# Two-sided Learning in New Keynesian Models: Dynamics, (Lack of) Convergence and the Value of Information[*]

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## Abstract

This paper investigates the role of learning by private agents and the central bank (two-sided learning) in a New Keynesian framework in which both sides of the economy have asymmetric and imperfect knowledge about the true data generating process. We assume that all agents employ the data that they observe (which may be distinct for different sets of agents) to form beliefs about unknown aspects of the true model of the economy, use their beliefs to decide on actions, and revise these beliefs through a statistical learning algorithm as new information becomes available. We study the short-run dynamics of our model and derive its policy recommendations, particularly with respect to central bank communications. We demonstrate that two-sided learning can generate substantial increases in volatility and persistence, and alter the behavior of the variables in the model in a significant way. Our simulations do not converge to a symmetric rational expectations equilibrium and we highlight one source that invalidates the convergence results of Marcet and Sargent (1989). Finally, we identify a novel aspect of central bank communication in models of learning: communication can be harmful if the central bank's model is substantially mis-specified.

Keywords: asymmetric information, learning, monetary policy

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# 1   Introduction

This paper studies the role of asymmetric information and learning in a New Keynesian framework in which both private agents and the monetary authority have imperfect knowledge about the true model of the economy. We focus on short run dynamics or, more generally, on the dynamics that can develop when beliefs have not yet converged to an asymmetric information rational expectations equilibrium. In this environment, a rich and complex variety of interactions between beliefs and actions can potentially emerge, with important consequences on the time series patterns of the variables of the model.

In a large number of situations and contexts, it is reasonable to assume that two or more interacting agents have asymmetric information about the environment in which they operate. In this work, we consider a model of the economy in which private agents observe neither policy shocks nor the monetary policy rules that are implemented by the central bank, whereas the monetary authority observes neither technology shocks nor the beliefs that private agents possess when forming their expectations. We assume that economic agents employ all of the information that they have available to estimate the aspects of the true data generating process which are unknown to them, and that they use a statistical learning algorithm to revise their beliefs as new data become available. In each period, these updated beliefs will be the basis for the decisions of both the policy maker and the private agents.

An extensive literature in economics focuses on the analysis of monetary policy in environments that are characterized by imperfect knowledge and learning. Some recent contributions to this literature stream include studies by Barnett and Ellison (2011), Bullard and Mitra (2002), Cho et al. (2002), Cogley et al. (2011), Evans and Honkapohja (2006), Honkapohja and Mitra (2005), Marcet and Nicolini (2003), Milani (2008), Orphanides and Williams (2005), and Bullard and Eusepi (2005).[1] A large part of the research in this area focuses on the conditions under which the economy converges to a determinate rational expectations equilibrium (REE) and on the role of monetary policy in achieving this result. [2] In this branch of the literature, two-sided learning was first studied in the seminal work of Marcet and Sargent (1989), who describe a general framework upon which most of the ensuing research (including this paper) is built. The previously published investigations that most closely approach the current study include a study by Bullard and Eusepi (2005),

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[1] For additional references, Evans and Honkapohja (2009) provide an extensive review of this literature.

[2] Bullard and Mitra (2002), for instance, investigate this issue for a variety of alternative policies under the assumption that the central bank adopts Taylor-type instrument rules. By contrast, Evans and Honkapohja (2003, 2006) focus on expectations-based targeting rules that are obtained from the optimization of the central bank's objective function (Svensson (2002) strongly argues that these rules are superior to Taylor-type policy rules because they reflect optimal behavior by the central bank).

who investigate two-sided learning in the context of a New Keynesian framework, but do not explicitly model the central bank's decision problem and endow both the central bank and private agents with the same information set. Honkapohja and Mitra (2005), study convergence to rational expectations equilibria under two-sided learning but use a different framework with respect to the perceived laws of motion of private agents and policymakers; moreover, they also do not explicitly model the central bank's decision problem. Evans and Honkapohja (2003) introduce an optimizing central bank but use substantially different assumptions about the perceived laws of motion of agents and their respective information sets (in particular, they do not consider structural shocks that are only observed by some agents in the economy). Some other interesting contributions are Barnett and Ellison (2011), who build two-sided learning into a version of Sargent (1999), and Giannitsarou (2005), who investigates convergence to rational expectations equilibria under different learning algorithms in a model that is similar to the framework of Marcet and Sargent (1989).

Our paper departs from most of the previous literature on two-sided learning with respect to the assumptions about policymakers' behavior and the focus on short- and medium-run dynamics. More specifically, our framework assumes that both private agents and the monetary authority have incomplete knowledge of the true data generating process and that they adopt the same approach to address their lack of full information: they form beliefs using the data that they have available in each period, and then reach optimal decisions on the basis of these beliefs. In addition, the substantially different assumptions about the central bank's knowledge and behavior in our framework also implies that the focus of our analysis is not on the ability of the monetary authority to enforce a particular equilibrium; instead, the focus of this paper is on the short-run dynamics that the interactions of beliefs and actions between private agents and policymakers can generate. To our knowledge, the study of optimal policymaking and two-sided learning in an environment characterized by asymmetric information has not yet received a great deal of attention, particularly in the context of the business cycle dynamics of a New Keynesian model.

With respect to learning, we assume that both private agents and the monetary authority use statistical models to estimate and predict the behavior of variables for which they do not know the true data generating process. They update their estimates as new data becomes available using a recursive learning algorithm. These assumptions follow the large branch of the learning literature that originates from Marcet and Sargent (1989), Cho et al. (2002), and Evans and Honkapohja (1998). However, similarly to Cogley et al. (2011), the agents' perceived laws of motion in our framework have the important feature of incorporating the cross-equation restrictions that originate from their respective beliefs. Because of the complex relationships between parameter updates and optimal decisions, the actual law

of motion for the variables in the framework that is examined in this study cannot be characterized analytically. For this reason, we study the impact of asymmetric information and learning in a number of simulations that are performed using standard parameter values for this model.

Our results indicate that two-sided learning can significantly alter the dynamics of the model. More specifically, we find that two-sided learning may generate large departures of the variables of interest from full information rational expectations equilibrium values and create changes in their behavior in terms of autocorrelations and correlations with the other variables. We find that convergence to an asymmetric rational expectations equilibrium may not occur in our type of framework. Our work also suggests that in this environment, the communication of information by the central bank does not appear to be effective in reducing the impact of asymmetric information and learning on the equilibrium dynamics. To the best of our knowledge, the fact that central bank communication can be harmful if the central bank has a mis-specified model of the economy is a novel result in the literature.[3]

The remainder of the paper is organized as follows. Section 2 presents the basic features of the New Keynesian model that is analyzed. Section 3 discusses the information that is available to agents, the learning procedure, and the decision-making approach of our framework. Section 4 derives agents' perceived laws of motion for the variables of interest and their implied actual law of motion. Section 5 describes the simulation exercises that we perform and presents their results. Section 6 concludes.

## 2    The true model of the economy

The true model of the economy is a standard New Keynesian framework, as developed by Gali (2008). We assume perfect indexation of prices that cannot be reset to past inflation, as in Christiano et al. (2001); this assumption ensures that the pricing equations are unaffected by the presence of positive trend inflation, which results in the steady state output level to be independent of the steady state inflation level.[4]

Given these assumptions, private agents' behavior in this economy can be described by the following equations:

$$y_t = E_t^P\left(y_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t^P\left(\pi_{t+1}\right) - r_t^n\right) \tag{1}$$

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[3]Eusepi and Preston (2010) study central bank communication in a scenario in which only the private sector is learning. We compare our results with their findings in section 3 of this paper.

[4]See Ascari (2004) for a discussion.

$$\pi_t = \frac{1}{(1+\beta)}\pi_{t-1} + \frac{\beta}{(1+\beta)}E_t^P(\pi_{t+1}) - \frac{\kappa}{(1+\beta)}(y_t - \overline{y}) \tag{2}$$

$$r_t^n = \overline{r} + u_t \tag{3}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{4}$$

where $y_t$ is output $\pi_t$ is the inflation rate, $i_t$ is the nominal interest rate, $r_t^n$ is the natural rate of interest and $\overline{y}$ is the steady state level of output (all the other steady variables drop out in the equations above). The variable $r_t^n$ is assumed to be the sum of the steady state real interest rate $\overline{r}$ and a technology shock $u_t$, or shock to the real side of the economy, which evolves according to the $AR(1)$ process described by (4). All variables are in logs. The parameters $\sigma$, $\beta$ and $\kappa$ have standard interpretations, and are obtained from the underlying problem of consumers and firms; see Gali (2008) for further details. Equations (1) and (2) have the standard interpretation of an IS equation and a Phillips curve equation. In contrast to the standard New Keynesian framework, the superscript $P$ in the expectation operator $E_t^P(\cdot)$ in (1) and (2) denotes the fact that at each time $t$ private agents will form these expectations based only on the information that they have available, which will generally be different from the information that is available to policymakers.

In addition to the private sector, the economy is populated by a central bank or public authority that is assumed to exert some degree of control over the nominal interest rate, and to use it as its policy instrument. More specifically, the central bank is assumed to be able to set the value of $i_t$ up to a monetary policy shock $v_t$. Let $x_t$ be the value of the policy instrument that is chosen by the central bank for time $t$, then the interest rate will be:

$$i_t = x_t + v_t \tag{5}$$

where $v_t$ is assumed to follow the AR(1) process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{6}$$

Neither private agents nor policymakers have full knowledge of the economy. In particular, the central bank does not observe the technology shock and does not know how private agents form expectations about the future values of the variables of interest and make decisions regarding prices and output. Conversely, private agents are not aware of the policy rule that the central bank uses to establish $x_t$ and do not observe the monetary policy shock. A more thorough definition of the information set available to each of these two sides is given next.

# 3    Information and decisions

The imperfect and asymmetric information that private agents and the monetary authority employ in the decision-making process are the central features that differentiate this work from the previous literature. However, these features require us to postulate certain additional assumptions about the way in which each side will use its specific knowledge to estimate the aspects of the economy that are not known, to make decisions in each period, and to update its information based on the new data that will be observed.

The state and noise vectors that include all the information available in the economy are:

$$z_t = \begin{bmatrix} y_t & \pi_t & i_t & u_t & v_t & 1 \end{bmatrix}' \tag{7}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^u & \varepsilon_t^v \end{bmatrix}' \tag{8}$$

As discussed above, the vectors $z_t$ and $\varepsilon_t$ are not perfectly observed. In particular, private agents do not observe the policy shock $v_t$, whereas the central bank does not observe the technology shock $u_t$. Thus, the vectors of variables that each side will use in their decision process can be written as follows:

$$z_t^P = \begin{bmatrix} y_t & \pi_t & i_t & u_t & 1 \end{bmatrix}' \tag{9}$$

$$z_t^{CB} = \begin{bmatrix} y_t & \pi_t & i_t & v_t & 1 \end{bmatrix}' \tag{10}$$

or:

$$z_t^P = M^P z_t$$

$$z_t^{CB} = M^{CB} z_t$$

where $M^P$ and $M^{CB}$ are simply selection matrices that choose the relevant variables from the overall state $z_t$.

Private agents use the vector $z_t^P$ to estimate the policymaker's interest rate rule and to predict the future values of the nominal interest rate. Similarly, the monetary authority employs $z_t^{CB}$ to approximate and predict the behavior of output and the inflation rate. We assume that agents make use of reduced-form regressions for this purpose, thereby estimating a simple linear relationship between the variables for which they have limited knowledge and the information that they observe.[5] Given this framework, the decision-making process can

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[5]In our model, agents estimate regressions only for variables that they can not control. This assumption contrasts with most of the previous literature on learning, in which agents estimate VARs on *all* the equilibrium variables, including their own decision variables.

be decomposed into two steps. First, private agents and policymakers use the available data to estimate the parameters of the model. Second, they employ their perceived model of the economy, together with their estimates of its parameters, to make their respective decisions. These steps are updated in each period according to the new information that is observed over time.

## 3.1 Estimation and learning

Private agents do not know the interest rate rule that the central bank uses to establish the value of the interest rate. However, they do know that the nominal interest rate affects output and the inflation rate through equations (1) and (2). For this reason, to form expectations about the future values of these two variables, they form conjectures about the relationship between $i_t$ and the variables that they can observe.

We assume that private agents behave like econometricians and estimate the following simple linear relationship:

$$i_t = \psi_t z^P_{t-1} + \omega^P_t$$

$$i_t = \psi_{0t} + \psi_{\pi t}\pi_{t-1} + \psi_{yt}y_{t-1} + \psi_{it}i_{t-1} + \psi_{ut}u_{t-1} + \omega^P_t \tag{11}$$

where the error term $\omega^P_t$ simply captures all the determinants of the nominal interest rate that are orthogonal to the information that is included in the state vector $z^P_{t-1}$.

The central bank has imperfect knowledge about the private side of the economy. However, the policy decision process requires the monetary authority to hold beliefs about the way in which the nominal interest rate affects the variables of interest. Similarly to the assumptions that we made for private agents, we also assume that policymakers behave as econometricians and estimate simple reduced-form relationships among $y_t$ and $\pi_t$ and the state vector $z^{CB}_{t-1}$ which includes the variables that they can observe:

$$y_t = c_{yt} z^{CB}_{t-1} + \omega^{CB}_{yt} \tag{12}$$

$$\pi_t = c_{\pi t} z^{CB}_{t-1} + \omega^{CB}_{\pi t} \tag{13}$$

As new data becomes available, private agents will update their estimates of the vector of coefficients $\psi$, and the central bank will update its estimates of $c_{yt}$ and $c_{\pi t}$. We assume that all of the agents in the economy use a standard recursive least squares algorithm (see, for instance, Evans and Honkapohja, 2001). We focus on the case of decreasing gains, in which the values of $\psi$, $c_{yt}$ and $c_{\pi t}$ converge to OLS estimates for appropriate gain sequences. Further details about the learning approach that we adopt in this work are provided in the

Appendix; the study of additional learning approaches is one of the directions of our future research.

We augment our learning algorithm with a projection facility. This projection facility ensures that parameter estimates remain within a predetermined region of values that we regard as suitable.[6] To be specific, we allow agents to make use of three types of projection facilities. The first type refers to the coefficients on the inflation rate and output in the perceived and actual policy rules, which are restricted to positive values. The second type of facility enables private agents to disregard estimates of the policy rule coefficients for which the solution of the expectational difference equation that they must solve in their decision process either does not exist or is not unique.[7] Finally, the third projection facility allows policymakers to rule out estimates of (12) and (13) that would cause the perceived law of motion of the variables of interest to be nonstabilizable. The actual projection facility that we use implies that if an estimate violates one of the restrictions that we impose, the relevant agents will construct an estimate by averaging the estimates from the previous two years (8 periods) and use this as their belief. A more formal description of the impact of projection facilities on the learning algorithm that is adopted in this paper is provided in the Appendix.

Projection facilities might rule out certain potentially interesting dynamics of the variables of interest. However, in the environment under analysis, which is characterized both by asymmetric imperfect information and by two-sided learning, it is important to endow agents with reasonable priors regarding the behavior of the other agents in the economy, because agents' decisions are based on beliefs, which are in turn affected by the other side's past decisions. It follows that it is possible for unreasonable beliefs with respect to the estimated coefficients to continue to reinforce each other instead of being redirected towards more sensible values. Regardless, the simulation section of this paper will provide further discussion about the role of projection facilities in our simulations.

Because we allow for the presence of trend inflation in (2), the true long-run level of inflation in this model is not known and is not constant. However, private agents can

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[6]For a more thorough discussion about the use of projection facilities in a number of learning algorithms, see Carceles-Poveda and Giannitsarou (2007).

[7]While it seems reasonable to assume that agents would rule out parameter estimates for which a stable pattern of the variables in $z_t^P$ does not exist, the case in which the solution is indeterminate is somewhat more complex. The analysis of learning in environments in which multiple equilibria could potentially arise requires not only to take a stand about the method by which one of the alternative solutions should be selected but also to model the manner in which agents should account for the indeterminacy of the equilibrium as they update their beliefs. The learning patterns that emerge in this environment could become very complicated. For this reason, we decided to begin with a simpler scenario in which private agents behave conservatively and disregard parameter estimates that would create indeterminate equilibria. Nonetheless, we do believe that the study of two-sided learning in the case of indeterminate equilibria is very interesting, and we intend to extend our research in this direction in the future.

estimate trend inflation in each period as a function of the estimated monetary policy rule and the steady state level of the real interest rate. It follows that this framework incorporates all the features that enable us to investigate the impact of uncertainty in the long-run level of inflation on current decisions, which is a direction we are currently pursuing in parallel work.

Before proceeding to a description of the decision process, we would like to address one issue that relates to the structure of the information set and the learning approach that we assume in this paper. One possible objection to the framework that we are adopting would be that because the central bank decision makers are private agents after all, it appears as though they should be able to know the learning problem of the other agents just by introspection. However, the model could be rewritten with the assumption that each private agent does not know a priori that all of the other agents use the same forecasting scheme. Instead, each household (or firm) $i$ could be endowed with a conditional expectations operator, $E_t^i$, that is indexed by $i$. If we assume that all of these expectation operators are indeed equal, we can integrate over $i$, and because we work with a linear model, this integration would still allow us to arrive at the standard aggregate equilibrium conditions that are presented above.

## 3.2   Policy decisions and the formation of expectations

The actual law of motion for the variables in the model is dependent on the decisions of private agents and policymakers. More specifically, private agents use their knowledge of the private side of the economy and their beliefs about the interest rate rule to form expectations, which in turn affect the behavior of $y_t$ and $\pi_t$ through (1) and (2). By contrast, the central bank uses its beliefs about the processes underlying $y_t$ and $\pi_t$ to set the policy instrument $x_t$.

With respect to the private sector, we assume that decisions follow the same timing that is adopted by Cogley et al. (2011). In particular, this timing involves the following steps. First, private agents estimate the parameters of the policy rule (11) using information up to and including time $t-1$. Then, they observe current period shocks and the value of the policy instrument and use them in combination with the previously available information to make decisions on actions. This approach implies that agents enter time $t$ with predetermined parameter estimates but subsequently use shock realizations from the current period to form expectations.

The central bank, on the other hand, has the power to decide the value of $x_t$ in (5). The

policy rule for $x_t$[8] is chosen by minimizing the expected discounted quadratic loss function:

$$E_{t-1}^{CB} \sum_{j=0}^{\infty} \beta^j [(\pi_{t+j})^2 + \lambda_y (y_{t+j})^2 + \lambda_i (i_{t+j} - i_{t+j-1})^2] \tag{14}$$

given (12) and (13), and the estimated values of $c_{yt}$ and $c_{\pi t}$. The parameters $\lambda_y$ and $\lambda_i$ represent the weights that are attached to the output variable relative to inflation and the relative cost of changing the nominal interest rate, respectively. The superscript in $E_{t-1}^{CB}$ indicates that expectations are formed with respect to the information set that is available to the central bank. We do not allow the central bank to react contemporaneously to monetary policy shocks because the bank could otherwise simply undo any effect of these shocks on the economy.

In their decisions, private agents and policymakers are assumed to behave as anticipated utility decision makers (Kreps, 1998); this assumption implies that all the agents will treat parameter estimates as true values, thus disregarding parameter uncertainty and the effects of learning. This assumption is common in the literature on learning in macroeconomics (for instance, see Evans and Honkapohja, 2001).

# 4    Model solution

Policymakers and private agents base their decisions on their respective perceived laws of motion (PLMs) for the variables of interest. However, their decisions will affect the true model of the economy, i.e., the actual law of motion (ALM) of these variables. This section provides more details about the agents' PLMs, their decision processes, and the resulting ALM.

## 4.1    The PLM for the central bank

The central bank's PLM for output and inflation is defined by equations (12) and (13). Using the vector $z_t^{CB}$, this PLM can be rewritten in state space form as follows:

$$A^{CB} z_t^{CB} = B^{CB} x_t + C_t^{CB} z_{t-1}^{CB} + D^{CB} \varepsilon_t^{CB} \tag{15}$$

The time subscripts in the matrices of parameters emphasize the fact that the estimates of $c_{yt}$ and $c_{\pi t}$ in (12) and (13) are updated over time, even if the aforementioned assumption

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[8] Only the current period policy recommendation of the calculated policy rule is actually implemented since the policy rule is updated every period.

of anticipated utility implies that policymakers will not account for these updates in their decision processes. The problem for the central bank is then to find the sequence $\{x_t\}$ that minimizes (14) subject to (15) under the assumption of constant parameter values. It is well known that, under standard conditions, the solution to this problem is linear in the state $z_{t-1}^{CB}$, i.e.:

$$
\begin{aligned}
x_t &= -F_t z_{t-1}^{CB} \\
&= f_{0t} + f_{\pi t}\pi_{t-1} + f_{yt}y_{t-1} + f_{it}i_{t-1} + f_{vt}v_{t-1}
\end{aligned}
$$

so that the expression for the nominal interest rate becomes:

$$
i_t = f_{0t} + f_{\pi t}\pi_{t-1} + f_{yt}y_{t-1} + f_{it}i_{t-1} + f_{vt}v_{t-1} + v_t \tag{16}
$$

If the matrices of parameters in (15) were constant over time, standard results in the optimal control literature would deliver a time-invariant optimal policy vector $F$. However, because the optimization problem in our setup is repeated in every period given updated values of $c_{yt}$ and $c_{\pi t}$ in (12) and (13), the optimal policy vector will be dependent on the current period estimates of these parameters.

Given estimates of the parameters for time $t$ and the chosen policy rule, the PLM for the central bank can be rewritten as follows:

$$
A^{CB}z_t^{CB} = \left(C_t^{CB} - B^{CB}F_t\right) z_{t-1}^{CB} + D^{CB}\varepsilon_t^{CB}
$$

or

$$
z_t^{CB} = \Phi_{1,t}z_{t-1}^{CB} + \Phi_2\varepsilon_t^{CB} \tag{17}
$$

where $\Phi_{1,t} = \left(A^{CB}\right)^{-1}\left(C_t^{CB} - B^{CB}F_t\right)$ and $\Phi_2 = \left(A^{CB}\right)^{-1}D^{CB}$.

The central bank implements the policy rule defined by (16); thus, the chosen value of the vector of coefficients $F_t$ will have an impact on the ALM of the nominal interest rate and, through this impact, will indirectly influence the ALMs of output and inflation.

## 4.2 PLM for private agents

The PLM for private agents can be obtained from equations $(1) - (4)$ and the perceived interest rate rule expressed by (11). In matrix form, this PLM may be written as follows:

$$
A^P z_t^P = B^P E_t^p\left(z_{t+1}^P\right) + C_t^P z_{t-1}^P + D^P\varepsilon_t^P \tag{18}
$$

where the time subscript in the matrix $C_t^P$ emphasizes the fact that the estimated coefficients of the perceived policy rule are updated over time. The matrices of coefficients $A^P$, $B^P$, $C_t^P$ and $D^P$ are specified in the Appendix.

Private agents use (18) as the basis for solving the expectation term $E_t^p \left( z_{t+1}^P \right)$. As discussed above, we use the same approach that is employed in Cogley et al. (2011). In more detail, in every period private agents estimate the coefficients of the perceived policy rule, and then solve the vector-valued expectational difference equation that features the equilibrium conditions, including the estimated policy rule. This approach has the important implication that agents account for cross-equations restrictions when forming forecasts. In addition, because of our assumptions, the PLM is just the reduced form VAR that is associated with (18), with reduced form coefficients that are time-varying because they depend on the estimates of the policy rule coefficients. For this reason, we can guess a solution of the following form:

$$z_t^P = \Gamma_{1,t} z_{t-1}^P + \Gamma_{2,t} \varepsilon_t^P$$

This solution can be substituted in for the expectation term to obtain the following equation:

$$(A^P - B^P \Gamma_{1,t}) z_t^P = C_t^P z_{t-1}^P + D^P \varepsilon_t^P$$

This equation, in turn, generates the following equations for the reduced form matrices:

$$\Gamma_{1,t} = (A^P - B^P \Gamma_{1,t})^{-1} C_t^P$$

$$\Gamma_{2,t} = (A^P - B^P \Gamma_{1,t})^{-1} D^P$$

We use Sims'(2001) Gensys program to find the values of $\Gamma_{1,t}$ and $\Gamma_{2,t}$. If $z_t^P$ is determinate, this program delivers the unique nonexplosive solution for these matrices of parameters.

As discussed above, we endow private agents with a projection facility that enables them to rule out coefficient estimates for which a stable solution does not exist. In addition, we let private agents employ an additional projection facility that allows them to disregard estimates of the parameters in $A^P$, $B^P$, $C_t^P$ and $D$ that would produce an indeterminate outcome for $z_t^P$. This facility has the consequence of preventing the Gensys program from randomly selecting a solution that private agents should implement.

## 4.3 The ALM

The ALM for the variables in the model involves equations $(1) - (4)$, which describe the true behavior of the private sector, together with the true interest rate rule that is expressed by

(16). In matrix form, this ALM can be written as follows:

$$Az_t = BE_t^p\left(z_{t+1}^P\right) + C_t z_{t-1} + D\varepsilon_t$$

The matrices $A$, $B$, $C$ and $D$ are defined in the Appendix. Notice that the matrix $C_t$ is time-variant because it includes the true policy coefficients, which the central bank will update in each period.

From the PLM for the private sector, we know that the following relationship holds:

$$
\begin{aligned}
E_t^p\left(z_{t+1}^P\right) &= \Gamma_{1,t} z_{t-1}^P \\
&= \Gamma_{1,t} M^P z_t
\end{aligned}
$$

which implies:

$$Az_t = B\Gamma_{1,t} M^P z_t + C_t z_{t-1} + D\varepsilon_t \tag{19}$$

It follows that the ALM of the model can be written as:

$$z_t = \Psi_{1,t} z_{t-1} + \Psi_{2,t}\varepsilon_t$$

where:

$$\Psi_{1,t} = (A - B\Gamma_{1,t} M^P)^{-1} C_t$$

$$\Psi_{2,t} = (A - B\Gamma_{1,t} M^P)^{-1} D$$

These last two expressions, which define the matrices of coefficients in the ALM of the economy, do not include the matrix $\Gamma_{2,t}$. Therefore, the inclusion of a perceived policy shock in the PLM of agents is de facto unnecessary.

# 5    Simulation Exercises

The model that we have described in the previous section involves complex interactions of beliefs and actions between private agents and policymakers that cannot be solved in closed form. In particular, although the learning procedure that agents use to update their beliefs has a recursive structure, the optimization approach of policymakers and the expectation formation process of private agents are highly nonlinear functions of their estimated parameters. For this reason, the equilibrium pattern that is implied by the assumed learning and decision sequence cannot be characterized analytically. Thus, the main goal of this section is to offer insights about the role of asymmetric information and two-sided learning in

the context of the New Keynesian framework that was described in the previous section by performing a Monte Carlo simulation.

We focus on the short-run (i.e., non-asymptotic) behavior of the endogenous variables in the model, and we investigate the patterns, magnitudes and durations of the deviations of these variables from their values in a rational expectation equilibrium. We define the REE as the equilibrium that emerges from an environment in which policymakers set a fixed policy rule for the instrument $x_t$ in (5) and maintain this policy for the entire simulation period. During this period, private agents continue to learn and compute expectations using the procedure that was described in the previous section. The fixed policy rule that we use in this case is a standard Taylor-type rule of the following form: $x_t = 0.5\bar{r} + 0.5y_{t-1} + 1.5\pi_{t-1} + 0.5i_{t-1}$. Although this specific policy rule is chosen arbitrarily, the lessons that we will obtain from comparing the model under learning with the rational expectations version would emerge under virtually any monetary policy rule.[9]

In addition to the benchmark scenario that incorporates learning and optimal decisions as described in the previous section, we also perform one additional exercise. We allow the central bank to communicate its perceived steady state value of inflation to private agents. More specifically, we assume that at the end of period $t$, the central bank announces $\bar{\pi}_t$, their time $t$ estimate of the long-run value of inflation, which can be calculated using the policy rule computed in period $t$. Private agents trust this announcement, and because of anticipated utility, they regard this announced level of the steady state inflation rate as fixed and use it in their regressions. In other words, they estimate the policy rule with the time $t$ left-hand-side variable being $i_t - \bar{i}_t$ and the right-hand-side variables being deviations from the implied steady state; $\bar{i}_t$ is the sum of the announced steady state inflation rate and $r$. When making their decisions in the next period, the private agents also use their knowledge of $\bar{\pi}_t$. Note that the central bank will update its estimate to $\bar{\pi}_{t+1}$ during this subsequent period. We believe that this setup is interesting because it allows us to study the impact that a reduction in the asymmetry of the information available to agents produces on the process of learning. However, because the central bank has imperfect information about the economy, its communication of $\bar{\pi}_t$ will not necessarily improve economic outcomes. Instead, policymakers' incorrect beliefs about the long-run value of the inflation rate could potentially destabilize the learning and decision process of private agents.

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[9]Ideally, we would like to compare our learning model with a corresponding rational expectations model featuring asymmetric information (instead of comparing it with a rational expectations model that features an arbitrary policy rule); however, as we will discover below, our simulations do not allow us to identify this corresponding asymmetric information rational expectations model. In addition, in our framework the differential equation approach of Marcet and Sargent (1989) does not provide us with any advantage over simply running simulations because we would have to solve the ODEs numerically as well, which would be cumbersome due to the non-linear mapping from parameter estimates to reduced form dynamics.

In all the simulations, we set the following parameters for the true model of the economy and the loss function of policymakers: $\sigma = 1$; $\kappa = 0.2$; $\beta^P = \beta^{CB} = 0.99$; $\bar{r} = 1/\beta^P - 1$; $\lambda_y = 1/16$ and $\lambda_i = 0.5$. For the real shock $u_t$ and the policy shocks $v_t$, we assume normal distributions with the following parameters: $\sigma_{\varepsilon^u}^2 = 0.008^2$; $\sigma_{\varepsilon^v}^2 = 0.008^2$; $\sigma_{\varepsilon^v}^2 = 0.008^2$; $\rho_u = 0$; $\rho_v = 0$.[10] To initialize the learning and decision process, we must set an initial value for the private agents' beliefs. We do this by using population regressions and population moments from the rational expectation solution of the model that is obtained by applying the same fixed policy rule adopted in the rational expectations scenario. Although the initial transition path in the simulations is affected by this choice, our conclusions are not.[11]

We report the results for the case of a recursive learning algorithm with decreasing gains, as described in the Appendix.[12] The value of $t_0$ was set equal to 12. In all of our exercises, we set the period length to $T = 1000$, and we performed $N = 1000$ simulations. We study the impact of asymmetric information and two-sided learning in the New Keynesian model under analysis by examining the distributions of the patterns for the variables of interest and the policy parameters that are obtained from the different simulations. We show the median, and 15th and 85th percentile bands of these distributions, and we report their relevant statistics.[13]

Figures $1-3$ and tables $1-2$ provide evidence of the impact of two-sided learning in this environment and demonstrate that this impact is significant. Figures $1-3$ show the median as well as the 15th and 85th percentiles of log output, annualized inflation and the annualized nominal interest rate over time. Compared to their distributions in the rational expectations case, all the variables are more volatile if asymmetric information and two-sided learning are included in the framework. This increase in volatility occurs for every variable and is consistent across all of the exercises that we performed. Interestingly, for the nominal interest rate in the benchmark case of two-sided learning, this increase is not particularly large, as the volatility of this rate only increases by 30 percent. By contrast, the magnitude of the volatility increase is quite large for all of the other examined variables. Table 1 shows that in both exercises, the median standard deviation for the output variable is more than

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[10]We are not advocating uncorrelated exogenous shocks as necessarily constituting the best option for fitting data; however, because we are interested in the relative volatility and persistence of the variables in our model, we choose to use this simplifying assumption. Our results are robust to the assumption of autocorrelated shocks (and actually become somewhat stronger under this assumption).

[11]Carceles-Poveda and Giannitsarou (2007) offer a discussion of the different methods that may be employed to obtain agents' initial beliefs in frameworks that are characterized by adaptive learning.

[12]We also experimented with constant gains, $g = 0.015$, and we found a much greater volatility for the variables of interest in this case. As mentioned above, the analysis of the learning patterns under alternative algorithms is one of the extensions that we intend to pursue.

[13]If we reported means instead of medians, the difference vis-a-vis rational expectations would be more striking; however, this result would be driven at least in part by a relatively small fraction of outliers.

twice its value in the rational expectations case, and the median standard deviation of the annualized inflation rate is more than 6 times its value in the rational expectations case. This table also highlights the fact that reducing asymmetric information through central bank communications does not decrease the volatility that is created by two-sided learning. On the contrary, the median standard deviations of log output and of the annualized nominal interest rate are higher under this assumption than in the benchmark case.

With respect to policy parameters, we report the distribution of the actual and estimated coefficients that are associated with the lagged inflation rate (figure 4).[14] If asymmetric information and learning are incorporated into the model, the convergence of these two sets of parameters to fixed values is not guaranteed. Because the private agents use different variables in their perceived policy rule than the variables that are actually utilized to set the monetary policy rule, the coefficients on lagged inflation (or other variables that appear in both the perceived and actual policy rules) in the perceived and actual policy rules do not have to converge to the same values. Thus, the main point worth emphasizing is that the distribution of the coefficients for both the perceived and the actual policy rules is non-degenerate even after 1000 periods rather than the simulated values for actual and perceived policy coefficients being different.[15] This result hints at either the possibility of a failure to converge to an asymmetric rational expectations equilibrium or the potential existence of multiple self-confirming equilibria in our model.[16] We will discuss below why the convergence theorems of Marcet and Sargent (1989) do not apply in our setup (and therefore why non-convergence is an issue that might arises within this framework).

It is interesting that communications from the central bank regarding the estimated long-run value of the inflation rates do not appear to help private agents learn the actual policy coefficients; as a result, these communications do not appear to reduce the volatility of policymakers' beliefs and policy decisions. As discussed above, the fact that the perceived long-run inflation rate is obtained through the use of a mis-specified model has the potential to destabilize rather than stabilize the learning and decision process, even if these communications actually reduce the asymmetry of information. This result is confirmed by figure 7, which depicts the absolute difference in the one-step-ahead inflation forecasts by the

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[14]We find that the other parameters in the policy rule exhibit patterns that are similar to the patterns that are reported in figure 4.

[15]We also find that the projection facility that requires the optimal policy parameters to assume reasonable values (in this case, a value greater than zero) is invoked in a non-negligible number of the simulations, particularly in the first periods of the learning and decision process.

[16]As discussed above, the framework that we are using, which essentially involves private agents using their beliefs to solve a rational expectations model every period, renders it infeasible for us to obtain any analytical results about the existence of multiple self-confirming equilibria using the standard ODE approach pioneered by Marcet and Sargent (1989).

central bank and private agents in the last period of the simulations.[17] This figure clearly demonstrates that compared with the baseline asymmetric information environment, this difference can be much larger in the scenario in which policymakers communicate their beliefs regarding long-run inflation. In a recent contribution, Eusepi and Preston (2010) discuss the conditions under which the central bank's communication of a (time-invariant) inflation target does not lead to convergence to the rational expectations equilibrium. Although the results of Eusepi and Preston (2010) have a similar flavor to the outcomes of our simulations, they are obtained by either letting one of the exogenous processes in their model become arbitrarily close to a random walk or by having price stickiness vanish, producing persistent exogenous processes. We, on the other hand, have i.i.d. disturbances and a fixed level of price stickiness (as encoded by the slope of the Phillips curve).[18]

One additional observation can be made from figure 7. The two panels of this figure emphasize the fact that two-sided learning has important consequences on the ability of agents to predict future values for the variables of interest, even for the very short run, i.e., for the one-period ahead forecast. Indeed, different knowledge and beliefs about the underlying true coefficients of the model and policy rules will cause private agents and policymakers to disagree about the predicted patterns of economic variables in the future; at times, these disagreements may be substantial.

In figure 5, we report the distribution of the autocorrelations of the variables of interest in the 3 alternative scenarios under analysis.[19] This figure indicates that two-sided learning can potentially increase the persistence of the variables in the model at all orders relative to the rational expectations case; this increase is often substantial. This result holds for all of the exercises that we performed and is considerably more pronounced for the scenario in which we allow the central bank to communicate its perceived long-run value of the inflation rate.[20]

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[17]To control for outliers, we report forecast differences in terms of annualized percentages up to the 80th percentile of the distribution.

[18]In the economic literature on information and signal extraction, a number of contributions originating from the seminal work of Morris and Shin (2002) have investigated the impact of transparency and public information on social welfare. In general, this research finds that more transparency by the central bank might be either welfare increasing or welfare decreasing, depending on the precision of the public signal relative to the private signal and on the weight that private agents attach to the departures of their individual actions from the aggregate. Our results regarding central bank communication are related to the conclusions of this literature; however, the focus of our paper is not on the analysis of the relationship between public information and social welfare; instead, we concentrate on the study of the changes in the dynamics of a standard New Keynesian model that are caused by the introduction of asymmetric information and learning.

[19]We report sample autocorrelations that are directly computed from the simulated data.

[20]Figure 5 somewhat underestimates the possible increase in autocorrelation because we report the median and 15th and 85th percentile bands. Although the results would look the same for the full information case if we used the mean autocorrelation, this measure is substantially higher than the median autocorrelation for the learning models that we consider. We choose not to report mean autocorrelations because these autocorrelations can be driven by outliers.

Tables $3-5$ report the correlations between variables for the different cases. The correlations between log output and inflation and between log output and the nominal interest rate do not appear to be affected by the introduction of two-sided learning. The only relationship that changes in a relevant manner is the relationship between the inflation rate and the nominal interest rate in the case involving asymmetric information and learning without central bank communication. In particular, the negative correlation between these two variables becomes much smaller in this scenario. This result suggests that in this case, the ability of the central bank to affect the inflation rate through its policy instrument might be reduced. On the whole, we can conclude that two-sided learning has the potential to affect the persistence of the variables in the models and to alter certain important contemporaneous relationships between these variables.

Finally, we study the stability properties of the analyzed models. Figure 6 reports the probability of bad outcomes, which we define as inflation rates higher than 20%, for the three alternative scenarios.[21] In the rational expectations model, in which policymakers set a fixed policy rule and private agents know this rule, this probability is zero. If asymmetric information and two-sided learning are introduced, the probability of inflation rates higher than 20% becomes positive, and it increases with the length of the sample period under consideration. In addition, if the central bank is allowed to communicate its beliefs about the long-run inflation rate, this probability becomes quite large, confirming our previous observation that "naive" communication of information that is based on incorrect beliefs about the true data generating process can have destabilizing effects on the economy, instead of facilitating learning and promoting the convergence of perceived and actual decisions. Figure 8 provides further support to this conclusion. The projection facilities that we implement ensure that private agents and policymakers rule out parameter estimates which, according to their respective PLMs, would generate explosive patterns of the endogenous variables in the model. However, as previously discussed, this condition is not sufficient to guarantee the stability of the ALM because in certain circumstances, decisions based on the individual PLMs could still produce a situation in which one or more eigenvalues of $\Psi_{1,t}$ are larger than one in absolute value. Figure 8 indicates that the probability of this type of event is nonzero, although this probability remains very small in the baseline scenario.[22] But when communication from the central bank to private agents is allowed, this probability becomes

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[21]This probability was computed as the fraction of times in which inflation was above the 20% value in the simulations that we performed.

[22]The scales in the figure for the baseline case and the scenario involving CB communication are very different. In particular, the scale for the CB communication case is much larger than the scale for the baseline case. One possible reason why we do not observe convergence in the baseline simulations in which no explosive eigenvalues occur is that the eigenvalues in those simulations are typically still very close to 1 in absolute value, thus effectively halting convergence.

considerably higher. To obtain further insight into the reasons underlying our results, it is instructive to compare our framework with the approach of Marcet and Sargent (1989). Marcet and Sargent (1989) require the ALM to be stable at all times to guarantee convergence to an asymmetric information rational expectations equilibrium. The existence of periods in our simulations in which the ALM is unstable implies that the convergence results in Marcet and Sargent (1989) are not applicable in our case, despite the fact that we endow agents with perceived laws of motion that are always stable[23]. The result of unstable dynamics coupled with stable perceived dynamics is also observed in models with one-sided learning, as discussed in Cogley et al. (2011).

Our exercises suggest that the impact of asymmetric information and two-sided learning in the context of a New Keynesian model of the economy is significant. We find that the pattern of the variables of the model can change considerably in terms of volatility and auto-correlations, that undesired outcomes are much more likely, and that significant differences between the forecasts of policymakers and private agents with respect to the future values of the variables of interest can potentially arise. We also show that the communication of information by the central bank does not necessarily improve private agents' learning of the true policy coefficients and therefore might not produce a more stable economic environment. In fact, in our simulations, the scenario in which policymakers disclose their beliefs about the long-run inflation rate is the one that exhibits the highest potential for extreme outcomes to emerge. This increase in volatility occurs because even though communication between agents reduces the asymmetry of the information that is employed by agents in their respective decision processes, these decisions will continue to be based upon each side's perceptions and imperfect knowledge regarding the true model of the economy.

# 6   Conclusions

This paper represents a first attempt to investigate the role of asymmetric information and two-sided learning in a New Keynesian model of the economy. The assumption that both monetary authorities and private agents have imperfect knowledge of the true data generating process, and that they attempt to learn over time from new information that becomes available, appear to be relatively accurate reflections of reality. For this reason, we believe that the study of the way in which this type of learning process can potentially alter the dynamics of a New Keynesian framework, which is often used as the basis for policy analysis,

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[23]One caveat is that Marcet and Sargent (1989) require the learning process to start in the domain of attraction of the ODE that governs convergence to their asymmetric rational expectations equilibrium. We can not guarantee that we are in this domain of attaraction. We have, however, started our simulations at substantially different beliefs and always found similar results.

is interesting and (hopefully) important. The results of our simulations support this idea by demonstrating that two-sided learning can cause beliefs and decisions to depart significantly from standard rational expectations values. We also emphasize a novel (at least to the best of our knowledge) aspect of central communication in learning models, namely, we show that if the central bank has a mis-specified model of the economy, central bank communications can generate substantially greater volatility, not only for an initial transition period, but for the entire time period considered in this study. This aspect of our analysis complements the research and conclusions of Eusepi and Preston (2010), who examine central bank communication under private sector learning.

The analysis in this paper can be extended in a number of directions. First, the impact of alternative assumptions regarding the learning approach of agents, such as the use of different learning algorithms, could be investigated. Second, we believe that it would be interesting to further analyze the effects that communication between the agents might produce on their process of learning about the true data generating process. In this work, we have studied the case in which policymakers disclose their perceived long-run inflation rates to private agents. Communications involving different types of information or communications from private agents to policymakers could also be investigated. Third, the framework that we employ in this paper allows us to investigate the impact of uncertainty about the long-run level of inflation on current beliefs and decisions, and this investigation also represents a direction that we are interested in continuing to pursue. Finally, one route that we would also like to explore is the estimation of our framework using real world data. This extension would allow us to employ this model to provide an interpretation of past events and to offer more cogent policy recommendations.

## Appendix

Learning algorithm

Let the equations to be estimated by agents be written in general terms as:

$$q_t = p'_{t-1}\phi_t + \eta_t$$

where $q_t$ is the dependent variable or a vector of dependent variables, $p_{t-1}$ a vector or matrix of regressors, $\eta_t$ the residual(s) and $\phi_t$ the vector of parameters of interest. In the case of private agents, this equation corresponds to (11), while for policymakers it encompasses (12) and (13). Using this notation, the learning algorithm can be written as:

$$R_t = R_{t-1} + g_t \left(p_{t-1}p'_{t-1} - R_{t-1}\right)$$

$$\phi_t = \phi_{t-1} + g_t R_t^{-1} p_{t-1} \left(q_t - p'_{t-1}\phi_{t-1}\right)$$

where $g_t$ represents the gain. In the simulations, we focus on Recursive Least Squares (RLS) learning, in which $g_t = \frac{1}{t_0+t}$.[24] However, we also perform some comparative exercises using Constant Gain (CG) learning, in which $g_t$ is a constant positive and small number, i.e. $g_t = g$, $0 < g < 1$. For a more thorough description of these learning algorithms and their properties, see Evans and Honkapohja (2001); for a discussion of their performance in a few standard macroeconomic models, see Carceles-Poveda and Giannitsarou (2007).

As mentioned in the main text, the basic learning algorithm will be augmented with a number of projection facilities. We assume that whenever the value of the estimated parameters by private agents and policymakers "hits" the projection facility, i.e. moves outside the predetermined parameter region $Q$, agents will use an average over past estimates as the current period estimate. Thus, the algorithm can be rewritten as:

$$\widehat{R}_t = \widehat{R}_{t-1} + g_t \left(p_{t-1}p'_{t-1} - \widehat{R}_{t-1}\right)$$

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + g_t \widehat{R}_t^{-1} p_{t-1} \left(q_t - p'_{t-1}\widehat{\phi}_{t-1}\right)$$

$$(\phi_t, R_t) = \begin{cases} \left(\widehat{\phi}_t, \widehat{R}_t\right) & \text{if } \left(\widehat{\phi}_t, \widehat{R}_t\right) \in Q \\ \left((1/N)\sum_{j=1}^{N} \phi_{t-j}, (1/N)\sum_{j=1}^{N} R_{t-N}\right) & \text{if } (\phi_t, R_t) \notin Q \end{cases}$$

The specific restrictions that we impose via projection facilities are discussed in the main text.

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[24] As mentioned in the main text, $t_0$ is set to 12 quarters in our simulations.

Matrices in the PLMs and ALM

The matrices of the PLM for the central bank can easily be obtained using the state space representation (15) and the policy rule (16) emerging as a result of the optimization problem. We have that:

$$A^{CB} z_t^{CB} = \left(C_t^{CB} - B^{CB} F_t\right) z_{t-1}^{CB} + D^{CB} \varepsilon_t^{CB}$$

or more explicitly:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_t \\ \pi_t \\ i_t \\ v_t \\ 1
\end{pmatrix}
=
\begin{pmatrix}
c_{1yt} & c_{2yt} & c_{3yt} & c_{4yt} & c_{5yt} \\
c_{1\pi t} & c_{2\pi t} & c_{3\pi t} & c_{4\pi t} & c_{5\pi t} \\
-f_{\pi t} & -f_{yt} & -f_{it} & -f_{vt} & -f_{0t} \\
0 & 0 & 0 & \rho_v & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ v_{t-1} \\ 1
\end{pmatrix}
+
$$

$$
+
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_{yt}^{CB} \\ \omega_{\pi t}^{CB} \\ \varepsilon_t^v
\end{pmatrix}
$$

The PLM for private agents is given by the true equations $(1) - (4)$ together with the perceived interest rate rule expressed by (11).

$$y_t = E_t^P\left(y_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t^P\left(\pi_{t+1}\right) - \bar{r} - u_t\right)$$

$$\pi_t = \frac{1}{(1+\beta)}\pi_{t-1} + \frac{\beta}{(1+\beta)}E_t^P\left(\pi_{t+1}\right) - \frac{\kappa}{(1+\beta)}\left(y_t - \bar{y}\right)$$

$$i_t = \psi_{0t} + \psi_{\pi t}\pi_{t-1} + \psi_{yt}y_{t-1} + \psi_{it}i_{t-1} + \psi_{ut}u_{t-1} + \omega_t^P$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

These equations can be rewritten in matrix form as:

$$
\begin{pmatrix} 1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & -\frac{1}{\sigma}\overline{r} \\ \frac{\kappa}{(1+\beta)} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ u_t \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{\sigma} & 0 & 0 & 0 \\ 0 & \frac{\beta}{(1+\beta)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} E_t^p \left[ \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ u_{t+1} \\ 1 \end{pmatrix} \right] +
$$

$$
+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(1+\beta)} & 0 & 0 & \frac{\kappa}{(1+\beta)}\overline{y} \\ \psi_{yt} & \psi_{\pi t} & \psi_{it} & \psi_{ut} & \psi_{0t} \\ 0 & 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ u_{t-1} \\ 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t^u \\ \omega_t^P \end{pmatrix}
$$

or

$$
A^P z_t^P = B^P E_t^p \left( z_{t+1}^P \right) + C_t^P z_{t-1}^P + D^P \varepsilon_t^P
$$

The ALM for the variables in the model can be obtained from the true equations $(1) - (4)$ together with the true interest rate rule expressed by $(16)$, and can be written in matrix form as:

$$
\begin{pmatrix} 1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & 0 & -\frac{1}{\sigma}\overline{r} \\ \frac{\kappa}{(1+\beta)} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ u_t \\ v_t \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{\sigma} & 0 & 0 & 0 \\ 0 & \frac{\beta}{(1+\beta)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} E_t^p \left[ \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ u_{t+1} \\ 1 \end{pmatrix} \right]
$$

$$
+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(1+\beta)} & 0 & 0 & 0 & \frac{\kappa}{(1+\beta)}\overline{y} \\ f_{yt} & f_{\pi t} & f_{it} & 0 & f_{vt} & f_{0t} \\ 0 & 0 & 0 & \rho_u & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_v & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} z_{t-1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t^u \\ \varepsilon_t^v \end{pmatrix}
$$

or:

$$
A z_t = B E_t^p \left( z_{t+1}^P \right) + C_t z_{t-1} + D \varepsilon_t
$$

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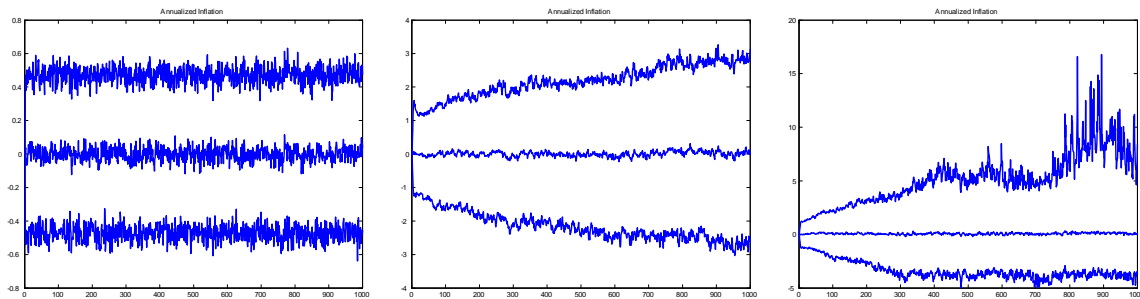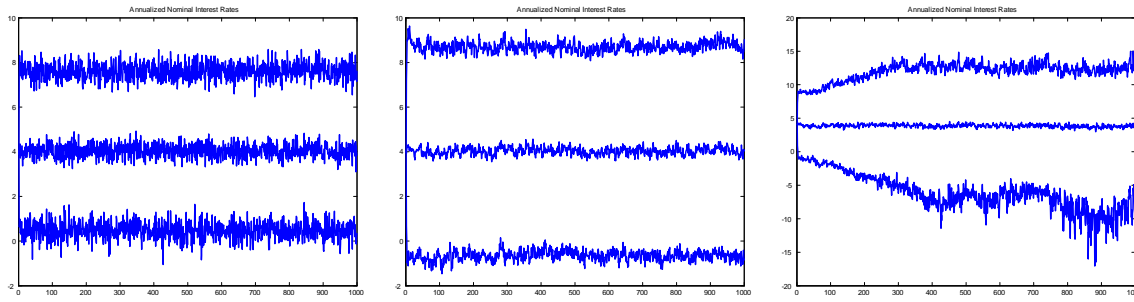
Figures and Tables

Figure 1 - Log output
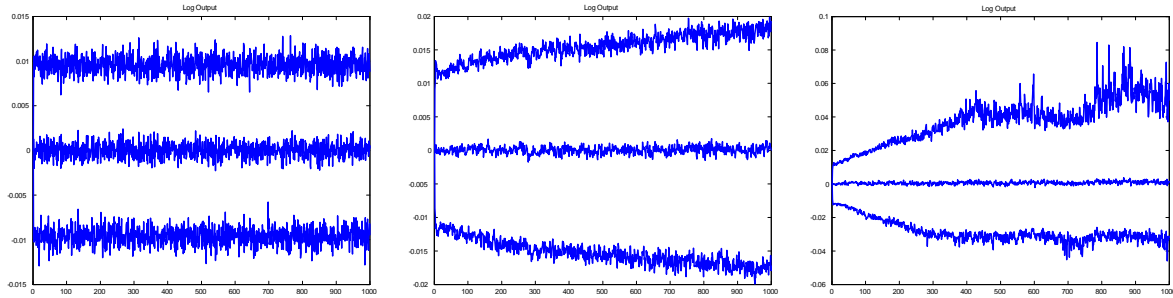


Note: Median value and 15th and 85th percentile bands of the simulated pattern for log output. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.
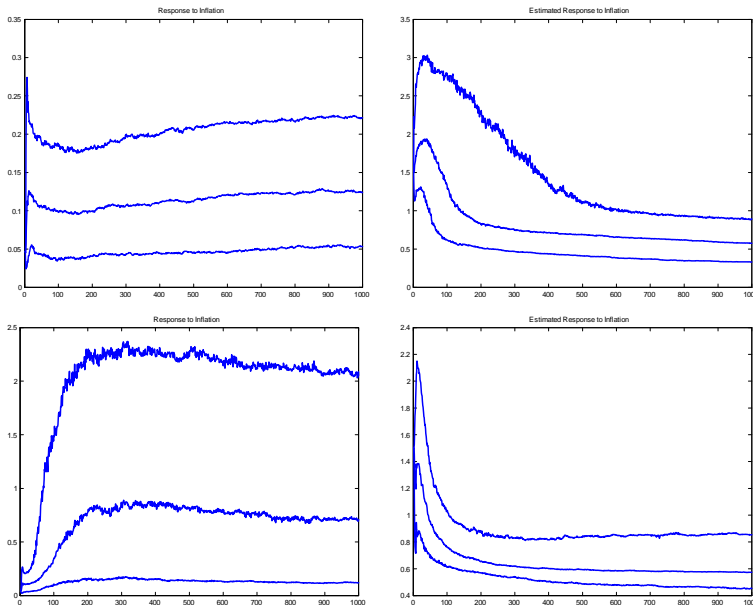
Figure 2 - Annualized inflation



Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized inflation rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 3 - Annualized nominal interest rate



Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized interest rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 4 - Selected policy coefficients - response to the inflation rate



Note: Median value and 15th and 85th percentile bands of the simulated actual and estimated policy response to the inflation rate. Each row reports a different scenario: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Figure 5 - Autocorrelations
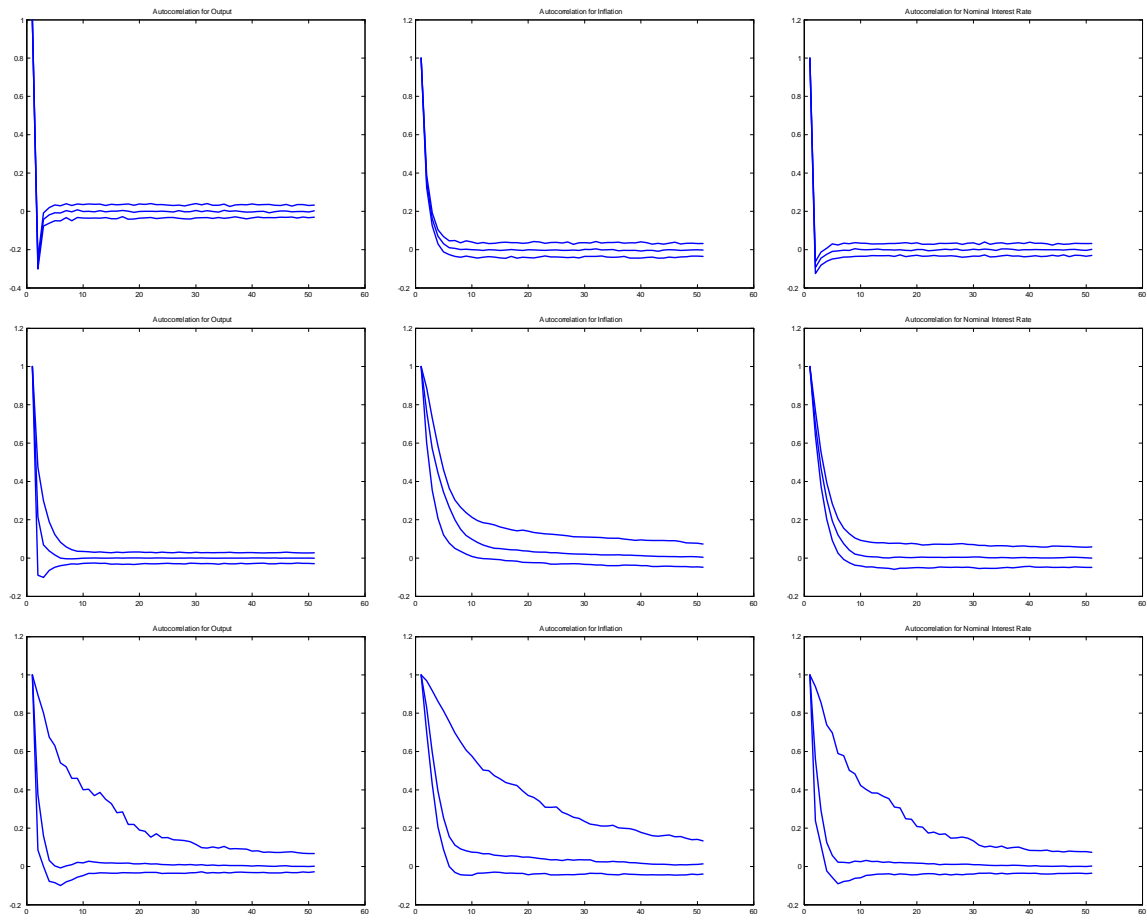


Note: Median value and 15th and 85th percentile bands of the autocorrelations of the variables of interest in the performed simulations. Each row reports a different scenario: 1) RE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

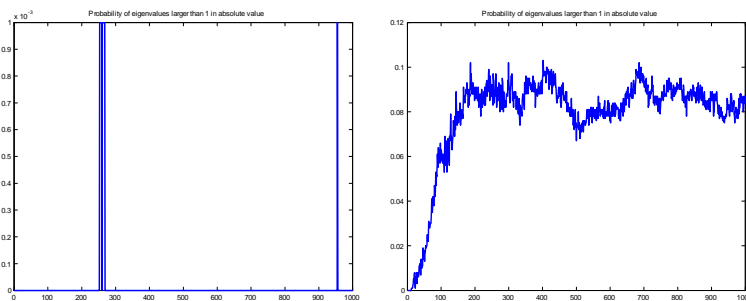Figure 6 - Probability of inflation larger than 20%
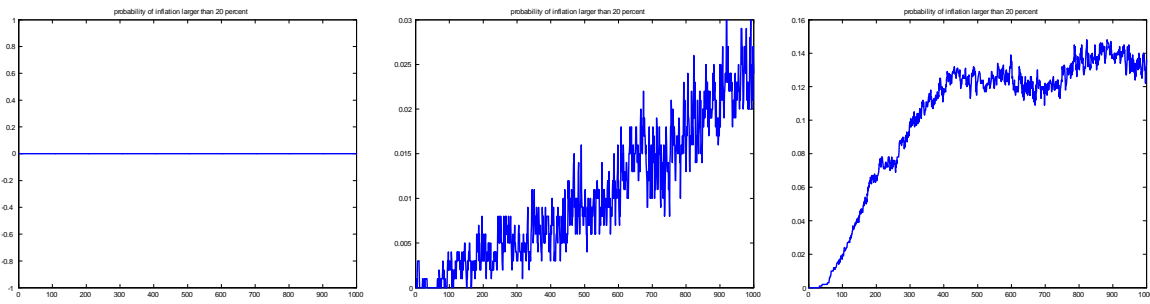


Note: Probability of inflation exceeding 20%. The panels report the following scenarios: 1) RE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 7 - Absolute difference inflation forecast
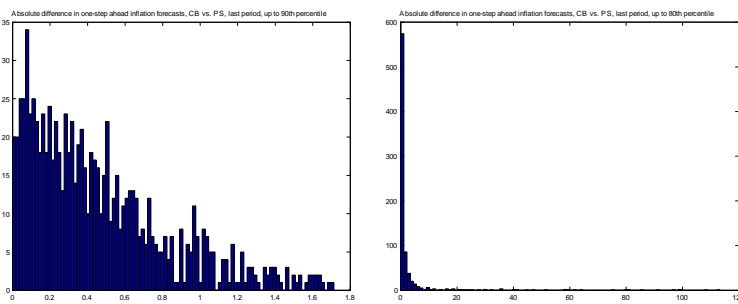


Note: Absolute difference in the one-step ahead inflation forecasted by the Central Bank and private agents. The panels report the following scenarios: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Figure 8 - Probability of explosive ALM



Note: Probability of eigenvalues larger than 1 in absolute value in the ALM. The panels report the following scenarios: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Table 1 - Median standard deviations relative to the RE case

| variable | benchmark | CB comm. |
|---|---|---|
| output | 2.3539 | 2.6698 |
| annualized inflation | 7.9535 | 6.1471 |
| annualized interest rate | 1.3287 | 2.0812 |

Table 2 - Standard deviations, RE case

| variable | standard deviation |
|---|---|
| output | 0.0092 |
| annualized inflation | 0.4537 |
| annualized interest rate | 3.4487 |

Table 3 - Correlations in the RE case

correlations - RE case

| | $y_t$ | $\pi_t$ | $i_t$ |
|---|---|---|---|
| $y_t$ | 1 | 0.7874 | -0.8461 |
| $\pi_t$ | 0.7874 | 1 | -0.7342 |
| $i_t$ | -0.8461 | -0.7342 | 1 |

Table 4 - Correlations in the benchmark case

| | $y_t$ | $\pi_t$ | $i_t$ |
|---|---|---|---|
| $y_t$ | 1 | 0.6474 | -0.6524 |
| $\pi_t$ | 0.6474 | 1 | -0.2876 |
| $i_t$ | -0.6524 | -0.2876 | 1 |

Table 5 - Correlations in the case of central bank communication

| | $y_t$ | $\pi_t$ | $i_t$ |
|---|---|---|---|
| $y_t$ | 1 | 0.7614 | -0.9621 |
| $\pi_t$ | 0.7614 | 1 | -0.8350 |
| $i_t$ | -0.9621 | -0.8350 | 1 |