The economic value of reciprocal bilingualism*

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Abstract

Some bilingual societies exhibit a distribution of language skills that cannot be explained by economic theories that portray languages as pure communication devices. Such distribution of skills are typically the result of public policies that promote bilingualism among members of both speech communities (reciprocal bilingualism). In this paper I argue that these policies are likely to increase social welfare by diminishing economic and social segmentation between the two communities. However, these gains tend to be unequally distributed over the two communities. As a result, in a large range of circumstances these policies might not draw sufficient support. The model is built upon the communicative value of languages, but also emphasizes the role of linguistic preferences in the behavior of bilingual individuals..

KEYWORDS: bilingualism, segmentation, linguistic preferences, network externalities

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1 Introduction

Language skills are typically attained in a sequential fashion. Individuals initially acquire a first language at home (*native language* or mother tongue) and later in their life they learn one or more second languages. It is well understood that the economic return on second language acquisition is particularly high in societies with multiple native languages. Indeed, a well functioning economy requires a common language, a *lingua franca,* and the majority language is usually the most efficient candidate. If size does matter for efficiency, what should be the role of a language with a relatively small number of native speakers in a multilingual environment? Indeed, some policies that promote the learning and use of minority languages are constantly scrutinized and often heavily criticized. Even more so in societies where all minority speakers are also competent in the majority language and hence the ability to communicate is not at stake. In this context, does it make any sense to promote the learning of a minority language among members of the majority speech community? Can such reciprocal bilingualism generate any economic gains, given that these additional skills are apparently redundant?

These are the main questions I address in this paper. I build on an extensive literature that has taken an economic approach to study languages. Since the distribution of language skills affects economic outcomes (see, for example, Mélitz (2008) on international trade, and Chiswick (2008) on labor market outcomes), economists have also paid attention to the incentives to acquire second languages. The standard game-theoretic analysis (Selten and Pool,1991; Church and King, 1993)[1] portrays different languages as alternative communication devices. Individuals decide whether or not to acquire a second language by comparing the learning costs (in terms of time and money) with the benefits (the ability to communicate and do business with members of other speech communities.) An important ingredient of this approach is the presence of network externalities: the value of knowing a language increases with the number of its speakers. When they take decisions in isolation, individuals do not internalize such externalities and hence, with respect to the social optimum, underinvest in language skills. In other words, the private return on second language acquisition is lower than the social return, which opens the door to efficiency-enhancing public policies.

[1] See also Güth et al.(1997), Gabszewicz et al. (2008) and Ortega and Tangerås (2008). Ginsburgh and Weber (2011) provide an excellent overview of the economic research on languages.

Models based exclusively on the communicative value of languages have significantly contributed to our understanding of the relation between language skills and economic outcomes. However, they are somehow put under stress when we examine bilingual societies where monoglots of the minority language practically do not exist and as a result communication is guaranteed by the universal knowledge of the majority language. Below I pay particular attention to three European regions that meet this criterion: Wales, the Basque Country and Catalonia. Regarding the acquisition of a second language, we observe that a fraction of the majority speech community do learn the minority language. On top of that, native speakers of the minority language, despite the fact that they are bilingual, transmit the minority language to the next generation.[2] Regarding economic outcomes, there is some empirical evidence indicating that bilingual individuals enjoy a comparative advantage with respect to monoglots of the majority language in labor markets.[3] Thus, markets seem to price positively the knowledge of the minority language for reasons beyond its communicative value.

In contrast to most of the economic literature, sociolinguists have always emphasized that a language is much more than a communication device. Language and culture are tightly interconnected and both impact on a person's identity. As a result, when people chooses which language to use, learn or transmit, they take into account not only relative communicative advantages but also their emotional attachment. Using the economics jargon we can say that individuals develop preferences about the language they use daily and eventually transmit to the next generation.

The notion of *linguistic preferences* has already been implicitly or explicitly used in a variety of contexts. Wickström (2005) builds a dynamic model to examine the

[2] The implications of the traditional approach on the pattern of intergenerational transmission of languages have not been completely spelled out. Suppose that bilingual parents can only transmit one language. If they care about the welfare of their children then they will prefer to transmit the majority language and minimize their children' subsequent learning costs. The problem becomes more complicated if bilingual parents can transmit both languages. However, provided that in this case they incur into an incremental transmission cost, then the minority language will be expected to gradually disappear.

John and Yi (2001) develop a dynamic model of language learning and transmission, focusing exclusively on the communicative value of languages. Unfortunately, decisions about intergenerational language transmission are not formalized as the solution of an optimization problem, but rather represented by a reduced form. Nevertheless, in their model the long-run survival of minority languages requires geographic isolation.

[3] In particular, bilingual individuals face better job opportunities and obtain higher earnings. See Drinkwater and O'Leary (1997) and Henley and Jones (2005) for the case of Wales, and Rendon (2007) and Di Paolo and Raymond (2012) for the case of Catalonia.

survival of minority languages. In his framework the intergenerational transmission of languages implicitly depends not only on the practical value of the language as means of communication, but also on the emotional attachment of the parents. However, linguistic preferences are not specified and decisions about language transmission are represented by a reduced form. Similarly, Grin (1992) presents a dynamic model to study the evolution of language use. Linguistic preferences are explicitly specified, but the set of decisions that determine the evolution of language skills are not (he also takes a reduced form approach).[4] In a different vein, Iriberri and Uriarte (2012) examine the role of the information structure in determining the use of minority languages in occasional social interactions. In their model language skills and preferences are fixed. Finally, in a previous paper (Caminal, 2010), I studied the provision of linguistic diversity in markets for cultural goods and media products. Like Iriberri and Uriarte, I assume a fixed distribution of language skills and preferences.[5]

In this paper I ignore the long-run dynamics of language use and transmission, and instead focus on the role of linguistic preferences in the feedback between the distribution of language skills and economic outcomes. I argue that the propensity to cooperate with members of other speech communities does not only depend on the ability to communicate but also on the potential conflict of interests over the language of use. In other words, while I build on the communicative value of languages I also emphasize that multilingual individuals are not indifferent about the language of use in different types of social and economic interactions. The model is static in the sense that linguistic preferences are exogenous, but language skills are endogenously determined. In combination with network externalities, linguistic preferences drastically change the welfare assessment of alternative distributions of language skills. In particular, I claim that policies that promote reciprocal bilingualism (everyone learning the other, non-native language), by limiting the scope of the conflict over the language of use, can improve the patterns of cooperation in society and generate significant welfare gains.[6]

[4] Consequently, neither of these two models is suitable for welfare analysis.

[5] In the context of a monopolistically competitive model I show that, with respect to the social optimum, market outcomes are biased in favor of stronger languages. In other words, markets provide too little linguistic diversity.

[6] Clearly, linguistic preferences have a social component closely related to ethnic or national identity. The analysis in this paper clearly falls short in this dimension. On top of the effects studied here, reciprocal bilingualism may reduce interethnic conflict by reducing observable differences. However, some groups may oppose policies that promote reciprocal bilingualism if they

The next section reviews a sample of bilingual countries and regions in Europe and North America. The sample is admittedly small, but nevertheless sufficiently rich to reflect a large variety of situations, both in terms of the relative strength of the two languages and the propensities of members of each community to become bilingual. It is emphasized that in some of these regions we observe a distribution of language skills that can not be explained by the communicative value of languages. However, it also becomes readily apparent that examples exhibiting high levels of reciprocal bilingualism are hard to find. Both observations are rationalized by the theoretical model.

Sections 3 and 4 examine a simple model that illustrates the main trade-offs. I consider the impact of alternative distributions of language skills in a bilingual society (the two languages are labeled S and W) where individuals obtain economic gains from cooperating with others. These cooperative tasks, or partnerships, require communication and hence the use of a particular language. Some high productivity partnerships include two individuals with different mother tongues. Hence, as in traditional models, cooperation requires partners to share a common language. In this framework the ability to communicate is not sufficient in order to materialize all potential gains from cooperation because individuals have linguistic preferences (although they differ in the intensity of those preferences.) Those individuals with a strong preference for using their mother tongue are willing to give up the extra economic gains from a high productivity partnership in exchange for a better linguistic match. More specifically, in the regime where only $W-$native speakers become bilingual (asymmetric bilingualism), then any high productivity match between two linguistically heterogeneous individuals must use language S. As a result, those $W-$native speakers with a strong preference for the use of their mother tongue will choose to quit the high productivity match and settle in a linguistically homogeneous, lower productivity match. The privately optimal separation rule is not socially optimal because the potential partner (in this case, a $S-$speaker) losses with the separation. Alternatively, in the regime where members of both speech communities become bilingual (reciprocal bilingualism) the fraction of inefficient separations is lower. The reason is twofold. First, the distribution of language skills is symmetric and cooperation can take place in either language. Hence, there is a higher chance that at least one of the members of the match feels less strongly

perceive them as a threat to their identity.

about the language of use and accepts conducting the tasks in her second language. Second, it becomes possible to adopt a more balanced pattern of language use (say, fifty-fifty), which again makes unilateral separations less likely.[7] As a result, provided learning costs are not too high, reciprocal bilingualism is socially optimal because it minimizes economic and social segmentation.[8]

The distributional implications of shifting from the first to the second regime are not trivial. It is clear that $W-$native speakers always gain since the frequency of inefficient separations decreases and, moreover, they enjoy a more favorable linguistic treatment. However, even abstracting from learning costs, the effect on $S-$native speakers is ambiguous. On the one hand, they benefit from less frequent inefficient separations, yet on the other hand lose their power to impose their preferred language.[9]

The model must be closed by specifying how learning decisions are taken. Several alternative scenarios are considered. In the first scenario (following the standard literature) learning decisions are taken by individuals. Such an assumption may not be very suitable in some cases. In particular, the examples reviewed in Section 2 indicate that public policies are essential in determining the distribution of language skills. Thus, we need to consider the case that learning patterns are the result of some kind of collective action. An important aspect is the choice of jurisdiction, at least in cases where speech communities are geographically segmented. Thus, I explore a second scenario where learning decisions are taken by two different bodies, each representing the interests of a speech community. These two scenarios do not necessarily provide the same answer, since individual decisions may also generate an externality on other members of their speech community. Finally, in the third scenario decisions are centralized. In this case the specific characteristics of the political system are crucial to predict how possible conflicts of interests may be

[7] In the present model differences in language skills prevent exploiting all the gains from cooperation. In the real world these differences may be also used to generate monopolistic rents by, for instance, restricting the access of certain jobs to speakers of a particular language. A model capturing such rent seeking activities would provide additional channels by which reciprocal bilingualism is bound to raise social welfare.

[8] There is a sizable literature on occupational segregation by race and ethnicity. See Patel and Vella (2007) or Hellerstein and Neumark (2008) for recent examples. Language seems an important part of these stories. However, the existing data does not allow us to separate the ability to communicate from linguistic preferences.

[9] Lazear (1999) examines the incentives of immigrants to form ghettos in order to avoid the costs of learning the majority language (or, more generally, the costs of assimilation). In my model segmentation occurs also because bilingual individuals have linguistic preferences.

resolved.[10]

Section 5 presents various extensions of the baseline model, including heterogeneous learning costs, the possibility of using monetary compensations in exchange for a more favorable language treatment, the size of cooperation groups, and the weight of speech communities. Finally, Section 6 presents some concluding remarks.

2 The distribution of language skills in bilingual societies

The coexistence of two (or more) languages in the same area is compatible with different distributions of language skills over the population. In particular, members of different speech communities may have different propensities to learn the other language. It will be useful to define three stylized scenarios. In the first, all individuals remain monolingual and hence communication across speech communities is impossible (Segmented Bilingualism, SB). A second clear-cut scenario is one where all native speakers of a particular language, say language W, learn language S, but all $S-$native speakers remain monolingual. In this case, the relative position of the two languages is highly asymmetric, with one strong language (S) that serves also as a lingua franca (in inter-community communication) and a weak language (W), used only in intra-community communication (Asymmetric Bilingualism, AB). In the third extreme scenario all individuals become bilingual and hence in principal there is no hierarchy between the two languages, since inter-community communication can take place in either language (Reciprocal Bilingualism, RB).[11] The chief focus of this paper is on the second and third benchmarks, but this section offers a broader perspective, since most real world examples tend to be more mixed. In order to consider intermediate cases, it will be useful to introduce two indices, an index of communication (due to Lieberson, 1964) and an index of reciprocity (a natural extension) between speech communities. In particular, if we denote by α^i the fraction of $i-$native speakers that are competent in $j, i, j = S, W, i \neq j$, then we can define the index of communication, C, as:

$$C = \alpha^S + \left(1 - \alpha^S\right) \alpha^W$$

[10] Under majority voting the relative size of speech communities plays a crucial role, However, other regimes may be able to implement more efficient outcomes.

[11] Linguists tend to classify bilingual societies according to the functional distribution of the languages. In fact our three scenarios are closely related to the three standard functional categories: *bipartlingualism*, *disglossia*, and *ambilingualism*.

This index takes values between 0 and 1, and measures the probability that in an heterogenous bilateral match (one S and on $W-$speaker) there is at least one common language, and hence subjects can communicate. We can also define the index of reciprocity, R, as

$$R = \alpha^W \alpha^S$$

Note that R also takes values between 0 and 1 and measures the fraction of redundancies. That is, in an heterogeneous bilateral match R measures the probability that both individuals are bilingual, and hence from a strictly communicative point of view one of the languages is redundant.[12]

Thus, in the first scenario (SB) we have $C = R = 0$, in the second (AB) $C = 1$, $R = 0$, and in the third (RB) $C = R = 1$.

Next, let us briefly discuss a sample of bilingual regions or countries and whether they exemplify the three extreme scenarios mentioned above. For each case I have relied on a different source and as a result there is no guarantee the values of the α's reported are comparable. However, comparability is not a major concern since the purpose here is purely illustrative. Also note that the political status of these geographical areas varies considerably. Again, my immediate concern is not to explain their differences, but rather to simply demonstrate the mere existence of alternative patterns of bilingualism.

The first benchmark (SB) is relatively uncommon since geographical proximity and/or political ties tend to generate strong incentives to overcome language barriers. Cyprus may perhaps be considered a sufficiently good example. For the last four hundred years the two main languages of the island have been Turkish and Greek. Historically, many Turkish-Cypriots also spoke Greek, while Greek-Cypriots almost always remained monoglots. However, as a result of the *de facto* political separation of the two communities in 1975, the number of bilingual Cypriots is nowadays negligible, especially among the younger generations (See, Özerk, 2001).

Many European regions where the vernacular language is still the mother tongue of a fraction of the population are fairly close to the second benchmark (AB). In a few cases minority languages have recently experienced a modest expansion. Two well-known examples are Wales and the Basque Country. In Wales Welsh is

[12]Note that these indices are independent of: (i) the relative size of speech communities and (ii) the distance between the two languages. Hence, they provide a rough measure of the average language skills of an heterogenous match, but they neither reflect the frequency of these matches nor the difficulty of learning the second language. .

spoken by about 20% of the population (2001 census), and practically everybody speaks English. The UK census does report knowledge of languages but not mother tongues, hence it is impossible to learn the fraction of English native speakers that are able to speak Welsh. However, the renaissance of Welsh in the main urban areas and among the 5-14 age group in the 1990's suggest that a non-negligible fraction of English native speakers became competent in Welsh (Aitchison and Carter, 2004). The education reforms in the late 1980's were a crucial driving force by making Welsh compulsory in schools up to age 16.

Data about mother tongues is more readily available for the Basque Country. Spanish is the native language of 78% of the population, while the other 22% report Basque as their mother tongue (Gobierno Vasco, 2006). As in the case of Wales, all Basque speakers know the dominant language (Spanish, in this case). However, a significant 13% of the native Spanish speakers also know Basque. In fact, Basque is compulsory in schools and recently has become the main language of instruction for two thirds of students in primary and secondary school.

Both Wales and the Basque Country seem to deviate from the AB scenario. Since all native speakers of the minority language do learn the dominant language, in terms of the ability to communicate native speakers of the majority language do not need to learn the minority language: their learning efforts appear somewhat redundant. Nevertheless, some of them do it under a policy scheme that can be interpreted as a gesture of reciprocity.

Let us turn now to case where a non-negligible fraction of both speech communities are monoglots: Quebec and Belgium. In Quebec there are two official languages, French and English. According to the 2006 Census, 8% of the population speak English and 79% speak French as their native language. Although French is more popular in the province, English is more valuable as a lingua franca outside Quebec. It turns out that 57% of native English speakers also acquire French as a second language, while 44% of native French speakers know English.[13] Thus, the incentives to become bilingual seem stronger for native English speakers: the local dominance of French apparently dominates the value of English as a lingua franca. If we take Canada a whole as the reference area then the relative size of these two

[13] These figures cannot be directly computed from census data. In order to pin down these coefficients I have assumed that those individuals whose mother tongue is a non-official language (12%) learn either French or English in proportion to the size of these two speech communities. If some members of this group learn them both then the real value of these two coefficients would be smaller.

coefficients would be reversed, since few English native speakers outside Quebec know French.

From a linguistic point of view Belgium can be divided into three regions: Flanders, populated mainly by Dutch native speakers (almost 60% of the Belgian population), Wallonia, populated mainly by French native speakers (more than 30% of the population) and Brussels, one of the capitals of the European Union, a melting pot of Belgian citizens and foreigners, where French is somewhat more predominant than Dutch. This is why I have chosen the entire country as our reference area, as otherwise we end up with linguistically homogeneous regions. According to Van Parijs (2007), just 15% of French native speakers acquire Dutch as a second language, while more than one half of Dutch speakers learn French. Thus, in this specific case the incentives to become bilingual seem to be much stronger for Dutch speakers, not because of domestic reasons (the French speaking community accounts for less than 40% of the total population) but rather due to the value of French as a lingua franca. The values of α's reported above are easily explained by the course offers of second languages, which are independently determined by each region (Desmet and Vermeire, 2000).[14]

The patterns of second-language learning in Belgium and Quebec are compatible with the traditional theory of languages as pure communication devices (Gabsewitz et al., 2008). In both cases a significant fraction of the two linguistic communities can afford to remain monolingual. However, many others do become fully bilingual. A positive level of reciprocity (as measured by our index) is consistent with the traditional theory since it is privately optimal that individuals learn a second language in order to gain access to the monoglots in the other community. In other words, from a purely communicative point of view their learning efforts may very well not be redundant in expectation.

However, the standard theory cannot explain the patterns of language acquisition in Wales and the Basque Country. Since everybody there speaks English or Spanish the learning efforts of the members of the dominant speech community seem redundant. However, the fraction of individuals with presumably *anomalous*

[14]Several former soviet republics exhibit language patterns similar to those in Belgium and Quebec. According to Ukraine's 2001 Census and Latvia's 2011 Census, about two thirds of the population consider the local languages (Ukrainian and Latvian) their mother tongue, and the other third report Russian. While two-thirds of native speakers of the local language also speak Russian, the fraction of Russian native speakers that also speak the local language vary from 12% in Ukraine to 60% in Latvia.

behavior is relatively small, and hence some observers may not value them as forceful counterexamples. In this respect our last example, Catalonia, is more striking. In this region the degree of reciprocal bilingualism is exceptionally high. In a recent survey (Generalitat de Catalunya, 2008), 32% of the population reported Catalan, while 57% reported Spanish as their mother tongue.[15] As in the Basque Country, all native Catalan speakers also know Spanish. What is more unusual is that about 75% of the Spanish native speakers also speak Catalan. Thus, the degree of reciprocal bilingualism is not too far from its maximum.

A crucial determinant of the widespread knowledge of Catalan among Spanish native speakers is the language policy of the regional government. The explicit aim is universal bilingualism and the main field of intervention is the school system where Catalan is the main language of instruction.[16] It is noteworthy that those policies have enjoyed a broad social consensus. Over the last thirty years policies that promote Catalan, and determine its privileged role in compulsory education, have only been challenged by small political parties that together never represented more than 15% of the seats in the regional parliament. In fact, casual evidence suggests that the language policy of the Catalan government has been contested much more strongly in the rest of Spain than in Catalonia.[17] Summarizing, the pattern of bilingualism in Catalonia is rather unique, at least in the context of Europe and North America. Both the distribution of language skills and the broad consensus over policies that generate such a distribution are somewhat puzzling for those who adopt the view that languages are basically neutral communication codes.

The next table reports the values of C and R for the regions discussed above, and provides a quantitative overview.

[15] The relative number of Catalan native speakers have drastically decreased over the last fifteen years as a result of very large immigration flows, specially from Latin America.

[16] Such an asymmetric language policy in favor of the weaker language produces balanced results: (i) At the end of compulsory education students' level of proficiency in Catalan and Spanish are similar (Consell Superior d'Avaluació del Sistema Educatiu, 2007). Moreover, the level of proficiency in Spanish of students coming out of Catalan schools is similar to the rest of Spain (Instituto de Evaluación, 2011).

[17] According to a recent survey (Fabà and Llaberia, 2011) the level of social conflict associated to the use of languages in Catalonia is very small. In particular, a large majority of respondents report no problem in using either language. Only 5% of respondents report trouble or inconvenience for having used Spanish, while using Catalan has been somewhat problematic for 15% of the respondents.

	C	R
Cyprus	small	very small
Wales	1.00	small
Basque Country	1.00	0.13
Quebec	0.76	0.25
Belgium	0.59	0.08
Ukraine	0.70	0.08
Latvia	0.90	0.44
Catalonia	1.00	0.75

Two main lessons can be drawn from the previous discussion. First, the cases of Wales, the Basque Country and especially Catalonia are difficult to reconcile with the traditional economic theory of second language acquisition. If a language was a neutral communication device and all members of the minority were bilingual, then native speakers of the dominant language would never be willing to incur the learning costs when the communicative value of their effort is null.[18] In other words, they would actively oppose public policies promoting reciprocal bilingualism. Second, examples with high levels of R are hard to find.

One could perhaps try to explain the first observation by invoking some kind of distortion in the political system. I doubt there exists a plausible political model that will be able to explain the broad political consensus in Catalonia. The second observation could perhaps be interpreted as an indication that reciprocal bilingualism is not able to generate sizable social gains. This paper offers an alternative view: reciprocal bilingualism may cause a relatively large increase in social welfare, but in many scenarios it might fail to gather sufficient political support because those gains tend to be unequally distributed across speech communities.

3 The benchmark model

Consider an economy populated by a mass one of individuals, whose first language is either S or W. For now suppose that the two speech communities have the same size. The first language or mother tongue will determine the linguistic preferences of bilingual individuals according to the pattern specified below.

[18]It is important to note than in these regions native speakers of minority languages learn the majority language and nevertheless they pass the minority language on to the next generation. See Fabà and Llaberia (2011) for the case of Catalonia. Linguistic preferences appear as an essential ingredient of any relevant explanation of such transmission pattern.

Learning a second language provides two types of return. First, it facilitates domestic interactions. This is the main focus of the analysis, and hence these type of benefits are endogenous. Second, the newly acquired language may also be useful in non-domestic interactions and/or may provide access to the stock of media and cultural goods supplied in that language. These latter benefits will be described by a fixed (exogenous) parameter, $B^i, i = S, W$. Without loss of generality, let $B^S \geq B^W$. In other words, despite of the fact the two communities have the same size, S is a relatively stronger language, because of its non-domestic status.[19]

Individuals play the following two-stage game. In the first stage, they may acquire a second language by incurring a cost F (same for both S and W). An individual that learns a second language becomes equally functional in both languages, but this does not imply that is indifferent about the language used in any social interaction. For the moment I assume that each individual pays its own learning cost.[20]

In the second stage, once the language skills of the population are fixed, individuals obtain the two types of returns on their learning efforts. Regarding the exogenous benefits I will first focus on the case $B^S > F$ and $B^W = 0$. The first assumption, $B^S > F$, implies that $W-$native speakers have always incentives to learn S: the only possible equilibrium scenarios are AB and RB. The second assumption, $B^W = 0$, is a normalization, since $S-$native speakers will take their learning decisions depending on whether the endogenous benefits from learning W are higher or lower than the "net costs", $F - B^W$. As for the endogenous benefits, I assume that each individual can potentially form a high productivity partnership only with a particular individual that is randomly selected from the entire population. Everyone has the same probability of becoming an individual's potential partner, independently of their language skills and preferences. A high productivity partnership can only be formed if partners can communicate with each other (if they share at least one language) and if they reach an agreement over the language of communication. If the partnership materializes then payoffs depend on the language used. An $i-$native speaker obtains 1 if i is adopted as the language of

[19] For instance, in the case of Belgium, French would be the stronger language, S, since it is more valuable than Dutch as an international lingua franca. Similarly, in the case of Catalonia it is also obvious that Spanish is the stronger language.

[20] Alternatively, we can best think of F as the total cost, including the time and effort that individuals must exert, but also the financial expenses (classrooms, books, etc.), which can perhaps be funded by general taxation. Such a distinction will become important when we examine the distributional effects of public policies.

communication. On the contrary, she obtains $1-\lambda$ if $j, j \neq i$, is adopted. λ is an independent realization of a random variable distributed over $\left[0, \overline{\lambda}\right]$ with density function $h(\lambda)$ and distribution function $H(\lambda)$. Let us denote by $\widehat{\lambda}$ the unconditional average. Summarizing, each partner obtains a monetary return of 1 from a high productivity partnership, and λ reflects the disutility (measured in monetary units) associated to the use of a second language.[21]

Thus, communication in a high productivity partnership can take place in S or in W. But it can also take place in both languages (on a fifty-fifty basis), a regime denoted by SW. In this case, each individual obtains $1-\mu\lambda$, where $\mu \leq \frac{1}{2}$ is an exogenous parameter. That is, the utility loss from using a second language half of the time is less or equal than one half the loss from using a second language all the time. If interpreted literally, this assumption indicates that the marginal disutility of using the second language for an additional unit of time (weakly) decreases with the amount of time using the second language. Alternatively, a fifty-fifty arrangement can also be interpreted as if both partners are in fact able to use their native language. In a two-person environment this may simply mean that individuals can send messages in their native language but they receive messages in their second language. A parameter μ lower than $\frac{1}{2}$ may reflect the fact that individuals care relatively more about the language of the messages they send than of the messages they receive.[22] Finally, $\mu \leq \frac{1}{2}$ may simply reflect equity considerations: Using a fair scheme may be jointly perceived as more desirable than any extreme pattern.

If the highly productive partnership does not form, then the expected payoff for each individual in the alternative activity is $1-\theta < 1$. Such a payoff can be interpreted as the monetary return obtained in a less productive activity that does not require the use of a language. Similarly, it may be the return from cooperating with other available members of the same speech community.

For simplicity, it is assumed that in the first stage individuals only know the distribution of λ (all individuals are ex-ante identical in this respect), and privately

[21] λ can represent indistinctively a pure preference for using the mother tongue or the limited competence in the second language, and hence the extra effort needed to communicate using that language. Vaillancourt (1982) shows that, in the case of francophones in Quebec, the preference for using their mother tongue in consumption activities weakens as their fluency in English improves, Nevertheless, those individuals with excellent knowledge of English still have a pure preference for using French.

[22] In the context of more complex organizations, multilateral communication can take place simultaneously in more than one language. Thus, an SW regime may simply mean that each individual can communicate with the center in her native language and the use of the second language is limited to interactions in small groups

learn their realization at the beginning of the second stage, right before they meet their potential partners. At this point, both their native language and knowledge of a second language are common knowledge. The only dimension that remains private information is their own realization of λ. Such asymmetry of information appears as a reasonable representation of the real world and will play a crucial role in the analysis. In Section 5 I comment on the effects of the imperfect observability of language skills and native languages.[23]

Also, for simplicity it is assumed that potential partners negotiate over the language of communication using a very simple bargaining protocol.[24] The three possible linguistic regimes are S, W or SW. When potential partners meet one of them is chosen with equal probability as the proponent, who makes a take-it-or-leave-it offer within the set (S, SW, W). The respondent either accepts or rejects the offer. If both members of the potential partnership belong to the same speech community there is obviously no obstacle in using the common native language as the language of communication. However, if they belong to different speech communities then the strategic incentives will depend on their language skills and their values of λ.

It is important to note that, for the moment, potential partnerships are not price mediated. In other words, the distribution of monetary surplus is fixed and unconditional to language skills or the language of communication. This is probably a good description of many types of social interactions where it is impossible to price discriminate on the basis of the individual's linguistic attributes. In any case price mediated agreements are examined in Section 5.

Imposing some restrictions on parameter values will simplify the presentation without significantly affecting the main economic insights. In order to interpret these restrictions it will be useful to outline some features of equilibrium behavior. If both potential partners of the productive match share the same mother tongue, then the match is formed with probability one, since the relationship can be conducted in their favorite language and everyone obtains a payoff equal to 1, which is higher than their opportunity cost, $1-\theta$. However, if one of them, say the $W-$native speaker, is bilingual but the $S-$speaker is monolingual, then, independently of who

[23] In a different setup, Iriberri and Uriarte (2012) show that asymmetric information about language skills reduces the use of the weak language below efficient levels.

[24] It is well known that under asymmetric information the optimal mechanism involves ex-post inefficiencies. Our particular protocol should be interpreted as a simple representation of these inefficiencies.

is the proponent, the partnership will materialize if and only if is profitable for the $W-$native speaker: if and only if $1 - \lambda \geq 1 - \theta$. Finally, if partners have different mother tongues but are both bilingual, then their optimal strategies are more complicated since all three regimes (S, SW, W) are feasible.

Assumption 1 $\overline{\lambda} \leq 2\theta$

Assumption 1 indicates that from a social point of view all feasible (at least one bilingual individual) high productivity partnerships should be formed. In other words, for all values of λ, total payoffs from forming the initial match, $2 - \lambda$, are higher than their opportunity cost, $2(1 - \theta)$. In other words, high productivity partnerships are always socially efficient.

Asumption 2 $\overline{\lambda} > \theta$.

Assumption 2 implies that there exist some bilingual individuals (with a sufficiently high value of λ) that will reject the efficient partnership if the language of communication is her second language. Assumptions 1 and 2 taken together imply that inefficient separations will occur with a positive probability when one of the communities is monolingual.

Note that $\mu\overline{\lambda} \leq \frac{1}{2}\overline{\lambda} \leq \theta$. The first inequality comes from the assumption that $\mu \leq \frac{1}{2}$, and the second follows from Assumption 1. Hence, for all values of λ bilingual individuals will willing to form the partnership if an SW policy is adopted.

Neither of these assumptions is essential in the derivation of the main qualitative results, but they greatly simplify the presentation by reducing the number of cases under consideration.

The model laid down in this section is static. A possible, and more elegant, alternative would be to introduce linguistic preferences in the standard dynamic model of the labor market with matching frictions. Suppose that coalitions collapse at some exogenous rate and freed individuals are matched with potential partners also at some exogenous rate. Matched individuals negotiate the conditions of cooperation. If they reach an agreement, which will depend on their linguistic preferences, then the coalition is formed, otherwise individuals remain "unemployed" and wait until the next opportunity arrives. In a model like this reciprocal bilingualism will also increase the probability that a match actually forms a coalition, cutting down "unemployment" spells. Such a model would certainly be more elegant, but much less tractable. On top of that I doubt it would be able to bring about additional insights. In other words, the proposed model can probably be thought of as a simple representation of a labor market subject to matching frictions.

4 Analysis of the benchmark model

Since $\beta^S > F$, $W-$ native speakers have always incentives to learn S, independently of the potential benefits arising from domestic interactions. Hence, the only two possible equilibrium scenarios are AB and RB.

4.1 Asymmetric bilingualism

Suppose that at the second stage $W-$speakers are bilingual but $S-$speakers remain monolingual. Half of the potential high productivity partnerships will be formed by linguistically homogeneous individuals who will face no obstacle reaching an agreement. However, the other half of the matches includes two individuals with different mother tongues. Obviously, the language of use must necessarily be S, and the partnership will materialize if and only if it is individually rational for the $W-$speaker; i.e., if and only if $1-\lambda \geq 1-\theta$, a condition that is independent of the identity of the proponent. As a result, the respective expected payoffs of S and W speakers will be given by:

$$U_{AB}^{S} = \frac{1}{2} + \frac{1}{2}\left\{H\left(\theta\right) + \left[1-H\left(\theta\right)\right]\left(1-\theta\right)\right\} = 1 - \frac{\theta}{2}\left[1-H\left(\theta\right)\right]$$

$$U_{AB}^{W} = \frac{1}{2}+\frac{1}{2}\left\{\int_{0}^{\theta}\left(1-\lambda\right)dH\left(\lambda\right) + \left[1-H\left(\theta\right)\right]\left(1-\theta\right)\right\} = 1-\frac{\theta}{2}\left[1-H\left(\theta\right)\right]-\frac{1}{2}\int_{0}^{\theta}\lambda dH\left(\lambda\right)$$

Thus, a fraction $1-H\left(\theta\right)$ of productive partnerships formed by linguistically heterogeneous individuals will fail, and a fraction $H\left(\theta\right)$ will succeed. In a successful partnership only the $W-$speakers bears the cost of using their second language. As a result $U_{AB}^{S} > U_{AB}^{W}$.

4.2 Reciprocal bilingualism

Suppose now that at the second stage both communities are bilingual. In linguistically heterogeneous matches, if the proponent is an $S-$speaker (the situation is perfectly symmetric if the proponent is a $W-$speaker) she anticipates that offering an SW policy will be accepted with probability one by the potential partner.

Hence, she has no incentive to offer W. If she offers SW then she obtains $1-\mu\lambda$. In the event she proposes S then she anticipates that with probability $H(\theta)$ the offer will be accepted, but with the complementary probability the potential partner will quit. Hence, she obtains a expected payoff equal to $1-[1-H(\theta)]\theta$. Therefore, she finds it optimal to offer SW if only if $\lambda \leq \Psi \equiv \frac{\theta}{\mu}[1-H(\theta)]$. Note that in principle Ψ may be higher or lower than $\overline{\lambda}$. If $\Psi \geq \overline{\lambda}$, then independently of the identity of the proponent, the partnership will materialize with probability one and the language policy will be SW. As a result $U_{RB}^{W} = U_{RB}^{S} = 1 - \frac{\mu\widehat{\lambda}}{2}$. A bit more complicated is the case where $\Psi < \overline{\lambda}$. With probability $H(\Psi)$, a linguistically heterogeneous partnership sticks together and conducts the relation under an SW policy. With probability $[1-H(\Psi)]H(\theta)$, the partnership is also formed but the respondent bears all the disutility derived from a monolingual policy. Finally, with probability $[1-H(\Psi)][1-H(\theta)]$ the partnership breaks down.

Consequently,

$$U_{RB}^{W} = U_{RB}^{S} = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}p + \frac{1}{2}r\right)$$

where p and r are the respective expected utility of the proponent and the respondent, which can be written as:

$$p = \int_0^{\Psi} (1-\mu\lambda)\, dH(\lambda) + [1-H(\Psi)]\{1-\theta[1-H(\theta)]\}$$

$$r = H(\Psi)\left(1-\mu\widehat{\lambda}\right) + [1-H(\Psi)]\left\{1-\theta[1-H(\theta)] - \int_0^{\theta} \lambda dH(\lambda)\right\}$$

Combining these expressions we obtain

$$U_{RB}^{W} = U_{RB}^{S} = 1 - \frac{\mu}{4}\int_0^{\Psi}\left(\lambda+\widehat{\lambda}\right) dH(\lambda) - \frac{1-H(\Psi)}{4}\left\{[1-H(\theta)]2\theta + \int_0^{\theta}\lambda dH(\lambda)\right\}$$

In contrast to the previous scenario, now speech communities are in a symmetric situation and hence all individuals obtain the same expected payoff.

4.3 Comparing alternative scenarios

The computations are presented for the case $\Psi < \overline{\lambda}$, but the qualitative results also hold in case $\Psi \geq \overline{\lambda}$.

Let us first focus on total surplus, TS. In this case the total gains derived from shifting from AB to RB, $\Delta T \equiv \left(U_{RB}^{W} + U_{RB}^{S}\right) - \left(U_{AB}^{W} + U_{AB}^{S}\right)$, can be written as:

$$\Delta TS = H(\Psi)\left\{\theta[1-H(\theta)] + \frac{1}{2}\int_0^{\theta}\lambda dH(\lambda)\right\} - \frac{\mu}{2}\int_0^{\Psi}\left(\lambda+\widehat{\lambda}\right) dH(\lambda) > 0$$

Let us compare the negotiation outcomes in heterogeneous matches across the two regimes. Let λ^S and λ^W be the realizations of the disutility parameter of the S and W speaker, respectively. In the AB regime the identity of the proponent is irrelevant, since all that matters is the realization of λ^W. Also, both speech communities have the same ex-ante preferences. Hence, the total expected surplus in the RB regime is also independent of the identity of the proponent. Thus, we only need to compare the negotiation outcomes in the two regimes when the $S-$speaker is the proponent. If $\lambda^S > \Psi$ then both the RB and AB regimes deliver the same outcome, i.e., the $S-$speaker offers to conduct the relationship in S and the respondent accepts if and only if $\lambda^W < \theta$. However, the two regimes differ if $\lambda^S < \Psi$. In this case, the match breaks down in the AB regime if and only if $\lambda^W > \theta$, but never breaks down in the RB regime. Thus, inefficient separations are less likely in the RB regime. Also, total costs of using a second language varies across regimes. If $\lambda^W < \theta$ the relation is then conducted in S in the AB regime and in SW in the RB regime. Hence, there could be realizations of λ^S and λ^W for which the total surplus generated in the AB regime may be higher than in the RB regime. However, even when μ takes its highest possible value ($\mu = \frac{1}{2}$), these realizations cannot affect the sign of the total change in expected surplus, which is mainly driven by the reduction in inefficient separations.

It is important to note that ΔT increases as μ decreases. In other words, if individuals value an SW policy (lower μ) more, then the social benefits of reciprocal bilingualism increase.[25]

Whether or not RB is socially optimal clearly depends on the difference between costs and benefits:[26]

Proposition 1 *Reciprocal bilingualism is socially optimal if and only if* $F \leq \Delta T$.

Let us now look at the distributional implications of shifting from AB to RB. We know that $U^S_{AB} > U^W_{AB}$ and $U^W_{RB} = U^S_{RB}$. Moreover, average welfare increases, which taken together with the previous results implies $W-$speakers must be better

[25] RB also raises welfare if the SW regime is not feasible and individuals must select either S or W.

[26] In our social welfare function all individuals have the same weight (the social planner maximizes total surplus). If, alternatively, the social planner is averse to inequality, then in principle it should promote RB more vigorously, since AB generates inequality by favoring the S-community. Or course, if we take into account other determinants of individual welfare (like monetary wealth) then distributional concerns may turn against RB if S-speakers are relatively poorer. In any case, I find the idea of using language policy as a redistribution device quite inappropriate.

off in the RB regime:

$$\Delta U^W \equiv U^W_{RB} - U^W_{AB} > 0$$

The reason is twofold. First, the frequency of inefficient separations is lower in the RB regime. Second, whenever high productivity partnerships succeed, $W-$speakers obtain a better linguistic treatment.

In contrast, the effect on $S-$speakers is much less straightforward. In particular:

$$\begin{aligned} \Delta U^S \equiv\; & U^S_{RB} - U^S_{AB} = \\ & \frac{H(\Psi)}{8}\left\{4\theta\left[1-H(\theta)\right] + 2\int_0^{\theta} \lambda dH(\lambda)\right\} - \mu\int_0^{\Psi}\left(\lambda+\widehat{\lambda}\right)dH(\lambda) - \int_0^{\theta}\lambda dH(\lambda) \end{aligned} \tag{1}$$

If μ is sufficiently low then the above expression is clearly positive. However, for relatively high values of μ the sign of ΔU^S is ambiguous. Suppose that $h(\lambda)$ is symmetric around the mean. Moreover, $\mu = \frac{1}{2}$, and $\theta = \widehat{\lambda}$. Hence, $H(\theta) = \frac{1}{2}$ and $\Psi = \theta$. In this case $\Delta U^S = \frac{1}{8}\left[\frac{\widehat{\lambda}}{2} - E\left(\lambda \mid \lambda \leq \widehat{\lambda}\right)\right]$. Thus, if the density function is uniform $U^S_{RB} = U^S_{AB}$, but if $h(\lambda)$ is increasing (decreasing) for $\lambda \leq \widehat{\lambda}$, then $U^S_{RB} < (>) U^S_{AB}$.

As we move from the AB to the RB regime, $S-$speakers also benefit from a lower frequency of inefficient separations, but they also lose their strategic advantage (their language monopoly power) in the negotiations. The result is that they suffer worse language treatment. It turns out that the second effect may actually dominate.

Even if $U^S_{RB} - U^S_{AB} > 0$, if $S-$speakers bear the full cost F, then their incentives to learn W will be insufficient from a social point of view. More specifically, if we let $F^S = U^S_{RB} - U^S_{AB}$, then we can state the main result as follows:

Proposition 2 *If $F \in \left[\max\left\{F^S, 0\right\}, \Delta T\right]$ and $S-$speakers bear the full cost of learning W, then they will prefer not to acquire a second language even though it would raise total surplus.*

Obviously, if some of the learning costs are covered by general taxation (and hence paid by $W-$speakers) then $S-$speakers incentives would improve. However, as shown above, even if $W-$speakers offer to fully compensate $S-$speakers for their learning costs, they would still refuse if $\Delta U^S < 0$.

5 Extensions

5.1 Heterogeneous learning costs

Suppose that individuals face different learning costs (like in Gabszewicz et al., 2008). Now the cost of learning a second language for an individual, F, is an independent realization of a random variable distributed over $\left[0, \overline{F}\right]$, with density function $j(F)$ and cumulative distribution $J(F)$. It is still assumed that $B^S > \overline{F}$ and hence all $W-$speakers learn S ($\alpha^W = 1$). However, $B^W = 0$ and in equilibrium a fraction α^S of $S-$speakers learn C.

5.1.1 Symmetric information on language skills

Suppose we maintain the assumption of the benchmark model according to which individuals can observe both the native language and the language skills of their potential partner and can condition their strategies on such information. The outcome of heterogeneous matches has been described in Section 4.1 if the $S-$speaker is monolingual and in Section 4.2 if she is bilingual. The incentives to learn W are still given by ΔU^S (equation 1), then $J\left(\max\left\{\Delta U^S, 0\right\}\right)$ is the fraction of $S-$speakers that become bilingual in equilibrium. This version of the model generates predictions about the differences in expected earnings by different linguistic groups. First, bilingual $S-$speakers obtain in average a higher monetary return than monolingual $S-$speakers. The reason is the probability that their match with a $W-$speaker fails is lower. Second, the expected monetary return of a $W-$speaker is intermediate between the bilingual and monolingual $S-$speakers, since they have a chance of meeting one type or the other.

Finally, the analog of Proposition 2 would be the following:

Remark 1 *If $\overline{F} > \Delta U^S$, the fraction of bilingual $S-$speakers is inefficiently low.*

Starting at the equilibrium value, a small exogenous increase in α^S generates second order losses on the additional bilingual $S-$speakers, but first order gains in $W-$speakers, who now face bilingual $S-$speakers more often.

5.1.2 Asymmetric information on language skills

An alternative, perhaps more realistic, scenario is one where heterogeneity of learning costs interacts with additional dimensions of asymmetric information and/or some kind of institutional inertia. More specifically, suppose that both native language and knowledge of the second language of potential partners are not observable, or at least they are not observable at the time of choosing strategies. The latter can be interpreted as a form of institutional inertia, where the language policy of various organizations is set by members of one of the two speech communities before they actually learn the identities of their potential partners. In other words, I assume the proponent sets language policy as a function of α^S but ignoring the identity of the potential partner; that is, a $W-$speaker ignores whether the potential partner is another $W-$speaker, a monolingual $S-$speaker, or a bilingual $S-$speaker. Similarly, an $S-$speaker does not know whether the potential partner is another $S-$speaker or a bilingual $W-$speaker.

The details of the analysis are postponed to the Appendix. Here I provide a brief summary of results. As expected, in this case the strategy of $W-$speakers as proponents varies with α^S. In particular, a higher value of α^S induces $W-$speakers to offer S less often and SW more often. As a result, a partnerships between a $W-$speaker and a monolingual $S-$speaker is less likely to succeed. Also, even though the probability of a success of a partnership between a W-speaker and a bilingual $S-$speaker does not change with α^S, the latter expects a worse language treatment. Summarizing, the learning decision of an individual $S-$speaker generates a negative externality on the rest of her speech community. However, the positive externality they generate on $W-$speakers is even larger, at least for a set of distribution functions that includes the uniform.

Remark 2 *The equilibrium value of α^S is likely to be inefficiently low, but it would be even lower if learning decisions where taken in order to maximize the welfare of the $S-$community.*

5.2 Price-mediated agreements

How do results change if the division of monetary surplus is not exogenous, but part of the negotiation? More specifically, suppose that the proponent makes a take-it-or-leave-it offer that includes not only a language policy but also a division of the monetary return: x for the respondent and $2-x$ for the proponent. The respondent must accept or reject the entire offer. The main insights of this exercise can be obtained by restricting our attention to the case where λ is uniformly distributed on $[0, 1]$.

Note that in a linguistically homogeneous match the average payoff of any individual is still equal to 1. The reason is that the proponent always offers to conduct the relation in the common native language and offers the respondent a compensation equal to $1-\theta$, which is always accepted and yields $1+\theta$ to the proponent.

The equilibrium outcome in heterogeneous matches will depend on the regime. Under AB S must be used in any heterogeneous partnership. If the $S-$speaker is the proponent, then she offers an agreement (S, x_S). The respondent accepts if and only if $x_S - \lambda \geq 1-\theta$. Thus, it will be accepted with probability $(x_S + \theta - 1)$. The proponent's optimal offer is the value of x_S that maximizes her expected payoff: $(x_S+\theta-1)(2-x_S)+(2-x_S-\theta)(1-\theta)$. Hence, the optimal offer is $x_S = 1$, and the associated probability of a breakdown is $(1-\theta)$, as in the case with exogenous division of surplus. However, if the proponent is a $W-$speaker, then she faces no uncertainty about the reaction of her potential partner and can capture all the potential rents. In other words, she offers $x_S = 1-\theta$ and captures $2-x_S-\lambda$ which is higher than $1-\theta$ for all values of λ. Hence, while half of the time the same rate of inefficient separations is obtained, in the other half ex-post efficiency results.

In contrast to the benchmark case where the sharing rule was exogenous, in this case it is not so obvious that $S-$speakers are in an advantageous position vis-a-vis $W-$speakers. Even though S is the only feasible language of communication, $W-$speakers can now enjoy some informational rents: when they are proponents they are the informed party (they know the total surplus, $2\theta - \lambda^W$), and hence they are able to appropriate all the surplus. In contrast, when the $S-$speaker is the proponent her ability to appropriate rents is limited by the asymmetry of information on λ^W. More specifically, the expected payoffs of S and $W-$speakers are given respectively by:

$$U_{AB}^S = 1 - \frac{1}{4}\theta(2-\theta)$$

$$U_{AB}^W = 1 - \frac{1}{4}\left(\frac{1-\theta^2}{2}\right)$$

By assumption (A.1) $\theta > \frac{1}{2}$, and hence U_{AB}^S is lower than U_{AB}^W. The commitment to use S is dominated by the informational rents that the better informed agents ($W-$ speakers) are able to extract.[27]

In the RB scenario then the proponent in a heterogeneous match (say the $W-$speaker) offers a menu where x varies according to the language policy: (S, x_S),

[27] Consequently, $W-$speakers are more than compensated for the use of their second language.

(SW, x_{SW}), (W, x_W). The respondent ($S-$speaker) will choose the optimal language-monetary return combination, provided it is higher than $1-\theta$. It turns out (details are provided in the Appendix) that if $\mu = \frac{1}{2}$, in equilibrium $W-$speakers offers $x_S = 1-\theta$, and $x_W = 1-\theta+\frac{\lambda^W}{2}$ (SW policy is never chosen in equilibrium) and the respondent chooses W if $\lambda^S \leq \frac{\lambda W}{2}$, and S otherwise. Hence, RB reduces the frequency of inefficient separations to zero. Moreover, it reduces total language costs by increasing the use of W as the ratio of $\frac{\lambda^S}{\lambda^W}$ decreases. Summarizing, total welfare is higher in the RB scenario and both communities are better off. More specifically,

$$U_{RB}^S = U_{RB}^W = \frac{43}{48} > U_{AB}^W > U_{AB}^S$$

and hence with respect to the benchmark model $S-$speakers' incentives to learn W improve.

5.3 Large cooperation groups

To what extent results change if we consider more complex organizations than two-person partnerships? Let us examine the polar extreme case in which efficient partnerships contain a continuum of agents. Since potential partners are still randomly selected, independently of their native language, then by the law of large numbers any high productivity partnership will have the same linguistic composition as the population at large. In this case we need to adapt the previous formulation of the gains from cooperation. Clearly, monetary returns must decrease with the fraction of potential members that refuse to participate in the partnership. In particular, if we let $\gamma \in \left[0, \frac{1}{2}\right]$ be the fraction of potential members that quit, then the monetary returns of every remaining individual amounts to $1-2\gamma\theta$. As in the benchmark model if nobody quits then each individual obtains a monetary return of 1, and if half of its potential members (all the members of one of the speech communities) quit, then the monetary payoff of the remaining individuals is $1-\theta$.

In the AB regime, any high productivity partnership must adopt S as the language of communication. Then a $W-$speaker quits if and only if $\lambda \geq (1-2\gamma)\,\theta$. Hence, there is a possible *bandwagon* effect. If individuals expect $\gamma = 0$, then those individuals with $\lambda > \theta$ will choose to quit, which reduces the gains for the remaining individuals. Soon, other individuals with values of λ below θ will also quit. And this process can go on ad infinitum. In fact, in equilibrium:

$$\gamma = \frac{1 - H\left[(1-2\gamma)\,\theta\right]}{2}$$

Clearly, $\gamma = \frac{1}{2}$ is always an equilibrium. Also, for a large class of distribution functions (including the uniform density case) this will be the only one.[28] In other words, in the AB regime cooperation among members of different speech communities is not feasible.

In the RB regime if an organization adopts a monolingual policy then the above result still holds (no cooperation). However, when a bilingual policy (SW) is adopted, an individual then will quit if and only if $\lambda \geq \frac{(1-2\gamma)\theta}{\mu}$. Hence, the equilibrium condition now becomes:

$$\gamma = 1 - H\left[(1-2\gamma)\frac{\theta}{\mu}\right]$$

Note that from assumption 1 (and since $\mu \leq \frac{1}{2}$) $\frac{\theta}{\mu} \geq \overline{\lambda}$ and hence $\gamma = 0$ is always an equilibrium.[29] Thus, in this simple specification of cooperation in large groups, fruitful interaction among different speech communities is only possible under RB. This simple example suggests that the economic value RB is enhanced if cooperation takes place in larger groups. Moreover, it also indicates that a balanced linguistic rule (SW) can become a necessary condition to support cooperation.[30]

5.4 Arbitrary exogenous benefits

If B^S is sufficiently lower than $\overline{F}$, there exist interior equilibria where both α^S and α^W take values between 0 and 1. Moreover, if B^S falls, then α^W also falls and α^S increases. Thus, for all practical purposes qualitative results are similar to those obtained in traditional models (Gabszewicz et al. 2008). Clearly, a lower value of B^S discourages $W-$speakers from learning S: lower α^W. As a result of the lower fraction of bilingual $W-$speakers, α^S increases. The reason for the strategic substitutability of the learning efforts of the two communities is the net result of two countervailing effects. As α^W falls, on the one hand, the communicative value of learning W increases (the probability of meeting a monoglot $W-$speaker increases and hence knowledge of a second language becomes more important to make cooperation feasible), and on the other hand, the additional benefits from

[28] For some distribution functions there may also exist an equilibrium with $0 < \gamma < \frac{1}{2}$, which will be ingnored in the main text..

[29] There is another rational expectations equilibrium with $0 < \gamma < \frac{1}{2}$.

[30] I have not discussed how language policy is chosen in multi-person organizations. Suppose an individual is chosen at random to select the language policy. She will anticipate that if she proposes S or W as the language of communication then cooperation is not feasible and will obtain $1 - \theta$. However, in the RB regime, if she proposes SW it will be accepted by everyone, and she will get $1 - \mu\lambda \geq 1 - \theta$.

reciprocal bilingualism decrease as the probability of having a partner that is a bilingual $W-$speaker falls. It turns out that the first effect dominates.

The main normative implications of the benchmark model also extend to the case of heterogeneous learning costs and low exogenous benefits (α^S and α^W below 1). Since individuals do not internalize the positive externalities generated by their learning decisions the social optimum is generally characterized by values of α^S and α^W higher than in equilibrium. In this case we can best describe these positive externalities as operating through two different channels. One is related to the communicative value of languages and the other to the enhanced cooperative effect of reciprocal bilingualism. A higher value of α^W implies: (i) an increase in the probability that a monolingual $S-$speaker meets a bilingual $W-$speaker (with whom communication is feasible), and (ii) an increase in the probability that a bilingual $S-$speaker meets a bilingual $W-$speaker, which facilitates cooperation. As a result, the entire $S-$community benefits from a higher value of α^W. The symmetric is true if α^S increases.

5.5 The relative size of speech communities

Suppose a fraction β of the population belongs to the $S-$community and a fraction $1-\beta$ to the $W-$community. The effect of changes in β are quite straightforward. There are two major reasons why as β increases above $\frac{1}{2}$ the social benefits generated by reciprocal bilingualism diminish. First, the frequency of linguistically heterogenous partnerships, $\beta(1-\beta)$, decreases. Second, the total learning costs increase as more individuals are asked to learn the weak language.

The effect of β on the equilibrium outcome is equally unsurprising. As β increases $S-$speakers have less incentives to become bilingual, as the resulting frequency of linguistically heterogeneous partnerships decreases.

Throughout this paper the "strong" language (S) has been defined in relation to its international status: $B^S > B^W$. In most of the examples the "strong" language is also the domestically dominant language. However, if β is sufficiently small, then W may be more frequently used and learnt more intensively than S, $\alpha^S > \alpha^W$. For example, this seems to be the case for the French language in Quebec. In other words, it may be possible that the "weak" language (W), according to our definition, is the dominant language in domestic interactions.

6 Concluding remarks

The main message of this paper is that reciprocal bilingualism is likely to raise social welfare by reducing social and economic segmentation. The underlying theory is based on three main principles. First, languages are communication devices and hence sharing one is a prerequisite for cooperation between members of different speech communities. Second, bilingual individuals are not indifferent about the language of use in social and economic interactions (linguistic preferences, of emotional attachment to one's native language). As a result, even if everyone is competent in a particular language, and hence the ability to communicate is not at stake, a certain degree of social and economic segmentation will remain. Third, learning a second language generates network externalities. Consequently, individuals do not internalize those externalities and hence public intervention is needed to implement the social optimum. As a result, policies that promote reciprocal bilingualism may increase social welfare by tempering the linguistic conflict of interest and enhancing cooperation.

The model is able to rationalize some empirical evidence that was puzzling for traditional theories. In particular, in bilingual societies where everyone is competent in the strong language, theories based exclusively on the communicative value of languages predict no differences in labor market outcomes between monolingual (only competent in the strong language) and bilingual workers (competent also in the weak language.) In contrast, the current model unambiguously predicts higher expected earnings for bilingual individuals (See Section 5.1). Also, if we were to introduce linguistic preferences in the standard dynamic labor model with search and matching frictions, one could anticipate that bilingual individuals would have higher chances of reaching an agreement when matched with appropriate partners, which would reduce their "unemployment" spells, and increase the chances of being employed. Both of these predictions are compatible with the empirical evidence for the Welsh and Catalan labor markets, where bilingual individuals enjoy higher rates of employment and wage premia (See the references in footnote 3.) Regarding the patterns of second language learning, the model provides an efficiency justification for language policies that promote the learning and use of minority languages in regions where communication is already guaranteed by the universal knowledge of the majority language.

Beyond aggregate welfare, the model also points out the distributional implications of various policies, which may be crucial to understand the effects of alternative jurisdictional arrangements and determine their feasibility in a centralized political system.

Suppose first that policy decisions are taken by bodies that represent the interests of each speech community. This is more likely when the two communities are geographically segmented, like in Belgium. In Section 5 I argue that individual decisions on second language learning may cause a negative externality on other members of their speech community. As a result, policies implemented independently by each community may result in outcomes that generate very poor social welfare results, even worse than under laissez-faire. More specifically, decision makers ignore the positive externality that their learning efforts would cause on the other community, and on top of that they do further restraint their learning efforts taking into account their internal negative externality, aggravating the underinvestment problem. In this sense, centralized policy making is a necessary condition for designing and implementing efficient policies. But it might not be a sufficient condition. It could be the case that reciprocal bilingualism involves higher total welfare but lower welfare for the speakers of the strong language. In fact, under majority voting the outcome will depend on the relative size of the two communities. Hence, it is possible that policies that promote reciprocal bilingualism do not receive sufficient political support, even though they maximize total welfare.[31] More generally, the implementation of the social optimum is likely to depend on whether or not speakers of the weak language are able to find a way to compensate speakers of the strong language. In other words, this is an instance of the political fragility of the social optimum, which may explain why societies with high levels of reciprocal bilingualism are relatively hard to find.

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8 Appendix

8.1 Heterogenous learning costs and unobservable language skills

In this Appendix we provide the analytical details of the discussion of Section 5.1.2. In principal such asymmetry of information could generate a coordination problem for linguistically homogeneous matches. However, this problem does not arise if we allow for a natural expansion of the strategy space. In particular, suppose that proponents choose one of the following strategies: (i) W preferred, but SW and S are also possible, (ii) W preferred, but SW is also possible, (iii) W only, (iv) S preferred, but SW and W are also possible, (v) S preferred, but SW is also possible, (vi) S only. Note that a $W-$speaker will never choose strategies

(iv) to (vi). Similarly, an $S-$speaker will never choose strategies (i) to (iii). If the proponent selects an option with a single language choice, then the respondent must accept or reject. If the offer includes more than one feasible language choice then, in case of acceptance, the respondent must choose a particular language policy. If the respondent rejects the offer the partnership breaks down. Under such a strategy space it is clear that proponents will choose their strategy aiming to maximize the expected payoff in heterogenous groups, since they anticipate that if both potential partners share the same mother tongue they will conduct their relation in their preferred language.[32]

If we let F_0 be the cost paid by the marginal individual, then in equilibrium $\alpha^S = J(F_0)$. The inverse function, $F_0(\alpha^S)$, satisfies $F_0'(\alpha^S) = \frac{1}{j(F_0)}$.

In a linguistically heterogenous partnership, if the proponent is a $W-$speaker then her expected payoffs associated to alternative offers are:

(i) W preferred but S accepted: $U^W = 1 - \lambda$

(ii) W preferred but SW accepted: $U^W = 1 - \alpha^S \mu\lambda - \theta(1-\alpha^S)$

(iii) W only: $U^W = 1 - \theta\left[1 - \alpha^S H(\theta)\right]$

The characterization of the optimal strategy depends on parameter values. In order to simplify the presentation let us assume that:

Assumption 3 $\mu \leq 1 - H(\theta)$

If we denote by $\lambda_1 \equiv \frac{\theta(1-\alpha^S)}{1-\alpha^S\mu} < \Psi$ then we obtain the follwing result:

Result 1 $W-$speakers' optimal strategy as proponents is: (i) if $\lambda \leq \lambda_1$, (ii) if $\lambda_1 \leq \lambda \leq \Psi$, and (iii) if $\lambda \geq \Psi$.

Obviously, a monolingual $S-$speaker will only accept option (i), and choose S, while a bilingual $S-$speaker will always accept (i) and (ii), in the first case choosing S and in the second SW. Finally, she will accept (iii) if and only if $\lambda^S \leq \theta$.

The $S-$speakers' optimal strategy in their role as proponents is analogous to that presented in Section 4. In particular, a bilingual $S-$speaker proposes: (v) if $\lambda \leq \Psi$, and (vi) if $\lambda > \Psi$. A monolingual $S-$speaker, obviously always proposes S. The $W-$speakers' acceptance rule also coincides with that of the benchmark model.

Result 1 follows from the direct comparison of these three payoffs, taking into account the $S-$speaker's optimal response. Hence, the $W-$speaker's expected payoff is:

$$\begin{aligned} U^W(\alpha^S) &= 1 - \frac{1}{4}\left(\int_0^{\lambda_1} \lambda dH(\lambda) + \int_{\lambda_1}^{\Psi} \left[\alpha^S\mu\lambda - \theta(1-\alpha^S)\right] dH(\lambda) + [1 - H(\Psi)]\,\theta\left[1 - \alpha^S H(\theta\right.\right. \\ &\quad -\frac{\alpha^S}{4}\left(H(\Psi)\,\mu\widehat{\lambda} + [1 - H(\Psi)]\left\{\theta[1 - H(\theta)] + \int_0^{\theta} \lambda dH(\theta)\right\}\right) + \\ &\quad -\frac{1-\alpha^S}{4}\left\{\theta[1 - H(\theta)] + \int_0^{\theta} \lambda dH(\theta)\right\} \end{aligned}$$

Similarly, the respective expected payoffs of a bilingual and a monolingual $S-$speaker are:

$$\begin{aligned} U^S_{bil}(\alpha^S) &= 1 - \frac{1}{4}\left([H(\Psi) - H(\lambda_1)]\,\mu\widehat{\lambda} + [1 - H(\Psi)]\left\{\theta[1 - H(\theta)] + \int_0^{\theta} \lambda dH(\theta)\right\}\right) - \\ &\quad -\frac{1}{4}\left(\int_0^{\Psi} \mu\lambda dH(\lambda) + [1 - H(\Psi)]\,\theta[1 - H(\theta)]\right) \end{aligned}$$

[32] It is important to note that under this expanded strategy space, the assumption in the benchmark model that individuals could observe the native language and language skills of their potential partners was deemed completely irrelevant since α^S was either 0 or 1.

$$U_{mon}^{S}\left(\alpha^{S}\right)=1-\frac{1}{4}\left(\theta\left[1-H\left(\lambda_{1}\right)\right]+\theta\left[1-H\left(\theta\right)\right]\right)$$

Note that both $U_{bil}^{S}\left(\alpha^{S}\right)$ and $U_{mon}^{S}\left(\alpha^{S}\right)$ decrease in α^{S}, while incentives to learn a second language, $U_{bil}^{S}\left(\alpha^{S}\right)-U_{mon}^{S}\left(\alpha^{S}\right)$, increase with α^{S} (at a decreasing rate). Moreover, $U_{bil}^{S}\left(\alpha^{S}=0\right)-U_{mon}^{S}\left(\alpha^{S}=0\right)>0$. Also, as expected, $U^{W}\left(\alpha^{W}\right)$ increases with α^{S}. If learning decisions are taken by individuals, then in equilibrium:

$$U_{bil}^{S}\left(\alpha^{S}\right)-U_{mon}^{S}\left(\alpha^{S}\right)=F_{0}\left(\alpha^{S}\right). \tag{2}$$

The following assumption guarantees the existence of a unique equilibrium.

Assumption 4 $j'(F)\leq 0$

This assumption implies that $F_{0}''\left(\alpha^{S}\right)\geq 0$. Since $U_{bil}^{S}\left(\alpha^{S}=0\right)-U_{mon}^{S}\left(\alpha^{S}=0\right)>F_{0}\left(\alpha^{S}=0\right)=0$, $U_{bil}^{S}\left(\alpha^{S}\right)-U_{mon}^{S}\left(\alpha^{S}\right)$ is increasing and concave, and $F_{0}\left(\alpha^{S}\right)$ is increasing and convex, then the left and right hand sides of equation (2) at most cross each other once. If they do not cross for values of $\alpha^{S}\in(0,1)$ then in equilibrium $\alpha^{S}=1$.

The optimal policy of the $S-$community consists of choosing α^{S} in order to maximize $\widehat{U}^{S}=\alpha^{S}U_{bil}^{S}\left(\alpha^{S}\right)+\left(1-\alpha^{S}\right)U_{mon}^{S}\left(\alpha^{S}\right)-\int_{0}^{F_{0}}FdJ(F)$. The first order condition of an interior solution is:

$$\left[U_{bil}^{S}\left(\alpha^{S}\right)-U_{mon}^{S}\left(\alpha^{S}\right)-F_{0}\left(\alpha^{S}\right)\right]+\left[\alpha^{S}\frac{dU_{bil}^{S}\left(\alpha^{S}\right)}{d\alpha^{S}}+\left(1-\alpha^{S}\right)\frac{dU_{mon}^{S}\left(\alpha^{S}\right)}{d\alpha^{S}}\right]=0$$

Since the second term is negative, then the first term must be positive. This implies that (under the conditions that guarantee a unique equilibrium) the optimal value of α^{S} is smaller than in equilibrium. Since $U^{W}\left(\alpha^{S}\right)$ is a strictly increasing function, the optimal value of α^{S} of the $W-$community is 1. Finally, since the $S-$community does not internalize the positive effect of α^{S} on U^{W}, it is obvious that the value of α^{S} that maximizes total welfare is lower than the value preferred by the $S-$community. However, it is much trickier to compare the social optimum with the equilibrium value under individual learning decisions, since total welfare is not generally concave in α^{S}.

Let us consider the following example: $\overline{\lambda}=1,\theta=\mu=\frac{1}{2},h(\lambda)=1$, for $\lambda\in[0,1]$. In this case, individual benefits from learning are:

$$U_{bil}^{S}\left(\alpha^{S}\right)-U_{mon}^{S}\left(\alpha^{S}\right)=\frac{1}{16\left(2-\alpha^{S}\right)}$$

The objective function of the $S-$community is invariant in α^{S}:

$$\widehat{U}^{S}\left(\alpha^{S}\right)=\frac{7}{8}$$

Finally, the welfare of the $W-$community is:

$$U^{W}\left(\alpha^{S}\right)=1-\frac{1}{4}\left[\frac{7}{8}-\frac{3\alpha^{S}}{8}+\frac{\alpha^{S}\lambda_{1}}{2}+\frac{\left(1-\alpha^{S}\right)\lambda_{1}^{2}}{4}\right]$$

where $\lambda_1 = \frac{1-\alpha^S}{2-\alpha^S}$. Note that $U^W\left(\alpha^S\right)$ is strictly increasing and convex in α^S. More specifically,

$$\frac{dU^W\left(\alpha^S\right)}{d\alpha^S} = \frac{1}{16}\left[\frac{1}{2} + (1-\lambda_1)^2\right] = \frac{1}{16}\left[\frac{1}{2} + \frac{1}{(2-\alpha^S)^2}\right]$$

Suppose learning costs are also uniformly distributed over the interval $\left[0, \overline{F}\right]$. Since the total benefits of the S community do not change with α^S, then obviously its optimal value of α^S is equal to zero. However, individual incentives are always positive and increasing in α^S. If $\overline{F} \leq \frac{1}{16}$, then the equilibrium value of α^S is equal to one. If $\overline{F} > \frac{1}{16}$, then the equilibrium value of α^S is a strictly decreasing function. Finally, if $\overline{F} \leq \frac{3}{32}$, then the socially optimal value of α^S is equal to 1. If $\overline{F} > \frac{3}{32}$, then the socially optimal value of α^S is a strictly decreasing function. As a result, the value of α^S chosen by the social planner is the highes, it takes a lower value when is the outcome of a game where with individual decision makers, and it is the lowest value if chosen according to the interests of the $S-$community.

8.2 Equilibrium strategies in the RB scenario with price-mediated agreements

Following the notation in the main text, suppose that the $W-$speaker is the proponent and the $S-$speaker the respondent. Without loss of generality we can focus on strategies that satisfy the following inequalities: $x_W \geq x_{WS} \geq x_S \geq 1-\theta$. If $x_W < x_{WS}$ then the respondent will never choose W, exactly as if $x_W = x_{WS}$. Similarly, if $x_{WS} < x_S$. Finally, if $x_S < 1-\theta$, then again the respondent will never choose S, and might break down the match for some realizations of λ^S. However, if $x_S = 1-\theta$, then this does not raise the $S-$speaker's opportunity cost of options W or SW, but now the $S-$speaker may sometimes choose S and increase $C-$speakers' payoff above the break down level, $1-\theta$. It is important to note that $x_S \geq 1-\theta$ guarantees that there are no inefficient separations.

For arbitrary values of (x_W, x_{SW}, x_S) that satisfy the above inequalities, the respondent must select her optimal response. W will be preferred to S if and only if $\lambda^W \leq \lambda_1 \equiv x_W - x_S$. Similarly, SW is preferred to S if and only if $\lambda^W \leq \lambda_2 \equiv 2\left(x_{SW} - x_S\right)$, and W is preferred to SW if and only if $\lambda^W \leq \lambda_3 \equiv 2\left(x_W - x_{SW}\right)$. There are two possible cases, depending on whether or not $\lambda_1 \geq \lambda_2$.

If $\lambda_1 < \lambda_2$ $\left(\text{which is equivalent to } x_{WS} > \frac{x_S + x_W}{2}\right)$ then the respondent chooses W if $\lambda^S \leq \lambda_3$, SW if $\lambda_3 \leq \lambda^S \leq \lambda_2$, and S if $\lambda^S > \lambda_2$. Hence, the proponent's expected payoff is:

$$U^W = \lambda_3\left(2 - x_W\right) + \left(\lambda_2 - \lambda_3\right)\left(2 - x_{WS} - \frac{\lambda^W}{2}\right) + \left(1 - \lambda_2\right)\left(2 - x_S - \lambda^W\right)$$

The optimal proponent's strategy in this region is the triple (x_W, x_{WS}, x_S) that maximizes the above payoff function, subject to $x_W \geq x_{WS} \geq x_S \geq 1-\theta$. It turns out, provided we are in this region, that:

$$\frac{dU^W}{dx_{WS}} = 4\left(x_W + x_S - 2x_{WS}\right) > 0$$

Hence, in the optimal policy $x_{WS} \leq \frac{x_W + x_S}{2}$ and SW is never chosen. In fact, the respondent chooses W if $\lambda^S \leq \lambda_1$, and S otherwise. Hence, the proponent's expected payoff is:

$$U^W = \lambda_1\left(2 - x_W\right) + \left(1 - \lambda_1\right)\left(2 - x_S - \lambda^W\right)$$

The optimal proponent's strategy is the pair (x_W, x_S) that maximizes the above payoff function subject to $x_W \geq x_S \geq 1-\theta$. From the first order condition for an interior solution with respect to x_W we obtain:

$$x_W - x_S = \frac{\lambda^W}{2} \tag{3}$$

Also,

$$\frac{dU^W}{dx_S} = -1 - \lambda^W + 2\left(x_W - x_S\right)$$

which evaluated at equation (3) is negative. Hence, the optimal proponent's strategy is $x_S = 1-\theta$ and $x_W = 1-\theta+\frac{\lambda^W}{2}$. Therefore, W is chosen if $\lambda^S \leq \frac{\lambda^W}{2}$, and S otherwise.

The symmetric is true when S is the proponent. As a result, the expected payoffs are $U^S_{RB} = U^W_{RB} = \frac{43}{48}$.