u^*(b,r;F,\varphi^*) = u(\mu(b,r,\varphi^*(b,r);F,\varphi^*)) - c(\varphi^*(b,r))$$

3.4. The Global Objective. We now define the objective of a social planner wanting to provide equality of opportunity. As mentioned in the introduction, this is a simple formalization of equality of opportunity, inspired by the Rawlsian tradition in political philosophy and formalized and discussed in Roemer (1998).

Definition 2 (equality of opportunity): Let (b^i, r^i, e^i) denote the characteristics of individual $i \in I$. A social mechanism μ provides *equality of opportunity* if for all $F \in \Omega$ and $\varphi^* \in \Delta$:

$$\forall i,k \in I,\ \ r^i = r^k \ \Rightarrow\ u^*(b^i,r^i;F,\varphi^*) = u^*(b^k,r^k;F,\varphi^*)$$

A social mechanism provides equality of opportunity if the welfare achieved by each individual varies with his relevant characteristics but not with his irrelevant characteristics. Here, the planner is concerned about indirect utility, and therefore should not be concerned about the effort exerted by individuals, but only about final welfare. This is consistent with the fact that effort as a choice variable has not been emphasized in the political philosophy literature.

If one policymaker had all information he could act as a social planner and design the social mechanism that guarantees equality of opportunity. But if information and decision making is dispersed the provision of equality of opportunity will be decentralized. In what follows we describe how the information and decision making is distributed in this model.

3.5. The local problem. There is one policymaker responsible for assigning outcomes for each activity j, who observes individual characteristics and effort exerted concerning activity j, and does not observe characteristics affecting the other activity. This implies that the local planner for activity j only knows the marginal distribution F_j, and φ_j. Given this information he chooses how to allocate outcomes for activity j.

Definition 3 (local mechanism): A *local mechanism* is a mapping ϕ_j that assigns to every marginal distribution F_j and φ_j a local outcome vector to each individual:

$$\phi_j : I\!R^3 \times \Omega_j \times \Delta_j \rightarrow I\!R$$

$$\phi_j : (b_j, r_j, e_j; F_j, \varphi_j) \to x_j$$

$$\text{s.t.} \int_{\boldsymbol{R}\times\boldsymbol{R}} \phi_j(b_j, r_j, e_j; F_j, \varphi_j)\, dF_j \le \int_{\boldsymbol{R}\times\boldsymbol{R}} a_j(b_j, r_j, e_j; F_j, \varphi_j)\, dF_j$$

A local mechanism assigns to every individual an outcome for a specific activity conditional on his characteristics and effort choices specific to that activity, and potentially on other individuals' characteristics and choices.

A pair of local mechanisms generates a social mechanism such that $\mu = (\mu_1, \mu_2)(b, r, e; F, \varphi) = (\phi_1(b_1, r_1, e_1; F_1, \varphi_1), \phi_2(b_2, r_2, e_2; F_2, \varphi_2))$. Instead of the mapping μ assigning an outcome vector to each individual we have (ϕ_1, ϕ_2), with ϕ_j assigning the outcome for activity j. Therefore, a pair of local mechanisms, $\{\phi_1, \phi_2\}$, provides *equality of opportunity* if the induced social mechanism $\mu = (\phi_1, \phi_2)$ provides equality of opportunity. Equivalently, we say that a pair of local mechanisms decentralizes equality of opportunity if the resulting social mechanism provides equality of opportunity.

We now define the rule that decentralizes the provision of equality of opportunity. In contrast to the equality of opportunity requirement, it introduces effort as an important variable that needs to be explicitly controlled for.

Definition 4 (local equality of rewards to effort): Let suppF_j be the support of F_j. And let $\text{Img}\varphi_j = \{e_j | \exists (b_j, r_j) \in F_j \text{ s.t. } \varphi_j(b_j, r_j) = e_j\}$, that is, the image of φ_j. A local mechanism provides *local equality of rewards to effort* if for all F_j and φ_j^*:

$$\forall e_j \in \text{Img}\varphi_j^* \;\; \forall i, k \in I$$

$$r_j^i = r_j^k = r_j \text{ and } e_j^i = e_j^k = e_j \;\Rightarrow\; \phi_j(b_j^i, r_j, e_j; F_j, \varphi_j^*) = \phi_j(b_j^k, r_j, e_j; F_j, \varphi_j^*)$$

Equivalently, if there exists w_j such that for all F_j and φ_j^*:

$$\forall e_j \in \text{Img}\varphi_j^* \;\; \forall b_j, r_j \in \text{supp}F_j \quad \phi_j(b_j, r_j, e_j; F_j, \varphi_j^*) = w_j(r_j, e_j; F_j, \varphi_j^*)$$

A local mechanism providing local equality of rewards to effort allows rewards to effort only to depend on relevant characteristics. In other words, two individuals with different irrelevant characteristics but the same relevant characteristics should get the same *if* they choose to do the same effort.[18]

[18]The rewards have to be the same only for the levels of effort that can be potentially

Theorem 1 *If the pair of local mechanisms (ϕ_1, ϕ_2) provides local equality of rewards to effort then it decentralizes equality of opportunity.*

Next we introduce conditions under which equalizing rewards to effort is not only sufficient but also necessary. We introduce notation that allows us to describe the set of efforts that the policymaker in activity j can expect an individual with characteristics (b_j, r_j) to choose in a given equilibrium. Given a pair of local mechanisms ϕ_1, ϕ_2 and the marginal distribution of characteristics in activity j and efforts in equilibrium, φ_j^*, let $E_j(b_j, r_j; F_j, \varphi_j^*)$ be the set of effort levels in activity j that an individual with characteristics (b_j, r_j) chooses for some $(b_{-j}, r_{-j}) \in \text{supp}F_{-j}$ under some distribution of characteristics and efforts in the other activity, $F_{-j} \in \Omega_{-j}$ and $\varphi_{-j}^* \in \Delta_{-j}$. Formally,

$$E_1(\hat{b_1}, \hat{r_1}; \hat{F_1}, \hat{\varphi_1^*}) = \{e_1 | \exists F_2 \in \Omega_2, \varphi_2^* \in \Delta_2 \text{ and } (b_2, r_2) \in \text{supp}F_2$$

$$\text{s.t. } e_1 = e_1^*(\hat{b_1}, b_2, \hat{r_1}, r_2; \hat{F_1}, F_2, \hat{\varphi_1^*}, \varphi_2^*)\}$$

$$E_2(\hat{b_2}, \hat{r_2}; \hat{F_2}, \hat{\varphi_2^*}) = \{e_2 | \exists F_1 \in \Omega_1, \varphi_1^* \in \Delta_1 \text{ and } (b_1, r_1) \in \text{supp}F_1$$

$$\text{s.t. } e_2 = e_2^*(b_1, \hat{b_2}, r_1, \hat{r_2}; F_1, \hat{F_2}, \varphi_1^*, \hat{\varphi_2^*})\}$$

The following condition on the mechanisms and the environment will force that policymakers equalize rewards to effort in order to provide equality of opportunity in the aggregate.

Definition 5 (richness): The mechanisms and the environment are *rich* if for all $F_j \in \Omega_j$, $\text{Img}\varphi_j^* \subseteq E_j(b_j, r_j; F_j, \varphi_j^*)$ for all $(b_j, r_j) \in \text{supp}F_j$.

The mechanisms and the environment are rich whenever the set of efforts observed by the planner in equilibrium, e_j^*, can be chosen by all individuals with $(b_j, r_j) \in \text{supp}F_j$ under some circumstance in the other activity $-j$. For example, if schools observe that individuals put from 1 to 10 hours of study, then there must be individuals of any characteristic in school willing to choose that effort for some circumstance outside of school. Therefore, whether you are in a good or bad school there should be circumstances outside

chosen by some individual ($e_j \in \text{Img}\varphi_j^*$). Whatever happens for levels of effort that are never chosen is not important. But for policy design the distinction is small and economically not important (since the difference is on levels of effort that are never chosen) so it simpler and practically equivalent to focus on making rewards to effort independent of irrelevant characteristics for all levels of effort.

of school that make you interested in any of the 1 to 10 hours of study. This does not mean that individuals with different characteristics have the same probability of choosing a given effort, but that there is an instance where every individual, as observed by the policymaker, is interested in choosing any of the efforts in equilibrium. This means that the policymaker cannot not rule out the possibility of any individual exerting a certain level of effort.[19]This is a condition on both the environment and the mechanisms since it requires enough heterogeneity in individual's characteristics and the outputs they get with those characteristics in each activity.

If richness is not satisfied then equalizing rewards to effort may not be a necessary condition for equality of opportunity to be decentralized. The following two examples illustrate how the violation of richness may allow for decentralization without equalizing rewards to effort. Example 1 shows a case where the mechanisms forces richness to be violated and example 2 one in which the environment does so.

Example 1 Let $u(x_1, x_2, e) = x_1 + x_2 - (e_1 + e_2)^2$. Suppose there are no relevant characteristics and let $a_1(b_1, e_1; F_1, \varphi_1) = e_1 b_1$, $a_2(b_2, e_2; F_2, \varphi_2) = K + e_2$. Suppose policymaker 2 imposes a local mechanism such that $\varphi_2(b_2, e_2; F_2, \varphi_2) = K$. Let policymaker 1 set $\phi_1(b_1, e_1; F_1, \varphi_1) = e_1 b_1 + T(b)$, where $T(b) = \int \frac{b^2}{4} dF_2 - \frac{b^2}{4}$. Individuals will choose $e_1 = \frac{b_1}{2}$ and $e_2 = 0$. Final welfare is constant for all individuals, $u^* = K + \int \frac{b^2}{4} dF_2$.

Richness is violated in this example because if $b_1^i \neq b_1^k$ then $e_1^*(b_1^i) \neq e_1^*(b_1^k)$ independently of what happens in the other activity. An individual with b_1^k will never choose $e_1^*(b_1^i)$. Policymaker 2 is not equalizing rewards to effort but equality of opportunity is provided.

Example 2 Let $u(x_1, x_2, e) = x_1 + x_2 - (e_1 + e_2)^2$. Let $b_1, b_2 \in (0, 1)$. Suppose there is perfect correlation between b_1 and b_2, say $b_1 = b_2$. Let $a_1(b_1, e_1; F_1, \varphi_1) = K + e_1$ and $a_2(b_2, e_2; F_2, \varphi_2) = K + e_2$. If each policymaker sets respectively $\phi_1 = K + e_1$ if $b_1 \leq \bar{b}$ and $\phi_1 = 0$ otherwise, and $\phi_2 = K + e_2$ if $b_2 > \bar{b}$ and $\phi_2 = 0$ otherwise. Each individual will specialize, if $b_1 \leq \bar{b}$ in activity 1 and in activity 2 otherwise. Final welfare will be

[19]One circumstance where this property would never hold, independently of the space of individual characteristics or distributions is in the case where the utility and the cost function are separable in activity 1 and 2, since then the choice e_j is just a function of (b_j, r_j). In that case it is clearly not necessary to equalize rewards to effort: equalizing partial utilities would be sufficient. But this assumes away any interaction between the cost of time devoted to different activities.

$u^*(b) = K + \frac{1}{4}$. Here, the environment is forcing richness to be violated (if $b_1 > \bar{b}$ the individual will never exert effort in activity 1) and decentralizing equality of opportunity does not require equalizing rewards to effort.

Overall, richness requires that there be little correlation between information in the different sectors in order to limit the capacity of a local planner to behave as a social planner.

We now present the second result of the paper. It states that equalizing rewards to effort is not only sufficient but necessary if richness of the mechanisms and the environment holds. We also impose differentiability on the mechanisms, which simplifies the proof of the theorem. This result shows that the objective of independent policymakers wanting to contribute to the provision of equality of opportunity should be equalizing rewards to effort.

Theorem 2: *If the pair of mechanisms and the environment are rich, and if the pair of local mechanisms* (ϕ_1, ϕ_2) *is differentiable, then it decentralizes equality of opportunity only if it provides local equality of rewards to effort.*

4 Concluding remarks

The allocation of rights and resources in a society is done in a decentralized manner, with each policy maker deciding how to allocate resources based on partial information. This paper interprets local distributive justice as decentralized global distributive justice. Thinking of local justice as decentralized global justice allows us to evaluate local interventions on the basis of their success in achieving global justice.

We analyze what local justice requires if we want to achieve the global justice goal of providing equality of opportunity. We show that under some conditions a collection of local mechanisms decentralizes the provision of equality of opportunity if and only if each mechanism provides local equality of rewards to effort. This implies that if a society is concerned about providing equality of opportunity, then its institutions and policymakers should be concerned about equalizing rewards to effort or equivalently giving the same output to individuals who exert the same amount of effort. In other words, the incentives individuals are given should not depend on irrelevant characteristics.

The result of the paper may indirectly provide a rationale for the difference between the distributive justice notions that political philosophers propose, i.e., normative distributive justice, and those that people express when voting for welfare programs or when stating their opinions in surveys or experiments, i.e., positive notions of distributive justice. Political philosophers overlook the role of effort because it is endogenous and ultimately a function of exogenous characteristics. Their debate is centered on what exogenous characteristics should be irrelevant.[20] Individuals, when asked in surveys or through experiments, also differ in which characteristics they consider to be irrelevant. However, they mostly agree that effort exerted of those on assistance programs is crucial to evaluate the fairness of a given allocation. For example, Bowles and Gintis (1998) report that "[T]he welfare state is in trouble not because selfishness is rampant (it is not), but because many egalitarian programs no longer evoke, and sometimes offend, deeply held notions of fairness [. . .] In a 1995 CBS/NYT survey, for example, 89% supported and mandated work requirement for those on welfare."[21] Yet, Konow (2003) in a survey on positive distributive justice states "[a] common view is that differences owing to birth, luck and choice are all unfair and that only differences attributable to effort are fair. A common finding (and claim) among social scientists is that individual effort affects the perceived fairness of allocations". We believe this difference between political philosophers and individuals in society may be the result of philosophers examining global distributive justice while individuals always being asked in the frame of a local distributive justice. If global equality of opportunity is desirable, then, even though we can ignore effort—or more generally endogenous choice variables— in the global distributive justice problem, we must control it in the local distributive justice problem. The difference between an individual in a survey and a philosopher then is the framework in which they express their opinions. There is not necessarily an inconsistency between what the two express when talking about justice.

[20]An example of a discussion on rewarding effort and justice is in Sen's article "Merit and Justice", in *Meritocracy and economic inequality*, where he states that "[A]n incentive argument is entirely "instrumental" and does not lead to any notion of intrinsic "desert". If paying a person more induces him or her to produce more desirable results, then an incentive argument may exist for that person's pay being greater. This is an instrumental and contingent justification—it does not assert that the person intrinsically "deserves" to get more[. . .]" Actions are rewarded for what they help to bring about but the rewarding is not valued in itself.

[21]Bowles and Gintis argue that the reason people care about the effort recipients of welfare exert is due to strong reciprocity. "[H]*omo reciprocans* is not committed to the abstract goal of equal outcomes but to a rough balancing out of burdens and rewards."

APPENDIX

Proof of Theorem 1: Every individual maximizes his utility given the social mechanism generated by the pair of local mechanisms and given the other individuals' choices. Since the local mechanisms provide local equality of rewards to effort we know that the social mechanism and therefore the choice problem faced by the individual has the following form:

$$\max_{e \in \boldsymbol{R}^2} u((w_1(r_1, e_1; F_1, \varphi_1^*), w_2(r_2, e_2; F_2, \varphi_2^*))) - c(e)$$

The effort level chosen will not depend on b since the utility function does not depend on b. The problem that two identical individuals face in equilibrium is identical. Even if the effort chosen is different the utility is still the same since in equilibrium for every effort level they get the same payoff. If one obtained a higher payoff that would contradict individuals behaving optimally. Individual final welfare is then a function only of the relevant characteristics r:

$$u^*(b, r) = v(r) = u((w_1(r_1, e_1^*(r)), w_2(r_2, e_2^*(r))) - c(e_1^*(r), e_2^*(r))$$

□

Proof of Theorem 2: After choosing the optimal level of effort the individual will have a utility of:

$$u^*(b, r) = u(\phi_1(b_1, r_1, e_1^*; F_1, \varphi_1^*), \phi_2(b_2, r_2, e_2^*; F_2, \varphi_2^*)) - c(e_1^*, e_2^*)$$

If the pair of local mechanisms provides equality of opportunity then the indirect utility needs to remain unchanged with respect to b, which implies that the following is true:

$$\forall F \in \Omega \; \forall b, r \in \text{supp} F \quad \frac{\partial u^*(b, r)}{\partial b_1} = \frac{\partial u^*(b, r)}{\partial b_2} = 0$$

Note that since each individual has zero mass, his actions will not change the distribution of efforts in equilibrium.[22] Therefore, differentiating the indirect utility with respect to b_j we obtain the following expressions:

[22] Continuity simplifies doing comparative statics on welfare when one individual's characteristics change. We analyze the effect on an individual's welfare of changing his characteristics within the same equilibrium. If the model had a discrete number of individuals, doing this comparative statics would involve understanding how the equilibrium correspondence changes with a change in one individual's characteristics, which could unnecessarily complicate the model and the analysis.

$$\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_1}(\frac{\partial \phi_1^*}{\partial b_1}+\frac{\partial \phi_1^*}{\partial e_1}\frac{\partial e_1^*}{\partial b_1})+\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_2}\frac{\partial \phi_2^*}{\partial e_2}\frac{\partial e_2^*}{\partial b_1}-\frac{\partial c(e^*)}{\partial e_1}\frac{\partial e_1^*}{\partial b_1}-\frac{\partial c(e^*)}{\partial e_2}\frac{\partial e_2^*}{\partial b_1}=0$$

Rewriting we obtain,

$$\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_1}\frac{\partial \phi_1^*}{\partial b_1}+(\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_1}\frac{\partial \phi_1^*}{\partial e_1}-\frac{\partial c(e^*)}{\partial e_1})\frac{\partial e_1^*}{\partial b_1}+(\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_2}\frac{\partial \phi_2^*}{\partial e_2}-\frac{\partial c(e^*)}{\partial e_2})\frac{\partial e_2^*}{\partial b_1}=0$$

Using the first order conditions of the individual we know that $\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_j}\frac{\partial \phi_j^*}{\partial e_j}-\frac{\partial c(e^*)}{\partial e_j}=0$ for $j=1,2$, which implies that $\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_1}\frac{\partial \phi_1^*}{\partial b_1}=0$. Similarly we can show that $\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_2}\frac{\partial \phi_2^*}{\partial b_2}=0$.

Therefore, since u is strictly increasing we have that:

$$\frac{\partial u(\phi_1^*,\phi_2^*)}{\partial \phi_j}\frac{\partial \phi_j^*}{\partial b_j}=0\Rightarrow \frac{\partial \phi_j}{\partial b_j}(b_j,r_j,e_j^*(b,r;F,\varphi^*))=0 \qquad (1)$$

Take $\bar{e_j}\in \text{Img}\varphi_j^*$. Let $(\bar{b_j},\bar{r_j})\in$ supp F_j be an individual optimally choosing $\bar{e_j}$. By (1) we know that for any $(b_j,\bar{r_j})\in$ supp F_j in an open ball around $(\bar{b_j},\bar{r_j})$ the local outcome given when effort is $\bar{e}$ remains unchanged. But we want this to be true for all $(b_j,\bar{r_j})\in$ supp F_j. By richness we know that for all $(b_j,r_j)\in$ supp F_j $\text{Img}\varphi^*\subseteq E_j(b_j,r_j;F_j,\varphi_j^*)$, which implies that for all $(b_j,\bar{r_j})\in$ supp F_j there exists $\hat{F_{-j}}$ and $(\hat{b_{-j}},\hat{r_{-j}})\in$ supp F_{-j} such that $e_j^*(b_j,\bar{r_j},\hat{b_{-j}},\hat{r_{-j}};F_j,\hat{F_{-j}},\varphi_j^*,\hat{\varphi_j^*})=\bar{e_j}$. Therefore local outcome assigned to effort $\bar{e_j}$ should remain unchanged around $(b_j,\bar{r_j})$ for every b_j, and therefore local outcome should be equal when b_j changes. This shows that the local mechanisms provides local equality of rewards to effort.

□

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