Oligopoly and consistent conjectural variations

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This article examines "consistent" conjectural variations in an oligopoly model with a homogeneous product. A conjectural variation is consistent if it is equivalent to the optimal response of the other firms at the equilibrium defined by that conjecture. When the number of firms is fixed, we find that competitive behavior is consistent when marginal costs are constant, but that when marginal costs are rising, the consistent conjectural variation will be between competitive and Cournot behavior. Finally, if we allow free entry and redefine consistency to account for such, then only competitive behavior will be consistent.

1. Introduction

■ The traditional criticism of industry models using conjectural variations is that each firm's conjecture about the output response of the other firms would not be confirmed if such a firm actually altered its output level from the equilibrium. As a result, there has recently been considerable interest in oligopoly models with "consistent" conjectural variations. A conjectural variation is consistent if it is equivalent to the optimal response of the other firms at the equilibrium defined by that conjecture.[1] In recent published work, Laitner (1980) has examined rationality in a duopoly model with a homogeneous product, and Bresnahan (1981) has considered consistency in a duopoly model with differentiated products. Consistency can be easily described in the duopoly context. Each firm's first-order condition defines its profit-maximizing output as a reaction function on (1) the output of the other firm and (2) the conjectural variation about the other firm's response. Thus, a conjectural variation by one firm about the other firm's response is consistent if it is equivalent to the derivative of the other firm's reaction function with respect to the first firm's output at equilibrium. Bramness (1979), Ulph (1981), and Geroski (1981a) have examined additional aspects of the duopoly case.[2] Finally, Capozza and Van Order (1980), Ulph (1980), Brander (1980), Kamien and Schwartz (1981),

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[1] The notion of consistency in oligopoly models has roots in other literatures. Originating from Muth (1961), there is the voluminous literature on rational expectations primarily focusing on macroeconomic models. In the general equilibrium literature, there is related work by Hahn (1978). In the game theory literature, Marschak and Selten (1978) define strategies in an inertia supergame which have a close correspondence to the static notion of consistency in this article. Finally, Porter and Spence (1978) define consistent expectations with respect to capacity expansion in a dynamic oligopoly model.

[2] All three of these articles focus on duopoly with differentiated products. Bramness (1979) defines the consistent conjectural variation in general. Ulph (1981) examines consistency in a linear model in which the two firms may face differing demand and cost conditions. Geroski (1981a) examines entry when the postentry equilibrium is consistent.

Holt (1980), and Farley (1980) each discuss some notion of consistency in models with more than two firms.[3]

In this article we provide a general treatment of consistent conjectural variations in an oligopoly model with a homogeneous product. In Section 2, we define the notion of consistency when there are more than two, but a fixed number, of firms. In Section 3, we examine what the consistent conjectural variation would be for various demand and cost functions. In terms of the structure-conduct-performance paradigm,[4] the model of Section 3 makes conduct (a consistent conjectural variation) endogenous with performance (output and price), but leaves structure (the number of firms) as yet exogenous. So in Section 4, we drop the assumption that the number of firms is fixed and allow free entry to determine the number of firms. As a result, consistency may also be redefined to take into account entry and exit. With this notion of "full" consistency, we obtain the striking result that the competitive equilibrium is the only one in which structure, conduct, and performance are simultaneously endogenous.

2. The model

■ Consider a homogeneous product X with an inverse demand function $P(X)$ and let there be m firms producing this product with a cost function $C(X_j)$. The profits of the jth firm are

$$\Pi_j(X_j) = P(X_j + \sum_{i \neq j} X_i) \cdot X_j - C(X_j). \tag{1}$$

In choosing X_j so as to maximize profits, firms form a conjectural variation about the combined output response of the other firms to a unit change in their own output level:

$$\frac{d(\sum_{i \neq j} X_i)}{dX_j} = \delta, \qquad \text{where} \qquad -1 < \delta < m - 1. \tag{2}$$

The conjectural variation is assumed to be the same for each firm and independent of both the output of the other firms and the number of other firms. An important advantage of an industry conjectural variation over conjectural variations on individual firms is that it requires no distinction between firms which are operating and mere potential entrants. The firm's perceived rate of change of profits can now be expressed as:

$$\frac{d\Pi_j}{dX_j} = P(X_j + \sum_{i \neq j} X_i) + (1 + \delta) \cdot P'(X_j + \sum_{i \neq j} X_i) \cdot X_j - C'(X_j). \tag{3}$$

In equilibrium, each firm perceives no incentive to change its output level. Because all firms are identical, we confine our attention to the symmetric equilibrium,

$$P(X) + (1 + \delta) \cdot P'(X) \cdot (X/m) - C'(X/m) = 0, \tag{4}$$

which defines the equilibrium industry output $X(m; \delta)$ as a function of the number of firms m and the conjectural variation δ.

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[3] Capozza and Van Order (1980) calculate a consistent price conjecture in a model of a price-setting firm with two neighbors. Holt (1980) also calculates a consistent price conjecture, but in a model with linearly differentiated products and constant marginal costs. Ulph (1980) examines a model with linearly differentiated products and quadratic costs. Brander (1980) and Farley (1980) both consider a general homogeneous product model, but their definitions of consistency fail to rationalize the full equilibrium response of the rest of the industry. Finally, Kamien and Schwartz discuss both homogeneous and differentiated product models with constant marginal costs, and in their revised version they employ the same definition of consistency used here.

[4] This paradigm was popularized by Bain in *Industrial Organization* (1959) and extensively employed by Scherer in *Industrial Market Structure and Economic Performance* (1970).

This conjectural variation model was introduced by Bowley (1924), and it explicitly incorporates the special cases of Cournot (1838), competitive, and collusive behavior.[5] If $\delta = 0$, the equilibrium defined by (4) is the familiar Cournot equilibrium (Nash equilibrium in quantities). If $\delta = -1$, each firm expects the rest of the industry to absorb exactly its output expansion by a corresponding output reduction. This means that each firm is a price-taker and the equilibrium is competitive. At the other end of the spectrum when $\delta = m - 1$, the equilibrium is collusive in that firms behave so as to maximize joint profits, given that there are m firms. Thus, any conjectural variation between -1 and $m - 1$ is potentially reasonable, and the goal is to narrow this range by imposing the condition that the conjectural variation must be locally equivalent to the actual response of the other firms.

To construct the consistent conjectural variation, we need to characterize what the equilibrium response of the rest of the industry would be to a change in the output of the jth firm. Consider the following equilibrium. Given X_j, the output of the jth firm, let X_o be the symmetric equilibrium output for the other $(m - 1)$ firms behaving under a conjectural variation δ. The equilibrium condition is similar to (4),

$$P(X_j + X_o) + (1 + \delta)\cdot P'(X_j + X_o)\cdot [X_o/(m - 1)] - C'(X_o/(m - 1)) = 0, \tag{5}$$

and defines the output of the other firms $X_o(X_j; m, \delta)$ as a function of the output of the jth firm.[6] We can now examine the actual equilibrium response of the other firms to a one unit change in the output of the jth firm evaluated at the symmetric equilibrium for all firms:

$$\frac{dX_o}{dX_j} = -\frac{P' + (1 + \delta)\cdot(X/m)\cdot P''}{[(m + \delta)/(m - 1)]\cdot P' + (1 + \delta)\cdot(X/m)\cdot P'' - [1/(m - 1)]\cdot C''(X/m)}. \tag{6}$$

For the conjectural variation δ to be consistent it must be equivalent to this local equilibrium response of the other firms at the overall symmetric equilibrium. Thus, since the equilibrium response is itself a function of the conjectural variation, consistent conjectural variations are the fixed points of (6):

$$\frac{dX_o\left(\dfrac{X}{m}; m, \delta\right)}{dX_j} = \delta. \tag{7}$$

A conjectural variation which satisfies condition (7) is consistent.[7] It would depend upon the number of firms m, but there may be more than one consistent conjecture for any given number of firms. Figure 1 illustrates two examples of (7), one in which there is a unique consistent conjectural variation between -1 and 0, and the other in which there are two consistent conjectural variations, -1 and one which is positive. Using (6), the consistency condition (7) can be written in terms of X, δ, and m as either:

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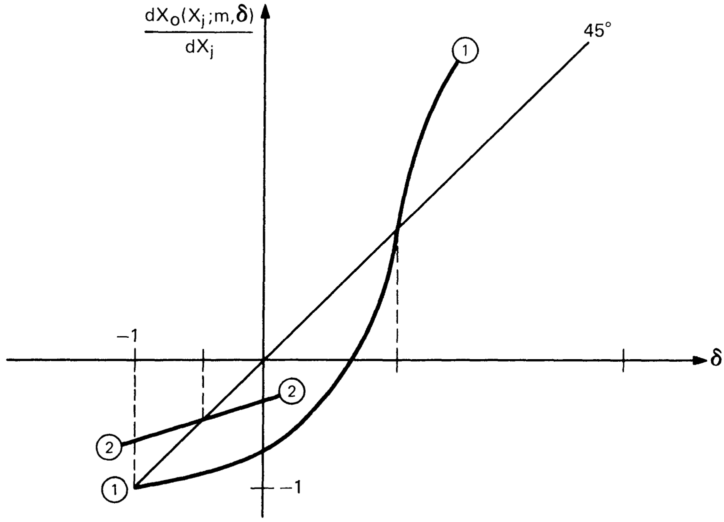
[5] See Perry (1979) or Seade (1980) for basic treatments of the conjectural variation model.

[6] Since δ is an industry conjectural variation rather than a firm conjectural variation, there is no logical inconsistency in defining the equilibrium of the rest of the industry in terms of both X_j and δ. The other firms need not have specific conjectures about the jth firm, or any other firm for that matter.

[7] The problem with the definitions of consistency in Brander (1980), Farley (1980), and the earlier version of Kamien and Schwartz (1981) is that they construct the industry response by summing over $i \neq j$ the direct effects of the change in X_j on the ith firm's response. This fails to take into account the sequence of indirect responses among $i \neq j$ which is fully incorporated into the equilibrium condition (5) defining $X_o(X_j; m, \delta)$. Thus, in equilibrium, a change in X_j would not confirm their definitions of a consistent conjectural variation. This problem does not arise in the duopoly case.

FIGURE 1

THE RESPONSE FUNCTION



$$\frac{(1+\delta)(m-1+\delta)}{m-1}\cdot P'(X)+\frac{(1+\delta)^2}{m}\cdot X\cdot P''(X)-\frac{\delta}{m-1}\cdot C''(X/m)=0, \quad (8a)$$

or

$$\frac{\delta}{m-1}\cdot\left[(1+\delta)\cdot P'(X)-C''\left(\frac{X}{m}\right)\right]+$$
$$(1+\delta)[P'(X)+(1+\delta)\cdot(X/m)\cdot P''(X)]=0. \quad (8b)$$

Expressions (8a) and (8b) will be useful in Section 3 for examining the type of consistent behavior which can arise under differing demand and cost conditions.

The conjectural variation model is a simple static representation of the potentially complex dynamics of an oligopoly, and consistency as defined by (8) is the simplest adequate static condition for rational behavior in such a model. Thus, the value of this notion of consistency in this oligopoly model rests upon the extent to which the results in Section 3 are reasonable and enlightening from a theoretical viewpoint or testable and refutable from an empirical viewpoint.[8]

A *consistent equilibrium* may be defined by combining the equilibrium condition (4) with the consistency condition (8). Such an equilibrium simultaneously determines industry output and behavior, given the number of firms. The output of each firm is such that no firm perceives an incentive to change its output based upon a conjectural variation which is a locally correct assessment of the response that would arise from the other firms. But before considering the results in Section 3 we must first outline the second-order and stability conditions for this system.

For each firm to be at a local profit-maximum, we require $d^2\Pi_j/dX_j^2 \le 0$ when evaluated at the symmetric equilibrium:

$$2\cdot(1+\delta)\cdot P'+(1+\delta)^2\cdot(X/m)\cdot P''-C''\le 0. \quad (9)$$

[8] Several authors have attempted to estimate conjectural variations for particular industries. See Iwata (1974) with respect to the Japanese flat glass industry, Anderson and Kraus (1978) with respect to the U.S. airline industry, and Gollop and Roberts (1978) and Geroski (1981b) with respect to the U.S. coffee roasting industry.

In addition, we require that the equilibrium output defined by (4) be unique and stable for given δ. If industry output is below (above) the symmetric equilibrium, firms will perceive an incentive to expand (contract):

$$(1 + \delta + m)\cdot P' + (1 + \delta)\cdot X \cdot P'' - C'' < 0. \tag{10}$$

The corresponding condition on the equilibrium with $(m - 1)$ firms, given X_j, requires that the denominator of expression (6) for dX_o/dX_j be negative. Under (10), multiple consistent equilibria arise solely because there may be more than one consistent conjectural variation.

For any solution to (4) and (8) to be a consistent equilibrium, firms must also earn nonnegative profits at the equilibrium firm output $X(m; \delta)$ under the consistent conjectural variation δ:

$$P(X(m; \delta))\cdot\left[\frac{X(m; \delta)}{m}\right] - C\left[\frac{X(m; \delta)}{m}\right] \geq 0. \tag{11}$$

If profits were negative (and no costs were sunk), firms would shut down and $X(m; \delta)/m$ would not be an equilibrium output.

Finally, we may also be interested in the stability of the conjectural variation. Stability requires that the response of the other firms is greater (less) than the conjectural variation when the conjecture is below (above) that which is consistent, i.e., $dX_o/dX_j \gtreqless \delta$ as $\delta \lesseqgtr \delta(m)$. Given the equilibrium output, stability of the conjectural variation can be obtained by requiring that the derivative of (8) with respect to δ be negative:

$$(2\delta + m)\cdot P' + 2(1 + \delta)\cdot(m - 1)\cdot(X/m)\cdot P'' - C'' < 0. \tag{12}$$

Joint stability of both the equilibrium output and the consistent conjectural variation would require the Jacobian of the system (4) and (8) to be negative semidefinite. Since there is little that can be said about the Jacobian, we shall not require that the consistent conjectural variation be stable. However, condition (12) can be useful in identifying consistent conjectural variations which are unstable, thereby narrowing the set of consistent equilibria to possibly one.

3. Consistent conjectural variations

■ In this section we consider what types of conjectural variations are consistent under condition (8) for differing specifications of demand and cost. In particular, when is competitive behavior consistent? The instability of certain consistent conjectural variations is examined. Finally, we discuss whether the consistent conjectural variation can indeed give rise to a consistent equilibrium. Proposition 1 handles the case of constant marginal costs while Proposition 2 deals with rising and falling marginal costs.

Proposition 1: Suppose marginal costs are constant ($C'' = 0$). (a) The competitive conjectural variation $\delta = -1$ is always consistent; any noncompetitive consistent conjectural variation would be unstable. (b) If there are only two firms and $-X\cdot P''/P' \neq 2$, then the competitive conjectural variation is the only consistent one. However, it is unstable from above when $-X\cdot P''/P' > 2$. If $-X\cdot P''/P' = 2$ (e.g., $P(X) = a\cdot X^{-1}$), then any conjectural variation is consistent. (c) If there are more than two firms and inverse demand declines at a linear or increasing rate ($P' < 0$ but $P'' \leq 0$), then the competitive conjectural variation is the only consistent one.

Proof: (a) The consistency of $\delta = -1$ is obvious from the fact that $(1 + \delta)$ is a factor of (8a) when $C'' = 0$. The instability of $\delta > -1$ is seen by substituting $X\cdot P''$ from (8a) into (12). This yields $-P'\cdot(m - 2)$ which is nonnegative. (b) When $m = 2$, (8a) becomes $(1 + \delta)^2\cdot[P'(X) + X\cdot P''(X)/2]$, and $\delta = -1$ is the only consistent conjectural variation as long as the bracketed term is not identically zero; otherwise, any conjectural variation is consistent. If $-X \cdot P''/P' > 2$, the bracketed term is positive, and thus the stability condition (12) is violated for $\delta \neq -1$. (c) When $m \geq 3$ and $C'' = 0$, the expression in (8a) must be negative for $\delta > -1$ when $P' < 0$ and $P'' \leq 0$. Thus, $\delta = -1$ is the only consistent conjectural variation in the relevant range.

These results indicate a strong relationship between constant marginal costs and the consistency of competitive firm behavior. If the rest of the industry were behaving competitively, a unit increase in X_j would shift the demand facing the other firms $P(X_j + X_o)$ inward by one unit and would result in exactly a one-unit contraction along the horizontal supply curve implied by constant marginal costs and competitive behavior.

Proposition 1(c) begs one to ask whether noncompetitive consistent conjectural variations can exist with constant marginal costs and inverse demand functions which decline at a decreasing rate, i.e., $P' < 0$ and $P'' > 0$. In Perry (1980) consistent conjectural variations were calculated for three such parameterizations of the inverse demand function $P(X)$. Those findings indicate that although noncompetitive consistent conjectural variations may exist, they often do not exist. And even if one existed, it would be unstable by Proposition (1a). Such a case is illustrated by the response function labelled 1-1 in Figure 1.

With constant marginal costs, the second-order conditions (9) and (10) are obviously satisfied for a competitive consistent conjectural variation. However, for the consistent equilibrium to exist, we obviously require $C(0) = 0$; otherwise the profitability condition (11) would be violated. If there were fixed costs, we would have to consider one of the unstable consistent conjectural variations to obtain a consistent equilibrium. Proposition (1b), (1c), and the examples mentioned indicate that we would not typically find such a consistent equilibrium.

Proposition 2: (a) If marginal costs are nonconstant ($C'' \neq 0$), then the competitive conjectural variation cannot be consistent. (b) If marginal costs are rising ($C'' > 0$) and if the inverse demand function $P(X)$ is such that $P'(X) + X \cdot P''(X) < 0$, then any consistent conjectural variation must be negative but not competitive, $-1 < \delta < 0$. In the limiting case of perfectly elastic demand ($P' = 0$), the consistent conjectural variation is Cournot, $\delta = 0$. (c) If marginal costs are falling ($C'' < 0$) and if the inverse demand is declining at a linear or increasing rate ($P' < 0$ and $P'' \leq 0$), then no consistent conjectural variation exists. If inverse demand is otherwise ($P' < 0$ and $P'' > 0$), any consistent conjectural variation which exists must be positive.

Proof: (a) This result follows immediately from the fact that if $C'' \neq 0$ but $\delta = -1$, condition (8a) could not be satisfied. (b) Since $C'' > 0$, the first bracketed term of (8b) is negative. The requirement $P' + X \cdot P'' < 0$ implies that *industry* marginal revenue shifts downward with *exogenous* output increases and guarantees that the perceived marginal revenue of each firm $i \neq j$ also shifts downward with increases in X_j, that is, $P' + (1 + \delta) \cdot (X/m) \cdot P'' < 0$ for $\delta \leq m - 1$. Thus, the second bracketed term of (8b) is negative, meaning that (8b) can be satisfied only if $-1 < \delta < 0$. When $P' = 0$, each firm chooses its output to equate marginal cost to the fixed price, irrespective of what other firms do. Thus, only $\delta = 0$ can be consistent. (c) For (8a) and (9) to be satisfied simultaneously, it must be that $(1 + \delta) \geq C''/P'$. This can be seen by substituting in either direction. Now, when C''/P' is substituted for $(1 + \delta)$ in the first term of (8a), the expression in (8a) becomes $C'' + (1 + \delta)^2 \cdot (X/m) \cdot P''$. This expression is negative since $C'' < 0$ and $P'' \leq 0$, and it exceeds the actual value of the left-hand side of (8a). Thus, condition (8a) could not be satisfied when (9) is required under these circumstances.

For the class of demands $P(X)$ for which $P' + X \cdot P'' < 0$, including all $P'' < 0$ and some $P'' > 0$, the consistent conjectural variation is between competitive and Cournot behavior whenever marginal costs are rising. Rising marginal costs dampen the response of the other firms away from the competitive behavior which was found to be consistent for constant marginal costs. A unit increase in the output of one firm is viewed by the other firms as an exogenous unit inward shift in demand which in turn shifts their perceived marginal revenue functions downward. As these firms contract output, their marginal costs fall, thereby reducing their incentive to contract. As a result, the overall reduction in the output of the other firms is less than unity so that $-1 < \delta < 0$. Figure 1 illustrates this case. The response function labelled 2-2 gives rise to a consistent conjectural variation between -1 and 0.[9] Under the demand and cost conditions of Proposition 2(b), the second-order conditions (9) and (10) are clearly satisfied. Thus, a consistent equilibrium would exist if the net revenues on inframarginal units exceed fixed costs.

When marginal costs are falling, the second-order condition (9) bounds the conjectural variation away from competitive or nearly competitive behavior; in particular, $(1 + \delta) \geq C''/P'$. But by Proposition 2(c), only a positive consistent conjectural variation could exist and only if $P'' > 0$. But even in this case, our experience with the examples in Perry (1980) for $C'' = 0$ gives rise to pessimism about the existence of a positive consistent conjectural variation or certainly a consistent equilibrium with such.

4. Free entry and the consistent conjectural variation

■ Propositions 1 and 2 pertain to the industry equilibrium when the number of firms is fixed. A consistent equilibrium made performance and conduct endogenous, given this industry structure. However, a consistent equilibrium only existed if firm profits were nonnegative. Thus, one way to make industry structure endogenous would be to impose the additional condition that the free entry number of firms $\bar{m}$ is the largest number for which profit condition (11) is satisfied. Exit occurs if the number of firms is greater than $\bar{m}$; whereas entry occurs if the number of firms is less than $\bar{m}$. Obviously, this scenario fails when demand and cost conditions are such that no consistent equilibrium exists for any number of firms. But otherwise, conditions (4), (8), and (11) would be a system defining a free entry consistent equilibrium. Although the system of conditions (4), (8), and (11) may define a reasonable free entry consistent equilibrium when the number of firms is small, the general problem with this approach is that the consistent conjectural variation defined by (7) would not be confirmed if entry or exit occurred. Thus, in this section, we define "full" consistency to take into account that shifts in the output of one firm alter the profitability of the industry and thereby affect the combined output of the other firms *via changes in the number of firms* as well as changes in the output per firm.

In redefining consistency to take free entry into account, we treat the number of firms m as a continuous variable. This is strictly inaccurate because entry and exit are discrete, discontinuous processes.[10] However, subject to the condition that we have a viable equilibrium with two or more symmetric firms, treating m as a continuous variable is a minor travesty in light of both the convenience and insight. To compute the equilibrium response dX_o/dX_j of the other firms when the number of firms in the industry is determined by free entry, we must take into account the simultaneous determination of the output of other firms X_o and the number of other firms $(m - 1)$ for a given output

[9] In Perry (1981a), I examined the case in which $C'' > 0$ and $C''' = 0$ so that $-1 < \delta(m) < 0$ and asked what happens to the consistent conjectural variation as the number of firms increases. As one would hope, consistent behavior becomes more competitive.

[10] Ulph (1981) illustrates the difficulties in defining consistency when one firm of a duopoly finds it unprofitable to operate. One of the firms has a cost disadvantage so that not only is its reaction function discontinuous at the point of shut-down, but also its equilibrium output may actually be zero. In this asymmetric context the discontinuities and resulting problems with defining consistency are more relevant and interesting than in the symmetric case. In Perry (1981b) I also discuss differential consistency when firms have differing costs.

level of the jth firm X_j. For a conjectural variation to then be fully consistent, it must be confirmed from the complete equilibrium system for the other firms. The symmetric first-order condition for output X_o and the free entry condition for the number of firms $(m - 1)$ are:

$$P(X_j + X_o) + (1 + \delta)\cdot P'(X_j + X_o)\cdot[X_o/(m - 1)] - C'(X_o/(m - 1)) = 0; \quad (13)$$

$$P(X_j + X_o)\cdot[X_o/(m - 1)] - C(X_o/(m - 1)) = 0. \quad (14)$$

Condition (13) is the same as (5), but in conjunction with (14), it defines a new reaction function $X_o(X_j; \delta)$ for the rest of the industry without the number of firms as an argument.[11] Assuming that there are at least initial scale economies from some source, e.g., fixed costs, (13) and (14) also define the free entry number of firms as a function of the jth firm's output, $m(X_j; \delta)$. We now state the following proposition:

Proposition 3: Suppose the industry equilibrium allows free entry. If $P' < 0$ and if there exists a fully consistent conjectural variation equilibrium at which the firm's second-order condition (9) is *strictly* satisfied, then this fully consistent conjectural variation is the competitive one.

Proof: With m solved simultaneously, the new reaction function $X_o(X_j; \delta)$ differs from the old reaction function $X_o(X_j; m, \delta)$. Thus, to examine $dX_o(X_j; \delta)/dX_j$, we must differentiate (13) and (14) with respect to X_o, m, and X_j. This yields the following differential system:

$$\begin{bmatrix}(m + \delta)\cdot P' + (1 + \delta)\cdot X_o\cdot P'' - C'' & -[X_o/(m - 1)]\cdot[(1 + \delta)\cdot P' - C''] \\ P + X_o\cdot P' - C' & -[X_o/(m - 1)]\cdot[P - C']\end{bmatrix}\cdot\begin{bmatrix}dX_o \\ dm\end{bmatrix}$$

$$= \begin{bmatrix}-(m - 1)\cdot P' - (1 + \delta)\cdot X_o\cdot P'' \\ -X_o\cdot P'\end{bmatrix}\cdot dX_j. \quad (15)$$

Abbreviate this matrix system as $A\cdot y = B\cdot z$, where $y = \begin{bmatrix}dX_o \\ dm\end{bmatrix}$ and $z = dX_j$. After grouping terms, imposing symmetry $X/m = X_o/(m - 1)$, and substituting $(P - C') = -(1 + \delta)\cdot(X/m)\cdot P'$ from the equilibrium condition (4), we find that

$$\det\begin{bmatrix}b_1 & a_{12} \\ b_2 & a_{22}\end{bmatrix} = -\det A$$

$$= \frac{(m - 1)\cdot X^2\cdot P'}{m^2}\left\{2\cdot(1 + \delta)\cdot P' + \frac{(1 + \delta)^2\cdot X\cdot P''}{m} - C''\right\}. \quad (16)$$

The term in braces is the second-order condition (9). Since $dX_o(X_j; \delta)/dX_j$ is the ratio of these two determinants, it equals minus one, i.e. only the competitive conjectural variation can be consistent when $P' < 0$ and condition (9) strictly negative.

Recall that when the number of firms was fixed, competitive behavior could be consistent only when marginal costs were constant. Proposition 3 now proves that with free entry, consistent competitive behavior will prevail for all demand and cost functions which allow the second-order conditions on the firm to be strictly satisfied. Although this excludes cases where marginal cost is constant or declining throughout, it certainly includes the standard case in which there are initial scale economies, say from fixed

[11] Note that no firm recognizes that it will of necessity make zero profits—only that all the others must. If firms foresaw zero profits, they would be indifferent among all output choices. Thus, this formulation preserves a nontrivial choice problem for each firm while symmetry and zero profits are imposed subsequent to the firm's choice problem.

costs, but eventual diseconomies from rising marginal costs. Moreover, this result obtains irrespective of the number of firms that actually arise in the equilibrium. It is the recognition of free entry, not the sheer number of firms, which generates competitive behavior. This is an important insight for two reasons. First, it obviates the reflex conclusion that imperfectly competitive behavior must be present when there is not a large number of firms in the industry. And second, it provides a substantive basis for the traditional presumption of competitive behavior when the number of firms is *not* small. There is no need to rely upon the limiting argument that competitive behavior arises because firms are trivial relative to the size of the market.

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