The Long Run Effect of Growth on Employment in a Labor Market with Matching Frictions: The Role of Labor Market Institutions.

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Abstract

In this paper, using an infinite horizon model with exogenous growth of labor productivity, we analyze the long run effect of growth on the employment rate in a labor market with matching frictions when there is either individual or collective wage setting and different timing for setting wages, labor and capital. We obtain that different labor markets institutions change the effect of growth on employment, however, if the coefficient of the constant relative risk aversion of the utility function is high, for almost all labor market institutions, growth is bad employment. On the contrary if this coefficient is small, for almost all labor market institutions, growth is good for employment.

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1 Introduction

In this paper we analyze the long run relationship between growth and unemployment in a labor market with matching frictions when there is individual and collective wage setting. Whereas in the short run both variables interact, that is, a higher rate of growth implies more capital accumulation and, then, more employment and a higher level of employment implies more income and, then, more capital accumulation, in the long run the effect changes. In infinite horizon (Ramsey) models with unemployment if the production function is neoclassical (neoclassical growth model) and there is exogenous labor augmenting technological progress, being the growth rate x, it turns out that in the long run steady state, output per efficient person is constant. This means that the long run rate of growth of income per capita is equal to the exogenous rate of technological progress x, that is, of course, independent of the long run employment level. The reason is that, independently of the effects of employment on growth, in the long run, due to the decreasing marginal product of capital, "growth always stops". Then, in papers with this set up, employment does not affect growth in the long run and the only thing to check is that if growth, that is the exogenous rate of technological progress x, affects employment.

This is done in a model without matching frictions in the labor markets by Raurich and Sorolla (2014). In this model wages are set by firms (efficiency wage model) and it turns out that without wage inertia growth does not affect employment in the long run but if we assume wage inertia growth has a positive effect on employment in the long run.

In models with matching frictions, Pissarides (1990, chapter 2), with individual wage setting, finds that an increase in the growth rate of productivity x reduces unemployment due to a capitalization effect of the firm when the interest rate r is exogenous (whereas Aghion and Hobbit (1991), adding "creative destruction" obtain a hump-shaped relationship). In order to endogenize the interest rate he uses a dynamic IS-LM model concluding that the effect of x on employment depends (as we will see) on the term $r - x$, and, with the dynamic demand postulated, this effect is difficult to analyze because it depends on the slopes of two curves (P.38). Eriksson (1997) endogenizes the interest rate using the neoclassical growth model and, with an ad hoc individual wage setting equation, obtains that the effect of growth on employment depends on the parameter of constant relative risk aversion σ of the utility function. This is because the long run interest rate is given by $r = \rho + \sigma x$, where ρ is the subjective discount rate. He obtains that if $\sigma > 1$ then growth has a negative effect on employment and the opposite occurs if $\sigma < 1$. Looking at these different effects of growth on employment one may ask if the difference is due to the different wage setting system used.

Moreover all these results are presented in the neoclassical set up that considers that wages, capital and labor are set simultaneously, but Stole and Zwiebel (1996) raise the case of the strategic behavior of the firm when it sets employment before wage setting.

This case has been embedded by Cahuc et al. in a labor market with frictions and capital (the Pissarides model with a large (multiworker) firm) in a series of papers (Cahuc and Wasmer (2001) and Cahuc et al. (2008)) showing that the specific timing may change optimal conditions for choosing capital, labor and wages. Under these circumstances one may suspect that the effect of growth on unemployment may also be different. The purpose of this paper is to analyze the effect of growth on employment for different wage setting systems (individual or collective) and different timing when choosing capital, labor and wages in a labor market with frictions.

When the production function is not finally neoclassical, that is, ends up being an AK function due to externalities or public capital, it may be the case, depending on the "educated" way of defining the externality, that employment affects growth in the long run. This happens for example in Bean and Pissarides (1993) with matching frictions, where matches last for one period, and an OLG model; Daveri and Tabellini (2000), without matching frictions and an OLG model; or Eriksson (1997) with matching frictions and an infinite horizon model. Here we concentrate on neoclassical production functions that do not generates endogenous growth. We focus on standard individual and collective wage setting systems deriving wage equations from optimization programs and, then, we improve Eriksson (1997), where he only analyzes individual wage setting using an "ad hoc" wage setting function. This paper also differs from Raurich and Sorolla (2014) because in here we assume matching frictions and individual and collective wage setting instead of no frictions and efficiency wages.

There are many papers that compare individual and collective wage setting in models with matching frictions without growth for analyzing other issues. Bauer and Lingens (2013) analyze if collective wage bargaining (compared to individual wage bargaining) restores efficiency. Ebell and Haefke (2006), in a model with imperfect competition in the goods market, study which bargaining regime emerges as the stable institution. De la Croix (2006), in a model with imperfect competition in the goods market, the effect of different wage setting systems on employment. García and Sorolla (2013) in a model with matching frictions where matches last for one period which wage setting system generates frictional unemployment and Ranjan (2013) the role of labor market institutions on offshoring.

Our results say that the effect of growth on the employment rate depends on the role of labor market institutions, specifically, on the timing of wage setting with respect to capital and labor, on the type of wage bargaining: individual or collective, and on the way of financing the unemployment benefit , having that an increase in x may increase, decrease or leave unaffected the long run rate of employment. However we obtain that for a high value of the constant relative risk aversion parameter of the individual utility function σ, for almost all labor market institutions, an increase in the long run rate of growth decreases the long run rate of employment, that is, growth is bad for employment.

On the contrary, if this coefficient is small, for almost all labor market institutions, growth is good for employment. These results based on a parameter of the utility function may seem surprising because the usual explanation for a positive effect of labor productivity on employment in the long run is that wages must increase less than labor productivity and it seems strange to have a wage equation where the rate of growth of wages depends on a parameter of the individual utility function. The explanation is that, in models with matching frictions, employment (and sometimes wages) also depends on the interest rate r due to the existence of labor turnover costs and this is the channel that introduces the effect of σ on employment.

The rest of the paper is organized as follows. In the next section we present the agents of the model. In section 3 we derive the full dynamic system, section 4 presents the steady state and section 5 computes the value functions in the steady state. Section 6 presents the results for the Nash solution, where capital, labor and wages are set simultaneously. Section 7 analyzes the Stole and Zwiebel case, where capital and labor are set simultaneously and before the wage. Section 8 the opposite case when the wage is set before capital and labor which are decided simultaneously. Section 9 concludes with a summary of the results in a table.

2 The Market Economy

2.1 Labor Market Flows

We assume matching frictions in the labor market where the matching function is $X(t) = m(V(t), U(t))$ where X are matches[1], V vacancies and U the amount of unemployment. Then $U = (N - L)$ where N is the total size of the work force that grows at the constant rate n and L is employment. We assume that m has constant returns to scale and $m_V > 0$ and $m_U > 0$. Then we define $\frac{X}{V} = m(1, \frac{1}{\frac{V}{U}}) \equiv q(\theta)$ where $\theta \equiv \frac{V}{U}$ is the degree of the labor market tightness, and one can show that $q' < 0$. Also we define $\frac{X}{U} = \frac{V}{U}\frac{X}{V} = \theta q(\theta)$ where one can show that $\frac{d(\theta q(\theta))}{d\theta} > 0$. Assuming that a proportion $0 < \lambda < 1$ of employed people loose the job, then employment flows are given by the differential equation:

$$\dot{L} = X - \lambda L = q(\theta)V - \lambda L = q(\theta)\frac{V}{U}U - \lambda L = q(\theta)\theta(N - L) - \lambda L. \quad (1)$$

2.2 The Family

We assume a big family that chooses the consumption of each member of the family C in order to maximize a CRRA utility function subject to the usual sequential budget constraint, where B is the total amount of assets, Π profits, b_0 the unemployment benefit,

[1] t is a continous variable and omitted when not necessary.

ω the real wage, r the real interest rate and VI the vacancy income that a firm pays and we assume goes to the consumer. The program is then to choose C in order to maximize:

$$\int_0^{\infty} e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} N dt$$

subject to:

$$\dot{B} = rB + (1-\tau_w)\omega L + b_0(N-L) + \Pi + VI - C.N$$

where $\sigma > 0$ is the constant relative risk aversion coefficient.

As usual the Euler equation is given by:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(r-\rho),$$

that rewritten in terms of consumption per efficient person $c \equiv \frac{C}{A}$, where A grows at the constant exogenous rate x, gives[2]:

$$\frac{\dot{c}}{c} + x = \frac{1}{\sigma}(r-\rho),$$

that is:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r-\rho-\sigma x).$$

2.3 The Multiple-Worker (Large) Firm

We assume a neoclassical production function $Y = F(K, AL)$ where A (the labor augmenting technological progress or "labor productivity"[3]), as we said, growths at the constant rate x. The firm chooses simultaneously L, K and V in order to maximize its value function V_F, that is, the sum of discounted profits along life,

$$V_F = \int_0^{\infty} e^{-rt} \left[F(K, AL) - (1+\tau_F)\,\omega L - \gamma_0 V - I\right] dt \tag{2}$$

subject to the employment flow equation given by (1):

$$\dot{L} = q(\theta)V - \lambda L$$

[2] In order to have the integral well defined we need to assume $\rho > (1-\sigma)x + n$, this means that $\rho - (1-\sigma)x > n$ and, in particular, $\rho + (\sigma - 1)x > 0$.

[3] We say "labor productivity" between quotes because the usual definition of labor productivity is $\frac{Y}{L} = F(\frac{K}{L}, A)$ that increases either if A increases or if $\frac{K}{L}$ increases.

and the capital accumulation equation:

$$\dot{K} = I - \delta K,$$

that is, the firm maximizes

$$V_F = \int_0^{\infty} e^{-rt} \left[F(K, AL) - (1 + \tau_F)\, \omega L - \gamma_0 \frac{\dot{L} + \lambda L}{q(\theta)} - \left(\dot{K} + \delta K \right) \right] dt$$

where I is investment, δ is the exogenous depreciation rate, τ_F a pay roll tax rate and γ_0 the cost of open a vacancy. If the cost of open a vacancy changes along time then we obtain the following first order conditions, given by the Euler equations $V_K - \frac{\partial V_{\dot{K}}}{\partial t} = 0$ and $V_L - \frac{\partial V_{\dot{L}}}{\partial t} = 0$, that are the capital demand equation:

$$F_K = r + \delta \tag{3}$$

and the employment equation:

$$F_L = (1 + \tau_F)\, \omega + \gamma_0 \frac{r + \lambda - \frac{\dot{\gamma}_0}{\gamma_0} + \frac{q'(\theta)\dot{\theta}}{q(\theta)}}{q(\theta)}. \tag{4}$$

where one interprets the term $\gamma_0 \frac{r + \lambda - \frac{\dot{\gamma}_0}{\gamma_0} + \frac{q'(\theta)\dot{\theta}}{q(\theta)}}{q(\theta)}$ as the turnover cost[4]. As one can see the assumption about how γ_0 changes is important because it is going to change the amount of employment decided by the firm. We assume that γ_0 is proportional to the wage that is $\gamma_0 = \gamma \omega$[5].

In this case the firm maximizes:

$$V_F = \int_0^{\infty} e^{-rt} \left[F(K, AL) - (1 + \tau_F)\, \omega L - \gamma \omega \frac{\dot{L} + \lambda L}{q(\theta)} - \left(\dot{K} + \delta K \right) \right] dt$$

And the first order conditions are the (exogenous wage) capital demand equation:

$$F_K = r + \delta \tag{5}$$

and the (exogenous wage) employment equation[6]:

[4]See, for example Cahuc et al. (2008).

[5]This assumption is standard in the literature (see for example Pissarides (1992)) but not neutral. For different assumptions about the evolution of $\gamma_0(t)$ along time ($\gamma_0(t) = \gamma$, $\gamma_0(t) = \gamma A(t)$, $\frac{\dot{\gamma}_0(t)}{\gamma_0(t)} = \gamma$,...) the main results about the long run relationship between growth and employment may change as we will mention later.

[6]We refer to these equations as the (exogenous wage) because the firm takes the wage as given. This will change later when we analyze strategic interactions.

$$F_L = (1+\tau_F)\,\omega + \gamma \frac{(r+\lambda-\frac{\dot{\omega}}{\omega}+\frac{q'(\theta)\dot{\theta}}{q(\theta)})}{q(\theta)}\omega = \left[(1+\tau_F)+\gamma\frac{(r+\lambda-\frac{\dot{\omega}}{\omega}+\frac{q'(\theta)\theta}{q(\theta)}\frac{\dot{\theta}}{\theta})}{q(\theta)}\right]\omega \quad (6)$$

Note that now the change of wages affect optimal employment, this is precisely one channel that implies that growth affect employment in the long run because in the steady state (the efficient wage $\frac{\omega}{A}$ is going to be constant and) the rate of growth of wages $\frac{\dot{\omega}}{\omega}$ is equal to the exogenous productivity rate x. Moreover the long run interest rate will also depend positively on x and then the crucial point is that if an increase of x increases or reduces the term $r-\frac{\dot{\omega}}{\omega}$ and, then, the turnover cost.

2.4 Equilibrium in the Output Market

Now we define $k=\frac{K}{AN}$ as capital per efficient person and we denote the employment rate as $l=\frac{L}{N}$. From the budget constraint of the family, assuming that the government has a balanced budget constraint and that the capital market is in equilibrium one obtains[7] the usual feasibility constraint/capital accumulation equation/equilibrium output market equation in terms of variables per efficient person:

$$\dot{k} = F(k,l) - c - (n+\delta+x)k.$$

3 The Dynamic Model

We can rewrite the labor market flows equation in terms of the employment rate as:

$$\dot{l} = q(\theta)\theta(1-l) - (\lambda+n)l. \quad (7)$$

The Euler equation is:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r-\rho-\sigma x). \quad (8)$$

If we define the capital per efficient unit of labor (the capital per efficient labor ratio) as $\hat{k}=\frac{K}{AL}$, the intensive production function as $f(\hat{k})=\frac{F(K,AL)}{AL}$ and the wage in efficiency units as $w=\frac{\omega}{A}$, then we know that $F_K=f(\hat{k})$ and $F_L=A\left[f(\hat{k})-\hat{k}f'\left(\hat{k}\right)\right]$ that substituted respectively in (5) and in (6) give the:

Capital demand:

[7] To obtain this equation is not trivial because one has to specify how the firm finances it investment: retaining profits, issuing bonds,... this is discussed in the seminal paper of Abel and Blanchard (1983) but at the end all the alternatives gives, as it should be, the feasibility constraint.

$$f'\left(\hat{k}\right) = r + \delta \tag{9}$$

and the Employment equation:

$$f(\hat{k}) - \hat{k}f'\left(\hat{k}\right) = \frac{\omega}{A}\left[(1+\tau_F) + \gamma\frac{(r+\lambda - \frac{\dot{\omega}}{\omega} + \frac{q'(\theta)\theta}{q(\theta)}\frac{\dot{\theta}}{\theta})}{q(\theta)}\right],$$

that is,

$$f(\hat{k}) - \hat{k}f'\left(\hat{k}\right) = w\left[(1+\tau_F) + \gamma\frac{(r+\lambda - \frac{\dot{w}}{w} - x + \frac{q'(\theta)\theta}{q(\theta)}\frac{\dot{\theta}}{\theta})}{q(\theta)}\right]. \tag{10}$$

Also we can rewrite the capital per efficient labor ratio as:

$$\hat{k} = \frac{K}{AL} = \frac{\frac{K}{AN}}{\frac{L}{N}} = \frac{k}{l}, \tag{11}$$

meaning that the capital per efficient labor ratio is, in fact, the amount of capital per efficient person divided by the employment rate. The capital accumulation equation is

$$\dot{k} = f(k,l) - c - (n+\delta+x)k. \tag{12}$$

Adding finally a general wage equation[8]

$$\omega = \tilde{\omega}(.), \text{ that is, } w = \frac{\tilde{\omega}(.)}{A} \tag{13}$$

the dynamic model is given by the seven equations: (7), (8), (9), (10), (11), (12), (13), being the endogenous variables r, $\hat{k}$, θ, l, w, k and c. Finally income per efficient person is given by:

$$y = F(k,l),$$

and product per efficient unit of labor:

$$\hat{y} = f(\hat{k}).$$

[8] The derivation of a specific wage equation outside the steady state is difficult to obtain if the program for its derivation is based on workers and firm value functions that take into acount discounted along life labor and firm income because they depend on $\dot{\theta}$ and $\dot{L}$. The only way to obtain a tractable wage equation in this situation is to use programs based on current wages and profits.

4 The Model in the Steady State for a General Wage Setting Equation

Assuming that the model converges to an steady state we have that in this steady state $\dot{k} = 0$ (that means $\dot{y} = \frac{Y}{AL} = 0$, that is $\frac{\left(\frac{Y}{L}\right)'}{\left(\frac{Y}{L}\right)} = x$, that is, as usual, the long rate of growth of income per capita is given by the exogenous rate of labor augmenting technological progress x), $\dot{w} = 0$, $\dot{c} = 0$, $\dot{\theta} = 0$ and $\dot{l} = 0$.

The endogenous variables are: r^*, $\hat{k}^*$, θ^*, l^*, w^*, k^* and c^* that are given by the:

Euler equation

$$r^* = \rho + \sigma x \tag{14}$$

which means that the long run interest rate is given by the subjective discount rate ρ plus the exogenous rate of technological progress (multiplied by $\sigma > 0$). Then an increase in x increases the long interest rate, decapitalizing the firm using Pissarides' words because discounted profits along life decrease.

Capital demand:

$$f'(\hat{k}^*) = r + \delta \tag{15}$$

Employment equation: now the employment equation becomes[9]:

$$F_L = \left[(1 + \tau_F) + \gamma \frac{(r^* + \lambda - x)}{q(\theta)}\right] \omega \tag{16}$$

where as in the usual case without vacancy costs ($F_L = (1 + \tau_F)\,\omega$) an increase in the wage reduces employment, the novelty here is that employment is also affected by labor market tightness via turnover costs having that an increase in θ increases employment. Note also that this is a crucial equation for the effect of growth on employment because r^* and x, the long run rate of growth appears in the expression and this concrete expression has been obtained assuming $\gamma_0 = \gamma\omega$[10]. As Pissarides (1990) says (P. 38) the long run effect of growth on employment depends on the term $r^* - x$. Pissarides (1990) finds r^* using a dynamic IS-LM model, but using the Ramsey model (as Eriksson (1997) does) the term $r^* - x$ becomes $\rho + (\sigma - 1)x$ having that an increase in x increases (decreases) the turnover costs if $\sigma > (<)1$ decreasing (increasing) then employment. This is the first channel for the effect of growth on employment: the employment equation effect that it is the unique channel that appears in Eriksson (1997) paper. The equation in terms of $\hat{k}$ is:

[9] This is equation (2.25) in Pissarides (1990) without τ_F.

[10] Without this assumption the employment equation becomes $F_L = (1 + \tau_F)\,\omega + \gamma_0 \frac{r + \lambda - \frac{\dot{\gamma}_0}{\gamma_0}}{q(\theta)}$. If we assume $\gamma_0 = \gamma A$ then we get a similar expression that is $F_L = (1 + \tau_F)\,\omega + \gamma A \frac{r + \lambda - x}{q(\theta)}$ with the complication that depends on A. But if we assume a constant γ_0 then we get $F_L = (1 + \tau_F)\,\omega + \gamma_0 \frac{r + \lambda}{q(\theta)}$ and in this case the employment equation does not depen directly on x.

$$f(\hat{k}^*) - \hat{k}^* f'\left(\hat{k}^*\right) = w^*\left[(1+\tau_F) + \gamma\frac{(r^* + \lambda - x)}{q(\theta^*)}\right] \tag{17}$$

Wage equation:

$$\omega^* = \tilde{\omega}(.) \text{ that is } w^* = \frac{\tilde{\omega}(.)}{A} \tag{18}$$

Equilibrium of labor market flows:

$$l^* = \left[\frac{\theta^* q(\theta^*)}{\lambda + n + \theta^* q(\theta^*)}\right] = \left[\frac{1}{1 + \frac{\lambda+n}{\theta^* q(\theta^*)}}\right] \tag{19}$$

It is interesting to note that $\theta^* q(\theta^*) = \frac{X}{U} \leqq 1$ which means that the equilibrium of the labor markets flows equation implies that as long as $0 < \lambda < 1$[11] the long run rate of employment $l < 1$, meaning that there is always unemployment in the long run. This equation also implies that when θ^* increases l^* increases.

Capital per efficient labor ratio $\hat{k}$ equation:

$$\hat{k}^* = \frac{k^*}{l^*} \tag{20}$$

Capital accumulation equation:

$$c^* = F(k^*, l^*) - (n + \delta + x)k^* \tag{21}$$

The solution of the model is recursive : $r^* = \rho + \sigma x$ is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the capital demand equation (15). If it turns out that the wage equation depends on $\hat{k}$, that is $w^* = \frac{\tilde{\omega}(.)}{A} = \hat{\omega}(\hat{k})$, then substituting it in the employment equation we have an equation that depends on $\hat{k}$, r, and θ[12] so the employment and wage equation gives θ^*, then the equilibrium of labor market flows gives l^*, the capital per efficient labor equality gives k^* and the capital accumulation equation c^*. Only in one of the cases presented the wage equation determines l^*, which implies that then the equilibrium of labor market flows gives θ^*, the employment equation gives w^*, the capital per efficient labor equality gives k^* and the capital accumulation equation c^*.

Finally

$$y^* = F(k^*, l^*)$$

and

[11] The case $\lambda = 1$, that means that matches last for one period, must be analyzed separed and this is done in García and Sorolla (2013).

[12] In fact, with almost all the specific wage equations derived we will have that the substitution of the wage equation on the employment equation gives an equation that depends only on r^* and θ because $\hat{k}$ vanishes.

$$\hat{y}^* = f(\hat{k}^*)$$

What we are going to do now is to derive specific wage equations in the steady state depending on the type (individual or collective) and timing of wage setting.

5 Value Functions in the Steady State

We denote the value function of an employed worker, that is, his expected discounted labor income along life that takes into account that he can change from employment to unemployment with the constant probability λ as V_E. Then, as usual, the following asset value equation holds:

$$rV_E = (1-\tau_w)\omega + \lambda(V_U - V_E). \tag{22}$$

We denote the value function of an unemployed worker as V_U and if θ is constant, that is, in an steady state, the following asset value equation holds:

$$rV_U = b_0 + \theta q(\theta)(V_E - V_U). \tag{23}$$

We know that the value function of the firm is

$$V_F = \int_0^\infty e^{-rt}\left[F(K,AL) - (1+\tau_F)\,\omega L - \gamma\omega V - \left(\dot{K} + \delta K\right)\right] dt$$

In an steady state $\dot{l} = 0$ and $\dot{k} = 0$ so we have to write the value function in terms of l and k. Multiplying and dividing by AN we obtain

$$V_F = \int_0^\infty e^{-rt}AN\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\,\frac{\omega L}{AN} - \gamma\omega\frac{\dot{L}+\lambda L}{q(\theta)AN} - \frac{\left(\dot{K}+\delta K\right)}{AN}\right] dt$$

we also know that $\frac{\dot{l}}{l} = \frac{\dot{L}}{L} - n$ that is $\dot{L} = \left(\frac{\dot{l}}{l} + n\right)L$ and $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n - x$ that is $\dot{K} = \left(\frac{\dot{k}}{k} + n + x\right)K$ that substituted in V_F gives

$$V_F = \int_0^\infty e^{-rt}AN\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\,\frac{\omega L}{AN} - \gamma\omega\frac{\left(\frac{\dot{l}}{l}+n+\lambda\right)L}{q(\theta)AN} - \frac{\left(\left(\frac{\dot{k}}{k}+n+x+\delta\right)K\right)}{AN}\right] dt$$

now making $\dot{l} = 0$ and $\dot{k} = 0$ and considering that $A = A_0e^{xt}$ and $N = N_0e^{nt}$ and that $A_0 = N_0 = 1$ we get

$$V_F = \int_0^{\infty} e^{-(r-(n+x))t} \left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN} - \gamma \frac{(n+\lambda)\,\omega L}{q(\theta)AN} - \frac{(n+x+\delta)\,K}{AN} \right] dt$$

so that means that in the steady state the real discount factor is $r-(n+x)$. Then the value asset equation implies

$$(r-(n+x))\,V_F = \left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN} - \gamma \frac{(n+\lambda)\,\omega L}{q(\theta)AN} - \frac{(n+x+\delta)\,K}{AN} \right]$$

that is

$$V_{FLR} = \frac{\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN} - \gamma \frac{(n+\lambda)\omega L}{q(\theta)AN} - \frac{(n+x+\delta)K}{AN} \right]}{r-(n+x)} \tag{24}$$

or

$$V_{FLR} = \frac{\left[F(k,l) - \left[(1+\tau_F) + \gamma \frac{(n+\lambda)}{q(\theta)} \right] wl - (n+x+\delta)\,k \right]}{r-(n+x)}$$

Note that in increases in x capitalizes the value of the firm and an increase in r decapitalizes it because the present value of profits along life is lower. Also we need to know the value function of a firm of hiring an extra worker $V'_{F,\cdot}$, then if the wage is taken as exogenous we have that the following asset value equation must hold

$$rV'_{F,N} = [F_L - (1+\tau_F)\,\omega] - \lambda V'_{F,N}$$

that is[13]

$$V'_{F,N} = \frac{F_L - (1+\tau_F)\,\omega}{r+\lambda} \tag{25}$$

note that we do not take into account the vacancy cost because we assume that at that point the cost of posting a vacancy has already been paid and hence it doesn't affect wage bargaining, this is a standard feature of the matching frictions model.

6 The Nash Solution

In this case L, K, and w are decided at the same time or they can be renegotiated at any time or set without commitment, this means, in particular, that investment is not an irreversible choice and can be changed at any time. One can show that the same solution

[13] Solving the dynamic program using the Bellman equation one gets directly this expression from the he derivative of the envelop condition in the steady state (see Cahuc et al. (2008) equation D.1).

holds if L is decided first and later K and w are chosen simultaneously (Cahuc (2014), 5.1.1), this second case is also justified when wages are being bargained simultaneously with the firm without the possibility of renegotiating employment.

6.1 Individual Wage Setting

When there is individual wage setting each individual worker bargains the wage with the firm, in this case when deciding the wage the function to maximize is

$$\beta \ln(V_E - V_U) + (1-\beta)\ln\left(V'_{F,N}\right) \tag{26}$$

where $(V_E - V_U)$ is the surplus that a worker gets if hired and $V'_{F,N}$ is the surplus that the firm gets if it hires an extra worker. This is the usual surplus sharing rule for individual wage setting, used normally in models with matching frictions. We substitute $V_E - V_U$ using (22)[14] that is $V_E - V_U = \frac{(1-\tau_w)\omega - rV_U}{r+\lambda}$ then the function to maximize is:

$$\beta \ln(\frac{(1-\tau_w)\,\omega - rV_U}{r+\lambda}) + (1-\beta)\ln\left(\frac{F_L(K, AL) - (1+\tau_F)\,\omega}{(r+\lambda)}\right) \tag{27}$$

that gives as a first order condition:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} rV_U + \frac{\beta}{(1+\tau_F)} F_L(K, AL)$$

Note that the wage setting rule says that the wage depends positively on the marginal product of labor.

Using (23) we get

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{(1-\beta)}{(1-\tau_w)} \theta q(\theta)(V_E - V_U) + \frac{\beta}{(1+\tau_F)} F_L(K, AL)$$

Note that the wage setting rule says that the wage depends positively on the unemployment benefit.

Calculating $(V_E - V_U)$ using Ranjan´s method[15] (Ranjan Online Appendix Part IV) we get (see appendix)

$$(V_E - V_U) = \frac{(1-\tau_w)}{r+\lambda} \frac{\beta}{1-\beta} \frac{\gamma(r+\lambda-x)}{(1+\tau_F)\,q(\theta)} \omega$$

and then the wage equation becomes

[14] Because the wage is bargained between an employed worker and the firm we substitute $V_E - V_U$ using only the asset value equation of an employed worker. In the collective wage setting case when a union represents both employed and unemployed workers we will use both asset value equations.

[15] This method uses the relationship between $V_E - V_U$ and $V'_{F,N}$ obtained in the first order condition to obtain an expression that depends on F_L and then we substitute F_L using the employment equation.

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\theta\gamma\beta(r+\lambda-x)}{(1+\tau_F)(r+\lambda)}\omega + \frac{\beta}{(1+\tau_F)} F_L(K, AL)^{16}.$$

Assuming also that $b_0 = b\omega$[17] , such that $b < (1-\tau_w)$ we get :

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b\omega + \frac{\theta\gamma\beta(r+\lambda-x)}{(1+\tau_F)(r+\lambda)}\omega + \frac{\beta}{(1+\tau_F)} F_L(K, AL)$$

that is

$$\left[1 - \frac{(1-\beta)}{(1-\tau_w)} b - \frac{\theta\gamma\beta(r+\lambda-x)}{(1+\tau_F)(r+\lambda)}\right]\omega = \frac{\beta}{(1+\tau_F)} F_L(K, AL) \tag{28}$$

and then the wage equation is

$$\omega = m_\omega F_L = \frac{\beta}{(1+\tau_F)\left[1 - \frac{(1-\beta)}{(1-\tau_w)} b - \frac{\theta\gamma\beta(r+\lambda-x)}{(1+\tau_F)(r+\lambda)}\right]} F_L(K, AL)^{18}$$

that is the wage is a proportion m_ω of the marginal product of labor that depends on θ, r and x having that an increase in x (decrease in θ) reduces the wage. This dependence of the wage equation on x does not appear neither in Pissarides (1990) nor in Eriksson (1997) and this the reason why the results obtained in this section are different from the ones presented in Eriksson's paper.

One can also compute $(V_E - V_U)$ using (22) and (23) which gives $(V_E - V_U) = \frac{(1-\tau_w)\omega - b_0}{r+\lambda+\theta q(\theta)}$ and then the wage equation becomes (see appendix):

$$\omega = \frac{1}{\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]} \frac{\beta(1-\tau_w)}{(1+\tau_F)} F_L(K, AL)$$

seeing in a more clearly way that an increase in x produces wage moderation. This is because an increase in x increases the long run $r = \rho + \sigma x$ decreasing $(V_E - V_U)$ and then the wage. Pissarides (1990) does not show this effect because the long run interest is derived using a IS-LM model and not the infinite horizon model.

The employment equation is given by (10), that is

$$F_L = \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda-x)}{q(\theta)}\right] \tag{29}$$

substituting (29) in (28) F_L and ω cancel and one gets the "equilibrium" labor market equation[19], that is, the employment equation when the wage setting equation is taken into

[17] One can use different ways for determining b_0, for example that the goverment has a balanced budget that is $b_0(N-L) = (\tau_w + \tau_F)\omega L$, or $b_0 = by$. We will use this simple method when it works.

[19] The equilibrium labor market equation says that m_ω times

account, that is:

$$\left[1-\frac{(1-\beta)}{(1-\tau_w)}b-\frac{\theta\gamma\beta(r+\lambda-x)}{(1+\tau_F)\,(r+\lambda)}\right]=\frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(r+\lambda-x)}{q(\theta)}\right] \tag{30}$$

Note that this equation holds for any neoclassical production function and that depends only on r and θ (not on $\hat{k}$ because F_L cancels). That is with this specific wage equation r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the capital demand equation (15), θ^* is given by the employment and the wage setting equation once one substitutes only r^* (not $\hat{k}^*$) in (30), l^* by the equilibrium of labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21).

In order to derive the effect of x on θ substitute $r = \rho + \sigma x$ in (30) to get

$$\left[1-\frac{(1-\beta)}{(1-\tau_w)}b-\frac{\theta\gamma\beta(\varrho+\lambda+(\sigma-1)x)}{(1+\tau_F)\,(\varrho+\lambda+\sigma x)}\right]=\frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)}\right] \tag{31}$$

And then an equilibrium in the labor market always exists and there exists $\sigma^* > 1$ such that if $\sigma > \sigma^*$ then $\frac{d\theta^*}{dx} < 0$ and then $\frac{dl^*}{dx} < 0$, if $\sigma = \sigma^*$ then $\frac{d\theta^*}{dx} = 0$ and then $\frac{dl^*}{dx} = 0$ and if $\sigma < \sigma^*$then $\frac{d\theta^*}{dx} > 0$ and then $\frac{dl^*}{dx} > 0$ (see appendix).

Note that this result modifies Eriksson´s (1997) result because he obtains that if $\sigma > 1$ ($<$) then $\frac{d\theta^*}{dx} < (>)0$ and then $\frac{dl^*}{dx} < (>)0$. This is due to the fact that the ad hoc wage equation that he assumes is not the one that results from the usual surplus sharing rule that produces that x affects also the wage equation.

The intuition for the result is the following: on the one hand an increase in x increases the long run interest rate and then reduces the mark up and the wage, on the other long run labor turnover costs depend on $r-x = \varrho+\lambda+(\sigma-1)x$ having that if $\sigma < 1$ and increase in x reduces turn over costs. Because total labor costs are given by $\omega\left[(1+\tau_F) + \text{turnover costs}\right]$ if both decrease then firms open more vacancies increasing θ and employment. But even for a "small" $\sigma > 1$ the increase now of the turnover cost is compensated by the the decrease of the wage increasing employment. Only if σ is high enough total labor cost increases reducing employment.

Finally, if the government has a balanced budget constraint, that is, $b_0(1-l) = (\tau_w + \tau_F)\omega l$ and if $\tau_w = \varphi\tau_F$, as shown in the appendix, the tax rates are given by

$$1+\tau_F = 1+\frac{b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

$$1-\tau_w = 1-\frac{\varphi b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

which means that now the equation that determines θ, (31) becomes more complicated, but as long as the effect of changing θ does not affect the slopes of the two sides of the equation the effect of x on θ is going to be the same, because the tax rates do not depend on x.

6.2 Collective Wage Setting

When there is collective wage setting we assume that a union that represents both employed and unemployed workers bargains the wage with the firm. In this case the function to maximize is[20]

$$\beta \ln \left\{ \left[\left(\frac{L}{N} (V_E - V_U) + \frac{(N-L)}{N} N \right) - V_U \right] N \right\} + (1-\beta) \ln (S_F) \tag{32}$$

where $\left(\frac{L}{N} (V_E - V_U) + \frac{(N-L)}{N} N \right)$ is the expected value function of a worker and then $\left(\frac{L}{N} (V_E - V_U) + \frac{(N-L)}{N} N \right) - V_U$ is the expected surplus of a worker. On the other hand S_F is the surplus that the firm gets when employing L workers.

Alternatively one may consider that in the collective bargaining a union that represents only employed workers bargains the wage with the firm, in this case the function to maximize is[21]

$$\beta \ln \left[(V_E - V_U) L \right] + (1-\beta) \ln (S_F) \tag{33}$$

Note that operating (32) gives also (33). The wage equation comes from maximizing (33) with respect to ω. Substituting $V_E - V_U$ from (22) and (23) as in Ranjan (2013) [22] we obtain $V_E - V_U = \frac{\omega - b_0}{r + \lambda + \theta q(\theta)}$ and S_F, the surplus of the firm in the steady state of employing all workers, is obtained deducting to (24) the cost of open a vacancy but not the cost of capital because we assume that investment is not an irreversible choice and if the firm does not employ the workers it invests nothing[23]. The objective function is:

$$\beta \ln \left[\left(\frac{(1-\tau_w)\omega - b_0}{r + \lambda + \theta q(\theta)} \right) L \right] + (1-\beta) \ln \left(\frac{\left[\frac{F(K,AL)}{AN} - (1+\tau_F) \frac{\omega L}{AN} - \frac{(n+x+\delta)K}{AN} \right]}{r - (n+x)} \right) \tag{34}$$

[20] This is the extension of the function proposed by Ranjan (2013) when the wage is negotiated.

[21] This is the function proposed by Ebell and Haefke (2006).

[22] The difference with the case in which the union cares only about employed workers is that in this case one computes $V_E - V_U$ only using (22).

[23] See for example Cahuc et al (2014) p. 447 or Anderson and Devereux (1988). If one wants to substract the cost of capital to the firm surplus one must asume that people that negotiates the wage think that investment is irreversible which is not the case. This may be justified when the wage is negotiated by the human resource department of the firm and investment decided by the financial department.

that gives the wage equation (see appendix):

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\beta}{(1+\tau_F)} \left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right]$$

Note that the wage setting rule says that the wage depends positively on the unemployment benefit and the average product of labor (labor productivity). Assuming also that $b_0 = b\omega$ the wage equation becomes:

$$\omega = \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]} \frac{\beta}{(1+\tau_F)} \left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right] \tag{35}$$

Having now that the wage depends on labor productivity and also depends negatively on x. Again the employment equation is given by:

$$F_L = \omega \left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right] \tag{36}$$

and substituting (35) in (36) one gets

$$F_L = \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]} \frac{\beta}{(1+\tau_F)} \left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right] \left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right]$$

that is

$$\frac{F_L}{\left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right]} = \frac{f(\hat{k}) - \hat{k} f'(\hat{k})}{f(\hat{k}) - (n+\delta+x)\hat{k}} = \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]} \frac{\beta}{(1+\tau_F)} \left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right] \tag{37}$$

that means that in this case, the employment and the wage equation given r and $\hat{k}$ gives θ. Now to analyze the effect of x on θ becomes more complicated because we have to take into account the indirect effect of x on $\hat{k}$. If the production function is Cobb-Douglas $(K^\alpha (AL)^{1-\alpha})$ it turns out that $f(\hat{k}) = \hat{k}^\alpha$ in which case the "equilibrium" labor market equation becomes

$$\frac{(1-\alpha)}{1-(n+\delta+x)\hat{k}^{1-\alpha}} \frac{[(1-\tau_w)-(1-\beta)b]}{(1-\tau_w)} = \frac{\beta}{(1+\tau_F)} \left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right] \tag{38}$$

That is, with this specific wage equation and a Cobb-Douglas production function r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the capital demand equation (15), θ^* is given by the employment and the wage setting equation once one substitutes r^* and $\hat{k}^*$in (38), l^* by the equilibrium of labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21). In order to analyze the effect of x on employment: substitute $\hat{k}^{1-\alpha} = \frac{\alpha}{r+\delta}$ and $r = \rho + \sigma x$ in (38) to get:

$$\frac{(1-\alpha)}{1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}} \frac{[(1-\tau_w)-(1-\beta)b]}{(1-\tau_w)} = \frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)}\right] \quad (39)$$

And then if $\alpha < \tilde{\alpha}$ an equilibrium in the labor market exists and there exists $\hat{\sigma} > 1$ such that if $\sigma > \hat{\sigma}$ then $\frac{d\theta^*}{dx} < 0$ and then $\frac{dl^*}{dx} < 0$, if $\sigma = \hat{\sigma}$ then $\frac{d\theta^*}{dx} = 0$ and then $\frac{dl^*}{dx} = 0$ and if $\sigma < \hat{\sigma}$ then $\frac{d\theta^*}{dx} > 0$ and then $\frac{dl^*}{dx} > 0$ (see appendix). The intuition for the result is the same that the one presented with individual wage setting: and increase in x decreases the wage and labor turnover costs when $\sigma < 1$. Then in the Nash situation, with individual or collective wage setting if $\sigma \leqq 1$ growth is good for employment and in both case there exists a value of $\sigma > 1$ where for a higher σ growth is bad for employment.

7 The Firm as the Stackelberg Leader (the "Stole and Zwiebel" Case)

We now analyze the case when the firm acts as a Stackelberg leader having that K and L are decided first and the wage is negotiated later[24] or K and L are set with commitment (K is irreversible) and the wage is negotiated without commitment. Now the wage equation depends on L and K that is $\omega = \tilde{\omega}(L,K)$ and the firm takes strategic advantage of this relation when setting K[25] and L. In this case the firm maximizes

$$V_F = \int_0^\infty e^{-rt}\left[F(K,AL) - (1+\tau_F)\,\tilde{\omega}(L,K)L - \gamma\tilde{\omega}(L,K)\frac{\dot{L}+\lambda L}{q(\theta)} - \left(\dot{K}+\delta K\right)\right] dt$$

And the first order conditions[26] are the sz (Stole and Zwiebel) capital demand equation:

$$F_K - L\frac{\partial\tilde{\omega}}{\partial K} = r + \delta$$

and the sz employment equation is:

[24] This is the case presented in Cahuc et al (2008), without technological progress. As we said before in Cahuc and Wasmer (2001) it is argued that the case where L is decided first and K and the wage negotiated later is equivalent to the Nash situation.

[25] In the Stole and Zwiebel (1996) paper there is no capital.

[26] These are presented in Cahuc et al (2008).

$$F_L = (1+\tau_F)\,\omega + (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L} + \gamma\frac{(r+\lambda+\frac{q'(\theta)\dot{\theta}}{q(\theta)})}{q(\theta)}\omega - \gamma\frac{L\frac{\partial\tilde{\omega}}{\partial L}\frac{\dot{L}}{L}}{q(\theta)} =$$

$$\left[(1+\tau_F) - \frac{\gamma\lambda}{q(\theta)}\frac{\dot{L}}{L}\right]L\frac{\partial\tilde{\omega}}{\partial L} + \left[(1+\tau_F) + \gamma\frac{(r+\lambda+\frac{q'(\theta)\theta}{q(\theta)}\frac{\dot{\theta}}{\theta})}{q(\theta)}\right]\omega \tag{40}$$

where, due to its endogeneity, $\frac{\dot{\omega}}{\omega}$ has disappeared from the expression in the turnover costs depending now only on r and not on $r - \frac{\dot{\omega}}{\omega}$ as in the Nash case.

In terms of $\hat{k}$ and l the sz capital demand is:

$$f'\left(\hat{k}\right) = r + \delta + L\frac{\partial\tilde{\omega}}{\partial K} \tag{41}$$

and the sz employment equation:

$$f(\hat{k}) - \hat{k}f'\left(\hat{k}\right) = \frac{1}{A}\left[(1+\tau_F) - \frac{\gamma\lambda}{q(\theta)}\left(\frac{\dot{l}}{l}+n\right)\right]L\frac{\partial\tilde{\omega}}{\partial L} + \left[(1+\tau_F) + \gamma\frac{(r+\lambda+\frac{q'(\theta)\theta}{q(\theta)}\frac{\dot{\theta}}{\theta})}{q(\theta)}\right]w \tag{42}$$

and in the steady state the sz employment equation is:

$$F_L = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right]L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right] \tag{43}$$

Note that now x only appears affecting indirectly $r\,(=\rho+\sigma x)$ and that an increase in x increases the turnover costs. The sz employment equation in terms of $\hat{k}$ is:

$$f(\hat{k}) - \hat{k}f'\left(\hat{k}\right) = \frac{1}{A}\left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right]L\frac{\partial\tilde{\omega}}{\partial L} + w\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]. \tag{44}$$

In this case the surplus of hiring an extra worker is given by

$$rV'_{F,SZ} = \left[F_L - (1+\tau_F)\left[\omega + L\frac{\partial\tilde{\omega}(L,K)}{\partial L}\right]\right] - \lambda V'_{F,SZ}$$

that is:

$$V'_{F,SZ} = \frac{\left[F_L - (1+\tau_F)\left[\omega + L\frac{\partial\tilde{\omega}(L,K)}{\partial L}\right]\right]}{r+\lambda} \tag{45}$$

7.1 Individual Wage Setting

Stole and Zwiebel (1996) were the first to consider that the firm may take into account the wage equation when deciding employment. As pointed out by them, as Ranjan (2012) in P. 6, says: "this results in overhiring by the firm because it recognizes that hiring an extraworker will reduce the marginal product of each worker and therefore, reduce the wage the firm will pay to each worker." Now the maximizing function for setting the wage is:

$$\beta \ln(\frac{(1-\tau_w)\,\omega - rV_U}{r+\lambda}) + (1-\beta)\ln\left(\frac{F_L(K,AL) - (1+\tau_F)\left[\omega + L\frac{\partial\tilde{\omega}}{\partial L}\right]}{(r+\lambda)}\right)$$

In this case the wage equation is (see appendix):

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} rV_U + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right].$$

Using (23) we get

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)(V_E - V_U) + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right].$$

Calculating $(V_E - V_U)$ using Ranjan method we get (see appendix)

$$(V_E - V_U) = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\gamma(r+\lambda)\omega - \gamma\lambda n L\frac{\partial\tilde{\omega}}{\partial L}}{(1+\tau_F)\,q(\theta)}\right]$$

and then the wage equation is

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\theta\beta\gamma\omega}{(1+\tau_F)} + \frac{\beta}{(1+\tau_F)}\left[F_L - \left(\frac{(1+\tau_F)\,(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right) L\frac{\partial\tilde{\omega}}{\partial L}\right]$$ [27]

And assuming $b_0 = b\omega$ the wage equation becomes

$$\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]\omega = \frac{\beta}{(1+\tau_F)}\left[F_L - \left(\frac{(1+\tau_F)\,(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right) L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

that is:

$$\omega = \frac{1}{\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]}\frac{\beta}{(1+\tau_F)}\left[F_L - \left(\frac{(1+\tau_F)\,(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right) L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

that if the production is Cobb-Douglas ($F(K,L) = A^{1-\alpha}K^{\alpha}L^{1-\alpha}$) gives the solution (see appendix):

$$\omega = \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)} \tag{46}$$

and note that again the wage is a proportion of $F_L = (1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}$ and that an increase in r ($\rho+\sigma x$ in the long run) reduces the wage[28]. Knowing the wage one can compute:

$$L\frac{\partial\tilde{\omega}}{\partial K} = \frac{\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)}A^{1-\alpha}K^{\alpha-1}L^{1-\alpha} \tag{47}$$

and

$$L\frac{\partial\tilde{\omega}}{\partial L} = \frac{-\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)}A^{1-\alpha}K^{\alpha}L^{-\alpha}. \tag{48}$$

Note that $L\frac{\partial\tilde{\omega}}{\partial K} = \alpha\omega\frac{L}{K}$ and $L\frac{\partial\tilde{\omega}}{\partial L} = -\alpha\omega$. Then the sz capital demand equation (41) becomes:

$$\left[\alpha - \frac{\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)-\theta\gamma\lambda n}{(r+\lambda)}\right)}\right]\hat{k}^{\alpha-1} = r+\delta \tag{49}$$

expression that, as noted by Cahuc et al. (2008), incorporates the hold up effect and the Stole and Zwiebel overemployment effect.

The employment equation given by (43) when the production function is Cobb- Douglas becomes:

$$(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha} = F_L = \left[(1+\tau_F)-\frac{\gamma\lambda n}{q(\theta)}\right]L\frac{\partial\tilde{\omega}}{\partial L}+\omega\left[(1+\tau_F)+\gamma\frac{(r+\lambda)}{q(\theta)}\right] \tag{50}$$

and substituting (46) and (48) in (50) the sz "equilibrium" labor market equation is:

$$1-\frac{(1-\beta)b}{(1-\tau_w)}-\alpha\beta-\frac{\beta\gamma}{(1+\tau_F)}\left(1+\frac{\alpha\beta n}{(r+\lambda)}\right)\theta = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)+\frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}\right] \tag{51}$$

Note that this equation holds only for the Cobb-Douglas production function and that depends only on r and θ (not on $\hat{k}$). That is with this specific wage equation r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given

[28] The same relationships are obtained if one substitutes $(V_E - V_U)$ by the equality $V_E - V_U = \frac{\omega - b_0}{r+\lambda+\theta q(\theta)}$ (see appendix).

by the sz capital demand equation (49), θ^* is given by the sz employment and the wage setting equation once one substitutes only r^*(not $\hat{k}^*$) in (51), l^* by the equilibrium of labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21). In order to analyze the effect of x on employment: substitute $r = \rho + \sigma x$ in (51) to get

$$1 - \frac{(1-\beta)b}{(1-\tau_w)} - \alpha\beta - \frac{\beta\gamma}{(1+\tau_F)}\left(1 + \frac{\alpha\beta n}{(\rho+\sigma x+\lambda)}\right)\theta \;=$$

$$\frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F) + \frac{\gamma(\rho+\sigma x+\lambda(1}{q(\theta)}\right.$$

And then an equilibrium in the labor market always exists and there exists $\bar{\sigma}$ such that if $\sigma > \bar{\sigma}$ then $\frac{d\theta^*}{dx} < 0$ and then $\frac{dl^*}{dx} < 0$, if $\sigma = \bar{\sigma}$ then $\frac{d\theta^*}{dx} = 0$ and then $\frac{dl^*}{dx} = 0$ and if $\sigma < \bar{\sigma}$ then $\frac{d\theta^*}{dx} > 0$ and then $\frac{dl^*}{dx} > 0$ (see appendix). The intuition for the result is easy to see looking at the employment equation when one substitutes $L\frac{\partial\tilde{\omega}}{\partial L}$ by $-\alpha\omega$ in (50) having

$$F_L = \omega\left[(1-\alpha)(1+\tau_F) + \frac{\gamma\lambda n}{q(\theta)} + \gamma\frac{(\rho+\lambda+\sigma x)}{q(\theta)}\right],$$

on the one hand an increase in x increases turnover costs by σx and reduces the wage ω then if σ is high enough the increase of turnover costs is greater the reduction of the wage and the the firm opens less vacancies having more unemployment.

7.2 Collective Wage Setting

Now, because capital is irreversible, the objective function to maximize when deciding wages is:

$$\beta \ln\left[\left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta))}\right)L\right] + (1-\beta)\ln\left(\frac{\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN}\right]}{r-(n+x)}\right)$$

where the surplus of the firm S_F in the steady state of employing all workers, is obtained deducting to (24) the cost of open a vacancy and the cost of capital because now investment is irreversible. The solution of the gives the wage equation:

$$\omega = \tilde{\omega}(K,L) = \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]}\frac{\beta}{(1+\tau_F)}\frac{F(K,AL)}{L},$$

where the wage set is a proportion of the average product of labor, that is, labor productivity but not on the interest rate. With the Cobb-Douglas production function the wage equation becomes:

$$\omega = \tilde{\omega}(K, L) = \frac{(1-\tau_w)}{[(1-\tau_w) - (1-\beta)b]} \frac{\beta}{(1+\tau_F)} A^{1-\alpha} K^{\alpha} L^{-\alpha}, \tag{53}$$

and then

$$L\frac{\partial \tilde{\omega}}{\partial K} = \frac{\alpha(1-\tau_w)}{[(1-\tau_w) - (1-\beta)b]} \frac{\beta}{(1+\tau_F)} A^{1-\alpha} K^{\alpha-1} L^{1-\alpha}. \tag{54}$$

$$L\frac{\partial \tilde{\omega}}{\partial L} = \frac{-\alpha(1-\tau_w)}{[(1-\tau_w) - (1-\beta)b]} \frac{\beta}{(1+\tau_F)} A^{1-\alpha} K^{\alpha} L^{-\alpha}. \tag{55}$$

Where again $L\frac{\partial \tilde{\omega}}{\partial L} = -\alpha\omega$. Then the sz capital demand equation is given by

$$\left[\alpha - \frac{\alpha(1-\tau_w)}{[(1-\tau_w) - (1-\beta)b]} \frac{\beta}{(1+\tau_F)}\right] \hat{k}^{\alpha-1} = r + \delta \tag{56}$$

that shows the hold up and the overemployment effect. The sz employment equation is given by:

$$(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha} = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial \tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right] \tag{57}$$

and substituting (53) and (55) in (57)we get (see appendix) the "equilibrium" labor market equation:

$$(1-\alpha)\left[1 - \frac{(1-\beta)b}{(1-\tau_w)}\right] = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F) + \frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}\right] \tag{58}$$

Note that this equation holds only for the Cobb-Douglas production function and that depends only on r and θ (not on $\hat{k}$).

That is with this specific wage equation r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the sz capital demand equation (56), θ^* is given by the sz employment and the wage setting equation once one substitutes only r^* (not $\hat{k}^*$) in (58), l^* by the equilibrium of labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21). In order to analyze the effect of x on employment: substitute $r = \rho + \sigma x$ in (58) to get:

$$(1-\alpha)\left[1 - \frac{(1-\beta)b}{(1-\tau_w)}\right] = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F) + \frac{\gamma(\rho+\sigma x+\lambda(1+\alpha n))}{q(\theta)}\right] \tag{59}$$

And then an equilibrium always exists and $\frac{d\theta^*}{dx} < 0$ and then $\frac{dl^*}{dx} < 0$ (see appendix). The intuition of the result is easy to see is writing again the employment equation as

$$F_L = \omega \left[(1-\alpha)(1+\tau_F) + \frac{\gamma \lambda n}{q(\theta)} + \gamma \frac{(\rho+\lambda+\sigma x)}{q(\theta)} \right].$$

As in the individual wage setting case and increases in x increases turnover cost but now does not decrease the wage meaning that the firm unambiguously opens less vacancies and there is more unemployment.

8 Setting the Wage Using the Employment Function

In this section we assume that the wage is decided first and then the firm decides K and L simultaneously. This means that in the wage bargaining it is known the capital demand equation:

$$F_K = (r+\delta),$$

that does not depend on ω, and the employment equation:

$$F_L = \omega \left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right],$$

that when the production function is Cobb-Douglas one can solve for employment, obtaining:

$$L = \tilde{L}(\omega) = \left[\frac{\frac{\omega}{A}\left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right]}{(1-\alpha)} \right]^{-\frac{1}{\alpha}} \frac{K}{A}, \tag{60}$$

or

$$l = \left[\frac{1-\alpha}{\frac{\omega}{A}\left[(1+\tau_F) + \gamma \frac{(r+\lambda-x)}{q(\theta)} \right]} \right]^{\frac{1}{\alpha}} k.$$

8.1 Individual Wage Setting

In this case the wage is set in order to maximize, as usual:

$$\beta \ln(\frac{\omega - rV_U}{r+\lambda}) + (1-\beta) \ln \left(\frac{F_L(K, AL) - (1+\tau_F)\,\omega}{(r+\lambda)} \right)$$

subject to (16), that gives:

$$\beta \ln(\frac{\omega - rV_U}{r+\lambda}) + (1-\beta) \ln \left(\frac{\gamma \frac{(r+\lambda-x)}{q(\theta)} \omega}{(r+\lambda)} \right)$$

in which case it is obvious that the wage set is going to be infinite wage because raising the wage increases the surplus of an employed worker and the value function for the firm of hiring an extra worker. Galí (1995) presents a more complicated case, where instead of having ω in the surplus function of an individual employed worker he introduces his utility function $U(C, L)$, having that when the individual sets unilaterally the wage, $\beta = 1$, and the production function is Cobb-Douglas, then the wage set by the individual is a mark up, that depends on the elasticity of the employment function, over the marginal rate of substitution.

8.2 Collective Wage Setting: the Union Monopoly Model

In this case the program for setting the wage is to choose ω in order to maximize

$$\beta \ln \left[\left(\frac{(1-\tau_w)\omega - b_0}{r(r+\lambda+\theta q(\theta))} \right) L \right] + (1-\beta) \ln \left(\frac{\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN} - \frac{(n+x+\delta)K}{AN} \right]}{r-(n+x)} \right)$$

subject to the capital demand and the employment equation. It is difficult to find and explicit solution for the wage in this case. Then, as in Ranjan (2012), we consider the limit case of the union monopoly model where the union sets the wage unilaterally, in which case the program is to choose ω in order to maximize

$$\left[\left(\frac{(1-\tau_w)\omega - b_0}{r(r+\lambda+\theta q(\theta))} \right) \tilde{L}(\omega) \right]$$

Then one gets the wage equation (see appendix) [29]:

$$\omega = \frac{1}{(1-\tau_w)(1-\alpha)} b_0 \tag{61}$$

In this case the wage is a mark-up over the unemployment benefit and one can check that this is the same wage equation that one gets in a model "without frictions"[30]. Note that the wage equation does no depend on the matching function $q(\theta)$ nor on K, the reason is that the wage set depends on the elasticity of the employment equation and, when the production function is Cobb-Douglas and $\gamma_0 = \gamma\omega$, the elasticity of the employment equation is constant and equal to $\frac{1}{\alpha}$ with or without frictions.[31] Now, because we are in a growth model, we have to endogenize b_0, the problem is that we can not use the assumption $b_0 = b\omega$ as we have done so far. The reason is that (61) becomes:

[29] The other more general way to proceed is to consider that when setting the wage agents think that investement is irreversible, depending, then, the function to maximize only on quasi rents $\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN}$ in which case one gets the more general wage equation $\omega = \frac{1}{(1-\tau_w)}\left(\frac{\beta}{(1-\alpha)} + (1-\beta)\right) b_0$.

[30] Correctly speaking in the model where the cost of open a vacancy is zero ($\gamma = 0$).

[31] This was shown in Garcia and Sorolla (2013) when matches last for one period that is $\lambda = 1$. Here this result is generalized for any $0 < \lambda < 1$.

$$\omega = \frac{1}{(1-\tau_w)(1-\alpha)} b\omega,$$

that is:

$$1 = \frac{1}{(1-\tau_w)(1-\alpha)} b,$$

meaning that only with this combination of parameters the wage equation and this way of setting the unemployment benefits are compatible for any wage ω set.

The "natural" alternative assumption to make is that the government increases the unemployment benefit with time, having that $b_0 = bA$ in which case (61) becomes:

$$\omega = \frac{b}{(1-\tau_w)(1-\alpha)} A,$$

that is,

$$w = \frac{b}{(1-\tau_w)(1-\alpha)}. \tag{62}$$

The substitution of the wage equation into the employment equation (60) gives the "equilibrium" labor market equation:

$$L = \left[\frac{b}{(1-\tau_w)}\left[(1+\tau_F) + \gamma\frac{(r+\lambda-x)}{q(\theta)}\right]\right]^{-\frac{1}{\alpha}} \frac{K}{A}$$

that is

$$\hat{k} = \left[\frac{b}{(1-\tau_w)}\left[(1+\tau_F) + \gamma\frac{(r+\lambda-x)}{q(\theta)}\right]\right]^{\frac{1}{\alpha}} \tag{63}$$

That is with this specific wage equation r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the capital demand equation (15), θ^* is given by the employment and the wage setting equation once one substitutes r^* and $\hat{k}^*$ in (63), l^* by the equilibrium of labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21). In order to analyze the effect of x on employment: one has to compute $\hat{k}^*$ using the capital demand equation, substitute it in (63) and consider that $r = \rho + \sigma x$ getting:

$$\left[\frac{\alpha}{(\rho+\delta+\sigma x)}\right]^{\frac{\alpha}{1-\alpha}} = \frac{b}{(1-\tau_w)}\left[(1+\tau_F) + \gamma\frac{(\rho+\lambda+(\sigma-1)x)}{q(\theta^*)}\right]$$

And then if $b < \tilde{b}$ an equilibrium always exists and there exists $\tilde{\sigma} < 1$ such that if $\sigma > \tilde{\sigma}$ then $\frac{d\theta^*}{dx} < 0$ and then $\frac{dl^*}{dx} < 0$, if $\sigma = \tilde{\sigma}$ then $\frac{d\theta^*}{dx} = 0$ and then $\frac{dl^*}{dx} = 0$ and if $\sigma < \tilde{\sigma}$ then $\frac{d\theta^*}{dx} > 0$ and then $\frac{dl^*}{dx} > 0$ (see appendix). The intuition now is that an increase in x increases the wage (because $A(t) = e^{xt}$ is higher), on the other hand, if $\sigma > 1$ it also increases the turnover cost having less employment, but if $\sigma < 1$ and increase in x decreases the turnover cost, if this decrease is high (small σ) the total effect on labor costs is negative opening more vacancies.

Alternatively, one can assume that the government sets the unemployment benefit in order to have a balanced budget that is $b_0(1-l) = (\tau_w + \tau_F)\omega l$ and that $\tau_w = \varphi\tau_F$ in which case one gets that the government's budget constraint is:

$$b_0 = (\tau_w + \tau_F)\frac{\omega l}{1-l} = (1+\varphi)\,\tau_F\frac{\omega l}{1-l}$$

and the wage equation (61) becomes:

$$\omega = \frac{1}{(1-\varphi\tau_F)(1-\alpha)}\,(1+\varphi)\,\tau_F\frac{\omega l}{1-l},$$

solving for the employment rate one gets:

$$l^* = \frac{1}{\left[1+\frac{(1+\varphi)\tau_F}{(1-\varphi\tau_F)(1-\alpha)}\right]}$$

which means that the wage equation adding the specific way of financing the unemployment benefit determines the equilibrium amount of employment[32].

That is with this specific wage equation r^* is given by the Euler equation (14), $\hat{k}^*$ (and then $\hat{y}^* = f(\hat{k}^*)$, that is labor productivity) is given by the capital demand equation (15), l^* is given by the wage setting equation (61), θ^* is given by the labor market flows equation (19), k^* by the capital per efficient labor equation (20) and c^* by the capital accumulation equation (21), the employment equation (16) gives the wage.

Note that in this case the long run employment rate does not depend on x that is $\frac{dl^*}{dx} = 0$ for any value of σ because employment is given by the wage equation that does not depend on x. So that means that there is no long run relationship between growth and unemployment. This is the same result that appears in Raurich and Sorolla (2014), where there are no frictions in the labor market and the wage is set using an efficiency wage model.

9 Conclusions

We conclude saying that in general the different timing for setting the wage, capital and labor, the type of wage setting: individual or collective and how the unemployment is financed, that is, different labor market institutions change the long run effect of exoge-

[32] A similar result is obtained if one does $b_0 = \upsilon y$, as in Daveri and Tabellini (2000), having $l = \frac{(1-\tau_w)}{\upsilon\left(\frac{\beta}{(1-\alpha)}+(1-\beta)\right)}$.

	INDIVIDUAL WAGE SETTING	COLECTIVE WAGE SETTING (Cobb-Douglas production function)
Nash (w, L and K are set simultaneously) $b_0=b\omega$	Wage equation: Wage is a proportion of marginal product and decreases with x. Turnover costs increases with x when $\sigma>1$. There is a $\sigma^*>1$ such that: $\sigma>\sigma^*$ $\frac{\partial l}{\partial x}<0$ $\sigma=\sigma^*$ $\frac{\partial l}{\partial x}=0$ $\sigma<\sigma^*$ $\frac{\partial l}{\partial x}>0$	Wage equation: Wage depends on labor productivity and decreases with on x. Turnover costs increases with x when $\sigma>1$. There is a $\hat{\sigma}>1$ such that: $\sigma>\hat{\sigma}$ $\frac{\partial l}{\partial x}<0$ $\sigma=\hat{\sigma}$ $\frac{\partial l}{\partial x}=0$ $\sigma<\hat{\sigma}$ $\frac{\partial l}{\partial x}>0$
Stole and Zwiebel (L and K simultaneously and first and w later) $b_0=b\omega$	Cobb-Douglas production function. Wage equation: Wage is a proportion of marginal product and decreases with x. Turnover costs increases with x. There is a $\bar{\sigma}$ such that: $\sigma>\bar{\sigma}$ $\frac{\partial l}{\partial x}<0$ $\sigma=\bar{\sigma}$ $\frac{\partial l}{\partial x}=0$ $\sigma<\bar{\sigma}$ $\frac{\partial l}{\partial x}>0$	Wage equation: Wage is a proportion of labor productivity and does not depen on x. Turnover costs increases with x. $\frac{\partial l}{\partial x}<0$
Wage setting knowing employment function (w first and K and L simultaneously and later) and $b_0=bA$	Infinite wage	Wage is a mark up over A and increases with x. Turnover costs increases with x when $\sigma>1$. There is a $\tilde{\sigma}<1$ such that: $\sigma>\tilde{\sigma}$ $\frac{\partial l}{\partial x}<0$ $\sigma=\tilde{\sigma}$ $\frac{\partial l}{\partial x}=0$ $\sigma<\tilde{\sigma}$ $\frac{\partial l}{\partial x}>0$
Wage setting knowing employment function (w first and K and L simultaneously and later) + government balanced budget constraint.	Infinite wage	Wage equation: wage does not depend on x and gives a constant employment rate. $\frac{\partial l}{\partial x}=0$

nous labor productivity growth, x, on employment. The reason is that the specific timing, the type of wage negotiation and the way of financing the unemployment benefit changes how the rate of growth affects the wage and turnover costs and then, total labor costs affecting vacancies and employment. The following table summarizes for each labor market institution the effect of x on the wage and the turnover cost and the final effect on employment.

One can see that, depending on the labor market institution, the effect of x on wages can be negative, zero or positive and that the effect on turnover costs positive or negative. However one obtains that for a higher constant relative risk aversion parameter of the individual utility function (σ) the effect of growth on employment is negative for all labor market institutions but one (the case of the union monopoly model when the government adjusts taxes for balancing its budget). This is because, for a higher σ, an increase in x implies a high increase in turnover costs via the long run interest rate ($r = \rho + \sigma x$ in the long run, where ρ is the individual discount rate), that can not be offset by the negative effect of x on wages that occurs with some labor market institutions. Moreover we also have that for a lower σ the effect of growth on employment is positive for all labor market institutions but two (the Stole and Zwiebel case with collective wage setting and the case of the union monopoly model when the government adjusts taxes for balancing its budget) because in this cases the reduction of wages offsets the increases in turnover costs.

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Appendix

This appendix presents the detailed (step by step) proofs of all the results presented in the text.

1.1) Computation of the Nash bargaining wage equation when there is individual wage setting:

The program is to choose ω in order to maximize:

$$\beta \ln(\frac{(1-\tau_w)\,\omega - rV_U}{r+\lambda}) + (1-\beta)\ln\left(\frac{F_L(K,AL)-(1+\tau_F)\,\omega}{(r+\lambda)}\right),$$

that gives as a first order condition:

$$\beta\frac{(1-\tau_w)}{((1-\tau_w)\,\omega - rV_U)} = (1-\beta)\frac{(1+\tau_F)}{F_L(K,AL)-(1+\tau_F)\,\omega},$$

that is:

$$\beta\,(1-\tau_w)\,(F_L(K,AL)-(1+\tau_F)\,\omega) = (1-\beta)\,(1+\tau_F)\,((1-\tau_w)\,\omega - rV_U), \tag{64}$$

and then:

$$\beta\,(1-\tau_w)\,F_L(K,AL) - \beta\,(1-\tau_w)\,(1+\tau_F)\,\omega = (1-\beta)\,(1+\tau_F)\,((1-\tau_w)\,\omega - rV_U)$$

or:

$$\beta\,(1-\tau_w)\,F_L(K,AL) = (1-\beta)\,(1-\tau_w)\,(1+\tau_F)\,\omega - (1-\beta)\,(1+\tau_F)\,rV_U + \beta\,(1-\tau_w)\,(1+\tau_F)\omega$$

that is:

$$(1+\tau_F)\,(1-\tau_w)\,\omega = \beta\,(1-\tau_w)\,F_L(K,AL) + (1-\beta)\,(1+\tau_F)\,rV_U$$

and finally:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}rV_U + \frac{\beta}{(1+\tau_F)}F_L(K,AL).$$

Using (23) we get:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)(V_E - V_U) + \frac{\beta}{(1+\tau_F)}F_L(K,AL).$$

1.2) Calculation of $(V_E - V_U)$ following Ranjan's method (using the employment equation).

Using (22) we get

$$(V_E - V_U) = \frac{1}{r+\lambda}\left((1-\tau_w)\,\omega - rV_U\right),$$

that is,

$$(r+\lambda)(V_E - V_U) = \left((1-\tau_w)\,\omega - rV_U\right),$$

then we can rewrite (64) as:

$$\beta\,(1-\tau_w)\,(F_L(K,AL) - (1+\tau_F)\,\omega) = (1-\beta)\,(1+\tau_F)\,(r+\lambda)(V_E - V_U),$$

that is:

$$(V_E - V_U) = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\frac{F_L(K,AL) - (1+\tau_F)\,\omega}{(1+\tau_F)} = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{F_L(K,AL)}{(1+\tau_F)} - \omega\right]$$

Now using the employment equation (16):

$$F_L = \left[(1+\tau_F) + \gamma\frac{(r+\lambda-x)}{q(\theta)}\right]\omega$$

we substitute F_L getting:

$$\begin{aligned}(V_E - V_U) &= \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\left[(1+\tau_F) + \gamma\frac{(r+\lambda-x)}{q(\theta)}\right]\omega}{(1+\tau_F)} - \omega\right] = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{[(1+\tau_F)\,q(\theta) + \gamma(r+ \ldots}{(1+\tau_F)\,q(\theta \ldots}\right. \\ &= \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{[(1+\tau_F)\,q(\theta) + \gamma(r+\lambda-x)]}{(1+\tau_F)\,q(\theta)} - 1\right]\omega = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\gamma(r+\lambda-x)}{(1+\tau_F)\,q(\theta)}\right]\end{aligned}$$

1.3) Computation of the Nash bargaining wage equation when there is individual wage wetting computing $(V_E - V_U)$ using (22) and (23).

Using (22) and (23) one gets:

$$(V_E - V_U) = \left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta))}\right)$$

and then the wage equation becomes:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta))}\right) + \frac{\beta}{(1+\tau_F)}F_L(K,AL)$$

assuming $b_0 = b\omega$ one gets:

$$\omega = \frac{(1-\beta)b\omega}{(1-\tau_w)} + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w) - b)\,\omega}{(r+\lambda+\theta q(\theta))}\right) + \frac{\beta}{(1+\tau_F)}F_L(K,AL)$$

that is:

$$\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega=\frac{\beta}{(1+\tau_F)}F_L(K,AL)$$

or:

$$\left[\frac{(1-\tau_w)}{(1-\tau_w)}-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega=\frac{\beta}{(1+\tau_F)}F_L(K,AL)$$

that is:

$$\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega=\frac{\beta\,(1-\tau_w)}{(1+\tau_F)}F_L(K,AL)$$

and simplifying:

$$\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]\omega=\frac{\beta\,(1-\tau_w)}{(1+\tau_F)}F_L(K,AL)$$

in which case the wage equation becomes:

$$\omega=\frac{1}{\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]}\frac{\beta\,(1-\tau_w)}{(1+\tau_F)}F_L(K,AL)$$

1.4) Existence of the Nash Equilibrium when there is individual wage setting.

The equilibrium labor market equation is:

$$\left[1-\frac{(1-\beta)}{(1-\tau_w)}b-\frac{\theta\gamma\beta(\varrho+\lambda+(\sigma-1)x)}{(1+\tau_F)\,(\varrho+\lambda+\sigma x)}\right]=\frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)}\right]$$

or

$$(1-\tau_w)-(1-\beta)b-(1-\tau_w)\left[\frac{\theta\beta\gamma(\varrho+\lambda+(\sigma-1)x)}{(\varrho+\lambda+\sigma x)\,(1+\tau_F)}\right]=\frac{\beta\,(1-\tau_w)}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\rho+\lambda+(\sigma-1)\,x)}{q(\theta)}\right]$$

that is

$$(1-\tau_w)-(1-\beta)b-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[\frac{(\varrho+\lambda+(\sigma-1)x)}{(\varrho+\lambda+\sigma x)}\right]\theta=\frac{\beta\,(1-\tau_w)}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\rho+\lambda+(\sigma-1)\,x)}{q(\theta)}\right]$$

then the left hand side is equal to $(1-\tau_w)-(1-\beta)b$ (>0 when $b<(1-\tau_w)$) when $\theta=0$ and is decreasing with θ. The right hand side is equal to $\beta(1-\tau_w)$when $\theta=0$ (lower than $(1-\tau_w)-(1-\beta)b$ if $b<(1-\tau_w)$) and is increasing with θ and then both lines intersect for a positive θ.

1.5) Calculation of $\frac{\partial\theta}{\partial x}$and σ^*.

From the equilibrium labor market equation we have:

$$G(\theta,x)=(1-\tau_w)-(1-\beta)b-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[\frac{(\varrho+\lambda+(\sigma-1)x)}{(\varrho+\lambda+\sigma x)}\right]\theta-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\left[(1+\tau_F)+\gamma\frac{(\varrho+\lambda+(\sigma}{q(\theta)}\right.$$

then, by the implicit function theorem:

$$\frac{\partial\theta}{\partial x}=-\frac{\frac{dG}{dx}}{\frac{dG}{d\theta}}=-\frac{\frac{(1-\tau_w)\theta\beta\gamma}{(1+\tau_F)}\frac{(\varrho+\lambda)}{(\varrho+\lambda+\sigma x)^2}-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\sigma-1)}{q(\theta)}}{-(1-\tau_w)\frac{\beta\gamma(\varrho+\lambda+(\sigma-1)x)}{(\varrho+\lambda+\sigma x)(1+\tau_F)}+\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\varrho+\lambda+(\sigma-1)x)q'(\theta)}{(q(\theta))^2}}$$

Note that $q'(\theta)<0$ and then the denominator is negative (remember that from the program of the family $\rho+(\sigma-1)x>0$) and then positive with the minus sign, so $\frac{\partial\theta}{\partial x}>0$ if

$$\frac{(1-\tau_w)\theta\beta\gamma}{(1+\tau_F)}\frac{(\varrho+\lambda)}{(\varrho+\lambda+\sigma x)^2}>\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\sigma-1)}{q(\theta)}$$

$$\theta\frac{(\varrho+\lambda)}{(\varrho+\lambda+\sigma x)^2}>\frac{(\sigma-1)}{q(\theta)}$$

Note that if $\sigma\leqq 1$ then $\theta\frac{(\varrho+\lambda)}{(\varrho+\lambda+\sigma x)^2}>\frac{(\sigma-1)}{q(\theta)}$ and then $\frac{\partial\theta}{\partial x}>0$. Therefore $\sigma^*>1$ is given by:

$$(\sigma^*-1)(\varrho+\lambda+\sigma^*x)^2=\theta q(\theta)(\varrho+\lambda).$$

1.6) Derivation of the tax rates with a balanced budget constraint:

We have that a balanced budget constraint means:

$$b_0(1-l)=(\tau_w+\tau_F)\omega l$$

and if $b_0=b\omega$ then

$$b(1-l)=(\tau_w+\tau_F)l$$

that is

$$(\tau_w+\tau_F)=\frac{1-l}{l}b$$

and if $\tau_w = \varphi\tau_F$

$$\tau_F = \frac{1-l}{l}\frac{b}{(1+\varphi)}$$

and then:

$$\tau_w = \frac{1-l}{l}\frac{\varphi b}{(1+\varphi)} = \left(\frac{1}{l}-1\right)\frac{\varphi b}{(1+\varphi)}.$$

On the other hand the equilibrium labor market flows equation is:

$$l = \left[\frac{1}{1+\frac{\lambda+n}{\theta^* q(\theta^*)}}\right]$$

or:

$$\left(\frac{1}{l}-1\right) = \frac{\lambda+n}{\theta^* q(\theta^*)}$$

And then:

$$\tau_F = \frac{b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

that is:

$$1+\tau_F = 1+\frac{b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

and

$$\tau_w = \frac{\varphi b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

that is:

$$1-\tau_w = 1-\frac{\varphi b}{(1+\varphi)}\frac{\lambda+n}{\theta^* q(\theta^*)}$$

2.1) Computation of the Nash bargaining wage equation when there is collective wage setting:

The program is to choose ω in order to maximize:

$$\beta \ln\left[\left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta)}\right)L\right] + (1-\beta)\ln\left(\frac{\left[\frac{F(K,AL)}{AN} - (1+\tau_F)\frac{\omega L}{AN} - (n+\delta+x)\frac{K}{AN}\right]}{r-(n+x)}\right)$$

that gives as first order condition:

$$\beta \frac{(1-\tau_w)}{((1-\tau_w)\omega - b_0)} = (1-\beta)\frac{(1+\tau_F)\,L}{[F(K,AL) - [(1+\tau_F)]\,\omega L - (n+\delta+x)K]}$$

that is:

$$\beta \frac{(1-\tau_w)\,[F(K,AL) - [(1+\tau_F)]\,\omega L - (n+\delta+x)K]}{(1+\tau_F)\,L} = (1-\beta)((1-\tau_w)\omega - b_0)$$

and then:

$$\beta \frac{(1-\tau_w)}{(1+\tau_F)} \frac{[F(K,AL) - (n+\delta+x)K]}{L} - \beta(1-\tau_w)\omega = (1-\beta)(\omega - b_0)$$

or:

$$(1-\tau_w)\omega - (1-\beta)b_0) = \beta \frac{(1-\tau_w)}{(1+\tau_F)} \left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right]$$

that is:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\beta}{(1+\tau_F)} \left[\frac{F(K,AL)}{L} - (n+\delta+x)\frac{K}{L} \right]$$

2.2) Existence of the Nash Equilibrium when there is Collective Wage Setting:

The equilibrium labor market equation is

$$\frac{[(1-\tau_w) - (1-\beta)b]}{(1-\tau_w)} \frac{(1-\alpha)}{1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}} = \frac{\beta}{(1+\tau_F)} \left[(1+\tau_F) + \gamma \frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)} \right]$$

or

$$[(1-\tau_w) - (1-\beta)b] \frac{(1-\alpha)}{1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}} = \beta \frac{(1-\tau_w)}{(1+\tau_F)} \left[(1+\tau_F) + \gamma \frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)} \right]$$

The left hand side is constant and positive when $b < (1-\tau_w)$ and $\rho > (1-\sigma)x+n$. The right hand side is equal to $\beta\,(1-\tau_w)$when $\theta = 0$ (lower than $[(1-\tau_w) - (1-\beta)b]\,\frac{(1-\alpha)}{1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}}$ when $\alpha < \tilde{\alpha} = \frac{(1-\tau_w)-(1-\beta)b-\beta(1-\tau_w)}{(1-\tau_w)-(1-\beta)b-\frac{(n+\delta+x)}{\rho+\delta+\sigma x}\beta(1-\tau_w)} < 1$) and is increasing with θ and then both lines intersect for a positive θ.

2.3) Calculation of $\frac{\partial\theta}{\partial x}$and $\hat{\sigma}$. From the equilibrium labor market equation we have

$$G(\theta,x) = [(1-\tau_w) - (1-\beta)b] \frac{(1-\alpha)}{1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}} - \beta \frac{(1-\tau_w)}{(1+\tau_F)} \left[(1+\tau_F) + \gamma \frac{(\varrho+\lambda+(\sigma-1)x)}{q(\theta)} \right] = 0$$

then, by the implicit function theorem

$$\frac{\partial\theta}{\partial x}=-\frac{\frac{dG}{dx}}{\frac{dG}{d\theta}}=-\frac{\left[(1-\tau_w)-(1-\beta)b\right]\frac{\alpha(1-\alpha)\left(\frac{(\rho-\sigma n+(1-\sigma)\delta}{(\rho+\delta+\sigma x)^2}\right)}{\left[1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}\right]^2}-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma(\sigma-1)}{q(\theta)}}{\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\varrho+\lambda+(\sigma-1)x)q'(\theta)}{(q(\theta))^2}}$$

Note that $q'(\theta)<0$ and then the denominator is negative and then positive with the minus sign then $\frac{\partial\theta}{\partial x}>0$ if :

$$\left[(1-\tau_w)-(1-\beta)b\right]\frac{\alpha\left(1-\alpha\right)\left(\frac{(\rho-\sigma n-(\sigma-1)\delta}{(\rho+\delta+\sigma x)^2}\right)}{\left[1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\sigma x}\right]^2}-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma(\sigma-1)}{q(\theta)}>0$$

Now the term $\rho-\sigma n-(\sigma-1)\delta>0$ if $\sigma<\frac{\rho+\delta}{n+\delta}=\breve{\sigma}>1$ because $\varrho>n$. Then if $\sigma\leqq 1$ we have that $\left[(1-\tau_w)-(1-\beta)b\right]\frac{(1-\alpha)\rho+\sigma x+\delta+\sigma(n+\delta+x)}{\alpha\left[1-(n+\delta+x)\frac{\rho+\sigma x+\delta}{\alpha}\right]^2}-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma(\sigma-1)}{q(\theta)}>0$ and then $\frac{\partial\theta}{\partial x}>0$.

If $\sigma>\breve{\sigma}>1$ then $\left[(1-\tau_w)-(1-\beta)b\right]\frac{(1-\alpha)\rho+\sigma x+\delta+\sigma(n+\delta+x)}{\alpha\left[1-(n+\delta+x)\frac{\rho+\sigma x+\delta}{\alpha}\right]^2}-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma(\sigma-1)}{q(\theta)}<0$ and then $\frac{\partial\theta}{\partial x}<0$. And finally there exists $1<\hat{\sigma}<\breve{\sigma}$ such that

$$\left[(1-\tau_w)-(1-\beta)b\right]\frac{\alpha\left(1-\alpha\right)\left(\frac{(\rho-\hat{\sigma}n-(\hat{\sigma}-1)\delta}{(\rho+\delta+\hat{\sigma}x)^2}\right)}{\left[1-\alpha\frac{(n+\delta+x)}{\rho+\delta+\hat{\sigma}x}\right]^2}=\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma(\hat{\sigma}-1)}{q(\theta)}.$$

3.1) Computation of the Stole and Zwiebel wage equation when there is individual wage setting.

The program is choose ω in order to maximize:

$$\beta\ln\left(\frac{(1-\tau_w)\,\omega-rV_U}{r+\lambda}\right)+(1-\beta)\ln\left(\frac{F_L(K,AL)-(1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}-(1+\tau_F)\,\omega}{(r+\lambda)}\right)$$

that gives as a first order condition:

$$\beta\frac{(1-\tau_w)}{((1-\tau_w)\,\omega-rV_U)}=(1-\beta)\frac{(1+\tau_F)}{F_L(K,AL)-(1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}-(1+\tau_F)\,\omega},\qquad(65)$$

that is,

$$\beta\,(1-\tau_w)\left(F_L(K,AL)-(1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}-(1+\tau_F)\,\omega\right)=(1-\beta)\,(1+\tau_F)\,((1-\tau_w)\,\omega-rV_U)$$

and then:

$$\beta\left(1-\tau_w\right)\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]-\beta\left(1-\tau_w\right)\left(1+\tau_F\right)\omega=(1-\beta)\left(1+\tau_F\right)\left(\left(1-\tau_w\right)\omega-rV_U\right)$$

or:

$$\beta\left(1-\tau_w\right)\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]=(1-\beta)\left(1-\tau_w\right)\left(1+\tau_F\right)\omega-(1-\beta)\left(1+\tau_F\right)rV_U+\beta\left(1-\tau_w\right)$$

that is:

$$\left(1+\tau_F\right)\left(1-\tau_w\right)\omega=\beta\left(1-\tau_w\right)\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]+(1-\beta)\left(1+\tau_F\right)rV_U$$

and solving for ω :

$$\omega=\frac{(1-\beta)}{(1-\tau_w)}rV_U+\frac{\beta}{(1+\tau_F)}\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

using (23) we get:

$$\omega=\frac{(1-\beta)}{(1-\tau_w)}b_0+\frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)(V_E-V_U)+\frac{\beta}{(1+\tau_F)}\left[F_L-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

3.2) Calculation of (V_E-V_U) following Ranjan's method (using the employment equation) and derivation of the wage equation substituting (V_E-V_U).

Using (22) we get:

$$(V_E-V_U)=\frac{1}{r+\lambda}\left(\left(1-\tau_w\right)\omega-rV_U\right)$$

that is:

$$\left(r+\lambda\right)\left(V_E-V_U\right)=(1-\tau_w)\omega-rV_U$$

Then we can write (65) as

$$\beta\frac{\left(1-\tau_w\right)}{\left(r+\lambda\right)\left(V_E-V_U\right)}=(1-\beta)\frac{\left(1+\tau_F\right)}{\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]-\left(1+\tau_F\right)\omega}$$

that is

$$(V_E-V_U)=\frac{\left(1-\tau_w\right)}{r+\lambda}\frac{\beta}{1-\beta}\frac{\left[F_L(K,AL)-\left(1+\tau_F\right)L\frac{\partial\tilde{\omega}}{\partial L}\right]-\left(1+\tau_F\right)\omega}{\left(1+\tau_F\right)}=\frac{\left(1-\tau_w\right)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\left[F_L(K,AL)-\right.}{(1+}\right.$$

and then using (43), one gets:

$$F_L = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

that is:

$$F_L - (1+\tau_F)\, L\frac{\partial\tilde{\omega}}{\partial L} = \left[-\frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

that substituted in $(V_E - V_U)$ gives:

$$(V_E - V_U) = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\left[-\frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]}{(1+\tau_F)} - \omega\right] = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\left[-\frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L}}{(1+}\right.$$

that is

$$(V_E - V_U) = \frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\gamma(r+\lambda)\omega - \gamma\lambda n L\frac{\partial\tilde{\omega}}{\partial L}}{(1+\tau_F)\, q(\theta)}\right]$$

substituting $(V_E - V_U)$ in the wage equation:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)(V_E - V_U) + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\, L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

one obtains:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\frac{(1-\tau_w)}{r+\lambda}\frac{\beta}{1-\beta}\left[\frac{\gamma(r+\lambda)\omega - \gamma\lambda n L\frac{\partial\tilde{\omega}}{\partial L}}{(1+\tau_F)\, q(\theta)}\right] + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\, L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

that is:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{\theta\beta}{r+\lambda}\left[\frac{\gamma(r+\lambda)\omega - \gamma\lambda n L\frac{\partial\tilde{\omega}}{\partial L}}{(1+\tau_F)}\right] + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\, L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

and:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{\theta\beta\gamma\omega}{(1+\tau_F)} - \left[\frac{\theta\beta\gamma\lambda n L\frac{\partial\tilde{\omega}}{\partial L}}{(r+\lambda)(1+\tau_F)}\right] + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\, L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

and:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\theta\beta\gamma\omega}{(1+\tau_F)} + \frac{\beta}{(1+\tau_F)} F_L - \left[\beta + \frac{\theta\beta\gamma\lambda n}{(r+\lambda)(1+\tau_F)}\right] L \frac{\partial\tilde{\omega}}{\partial L}$$

and:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\theta\beta\gamma\omega}{(1+\tau_F)} + \frac{\beta}{(1+\tau_F)} F_L - \left[\beta \left(1 + \frac{\theta\gamma\lambda n}{(r+\lambda)(1+\tau_F)}\right)\right] L \frac{\partial\tilde{\omega}}{\partial L}$$

and finally:

$$\omega = \frac{(1-\beta)}{(1-\tau_w)} b_0 + \frac{\theta\beta\gamma\omega}{(1+\tau_F)} + \frac{\beta}{(1+\tau_F)} \left[F_L - \left(\frac{(1+\tau_F)(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right) L \frac{\partial\tilde{\omega}}{\partial L}\right]$$

Assuming that $b_0 = b\omega$ and solving for ω one gets:

$$\omega = \frac{1}{\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]} \frac{\beta}{(1+\tau_F)} \left[F_L - \left(\frac{(1+\tau_F)(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right) L \frac{\partial\tilde{\omega}}{\partial L}\right]$$

We can write this equation as:

$$\omega = a \left[F_L - bL \frac{\partial\tilde{\omega}}{\partial L}\right]$$

that is:

$$\omega = ab \left[\frac{1}{b} F_L - L \frac{\partial\tilde{\omega}}{\partial L}\right]$$

and if the production function is Cobb-Douglas one can check that the solution is:

$$\omega = a \frac{(1-\alpha) A^{1-\alpha} K^{\alpha} L^{-\alpha}}{(1-\alpha ab)}$$

substituting a and b one gets:

$$\omega = \frac{\beta}{(1+\tau_F)\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]} \frac{(1-\alpha) A^{1-\alpha} K^{\alpha} L^{-\alpha}}{\left[1 - \alpha \frac{\beta}{(1+\tau_F)\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]} \left(\frac{(1+\tau_F)(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right)\right]}$$

and simplifying the wage equation is

$$\omega = \frac{(1-\alpha) A^{1-\alpha} K^{\alpha} L^{-\alpha}}{\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha \left(\frac{(1+\tau_F)(r+\lambda) + \theta\gamma\lambda n}{(r+\lambda)}\right)}$$

and note that again the wage is a mark up on $F_L = (1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}$ and that an increase in r reduces the wage because $\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)} = (1+\tau_F) + \frac{\theta\gamma\lambda n}{(r+\lambda)}$ decreases.

3.3 Computation of the Stole and Zwiebel wage equation when there is individual wage setting computing $(V_E - V_U)$ using (22) and (23).

We know that:

$$(V_E - V_U) = \left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta))}\right)$$

and substituting $(V_E - V_U)$ in the wage equation

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0 + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{(1-\tau_w)\omega - b_0}{(r+\lambda+\theta q(\theta))}\right) + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

assuming $b_0 = b\omega$ one gets

$$\omega = \frac{(1-\beta)b\omega}{(1-\tau_w)} + \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w)-b)\,\omega}{(r+\lambda+\theta q(\theta))}\right) + \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

that is

$$\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega = \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

or

$$\left[\frac{(1-\tau_w)}{(1-\tau_w)} - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{(1-\beta)}{(1-\tau_w)}\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega = \frac{\beta}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

and

$$\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\theta q(\theta)\left(\frac{((1-\tau_w)-b)}{(r+\lambda+\theta q(\theta))}\right)\right]\omega = \frac{\beta\,(1-\tau_w)}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

and

$$\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]\omega = \frac{\beta\,(1-\tau_w)}{(1+\tau_F)}\left[F_L - (1+\tau_F)\,L\frac{\partial\tilde{\omega}}{\partial L}\right]$$

in which case the wage equation becomes:

$$\omega = \frac{1}{\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]} \frac{\beta\,(1-\tau_w)}{(1+\tau_F)} \left[F_L - (1+\tau_F)\, L \frac{\partial \tilde{\omega}}{\partial L}\right]$$

and then the solution is:

$$\omega = \frac{1}{\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]} \frac{\beta\,(1-\tau_w)}{(1+\tau_F)} \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{(1-\alpha \frac{1}{\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]} \frac{\beta(1-}{(1+\tau}}$$

that is:

$$\omega = \frac{1}{\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]} \frac{\beta\,(1-\tau_w)}{(1+\tau_F)} \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\left(\frac{\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]-\alpha\beta(1-\tau_w)}{\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]}\right.}$$

and then:

$$\omega = \frac{\beta\,(1-\tau_w)}{(1+\tau_F)} \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\left(\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right] - \alpha\beta\,(1-\tau_w)\right)}$$

or

$$\omega = \frac{1}{\frac{(1+\tau_F)}{\beta(1-\tau_w)}} \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\left(\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right] - \alpha\beta\,(1-\tau_w)\right)}$$

or

$$\omega = \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\left(\frac{(1+\tau_F)}{\beta(1-\tau_w)}\left[(1-\tau_w) - (1-\beta)b - (1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right] - \alpha\,(1+\tau_F)\right)}$$

having finally:

$$\omega = \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\left(\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{(1-\beta)}{(1-\tau_w)}\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right] - \alpha\,(1+\tau_F)\right)}$$

and then again an increase in r diminishes the wage.

We can compare it with the one obtained in the Nash equilibrium that is:

$$\omega = \frac{\beta\left(1-\tau_w\right)}{\left(1+\tau_F\right)} \frac{F_L(K, AL)}{\left[(1-\tau_w)-(1-\beta)b-(1-\beta)\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]}$$

that is

$$\omega = \frac{F_L(K, AL)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{(1-\beta)}{(1-\tau_w)}\left(\frac{((1-\tau_w)-b)}{(\frac{r+\lambda}{\theta q(\theta)}+1)}\right)\right]}$$

seeing that in the Stole and Zwiebel wage equation the wage is higher than the Nash wage equation.

We can also compare it with wage equation obtained when computing $(V_E - V_U)$ using Ranjan's method that is:

$$\omega = \frac{(1-\alpha)A^{1-\alpha}K^{\alpha}L^{-\alpha}}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)-\theta\gamma\lambda n}{(r+\lambda)}\right)}$$

3.4) Calculation of the equilibrium capital demand and the equilibrium employment equation.

From the wage equation (46) one gets

$$L\frac{\partial\omega}{\partial K} = \frac{\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)}A^{1-\alpha}K^{\alpha-1}L^{1-\alpha}$$

and

$$L\frac{\partial\omega}{\partial L} = \frac{-\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)}A^{1-\alpha}K^{\alpha}L^{-\alpha}.$$

The capital demand equation is

$$f'\left(\hat{k}\right) = r+\delta+L\frac{\partial\tilde{\omega}}{\partial K}$$

that is

$$\left[\alpha-\frac{\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\theta\beta\gamma}{(1+\tau_F)}\right]-\alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)}\right]\hat{k}^{\alpha-1} = r+\delta$$

The employment equation is

$$F_L = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

and with the Cobb-Douglas production function:

$$(1-\alpha)AK^{\alpha}L^{-\alpha} = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

3.5) Existence of equilibrium in the Stole and Zwiebel case when there is individual wage setting: Substituting the wage equation and $L\frac{\partial\omega}{\partial L}$ in the employment equation on gets

$$(1-\alpha)AK^{\alpha}L^{-\alpha} = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] \frac{-\alpha(1-\alpha)}{\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)} AK^{\alpha}L^{-\alpha} + \frac{\cdots}{\frac{(1+\tau\cdots}{\beta}}$$

$$1 = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right] \frac{-\alpha}{\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right)} + \frac{1}{\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right]\cdots}$$

$$\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right) = \left[(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right] - \alpha\left[(1+\cdots\right.$$

$$\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right) = (1-\alpha)(1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)} + \alpha\frac{\gamma\lambda\cdots}{q(\theta\cdots}$$

$$\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left(\frac{(1+\tau_F)(r+\lambda)+\theta\gamma\lambda n}{(r+\lambda)}\right) = (1-\alpha)(1+\tau_F) + \frac{\gamma(r+\lambda(1+\alpha n\cdots}{q(\theta)}$$

$$\frac{(1+\tau_F)}{\beta}\left[1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)}\right] - \alpha\left((1+\tau_F) + \frac{\theta\gamma\lambda n}{(r+\lambda)}\right) = (1-\alpha)(1+\tau_F) + \frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}$$

$$1 - \frac{(1-\beta)b}{(1-\tau_w)} - \frac{\theta\beta\gamma}{(1+\tau_F)} - \alpha\frac{\beta}{(1+\tau_F)}\left((1+\tau_F) + \frac{\theta\gamma\lambda n}{(r+\lambda)}\right) = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F) + \frac{\gamma(r+\lambda(1+\cdots}{q(\theta)}\right.$$

$$1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\beta\gamma}{(1+\tau_F)}\left(1+\frac{\alpha\beta n}{(r+\lambda)}\right)\theta-\alpha\beta=\frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)+\frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}\right]$$

$$1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\beta\gamma}{(1+\tau_F)}\left(1+\frac{\alpha\beta n}{(r+\lambda)}\right)\theta-\alpha\beta=\frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)+\frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}\right]$$

Substituting x one gets

$$1-\frac{(1-\beta)b}{(1-\tau_w)}-\frac{\beta\gamma}{(1+\tau_F)}\left(1+\frac{\alpha\beta n}{(\rho+\sigma x+\lambda)}\right)\theta-\alpha\beta=\frac{\beta}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)+\frac{\gamma(\rho+\sigma x+\lambda(1+\alpha n}{q(\theta)}\right.$$

and then the equilibrium employment equation in the Stole and Zwiebel case is:

$$(1-\tau_w)(1-\alpha\beta)-(1-\beta)b-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[1+\frac{\alpha\beta n}{(\varrho+\sigma x+\lambda)}\right]\theta=\frac{\beta(1-\tau_w)}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)+\frac{\gamma(\varrho+\sigma}{}\right.$$

that one can compare with the one obtained in the Nash case that is:

$$(1-\tau_w)-(1-\beta)b-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[\frac{(\varrho+\lambda+(\sigma-1)x)}{(\varrho+\lambda+\sigma x)}\right]\theta=\frac{\beta(1-\tau_w)}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(\rho+\lambda+(\sigma-1)x)}{q(\theta)}\right]$$

then the left hand side is equal to $(1-\tau_w)(1-\alpha\beta)-(1-\beta)b$ (>0 when $b<(1-\tau_w)$) when $\theta=0$ and is decreasing with θ. The right hand side is equal to $\beta(1-\alpha)(1-\tau_w)$when $\theta=0$ (lower than $(1-\tau_w)(1-\alpha\beta)-(1-\beta)b$ when $b<(1-\tau_w)$) and is increasing with θ and then both lines intersect for a positive θ. So an equilibrium always exists.

3.3) Calculation of $\frac{\partial\theta}{\partial x}$and $\bar{\sigma}$. From the equilibrium labor market equation we have:

$$G(\theta,x)=(1-\tau_w)(1-\alpha\beta)-(1-\beta)b-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[1+\frac{\alpha\beta n}{(\varrho+\sigma x+\lambda)}\right]\theta-\frac{\beta(1-\tau_w)}{(1+\tau_F)}\left[(1-\alpha)(1+\tau_F)-\right.$$

then, by the implicit function theorem

$$\frac{\partial\theta}{\partial x}=-\frac{\frac{dG}{dx}}{\frac{dG}{d\theta}}=-\frac{\frac{(1-\tau_w)\theta\beta\gamma}{(1+\tau_F)}\frac{\alpha\beta n\sigma}{(\varrho+\lambda+\sigma x)^2}-\frac{(1-\tau_w)\beta}{(1+\tau_F)}\frac{\gamma\sigma}{q(\theta)}}{-\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\left[1-\frac{\alpha\beta n}{(\varrho+\sigma x+\lambda)}\right]+\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\varrho+\sigma x+\lambda(1+\alpha n))q'(\theta)}{(q(\theta))^2}}$$

Note that $q'(\theta)<0$ and then the denominator is negative and then positive with the minus sign, and the numerator is positive when

$$\frac{(1-\tau_w)\theta\beta\gamma}{(1+\tau_F)}\frac{\alpha\beta n\sigma}{(\varrho+\lambda+\sigma x)^2}-\frac{(1-\tau_w)\beta}{(1+\tau_F)}\frac{\gamma\sigma}{q(\theta)}>0$$

that is:

$$\frac{(1-\tau_w)\,\theta\beta\gamma}{(1+\tau_F)}\frac{\alpha\beta n\sigma}{(\varrho+\lambda+\sigma x)^2} > \frac{(1-\tau_w)\,\beta}{(1+\tau_F)}\frac{\gamma\sigma}{q(\theta)}$$

or

$$\frac{\alpha\beta n\theta}{(\varrho+\lambda+\sigma x)^2} > \frac{1}{q(\theta)}$$

that is when

$$\alpha\beta n\theta q(\theta) > (\varrho+\lambda+\sigma x)^2$$

and then

$$\sigma < \frac{\sqrt{\alpha\beta n\theta q(\theta)}-\varrho+\lambda}{x} = \bar{\sigma}$$

4.1) Computation of the Stole and Zwiebel wage equation when there is collective wage setting.

The function to maximize is:

$$\beta\ln\left[\left(\frac{(1-\tau_w)\omega-b_0}{r(r+\lambda+\theta q(\theta))}\right)L\right]+(1-\beta)\ln\left(\frac{\left[\frac{F(K,AL)}{AN}-(1+\tau_F)\,\frac{\omega L}{AN}\right]}{r-(n+x)}\right)$$

that gives as first order condition:

$$\beta\frac{(1-\tau_w)}{((1-\tau_w)\omega-b_0)} = (1-\beta)\frac{(1+\tau_F)\,L}{[F(K,AL)-[(1+\tau_F)]\,\omega L]}$$

that is

$$\beta\frac{(1-\tau_w)\,[F(K,AL)-[(1+\tau_F)]\,\omega L]}{(1+\tau_F)\,L} = (1-\beta)((1-\tau_w)\omega-b_0)$$

and then

$$\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{[F(K,AL)]}{L}-\beta(1-\tau_w)\omega = (1-\beta)(\omega-b_0)$$

or

$$(1-\tau_w)\omega-(1-\beta)b_0) = \beta\frac{(1-\tau_w)}{(1+\tau_F)}\left[\frac{F(K,AL)}{L}\right]$$

that is

$$\omega = \frac{(1-\beta)}{(1-\tau_w)}b_0+\frac{\beta}{(1+\tau_F)}\left[\frac{F(K,AL)}{L}\right]$$

4.2) Existence of equilibrium in the Stole and Zwiebel case when there is collective wage setting:

The wage equation is:

$$\omega = \tilde{\omega}(K,L) = \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]}\frac{\beta}{(1+\tau_F)}AK^{\alpha}L^{-\alpha}$$

and then:

$$L\frac{\partial\tilde{\omega}}{\partial L} = \frac{-\alpha(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]}\frac{\beta}{(1+\tau_F)}AK^{\alpha}L^{-\alpha}$$

as usual the employment equation is given by:

$$(1-\alpha)AK^{\alpha}L^{-\alpha} = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right]L\frac{\partial\tilde{\omega}}{\partial L} + \omega\left[(1+\tau_F)+\gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

and substituting the wage equation and $L\frac{\partial\tilde{\omega}}{\partial L}$ the equilibrium employment equation is:

$$(1-\alpha) = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right]\frac{-\alpha(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]}\frac{\beta}{(1+\tau_F)} + \frac{(1-\tau_w)}{[(1-\tau_w)-(1-\beta)b]}\frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\right.$$

that is:

$$(1-\alpha)\left[1-\frac{(1-\beta)b}{(1-\tau_w)}\right] = \left[(1+\tau_F) - \frac{\gamma\lambda n}{q(\theta)}\right]\frac{-\alpha\beta}{(1+\tau_F)} + \frac{\beta}{(1+\tau_F)}\left[(1+\tau_F)+\gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

and:

$$(1-\alpha)\left[1-\frac{(1-\beta)b}{(1-\tau_w)}\right] = \frac{\beta}{(1+\tau_F)}\left[-\alpha\left(1+\tau_F\right) + \frac{\alpha\gamma\lambda n}{q(\theta)} + (1+\tau_F) + \gamma\frac{(r+\lambda)}{q(\theta)}\right]$$

and finally:

$$(1-\alpha)\left[1-\frac{(1-\beta)b}{(1-\tau_w)}\right] = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)\left(1+\tau_F\right) + \frac{\gamma(r+\lambda(1+\alpha n))}{q(\theta)}\right]$$

substituting r one gets:

$$(1-\alpha)\left[1-\frac{(1-\beta)b}{(1-\tau_w)}\right] = \frac{\beta}{(1+\tau_F)}\left[(1-\alpha)\left(1+\tau_F\right) + \frac{\gamma(\rho+\sigma x+\lambda(1+\alpha n))}{q(\theta)}\right]$$

that is

$$(1-\alpha)\left[(1-\tau_w)-(1-\beta)b\right]=\beta\frac{(1-\tau_w)}{(1+\tau_F)}\left[(1-\alpha)\left(1+\tau_F\right)+\frac{\gamma(\rho+\sigma x+\lambda(1+\alpha n))}{q(\theta)}\right]$$

then the left hand side is constant and positive when $b<(1-\tau_w)$ when $\theta=0$. The right hand side is equal to $(1-\alpha)\beta\,(1-\tau_w)$when $\theta=0$ (that is lower than $(1-\alpha)\left[(1-\tau_w)-(1-\beta)b\right]$ when $b<(1-\tau_w)$ and is increasing with θ and then both lines intersect for a positive θ.

4.3) Calculation of $\frac{\partial\theta}{\partial x}$. From the equilibrium labor market equation we have:

$$G(\theta,x)=(1-\alpha)\left[(1-\tau_w)-(1-\beta)b\right]-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\left[(1-\alpha)\left(1+\tau_F\right)+\frac{\gamma(\rho+\sigma x+\lambda(1+\alpha n))}{q(\theta)}\right]=0$$

then, by the implicit function theorem

$$\frac{\partial\theta}{\partial x}=-\frac{\frac{dG}{dx}}{\frac{dG}{d\theta}}=-\frac{-\beta\frac{(1-\tau_w)}{(1+\tau_F)}\frac{\gamma\sigma}{q(\theta)}}{\frac{(1-\tau_w)}{(1+\tau_F)}\beta\gamma\frac{(\rho+\sigma x+\lambda(1+\alpha n))q'(\theta)}{(q(\theta))^2}}$$

Note that $q'(\theta)<0$ and then the denominator is negative and then positive with the minus sign, and the numerator is negative when $\sigma>0$.

5.1) Computation of the wage equation when setting the wage knowing the employment function when there is collective wage setting.

The program is to choose ω in order to

maximize

$$\beta\ln\left[\left(\frac{(1-\tau_w)\omega-b_0}{r(r+\lambda+\theta q(\theta))}\right)L\right]$$

subject to

$$L=\left[\frac{\omega\left[(1+\tau_F)+\gamma\frac{(r+\lambda-x)}{q(\theta)}\right]}{(1-\alpha)A^{1-\alpha}}\right]^{-\frac{1}{\alpha}}K=\left[\frac{\omega c}{(1-\alpha)A^{1-\alpha}}\right]^{-\frac{1}{\alpha}}K=c^{-\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}A^{\frac{1-\alpha}{\alpha}}K\omega^{-\frac{1}{\alpha}}=X\omega^{-\frac{1}{\alpha}}$$

that is, to maximize

$$\beta\ln\left[\left(\frac{(1-\tau_w)\omega-b_0}{r(r+\lambda+\theta q(\theta))}\right)X\omega^{-\frac{1}{\alpha}}\right]$$

The optimal wage will be the same than the one obtained maximizing the function:

$$\left[\left((1-\tau_w)\omega-b_0\right)\omega^{-\frac{1}{\alpha}}\right]$$

The first order condition is:

$$\left[(1-\tau_w)\omega^{-\frac{1}{\alpha}} - \frac{1}{\alpha}\left((1-\tau_w)\omega - b_0\right)\omega^{-\frac{1}{\alpha}-1}\right] = 0$$

or:

$$\left[(1-\tau_w)\omega^{-\frac{1}{\alpha}} - \frac{1}{\alpha}\frac{((1-\tau_w)\omega - b_0)}{\omega}\omega^{-\frac{1}{\alpha}}\right] = 0$$

that simplifying gives:

$$(1-\tau_w) = \frac{((1-\tau_w)\omega - b_0)}{\alpha\omega}$$

that is

$$(1-\tau_w)\alpha\omega = ((1-\tau_w)\omega - b_0)$$

or

$$(1-\tau_w)(1-\alpha)\omega = b_0$$

and then the wage is:

$$\omega = \frac{b_0}{(1-\tau_w)(1-\alpha)}$$

5.2) Existence of equilibrium when setting the wage using the employment function when there is collective wage setting.

The equilibrium labor market equation is:

$$\left[\frac{\alpha}{(\rho+\delta+\sigma x)}\right]^{\frac{\alpha}{1-\alpha}} = \frac{b}{(1-\tau_w)}\left[(1+\tau_F) + \gamma\frac{(\rho+\lambda+(\sigma-1)x)}{q(\theta^*)}\right]$$

Then the left hand side is constant and positive. The right hand side is equal to $\frac{b(1+\tau_F)}{(1-\tau_w)}$when $\theta = 0$ (that is lower than $\left[\frac{\alpha}{(\rho+\delta+\sigma x)}\right]^{\frac{\alpha}{1-\alpha}}$ when $b < \tilde{b} = \frac{(1-\tau_w)}{1+\tau_F}\left[\frac{\alpha}{(\rho+\delta+\sigma x)}\right]^{\frac{\alpha}{1-\alpha}}$ and is increasing with θ and then both lines intersect for a positive θ.

5.3) Calculation of $\frac{\partial\theta}{\partial x}$ and computation of $\tilde{\sigma}$. From the equilibrium labor market equation we have:

$$G(\theta, x) = \left[\frac{\alpha}{(\rho+\delta+\sigma x)}\right]^{\frac{\alpha}{1-\alpha}} - \frac{b}{(1-\tau_w)}\left[(1+\tau_F) + \gamma\frac{(\rho+\lambda+(\sigma-1)x)}{q(\theta^*)}\right] = 0$$

then, by the implicit function theorem

$$\frac{\partial\theta}{\partial x} = -\frac{\frac{dG}{dx}}{\frac{dG}{d\theta}} = -\frac{-\frac{\alpha}{1-\alpha}\left(\frac{\alpha}{(\rho+\delta+\sigma x)}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\alpha\sigma}{(\varrho+\delta+\sigma x)^2} - \frac{b}{(1-\tau_w)}\frac{\gamma(\sigma-1)}{q(\theta)}}{-\frac{b}{(1-\tau_w)}\frac{(\rho+\lambda+(\sigma-1)x)q'(\theta)}{(q(\theta))^2}}$$

Note that $q'(\theta) < 0$ and then the denominator is negative and then positive with the minus sign. If $\sigma \gtreqqless 1$ then the numerator is negative and then there exists $\tilde{\sigma} < 1$ such that $-\frac{\alpha}{1-\alpha}\left(\frac{\alpha}{(\rho+\delta+\tilde{\sigma}x)}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\alpha\tilde{\sigma}}{(\varrho+\delta+\tilde{\sigma}x)^2} - \frac{b}{(1-\tau_w)}\frac{\gamma(\tilde{\sigma}-1)}{q(\theta)} = 0$ and then if $\sigma < \tilde{\sigma}$ and the numerator is positive.

5.4) Derivation of the wage equation when the government has a balanced budget constraint. Eliminating ω one gets:

$$1 = \frac{(1+\varphi)\,\tau_F}{(1-\varphi\tau_F)(1-\alpha)}\frac{l}{1-l}$$

that is:

$$1-l = \frac{(1+\varphi)\,\tau_F}{(1-\varphi\tau_F)(1-\alpha)}l$$

or

$$1 = \left[1+\frac{(1+\varphi)\,\tau_F}{(1-\varphi\tau_F)(1-\alpha)}\right]l$$

and then:

$$l = \frac{1}{\left[1+\frac{(1+\varphi)\tau_F}{(1-\varphi\tau_F)(1-\alpha)}\right]}.$$