Immigration and the School System∗

Facundo Albornoz† Antonio Cabrales‡ Esther Hauk§

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Abstract

Immigration is an important problem in many societies, and it has wide-ranging effects on the educational systems of host countries. There is a now a large empirical literature, but very little theoretical work on this topic. We introduce a model of family immigration in a framework where school quality and student outcomes are determined endogenously. This allows us to explain the selection of immigrants in terms of parental motivation and the policies which favor a positive selection. Also, we can study the effect of immigration on the school system and how school quality may self-reinforce immigrants’ and natives’ choices.


Key-words: education, immigration, school resources, parental involvement, immigrant sorting.

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†Department of Economics, Birmingham University; email: f.albornoz@bham.ac.uk

‡Department of Economics, Universidad Carlos III de Madrid; email: acabrale@eco.uc3m.es

§Instituto de Análisis Económico (IAE-CSIC) and Barcelona Graduate School, Campus UAB, Bellaterra (Barcelona); email: esther.hauk@iae.csic.es.
1 Introduction

Many countries host a growing number of immigrants, many of whom are children. For example, The US Census Bureau estimated in 2000, that 34% of all youth aged 15–19 were from minority groups and one in five school-age children live in immigrant families (Kao and Thompson, 2003). According to the Innocenti Research Center, in 2009 almost a quarter of children are immigrants in the Netherlands, Germany, Sweden and the United States. This proportion is about one-sixth in France and Great Britain (Alba, Sloan, and Sperling, 2011a). The future of these societies clearly depends on these children. They certainly affect a country’s human capital through at least two channels. First, these children have to be schooled, and by changing the school composition they have a large effect on school quality and the performance of native children.\textsuperscript{1} Also, many of them will stay, so their educational attainment will affect a substantial part of the human capital of host countries.

Not surprisingly, the educational achievement of immigrant children and their effects on the schooling system in the host country is a core concern of policy makers. Studies like those conducted by PISA\textsuperscript{2} and other international organizations have allowed for the empirical analysis of immigrant educational success and the externalities imposed on natives. In many countries, a large fraction of immigrant children face substantial disadvantages in reaching educational parity with native children\textsuperscript{3} (Heath, Rothon, and Kilpi1, 2008; Anghel and Cabrales, 2010), but it is also not at all rare for some immigrant students to be top of the class.\textsuperscript{4} Researchers by now agree that immigrant students perform differently by origin group (Levels, Dronkers, and Kraaykamp, 2008)

\textsuperscript{1}See e.g. Albornoz, Berlinski, and Cabrales (2011) and references therein. A good review of the literature on peer effects in education is provided by Sacerdote (2010). See also Hanushek and Woessmann (2010) for a more general discussion about the educational production function.

\textsuperscript{2}Programme for International Student Assessment, http://www.pisa.oecd.org/. Another good example is the Trends in International Mathematics and Science Study (TIMSS) and Progress in International Reading Literacy Study (PIRLS) conducted by Boston College. http://timss.bc.edu/index.html

\textsuperscript{3}Australia and Canada are the big exceptions where immigrants often outperform natives before controlling for individual characteristics (Schnepf, 2004).

\textsuperscript{4}Card (2005) shows that of the 39 largest country of origin groups in the US, 33 groups for sons and 32 groups for daughters have higher average educational attainment than children of natives. Dustmann and Theodoropoulos (2010) and Dustmann, Frattini, and Theodoropoulos (2010) show for Britain that at the end of compulsory schooling the second generation of Indian and Chinese pupils outperform While British pupils by more than 30% of a standard deviation in both English and mathematics. See also Dustmann and Glitz (2010) for an overview on migration and education.
and (Levels and Dronkers, 2008) and cross-nationally (Marks, 2005). Even immigrants from the same origin perform different according to their destination country (Bertoli, Fernandez-Huertas Moraga, and Ortega, 2010, forthcoming). Moreover, the immigration mix differs considerably across countries, which is only partially due to colonial links. Policy variables in the host countries, like the ease of immigrating and naturalization policies, seem to be important to explain the different immigrant mix and the educational success of immigrant children (Dronkers and Fleischmann, 2010).

In spite of the importance of the problem, and the abundant empirical literature on the topic, there is relatively little theoretical work aimed at producing a framework for the understanding of this phenomenon. Our paper attempts to bridge this gap. The main novelty of our work is that we introduce a model of family immigration in a framework where school quality and student outcomes are determined endogenously. This allows us to address two different but related research questions. First, we can explain the selection of immigrants in terms of parental motivation and discuss how different types of immigrants are selected according to different immigration policies. Second, we can study the effect of immigration on the different dimensions of the school system, such as student effort, parental involvement, school incentives and resources. Finally, we can also study how the endogenous response of the school system to immigration is interrelated with both immigrants’ and natives’ educational choices.

For this purpose we develop a new model of the school system, inspired by Albornoz, Berlinski, and Cabrales (2011). In our model, children are shortsighted and need to be motivated to study. Parents divide their time between working and motivating their children, and they decide whether or not to emigrate. Schools provide additional incentive schemes to enhance children’s

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5In the Netherlands, e.g. two-thirds of immigrant-origin children have non-western origins with a Moroccan, Turkish and post-colonial majority (Statistic Netherlands, 2010). In the US, half of immigrant children have a Latin American and a tenth a Caribbean background while in France a half of the immigrant children are from former French African colonies. These numbers are taken from Alba, Sloan, and Sperling (2011b), based on Kirszbaum, Brinbaum, and Simon (2009).

6While we are not the first paper looking at emigration as a family decision, the existing literature mainly looks at a family consisting of a husband and wife (Mincer, 1978; Borjas and Bronars, 1991) but not at the children. A noticeable exception is Caponi (2009), who studies the migration decision from Mexico to the US. He distinguishes two functions of human capital: it is used to generate earnings but also transferred to future generations determining their future earnings. These functions are differently affected by the decision to emigrate. Unlike the present paper, Caponi (2009) does not look at the interaction of immigrants with the school system.
learning effort. The effect of these schemes may depend on school resources, which are decided by the education policy. The economic future of the children depends on the learning effort chosen while at school. This effort determines their probability to get a skilled job in lieu of an unskilled one. Parents are altruistic, and the concern for their children’s future is modeled by assuming their utility is a weighted average of their own material well-being and that of their children in their future.

We first look at the the case of classroom homogeneity, where children and families are perfectly assorted into schools, but countries might differ in their wage structure or their exogenous school quality. In this setup, we determine the optimal incentives schemes put into place by parents and teachers and we study the immigration decision. We show that whether or not the more highly motivated parents are more likely to emigrate crucially depends on two factors: the exogenous quality of the school system, and on the skill premium in the host and the origin countries. As in many standard immigration models, a sufficiently high skill premium abroad is likely to lead to positive immigrant selection. There are however two crucial differences in our model. First, positive selection occurs through the future benefits to children. Secondly, if parental wages abroad are too high compared to wages at home, selection falls on the intermediate range of parental motivation. If the skill premium at home is bigger than the one abroad, immigrant selection might even be negative. This result is crucial to understand under which circumstances increasing immigration costs will lead to a better selection of immigrants. Although these results are derived for homogeneous classrooms, we show that the same logic can be generalized to a more general model where heterogeneous parents differ in their education concerns and talent.

One advantage of our framework is tractability. We can qualify a rich variety of policy strategies according to their capacity to attract more motivated immigrants. More specifically, we study policies such as allowing immigrant children to naturalize and the effects of unanticipated family reunification program for temporary immigrants. We also analyze the possibility that parents differ in their cultural values. This implies different preferences about the values transmitted by the school. As a consequence, cultural alienation may emerge for those parents/children with values substantially differing from those transmitted by the school. We show that school cultural orientation

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7When looking into the literature on immigrant self-selection and multiple destinations the standard approach seems to be to use a variation of the Roy (1951) model which was introduced into the literature on international migration by Borjas (1987). This literature does not consider the interaction of endogenous school quality with immigration decisions.

8As happened in the case of many guest worker programs around the world.
is a crucial dimension to understand how immigrants self-select according to their school concerns. We show, for example, that cultural alienation may lead to negative selection. This suggests that school flexibility to incorporate new foreign values is a relevant characteristic to favor the arrival of the most motivated immigrants.

We then turn to the question of the school effects of immigration. Once we allow for classroom heterogeneity, we show that the overall effect of immigrant children depends on the characteristics of the average native compared to the average immigrant parents. We establish first that children’s learning effort increase in parental motivation. This result links the effects of immigration on schools to immigrant self-selection and how immigration affects classroom composition. Of course, more motivated immigrants would involve more positive effects on the host country school system, but these effects are mediated by the characteristics of both the native parents and the pre-immigration school system. We can show, for example, that, although a negative selection of immigrant parents reduces the school effort of native students, this particularly hits native students with relatively low parental motivation; a result that has been uncovered as a regularity in many empirical studies (Gould, Lavy, and Paserman, 2004).

Finally, we look at the effect of immigration on school resources in a world with public schools to be financed by parents through taxes. We assume that the policy maker maximizes the utility of the median voter parent, and show that school resources increase in immigrant motivation. Hence, a negative selection in parental motivation hits the native students directly through the reaction of teachers and indirectly through a reduction in school resources by the policy maker. This suggests that part of the potential deterioration of schools with increasing number of immigrant children may be (at least partially) explained by the response of the education policy to immigration, not to the presence of immigrants itself.

The remainder of the paper is organized as follows. Section 2 introduces the model of parental motivation and the school system. In section 3, we study immigrant selection in a homogeneous classrooms and discuss under which circumstances higher emigration costs can improve parental selection. Section 4 turns to the question of naturalization policies and what type of parent would emigrate if children were not able to come. We also study how the cultural orientation at school might interfere with immigrant selection. In section 5, we prepare the ground for analyzing the school effects of immigration by introducing heterogeneity among parents. This section also serves as a robustness exercise for our predictions on immigration policies. In section 6, we study the effects of immigration on school system. Section 7 discusses
further implications of our model and concludes. After each prediction of our model, we document supportive empirical evidence. All proofs not in the main text are gathered in a technical appendix.

2 Parental concern and the school system

In this section, we develop the basic model of the school system, assuming that the individuals who go to each school have the same characteristics. In other words, there is perfect assortative matching of students and families into schools. This assumption simplifies the exposition considerably. Later on, in section 5 we relax the assumption and allow for heterogeneity within schools.

Our model of the school system is inspired by the one in Albornoz, Berlinski, and Cabrales (2011), where the school system results from the interaction of students (children, who need incentives to put effort on learning), parents (who work and set up costly incentives schemes for students), and teachers/headmasters (who decide on the incentive scheme provided at schools). We now describe our different actors in detail. We assume that every parent has one child.

The students:

The students are children who live the present and do not worry about the future. They perceive learning as costly, because they would rather play, and do not internalize the future benefits of studying today. As a consequence, students need to be motivated to exert learning effort. The incentive scheme is put into place by parents and the school. Let $c_1$ be the parents’ reward for every unit of effort $e$ and $c_2$ be the school’s reward. Then, children choose $e$ to maximize their short-term utility:

$$U^c = (c_1 + c_2) e - \frac{1}{2} e^2,$$  \hspace{1cm} (1)

where $\frac{1}{2}e^2$ is the cost of learning. Notice that we assume that parents and school incentives are substitutes. No qualitative change ensues if we assume the incentives to be complementary.\footnote{This is because the substitutability at the children’s utility level is mitigated by complementarities elsewhere. Albornoz, Berlinski, and Cabrales (2011) discuss this issue in depth.}

The optimal effort decision by the children is given by

$$\frac{\partial U^c}{\partial e} = 0 = c_1 + c_2 - e$$

$$e = c_1 + c_2.$$  \hspace{1cm} (2)
Hence, children’s effort is simply the sum of parental and school incentives.

**The parents:**

Unlike children, parents understand the long-term consequences of their children’s choices today, namely how the child’s learning effort when young influences the child labor market prospects in the future. In particular, the probability that the child will get a high-skilled job equals the child’s learning effort \( e \), while the child becomes an unskilled worker with probability \( (1 - e) \). Skilled jobs and unskilled jobs are paid at a base rate of wages per efficiency unit of \( \phi^S \) and \( \phi^U \) respectively. These base rates of wages can differ across countries as well as between natives and immigrants. Hence a child’s future labor market prospect is given by \( (\phi^S e + (1 - e) \phi^U) \).

A parent has to split her total time \( T \) between working and providing incentives to her child. How much time a parent dedicates to generating educational incentives for her child depends on parental motivation and the cost of generating the reward. Parental motivation is modeled as the weight \( \theta \) a parent gives to child’s labor market prospect in her utility function. Parents might care about their children because of intrinsic motivation or because of their perceptions about their child’s talent. The cost of generating incentives for their child is the foregone parental wage, given by:

\[
w' = v' \phi',
\]

where \( v' \) is parental talent and \( \phi' \) is the parental base rate of wages per efficiency unit. Parental talent does not only increase wages, it also decreases the time parents need to spend for generating their child’s incentive reward. This time is given by \( c_1 e/2v' \). Hence, the parental utility function is given by the expression

\[
U^P = \theta \left( \phi^S (c_1 + c_2) + (1 - (c_1 + c_2)) \phi^U \right) + \left( T - \frac{1}{2} \frac{c_1 e}{v'} \right) w'.
\]

Taking the optimal effort decision of children (2) into account, the parent’s utility can be rewritten as

\[
U^P = \theta \left( \phi^S (c_1 + c_2) + (1 - (c_1 + c_2)) \phi^U \right) + \left( T - \frac{1}{2} \frac{c_1}{v'} \left( c_1 + c_2 \right) \right) w'.
\]

With this formulation, the parent’s choice problem reduces to the incentive scheme \( c_1 \), which determines the amount of time dedicated to their children so that we have

\[
\frac{\partial U^P}{\partial c_1} = \theta \left( \phi^S - \phi^U \right) - \left( c_1 + \frac{1}{2} c_2 \right) \frac{w'}{v'} = 0,
\]
leading to the optimal parental choice after using (3), which can be written as
\[ c_1 = (\phi^S - \phi^U) \frac{\theta}{\phi^T} - \frac{1}{2} c_2. \] (4)

Optimal parental incentives clearly depend on the school incentives, in fact they are decreasing in school incentives. We now turn to the school’s problem.

The school:
Schools/teachers also fully understand and care about the future job perspectives of their students, assigning weight $\theta^T$ to students’ success. The teachers have to decide how much of the time $T_T$ that remains after teaching their compulsory hours they will use to motivate their students (such as training or preparing learning activities), and how much they will use for outside job opportunities (such as private tuition) which are paid at wage rate $\Psi^T$. For simplicity, we normalize teacher’s talent to 1 (which is equivalent to abstracting from teacher’s talent), hence the teacher’s time spent generating the reward $c_2$ is equal to $\frac{1}{2} c_2 e$. Taking the optimal effort decision of children (2) into account, the teacher’s problem is to choose the level of $c_2$ that maximizes

\[ U^{HM} = \theta^T (\phi^S (c_1 + c_2) + (1 - (c_1 + c_2) \phi^U)) + \left( T_T - \frac{1}{2} (c_1 + c_2) c_2 \right) \Psi^T. \]

The first order conditions for this problem is:
\[ \frac{\partial U^{HM}}{\partial c_2} = \theta^T (\phi^S - \phi^U) - \left( c_2 + \frac{1}{2} c_1 \right) \Psi^T = 0, \]
leading to the optimal choices
\[ c_2 = \frac{\theta^T (\phi^S - \phi^U)}{\Psi^T} - \frac{1}{2} c_1. \] (5)

Introducing (5) into (4)
\[ c_1 = (\phi^S - \phi^U) \frac{\theta}{\phi^T} - \frac{1}{2} \left( \frac{\theta^T (\phi^S - \phi^U)}{\Psi^T} - \frac{1}{2} c_1 \right), \]
we can express parental incentives $c_1$ as
\[ c_1 = \frac{4}{3} (\phi^S - \phi^U) \left( \frac{\theta}{\phi^T} - \frac{1}{2} \frac{\theta^T}{\Psi^T} \right). \] (6)
It is easy to see that the incentives set by parents $c_1$ increase in $\theta$ and in $(\phi^S - \phi^U)$, decrease in parent’s base wages $\phi'$ and increase as well in teacher’s outside opportunities $\Psi^T$, i.e. if teacher’s have less incentive to dedicate time to students because it is more costly for them, parents compensate for this. Similarly, $c_1$ decreases with teacher’s motivation $\theta^T$.

Using (6) in (5) and simplifying gives us the incentives set by school

$$c_2 = \frac{4}{3} (\phi^S - \phi^U) \left( \frac{\theta^T}{\Psi^T} - \frac{1}{2} \frac{\theta}{\phi'} \right).$$

(7)

Given that children’s effort is the sum of (6) and (7), we obtain:

$$e = \frac{2}{3} (\phi^S - \phi^U) \left( \frac{\theta}{\phi'} + \frac{\theta^T}{\Psi^T} \right).$$

(8)

The effort of children is therefore increasing in the difference in the base rate of wages $(\phi^S - \phi^U)$, increasing in both parental and teacher’s motivation, and decreasing in parental base wage and teachers’ wages for jobs outside their teaching obligations.

The following assumptions are sufficient to abstract from corner solutions:

**Assumption 1** $\frac{2\theta^T}{\Psi^T} \geq \frac{\theta}{\phi'} \geq \frac{1}{2} \frac{\theta^T}{\Psi^T}$

**Assumption 2** $\frac{2}{3} (\phi^S - \phi^U) \left( \frac{\theta}{\phi'} + \frac{\theta^T}{\Psi^T} \right) \leq 1$

Assumption 1 guarantees that $c_1 \geq 0$ and $c_2 \geq 0$ while assumption 2 ensures $e \leq 1$

Introducing the optimal incentive schemes provided by parents (6) and schools (7) into parent’s utility (2) and simplifying allows us to calculate parent’s utility as

$$U^p = Tw' + \theta \phi^U + \frac{2}{9} (\phi^S - \phi^U)^2 \left( \frac{\theta^2}{\phi'} + 2\theta \frac{\theta^T}{\Psi^T} + \phi' \left( \frac{\theta^T}{\Psi^T} \right)^2 \right).$$

(9)

The first term $Tw'$ corresponds to the maximum earnings from working (what a parent can get by working all the time), while the second term $\theta \phi^U$ gives the parental utility if the child does not make any educational effort. Providing incentives to children increases the parental utility whenever skilled jobs are better paid than unskilled jobs; that if if $\phi^S > \phi^U$.  

\[ \text{Notice that the third term of (9), namely } \frac{2}{9} (\phi^S - \phi^U)^2 \left( \frac{\theta^2}{\phi'} + 2\theta \frac{\theta^T}{\Psi^T} + \phi' \left( \frac{\theta^T}{\Psi^T} \right)^2 \right) \text{ is equal to } \frac{\phi'}{2} e^2 \text{ where } e \text{ refers to the optimal effort determined by (8)} \]
3 Immigrant Self-Selection

There are two countries: Home (the source or origin) and Abroad (the destination or host). Both may differ in various dimensions but two matter most for potential immigrants: (i) the quality of the school system and (ii) economic opportunities. We study the role of each dimension separately.

3.1 School quality

The quality of the school system is captured by $\frac{\theta^T}{\Psi^T}$. Assume for now that this is the only difference between $H$ (Home) and $A$ (Abroad), and that school quality is better abroad. This requires $\frac{\theta^T_A}{\Psi^T_A} > \frac{\theta^T_H}{\Psi^T_H}$. In this case, emigration occurs if the gains from emigration are bigger than emigration costs $F$, that is if:

$$\frac{2}{9} (\phi^S - \phi^U)^2 \left( 2\theta \left( \frac{\theta^T_A}{\Psi^T_A} - \frac{\theta^T_H}{\Psi^T_H} \right) + \phi' \left( \left( \frac{\theta^T_A}{\Psi^T_A} \right)^2 - \left( \frac{\theta^T_H}{\Psi^T_H} \right)^2 \right) \right) > F. \quad (10)$$

which after examination implies the following result:

Proposition 1 The cost that a parent is willing to pay to immigrate increases in school quality, but it increases proportionally more for parents with higher motivation.

Proof. It is easy to see that the cross derivative of left hand side of (10) with respect to $\theta$ and $(\frac{\theta^T_A}{\Psi^T_A} > \frac{\theta^T_H}{\Psi^T_H})$ is positive.

In other words, if one increases immigration costs, but at the same time increases school quality, the selection of immigrants should improve since those that get discouraged with the higher costs are more likely to be those for whose the increase in school quality matters less. A testable implication of this proposition is then that the school performance of immigrant children should be better in countries with higher immigrations costs and high quality (public) schools. In this respect, Gibson and McKenzie (2011) show that the quality of Australian schools is a key pull factor for most of the qualified immigrants arriving from New Zealand, Tonga and New Guinea Papua.

3.2 Economic opportunities

We assume now that school systems are identical but economic opportunities are different. These are captured by differences in $\phi'$ (base rate of wages per
efficiency unit), $\phi^S$ and $\phi^U$. We assume that wages abroad are at least as high as wages at home. It can be shown that emigration from $H$ to $A$ occurs if

$$Tv'(\phi'_A - \phi'_H) + \theta (\phi'_A - \phi'_H) + \frac{2}{9} \theta^2 \left( \frac{(\phi^S_A - \phi^U_A)^2}{\phi'_A} - \frac{(\phi^S_H - \phi^U_H)^2}{\phi'_H} \right) +$$

$$\frac{4}{9} \theta \frac{\theta^T}{\Psi^T} \left( (\phi^S_A - \phi^U_A)^2 - (\phi^S_H - \phi^U_H)^2 \right) +$$

$$\frac{2}{9} \left( \frac{\theta^T}{\Psi^T} \right)^2 \left( (\phi'_A - \phi'_A) \phi'_A - (\phi'_H - \phi'_H)^2 \phi'_H \right) \right) \right)$$

$$> F$$

We first examine how gains from emigration change with parental motivation $\theta$. Therefore, we need to explore the derivative of (11) with respect to $\theta$, which is

$$\frac{4}{9} \left( \theta \left( \frac{(\phi^S_A - \phi^U_A)^2}{\phi'_A} - \frac{(\phi^S_H - \phi^U_H)^2}{\phi'_H} \right) + \theta^T \left( (\phi^S_A - \phi^U_A)^2 - (\phi^S_H - \phi^U_H)^2 \right) \right) +$$

$$\left( \phi'_A - \phi'_H \right)$$

$$> F$$

By our assumption that wages abroad are at least as high than at home, the last term of (12) is positive. Notice as well that the first two terms of (12) depend on the relative wage difference between skilled and unskilled workers at home and abroad. Additionally, a positive first term requires the home wage not to be too low compared to the wage abroad. The next proposition clarifies these conditions:

**Proposition 2** Assume that base line wages abroad are at least as high as in the home country and that $\phi^S_i \geq \phi^U_i$ for $i = A, H$.

1. A sufficient condition for higher $\theta$ parents to be more likely to emigrate is given by:

$$\frac{\phi^S_A - \phi^U_A}{\phi'_A} > \frac{\sqrt{\phi^S_H}}{\sqrt{\phi'_A}}.$$  

2. Immigrant selection will fall on intermediate-$\theta$-parents if (13) is violated and

$$\left( \phi'_A - \phi'_H \right) + \frac{4}{9} \frac{\theta^T}{\Psi^T} \left( (\phi^S_A - \phi^U_A)^2 - (\phi^S_H - \phi^U_H)^2 \right) > 0,$$
where the smaller \((\phi_A^U - \phi_H^U) + \frac{4}{9} \frac{\theta^r}{\theta^t} \left( (\phi_A^S - \phi_A^U)^2 - (\phi_H^S - \phi_H^U)^2 \right)\), the lower the \(\theta\) associated with parent that decides to migrate.

3. The lowest \(\theta\) parents are the most likely to emigrate if both (13) and (14) are violated.

**Proof.** The last term of (12) is nonnegative by assumption.

1. Condition (13) makes the first term of (12) positive. By assumption \(\phi'_A \geq \phi'_H\), so (13) implies \(\phi_A^S - \phi_A^U > \phi_H^S - \phi_H^U\) which guarantees that the second term of (12) is positive.

2. Condition (14) makes the first term negative and the sum of the second and the third terms of (12) positive, while the violation of (13) makes the third term negative. Hence (12) is positive for \(\theta\) sufficiently small and the \(\theta\) that benefits most from emigration is given when (12) is zero.

3. The sum of the second and the third terms of (12) is now negative, which is only possible if \(\phi_A^S - \phi_A^U << \phi_H^S - \phi_H^U\). This implies that the first term of (12) is also negative. Hence (12) is always negative. So if there was emigration, it is the worst types, in terms of motivation, who decide to emigrate.

Proposition 2 sheds light on how immigration costs \(F\) influence the selection of immigrants and consequently the educational performance of immigrant children, which is increasing in parental motivation. The effect of immigration costs clearly depends on whether or not conditions (13) and/or (14) are satisfied.

When condition (13) is satisfied Consider first different host and origin countries for which condition (13) is satisfied. This implies that more highly motivated parents have higher benefits from emigrating, and therefore selection improves with higher emigration costs \(F\). This explains why in destination countries where (13) is satisfied:

(i) for a given origin country, immigrant children perform better in host countries for which the emigration costs are higher,

(ii) for a given host country, the immigrant children who perform better are those whose parents faced the higher emigration costs.
It is easy to think of examples where case (i) holds. To begin with, the condition imposed by (13) should be satisfied for countries as different as Spain and U.S.. Also, based on cultural reasons, it should also be clear that emigrating to Spain involves relatively lower costs than settling in the U.S. Although not directly related to parents’ selection as in our model, Bertoli, Fernandez-Huertas Moraga, and Ortega (forthcoming) show that Ecuadorian immigrant selection to the U.S. is better than for immigrants coming to Spain.

Spain as a host country also provides an example for case (ii). Given its language and the pre-existence of an important and organized Ecuadorian community, migrants from Ecuador incur in lower immigration costs than, for example, immigrants from Romania. Our model then can explain why Romanian children do better at school than Ecuadorians, conditional on observables socioeconomic background, to the point of getting higher scores than them in Spanish language (Anghel and Cabrales, 2010).

When condition (13) is not satisfied The implications of the model change considerably if we look at host and origin countries where condition (13) is violated. This happens for example if it is mainly the unskilled jobs that are better paid in the destination country than in the origin country. An example of this situation is given by the immigrants hosted in Argentina from countries like Bolivia, Peru or Paraguay (Gasparini, Cruces, and Tornarolli, 2009). These origin countries are characterized by a very high skill premium, certainly as high as in Argentina. Also, the wages in Argentina are not that much higher. In this case, the positive relationship between $F$ and $\theta$ no longer holds. As a consequence, in a case like this it makes sense for the destination country to adopt policies to reduce $F$ in order to attract immigrants with a high parental motivation, irrespectively of their level of skills. This gives some theoretical support to the strategy of Argentina, which has one of the most generous immigration laws in the world (Albarracín, 2004).

4 Immigration and government policies

In this section we discuss how different government policies can affect the selection of immigrants in terms of the importance they attribute to education. Most of these policies are taken for other reasons, so this discussion should not be viewed as providing policy implications, and it is also very hard to do that reasonably in the absence of a general equilibrium model. So our general aim here is to study the educational side-effects of different policies which affect immigration. Moreover, they often provide empirical implications which help
assess the descriptive validity of our model.

4.1 Naturalization of immigrants

An important issue is whether or not to allow immigrants, and especially their children, to naturalize. Naturalization typically means easier access to better jobs in the future. Hence, naturalization implies that immigrant children will have a higher base-wage rate for high skill jobs. This does not hold not for their parents, and therefore it increases the range of parameters for which condition (13) holds so that only highly motivated immigrant parents are attracted. In other words, naturalization favors the selection of highly motivated immigrant parents and leads to better school performance of immigrant children. This prediction in consistent with Dronkers and Fleischmann (2010) who study immigration in 13 EU countries and find that a significant macro-characteristic for the educational performance of immigrant children is the destination country’s naturalization policy. In particular, the more generous the naturalization policy, the higher the educational attainment of immigrant children.

4.2 Family migration versus migration of the head of the family only

Many countries have tried to restrict the rights of immigrants, in particular of temporary immigrants. Specially designed guest worker programs limit family immigration or encourage immigrants to leave their children behind. Israel recently passed a law dictating that migrant workers are not allowed to have relationships, let alone children in Israel. Furthermore, children of migrant workers already in the country are supposed to be deported. The guest worker program proposed by president Bush (but not approved by the US congress) was heavily debated. An important part of the discussion concerned

\[11\] Bratsberg, Ragan, and Nasir (2002) provide evidence that in the U.S. naturalized immigrants have a more favorable job distribution and higher wages than non-naturalized immigrants. Moreover, naturalization leads to further wage growth. It allows entry into certain jobs that are reserved to nationals only, but also gives advantages in terms of signaling long term commitment and the flexibility to travel. The same results are found by Steinhardt (2008) for Germany and Fougère and Fougère and Safi (2008) for France.

the pros and cons of opening the doors to the immigrant’s family.\textsuperscript{13}

From our point of view, an interesting question is how are the immigrant parents of children left behind selected. Immigrants who have to leave their children behind cannot spend time with them to motivate their learning effort. Schools may partially compensate for the missing parental incentives: recall that $c_2$ is decreasing in $c_1$. As a benchmark case, assume that the best case scenario for the children of immigrants: their school will motivate them maximally by setting $c_1 = 0$ (as if both parents had emigrated).\textsuperscript{14} Assume again that all parents are equal. Then, the effort of the children left behind would be $e = c_2 = \theta^T \left( \phi^S_H - \phi^U_H \right) / \Psi^T$ and a parent’s utility from emigrating would be:

$$U^P_A = \theta \left( \frac{\theta^T \left( \phi^S_H - \phi^U_H \right)^2}{\Psi^T} + \phi^U_H \right) + Tw'_A - F,$$

while the payoff from staying at home would be:

$$U^P_H = Tw'_H + \theta \phi^U_H + \frac{2}{9} \left( \phi^S_H - \phi^U_H \right)^2 \left( \frac{\theta^2 v'_H}{w'_H} + 2 \theta \frac{\theta^T}{\Psi^T} + \frac{w'_H}{v'} \left( \frac{\theta^T}{\Psi^T} \right)^2 \right).$$

Hence emigration would occur if $U^P_A > U^P_H$, which requires

$$Tv' \left( \phi'_A - \phi'_H \right) + \left( \phi^S_H - \phi^U_H \right)^2 \frac{1}{9} \left( 5 \theta^T - 2 \theta^2 \phi'_H - 2 \phi'_H \left( \frac{\theta^T}{\Psi^T} \right)^2 \right) > F.$$ 

The derivative of this expression with respect to parental motivation is

$$\left( \phi^S_H - \phi^U_H \right)^2 \frac{1}{9} \left( 5 \frac{\theta^T}{\Psi^T} - 4 \frac{\theta}{\phi'_H} \right),$$

which is positive for $\theta < \theta^T \phi'_H / 4 \Psi^T$. Hence the individuals with intermediate values of $\theta$ are more likely to emigrate. We can no longer obtain positive selection: the most motivated parents are unlikely to leave since emigration harms the future perspectives of their children.\textsuperscript{15}

\textsuperscript{13}Critics perceived the program as a way of legalizing illegal immigrants from Mexico, which would lead to a huge inflow of their children.

\textsuperscript{14}This assumption might be less extreme than it seems. Remittance by immigrants is often meant to keep their children in school or to pay for a better education by schools.

\textsuperscript{15}Antman (2011) provides evidence that parental migration from Mexico to US reduces the time the children left behind allocate to studying, especially for sons. Also for Mexico, McKenzie and Rapoport (2011) find evidence of a significant negative effect of migration on schooling attendance and attainment.
One might argue that if children are not allowed to come, parental motivation should not be an important selection criterion, as the effects of their children at school is the only externality generated by motivation. However, there are several reasons why parental motivation should be taken into account. One is that parental motivation is likely to be positively correlated with work ethic. In other words, more motivated parents are likely to have a higher work ethic too.\textsuperscript{16} This is reinforced by the fact that laws can change over time and immigrants who were not allowed to bring their children might later be allowed to reunite.\textsuperscript{17} Unless this is fully anticipated and temporary family separation involves sufficiently low cost, our analysis would suggest that a host country can get a better immigrant selection if family immigration is facilitated from the beginning.

### 4.3 The role of culture orientation at school

Countries differ in their cultures. As a consequence, the values transmitted at schools are likely to be different across countries. In order to consider the effect of school cultural differences in the decision to migrate, we assume parents care about the school orientation. We describe their utility by:

\[
U^P = \theta \left( \phi S e + (1 - e) \phi U \right) \Delta + \left( T - \frac{1}{2} \frac{c_1 e}{w'} \right) w',
\]

where $\Delta$ captures the cultural differences between parents and the school. To be more precise, let $\Delta$ be

\[
\Delta = 1 - (\Phi - \tau)^2.
\]

\textsuperscript{16} Although this link is not captured in the present model, it is easy to extend the model to incorporate work ethic by letting parents allocate their time between leisure, education and work and assuming that the same parameter affects the weight given to education and inversely the enjoyment of leisure. This specification was used in a former version of the model leading to qualitatively similar results.

\textsuperscript{17} There are many historical examples of this possibility. Guest worker programs all over the world served to establish permanent immigrant minorities. Consider for example Germany, which signed a guest worker program with Turkey in 1961, allowing for temporary immigration only. While many Turkish guest workers returned when they were supposed to return, the agreement between Germany and Turkey ended in 1973 and many Turkish guest workers established themselves permanently, bringing their families later on. However, these parents were not positively selected, which might explain the bad school performance of the children of German guest workers (Dronkers and de Heus, 2010). There are good reasons to believe that new temporary immigrant programs are likely to lead to the same result, since the pressure toward granting immigrants more rights and at least basic family rights has increased. The United Nations and the International Labor Organization have enacted a number of international conventions in this direction (Weissbrodt, 2003).
In this expression, $\Phi$ and $\tau$ summarize the culture orientation of the school and parent respectively. Observe that when they coincide, $\Delta = 1$ and thus $U^p$ is the same as before.

To see how cultural differences affect immigrant self-selection, we assume as before that the source and destination countries differ in wages. We also assume that there is no within country cultural heterogeneity, and that schools respond to this situation by providing the cultural orientation that parents desire. From the perspective of the potential immigrant, this assumption implies that she/he feels some degree of cultural alienation only when living abroad. That is we assume $\Delta_H = 1$ and $\Delta_A \leq 1$. We can now state the main result of this section:

**Proposition 3** If wages abroad are at least as high as wages at home

1. Selection is increasing in $\theta$ for

$$\frac{\Delta (\phi^S_A - \phi^U_A)}{\phi^S_H - \phi^U_H} > \sqrt{\frac{\phi^A}{\phi^H}},$$  

(15)

and

$$\left(\Delta \phi^U_A - \phi^U_H\right) + \frac{\theta^T}{\Psi^T} \left(\Delta \left(\phi^S_A - \phi^U_A\right)^2 - \left(\phi^S_H - \phi^U_H\right)^2\right) > 0.$$  

(16)

2. Parents with intermediate values of $\theta$ are selected, if (15) is violated and (16) holds

3. Parents with the lowest $\theta$ are most likely to emigrate, if both (15) and (16) are violated.

4. If (15) holds and (16) is violated, there exists a threshold $\hat{\theta}$ such that (43) is positive for $\theta > \hat{\theta}$ and negative for $\theta < \hat{\theta}$.

Observe that conditions (15) and (16) are violated for low $\Delta$ even if condition (13) holds, which is the corresponding condition for positive immigrant selection in parental motivation in the absence of cultural concerns. The possibility of cultural alienation scares away the most motivated immigrants and may even lead to negative selection. This imposes an important policy trade-off for the destination country. The fact that school orientation may affect selection implies that flexibility on the school orientation and incorporation of some foreign values at schools could favor the attraction of more motivated immigrants.
This implication of our model might throw some light on recent empirical findings by Dronkers (2010). In a cross-country comparison of language skills using the Pisa data, Dronkers (2010) found that pupils from Islamic countries have a substantial disadvantage in language scores compared to immigrant pupils from other countries of origin, which cannot be explained on the basis of individual socioeconomic backgrounds, school characteristics or the education system’s characteristics. Given that the Muslim culture differs considerably from the mainstream culture of most immigrant receiving countries, and that Muslims have strong cultural concerns, our model predicts that in their case it is the less motivated parents who emigrate. The educational effort of Muslim children, as given in appendix, is lower than that of other immigrants through two channels. On the one hand, they are less stimulated by their parents, who care relatively less about their education. On the other hand, cultural alienation reinforces this lack of concern even further. This problem could be mitigated by allowing for Muslim schools. Indeed, Dronkers (2010) provides evidence that a higher share of pupils with an immigrant background in a school hampers educational performance (of all students), but if these pupils have the same regional origin (Islamic countries; non-Islamic Asian countries), a higher share of pupils with an immigrant background at that school promotes educational performance.

5 Heterogeneous classrooms

The effect of immigration on the school system is multidimensional. However, the key common mechanism involves changes in classroom composition. To analyze this, we need to introduce heterogeneity across parents/students within classrooms. As a consequence, we allow parents \( i \) to differ in parental motivation \( \theta_i \) and talent \( \nu'_i \). How do immigrants affect schooling in the host country?

In the first place, immigrants bring a different mix of parental motivation to the classroom. This directly affects the school incentive schemes. To see this, we consider that school motivation is reinforced by the average parental involvement in their children’s education. In this way, we capture situations where teachers’ incentive are encouraged by interacting with highly motivated parents. This assumption also captures the fact that it is demoralizing for teachers to deal with disinterested parents or, more generally, with student apathy. To capture this link formally, we postulate:

\[ 18 \text{As captured by the degree of differentiation in secondary education.} \]
Assumption 3 $\theta^T$ depends on the average parental motivation. That is,

$$\theta^T = k\bar{\theta} = \frac{k}{M} \sum_{i=1}^{M} \theta_i,$$

where $M$ is the number of parents affecting the education of a particular school class of children and $k$ indicates the exogenous weight that the school assigns to the future perspectives of their students.

Second, as parental involvement is endogenous and depends on peer effects and school incentives, immigration affects involvement by native parents in the learning process.

Finally, native parents can also react to new school incentives by modifying their demands for school resources. This means that there will be an endogenous response to immigration through the political process mediated by school resources.

5.1 Optimal incentives and learning efforts

We start by establishing the post-immigration equilibrium levels of $c_1$ and $c_2$. To do so, consider first the utility of a headmaster in a school with heterogeneous students,

$$U_{HM}^A = \theta^T_A \sum_{i=1}^{N_j} (\phi_i^S \epsilon_i + (1 - \epsilon_i) \phi_i^U) + \left( T - \frac{1}{2} \sum_{i=1}^{N_j} \epsilon_i c_{2}^A \right) \gamma_{HM}^A.$$

where $\gamma_{HM}^A$ represents the wage rate for jobs not related to the school work and $T$ is the total time left for the teacher after completing the minimum job requirement of the teaching job. This formulation assumes that each teacher is in charge of $N_j$ children. The composition of $N_j$ is $N_N + N_A$ where $N$ and $A$ stand for native and immigrant children respectively. Thus, (18) can be rewritten as:

$$U_{HM}^A = \theta^T_A \left( \sum_{k=1}^{N_N} (\phi_k^S \epsilon_k^N + (1 - \epsilon_k^N) \phi_k^U) + \sum_{l=1}^{N_A} (\phi_l^S \epsilon_l^A + (1 - \epsilon_l^A) \phi_l^U) \right)$$

$$+ \left( T - \frac{1}{2} c_2^A \left( \sum_{k=1}^{N_N} \epsilon_k^N + \sum_{l=1}^{N_A} \epsilon_l^A \right) \right) \gamma_{HM}^A.$$
In order to save some notation let us define
\[ \psi_i = \frac{\theta_i v'_i}{w'_i} = \frac{\theta_i}{\phi'_i}, \] (19)
that is, as the ratio of parental motivation to base line wage.

Also, as we will see school incentives depend on the average level of parental incentives chosen by both native an foreign parents, so we define:
\[ \Omega_I = \frac{1}{N_I} \sum_{i=1}^{N_I} \theta_i v'_i w'_i = \frac{1}{N_I} \sum_{i=1}^{N_I} \theta_i \phi'_i = \frac{1}{N_I} \sum_{i=1}^{N_I} \psi_i \] for \( I = N, A. \) (20)

Notice that \( \Omega_N / \Omega_A \) expresses the average motivation to parental base-wage ratio among the native and foreign population. Using this, it follows:

Following the steps in section 2, the optimal levels of incentives and the learning effort of immigrant and a native children are:

**Lemma 1**

\[ c^A_{1i} = (\phi^S - \phi^U) \left( \psi^A_i - \frac{2}{3} \frac{\theta^T_A}{\gamma^A_{HM}} + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \] (21)

\[ c^N_{1i} = (\phi^S - \phi^U) \left( \psi^N_i - \frac{2}{3} \frac{\theta^T_A}{\gamma^A_{HM}} + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \] (22)

\[ c^A_{2j} = \frac{2}{3} (\phi^S - \phi^U) \left( \frac{2}{3} \frac{\theta^T_A}{\gamma^A_{HM}} - \frac{N_A \Omega_A + N_N \Omega_N}{(N_N + N_A)} \right). \] (23)

\[ e^A_i = (\phi^S - \phi^U) \left( \psi^A_i + \frac{2}{3} \frac{\theta^T_A}{\gamma^A_{HM}} - \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right) \] (24)

\[ e^N_i = (\phi^S - \phi^U) \left( \psi^N_i + \frac{2}{3} \frac{\theta^T_A}{\gamma^A_{HM}} - \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \] (25)

**Proof.** See the appendix. ■

It is clear now that the learning effort of an immigrant child relative to that of a native child depends on their parents’ characteristics, such as the base wage per efficiency unit or work ethic. To see this, we have to consider

\[ e^A_i - e^N_i = (\psi^A_i - \psi^N_i) (\phi^S - \phi^U) = \left( \frac{\theta^A_i}{\phi^A_i} - \frac{\theta^N_j}{\phi^N_j} \right) (\phi^S - \phi^U), \]
which implies that for immigrant children to make greater effort in average than the natives, the following condition has to be satisfied:

\[
\frac{1}{N_A} \sum_{i \in N_A} e_i^A > \frac{1}{N_N} \sum_{i \in N_N} e_i^N \text{ whenever } \frac{1}{N_A} \sum_{i \in N_A} \frac{\theta_i^A}{\phi_i^A} > \frac{1}{N_N} \sum_{i \in N_N} \frac{\theta_i^N}{\phi_i^N}.
\]

Having established this, the next proposition follows immediately:

**Proposition 4** The children of immigrants exert more effort at school than natives if and only if the average of the ratio of parental motivation to wage of an efficiency unit in the immigrant parents’ group is larger than that of native parents.

Clearly, immigration affects education by changing average parental motivation at the classroom level. Therefore, we have to (1) clarify the decision to immigrate of heterogeneous potential immigrants and (2) establish how the variables of interest (school and parental incentives, resources, student effort and education outcomes) are determined by parental motivation.

Of course, the result holds when the environments of immigrants and natives is the same. This clearly holds at the school level. In the country as a whole it would only hold if all schools are the same and immigrants and natives are equally distributed among schools.

### 5.2 Decision to emigrate in a heterogeneous world

In this section we look at the emigration decision in a heterogeneous world. This is mainly meant as a robustness exercise in order to show that our policy analysis for the case with homogeneous classrooms extends to situations where heterogeneous children school together. In particular, we will derive the conditions for positive selection in parental motivation and argue when they are likely to coincide with results obtained in the homogeneous case.

A parent \( i \) will emigrate from country \( H \) to country \( A \) if \( U_{\pi_i}^A > U_{\pi_i}^H \). In the appendix we show that letting \( \Omega_{NA} = (N_A\Omega_A + N_N\Omega_N) / (N_N + N_A) \) and using optimal parental and school incentives:
Lemma 2 $U^A_{Pi} > U^H_{Pi}$ if and only if

$$Tv' (\phi^A_i - \phi^H_i) + \theta_i (\phi'_i - \phi'^H_i) + (\phi^S_A - \phi^U_A)^2 \left( \frac{\theta_i}{3} + \frac{2}{3} \frac{\theta^T_A}{\gamma^{HA}_H} - \frac{1}{3} \Omega^{NA}_A \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \frac{\theta^T_A}{\gamma^{HA}_H} - \frac{1}{3} \Omega^{NA}_A \right) \phi^A_i \right)$$

$$- (\phi^S_H - \phi^U_H)^2 \left( \frac{\theta_i}{3} + \frac{2}{3} \frac{\theta^T_H}{\gamma^{HA}_H} - \frac{1}{3} \Omega^{HA}_H \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \frac{\theta^T_H}{\gamma^{HA}_H} - \frac{1}{3} \Omega^{HA}_H \right) \phi^H_i \right)$$

> $F_i$.

Suppose the heterogeneity is such that the vector of variables

$$\xi_i \equiv (\theta_i, v'_i, \phi^A_i, \phi^H_i) \in \Xi,$$

characterizing each individual belongs to a finite set of types $\Xi$. At the same time the variable $F_i$ follows the distribution $F(.)$ in the compact interval $[0, A]$. Note that according to equation (26) if an individual with type $\xi_i$ and value of cost of moving $F_i$ wants to move, another individual with type $\xi_j = \xi_i$ and $F_j > F_i$ also wants to move. Hence, the equilibrium can be characterized by a set of thresholds. For each type $\xi \in \Xi$ there is some $F_{\xi}$ such that for all $i$ with $\xi_i = \xi \in \Xi$ the individual moves to $A$ if and only if $F_i < F_{\xi}$.

**Proposition 5** An equilibrium in entry decisions characterized by thresholds always exist.

**Proof.** See the appendix. ■

In order to understand the effects of differences in parental motivation on the receiving and sending countries we now use the link of school motivation to parental motivation stipulated in (17).

**Proposition 6** Assume that that immigrants are a sufficiently small part of the population both on origin and destination, so that

$$\frac{\partial \Omega^{NA}}{\partial \theta_\xi} = \frac{\partial \Omega^{HA}}{\partial \theta_\xi} = 0$$

and that baseline wages are at least as high abroad as at home. Assume also that for all $i, i' \in N$, we have that $\phi^A_i = \phi^A_{i'}$ and $\phi^H_i = \phi^H_{i'}$. Then, if we have two types $\xi$ and $\xi'$ such that $\theta_\xi > \theta_{\xi'}$ then under the following conditions we
have that in equilibrium the proportion of individuals with type $\theta_\xi$ that decides to emigrate is larger than the proportion of emigrants with type $\theta_{\xi'}$:

$$\left(\phi^S_A - \phi^U_A\right)^2 \geq \left(\phi^S_H - \phi^U_H\right)^2$$  \hspace{1cm} (27)

$$\frac{2k_A}{\gamma_{HM}^{NA}} - \frac{2k_H}{\gamma_{HM}^{H}} \geq \frac{\Omega_{NA}}{\Omega_{H}}$$  \hspace{1cm} (28)

**Proof.** See the Appendix. ■

The first condition of proposition 6 given by (27) is equivalent to our condition for positive selection in parental motivation in the homogeneous case, namely condition (13). The second condition given by (28) is really endogenous and it points to the differences that arise due to heterogeneity. To get an intuition for its meaning it will be helpful to consider a situation with two countries that are identical except for their wage structure. More technically, assumption 4 holds in both countries, that is: both countries have the same exogenous school qualities, $2k_A/\gamma_{HM}^{A} = 2k_H/\gamma_{HM}^{H}$; the same initial distribution of parental motivation; the same distribution of parental motivation among skill groups; and the same proportion of people in skilled employment. Under these conditions, inequality (28) definitely holds. To see this, notice that due to the equality in exogenous school quality, condition (28) reduces to $\Omega_{NA} \leq \Omega_{H}$, or equivalently $\frac{1}{N_H} \sum \theta_i/\phi^H_i \geq \frac{1}{N_{NA}} \sum \theta_j/\phi^{NA}_j$. This is true since wages in country $A$ are at least as high as wages in country $H$, and the distribution of parental motivation among skill groups is identical. This argument shows that the first wave of immigrants is likely to attract highly motivated parents under the same conditions as in homogeneous classrooms. Obviously if the exogenous school quality is better abroad than at home so that $2k_A/\gamma_{HM}^{A} > 2k_H/\gamma_{HM}^{H}$, condition (28) is relaxed.

6 Heterogeneous parental motivation and its implications

6.1 Motivation, school quality and student effort

Immigration affects the composition of a class and therefore its impact on results by changing the averages of motivation across classrooms. To show this, we have to establish the condition under which school quality and student effort depend on parental motivation.
Using assumption (3) and equation (23), we can express the school incentives in a more general way:

\[ c_2 = \frac{2}{3N} \sum_{i=1}^{N_l} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i, \tag{29} \]

where the index \( l \) refers to the country in question and \( N_l \) refers to the total number of students at a school in country \( l \).

Since

\[ \text{sign} \frac{\partial c_2^A}{\partial \theta_i} = \text{sign} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right), \]

the incentives provided by schools (29) increase in parental motivation for parents whose wages are such that \( 2k/\gamma_{HM}^l > 1/\phi_i^l \) and decrease in parental motivation for parents with wages such that \( 2k/\gamma_{HM}^l < 1/\phi_i^l \).

On the other hand, schools will only provide positive incentives if

\[ \sum_{i=1}^{N_l} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i > 0. \]

Clearly, in a world with unskilled and skilled parents, where wages are described by \( \phi_U \) and \( \phi^S = \alpha \phi_U \), with \( \alpha > 1 \), the following condition is sufficient to guarantee that school incentives increase in average parental involvement:

**Assumption 4** \( 2k/\gamma_{HM}^l > 1/\phi_i^l \) for all \( i \).

Assumption 4 can be rewritten as \( 2\phi_i^l > \gamma_{HM}^l/k \) for all \( i \) where \( \gamma_{HM}^l/k \) is the inverse of school quality. The assumption therefore states that parental baseline wages cannot be too low compared to the ratio of school’s opportunity cost of providing students incentives to the weight schools give to the future performance of the children. This assumption will hold for high \( k \) schools (schools highly concerned with their students’ future) and for countries where \( \gamma_{HM}^l \) is low. Arguably, assumption 4 is relatively mild for countries targeted by migration.

Assumption 4 allows for positive school incentives both when children are schooled according to their parental skill levels and when parental skill levels are mixed across schools. In the latter case, it is still possible to guarantee that school incentives are positive even if assumption 4 is violated. This is possible if:

\[ \frac{1}{N} \sum_{i \in N} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i > 0. \]
This can be nicely illustrated in a world where parents either have a skilled or an unskilled wage and let \( \phi^S = \alpha \phi^U \) where \( \phi^U < \gamma_{HM}^l/k < \alpha \phi^U \). Then schools set positive incentives if

\[
\frac{1}{N} \sum_{i \in N} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i = \frac{1}{N^S + N^U} \sum_{i \in N^S \cup N^U} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i > 0 \quad (30)
\]

where \( N^S/N^U \) is the number of skilled/unskilled workers respectively.

Condition (30) is equivalent to:

\[
\frac{\sum_{i \in N^S} \theta_i}{\sum_{i \in N^U} \theta_i} > \frac{\phi^U - \frac{2k}{\gamma_{HM}^l}}{\frac{2k}{\gamma_{HM}^l} - \frac{1}{\alpha \phi^U}} = \frac{\alpha \left( \gamma_{HM}^l - 2k \phi^U \right)}{2k \alpha \phi^U - \gamma_{HM}^l}.
\]

Define \( \beta = \sum_{i=1}^{N^S} \theta_i / \sum_{i=1}^{N^U} \theta_i \); that is, the total parental motivation is higher for skilled parents than for unskilled parents if and only if \( \beta > 1 \). Then we obtain

\[
\frac{1}{N^S + N^U} \sum_{i=1}^{N^S + N^U} \left( \frac{2k}{\gamma_{HM}^l} - \frac{1}{\phi_i^l} \right) \theta_i > 0 \iff \phi^U > \left( \beta + \alpha \right) \frac{\gamma_{HM}^l}{2 (1 + \beta) k \alpha}.
\]

Notice that the right hand side is decreasing in \( \beta \). Therefore, we obtain a weaker condition, which may be satisfied if the total parental motivation is lower for unskilled than skilled workers. We can state now the following result:

**Proposition 7**

1. Under assumption 4 overall school incentives increase in parental motivation.

2. Consider a world where parents either have a skilled or unskilled job with wages \( \phi^S = \alpha \phi^U \) and \( \phi^U < \gamma_{HM}^l/k < \alpha \phi^U \). Hence assumption 4 is violated because it does not hold for unskilled workers. School incentives are positive if

\[
\frac{(\beta + \alpha) \gamma_{HM}^l}{2 (1 + \beta) k \alpha} < \phi^U < \frac{\gamma_{HM}^l}{2k} < \alpha \phi^U.
\]

in which case school incentives decrease in parental motivation of unskilled parents and increase in parental motivation of skilled parents.

Condition (31) is more easily satisfied for high levels of \( \alpha \); that is, in countries where the skill gap between high and low skilled jobs is big. The term \( \gamma_{HM}^l/k \) captures the inverse of exogenous school quality. A fall in exogenous school quality will eventually lead to a violation of this condition, pushing in turn school incentives to zero.

\[^{19}\]The parameter \( \alpha > 1 \) captures how beneficial skilled jobs are relative to unskilled jobs.
6.2 Motivation and effort

School incentives are difficult to observe. For this reason, student outcomes constitute a typical empirical measure of school quality. We therefore need to examine children’s learning effort in more detail. Such effort may be expressed as

\[ e_i = \left( \phi_i^S - \phi_i^U \right) \left( \frac{\theta_i}{\phi_i'} + \frac{1}{3N_i} \sum_{i=1}^{N_i} \left( \frac{2k}{\gamma_i'H_M} - \frac{1}{\phi_i'} \right) \theta_i \right), \quad (32) \]

From this equation it is straightforward to establish:

**Proposition 8** *Children’s learning effort is always increasing in parental motivation.*

**Proof.** It follows from \( \frac{\partial e_i}{\partial \theta_i} > 0 \)

Although fairly simple, this result allows us to establish some interesting implications.

**The channels through which motivation affects performance** First, even in situations where school incentives decrease in parental motivation, the direct effect of rising parental involvement on student effort offsets its negative impact on the school. Hence, the greater learning effort of children from highly motivated parents must come because of the parents’ higher demands. The empirical evidence of pushy immigrant parents is vast in the case of immigration to the US. As shown by Glick and White (2004) and Hao and Bonstedt-Bruns (1998), immigrant parents are associated with greater demands on their children in terms of school engagement and academic achievement. Keller and Tillman (2008) find that both parental and self-reported expectations have significant direct effects on college attendance. Goyette and Xie (1999) provide evidence that in the US the behaviors and expectations of Asian immigrant parents’ tend to raise their children’s school attendance above the average.

**Effects on natives and immigrants** Turning to the effect of immigration on schooling, proposition 8 implies that this effect is mediated by parental characteristics and the way immigrants are schooled. To see this more clearly, we can rewrite (32) and obtain:

\[ e_i = \left( \phi_i^S - \phi_i^U \right) \frac{\theta_i}{\phi_i'} + \frac{1}{2} \theta_i. \quad (33) \]
This expression allows us to analyze how immigration affects the performance of native pupils. For a given school, the relative effect of immigration on native children varies across their parent characteristics, which are captured by \( \psi_i = \theta_i / \phi_i' \). A change in \( c_2 \) simply shifts the initial \( c_1 \) up (if immigrants are better in average) or down (otherwise), and therefore the effect on \( e_i \) is lower the higher the initial \( c_1 \) or equivalently, for children associated with a higher \( \psi_i \). In other words, the performance of disadvantaged children (low \( \psi_i \) parents) is more affected by immigration than their advantaged classmates (high \( \psi_i \) parents). The evidence for this is very strong. Gould, Lavy, and Paserman (2009) provide very suggestive evidence for this prediction. Focusing on the mass migration wave from the former Soviet Union to Israel in the early 1990s, they find a negative effect of immigrants on native outcomes which is larger for natives from a more disadvantaged social background. Similarly, Betts (1998) shows that immigration reduces the probability of completing high-schools for American-native minorities (Blacks and Hispanics). No negative effect of immigrants is found for non minority groups. Finally Brunello and Rocco (2011) study whether a higher share of immigrant pupils affects the school performance of natives using aggregate multi-country data from PISA. They find evidence of a negative and statistically significant relationship but the size of the estimated effect is small and it is bigger for natives with a relatively disadvantaged parental background.

Expression (33) also allows us to examine the effect of schools on immigrant performance. A typical measure of school quality is the the pre-immigration performance or general performance of its native pupils. As discussed above, overall native performance is partly driven by \( c_2 \). According to (33), a higher level of \( c_2 \) would benefit all children at the school, and hence this would include the immigrant children. This is consistent with the vast evidence suggesting that better schools benefit immigrants (Dronkers and Fleischmann, 2010). The “Operation Solomon” provides a natural experiment for this result. This refers to the exodus of 15,000 Ethiopian immigrants, who were airborne to Israel within 36 hours in May 1991. Importantly, they were randomly sorted across the country. According to our model the average performance of those immigrants who were randomly placed into better schools should be higher. As shown by Gould, Lavy, and Paserman (2004), this was exactly the case: those Ethiopians who were assigned to better elementary schools had better results in high school.

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20The measure of better elementary schools used by Gould, Lavy, and Paserman (2004) was the average standardized maths scores before Ethiopian entered or other environmental measures such as welfare rate and average high school matriculation rate.
The impact on segregation and of segregation. So far, we have always considered exogenous classroom composition. But a corollary of the previous point is that the selection of immigrants can have important implications on school segregation. If the selection of immigrants is negative, or even if positive, it involves mainly unskilled workers, this can easily lead to a flight from some schools into others. In many countries this implies a flight to the private schools sector. Indeed, Betts and Fairlie (2003) find that American native students fly toward private secondary schools in response to the influx of immigrants into public institutions. Also, Berniell (2010) discussing the massive recent flow of immigrants into Spain shows that “in 1998-99, when the fraction of immigrants in Madrid was only 2.6%, about 59% of natives were attending public schools, while one decade later -when immigrants comprised 17% of total population roughly 50% of natives chose public institutions. On the other hand, in 1998-99 only 68% of immigrant parents were choosing public schools, while in 2008-09 this number raised to 77%.”

In a world with skilled and unskilled workers school, incentives can also be rewritten as

$$c_2 = \frac{2 (\phi_i^S - \phi_i^U)}{3 (N^U + N^S)} \left( \left( \frac{2k}{\gamma_{HM}} - \frac{1}{\phi_i^U} \right) N^U \bar{\theta}^U + \left( \frac{2k}{\gamma_{HM}} - \frac{1}{\alpha \phi_i^U} \right) N^S \bar{\theta}^S \right).$$  \hspace{1cm} (34)

Consider situations where schools are segregated by the skill level of parents, i.e. children of unskilled workers are schooled together and so are children of skilled workers, the natives always benefit if immigrants have a high parental motivation, and they suffer otherwise. In countries where children of skilled and unskilled parents are schooled together and randomly assigned to schools, immigration is likely to change the skilled/unskilled composition of the classroom. If immigrants are positively selected according to parental motivation and are only high-skilled workers matched to high-skilled jobs, the effect on native student’s effort is positive. If, however, immigrants are all positively selected but unskilled and the overall classroom size is constant, then selection has to be extremely restrictive in the sense that only immigrants with the highest motivation are admitted for the overall effect on school incentives to be positive. Similarly, a negative selection of only unskilled immigrants will always affect natives negatively, while a negative selection of skilled immigrants has to be extremely negative to have the same effect. If skilled and unskilled immigrants come in the same proportion than skilled and unskilled natives, a positive (negative) selection in parental motivation will always benefit (harm) native children.
6.3 Motivation and school resources.

We are now going to endogenize school resources to see whether some new effects arise from the feedback between immigrants’ ethos and resource provision. Let us denote by $r_j$ the amount of resources an administration gives to a particular school. This could be thought of as class size (or teacher student ratio) as well as other resources, such as support to teaching staff, computers and other means of making the provision of incentives easier for teachers. The levels of resources, $r_j$ is announced by the policymaker before parents and headmasters decide on the level of incentives, so they take $r_j$ as given when they make their decisions. Given $r_j$ the utility of a headmaster is now:

$$U_{HM}^A = \theta_T^A \sum_{i=1}^{N_j} \left( \phi^S e_i + (1 - e_i) \phi^U \right) + \left( T - \frac{1}{2r_j} \sum_{i=1}^{N_j} e_i c_{A2}^j \right) \gamma_{HM}^A. \tag{35}$$

Following the previous analysis, we can obtain the equilibrium values of the key variables of the school system:

**Lemma 3**

$$c_{1l}^A = (\phi^S - \phi^U) \left( \psi_{1l}^A - \frac{2}{3} r_j \theta_T^A + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \tag{36}$$

$$c_{1l}^N = (\phi^S - \phi^U) \left( \psi_{1l}^N - \frac{2}{3} r_j \theta_T^A + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \tag{37}$$

and for school incentives:

$$c_{A2}^j = \frac{2}{3} \left( \phi^S - \phi^U \right) \left( \frac{2r_j \theta_T^A}{3 \gamma_{HM}^A} - \frac{N_A \Omega_A + N_N \Omega_N}{(N_N + N_A)} \right). \tag{38}$$

The learning effort of an immigrant child and a native child given by (2) are therefore

$$e_i^A = (\phi^S - \phi^U) \left( \psi_i^A + \frac{2}{3} r_j \theta_T^A + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \tag{39}$$

$$e_i^N = (\phi^S - \phi^U) \left( \psi_i^N + \frac{2}{3} r_j \theta_T^A + \frac{N_A \Omega_A + N_N \Omega_N}{3(N_N + N_A)} \right). \tag{40}$$
Proof. See the appendix. ■

Now we introduce the utility of the policymaker who decides the level of resources for the schools. The policymaker maximizes the complete utility of the (median-voter) parent (denoted by $P_i$) which requires adding the cost of the school resources ($r_j$). This median-voter is a native, and in fact we are choosing him as the median of the natives, as in most countries first-generation immigrants do not get the right of vote, or they get it when they are naturalized at which point most of their children will have already gone (at least partially) through the education system.\textsuperscript{21} The costs of resources $r_j$ are paid by parents through general taxation, which parents care about, and are internalized by the policymaker when deciding $r_j$. We assume that all schools have the same resources so that $r_j = r$ for all $j$. Resource costs are assumed to be quadratic.\textsuperscript{22}

Thus, we can represent the policymaker’s preferences as,

$$U_{PM} = U_{PM} - \frac{\rho}{2} r^2,$$

where $\rho$ is a constant parameter summarizing the cost of resources. Our formulation assumes that schools are financed out of lump sum taxation and the government keeps a balanced budget.

Substituting (40) and (37) into (41), and then optimizing $U_{PM}$ over $r$ we obtain:

$$r = \left(\phi_S - \phi^U\right)^2 \frac{2}{3} \frac{\theta^T_A}{\gamma_{HM}} \left(\theta_{IM} + \phi_{IM} \left(\frac{N_{NA} + N_{NN}}{3(N_N + NA)}\right)\right)$$

$$\rho - \phi_{IM} \left(\left(\phi_S - \phi^U\right)^2 \left(\frac{2}{3} \frac{\theta^T_A}{\gamma_{HM}}\right)^2\right)$$

Note that resources increase in the motivation of the immigrant populations through two sources. First $r$ is increasing in $\theta^T_A$ which by assumption (17) depends on the average motivation of the student parents. Secondly, it also depends positively on the motivation of immigrants through $\Omega_{A}$. Hence, the motivation of immigrants reinforces the effects of immigrants selection that happen through $c_2$, which we already discussed in section 6.1. Thus, a poorly

\textsuperscript{21}To become a US citizen an immigrant must have been a permanent resident for at least five years. Becoming a permanent resident also takes a few years, and we are considering immigrants who already have children at the time they emigrate.

\textsuperscript{22}This can be justified by taking into consideration that the state has monopsony power in the market for teachers and faces a marginal cost function that increases in the number of teachers hired. This is so, for example, because to attract one more teacher the monopsonist has to pay an extra cost, since the marginal potential teacher needs a higher reward to be attracted to the profession.
selected immigrant population hits the native students (and the more motivated immigrants) directly through school incentives, and indirectly through a reduction in school resources by the policymakers.

Several authors have found evidence that bad immigrant selection leads to a reduction in public spending on schooling. Using a quantitative model of school choice and voting over public education Coen-Pirani (Forthcoming) shows that education spending per student in California would have been 24 percent higher in the year 2000 if U.S. immigration had been restricted to its 1970 level. As our approach Coen-Pirani (Forthcoming) abstracts from illegal immigration and allows only native households to vote. His calibrated parameters reveal that immigrants in California care relatively less for education than natives, hence our model provides an alternative explanation for his findings. The relationship between resources dedicated to public schools and immigration is also examined by Dottori and Shen (2008). They provide cross-country evidence (e.g. a mean-difference test) that countries that experience negative changes in public expenditure per pupil from 1990 to 2004 (Docquier and Marfouk (2006) data set) are those with larger increases in the low-skilled immigrants’ share of the population (UNESCO data). This finding is consistent with our model, if low-skilled immigrants are also less concerned about education on average than high skilled immigrants. Indeed, this negative correlation disappears when Dottori and Shen (2008) look at changes in the share of immigrants with tertiary education and lagged changes in public expenditure per pupil. As we also discussed in section 6.1, these effects will be reinforced if, in addition, there is a flight of natives away from public schools into private ones, as Berniell (2010) documents has happened in Spain recently, for example.

There is possibly one more channel for immigrants’ motivation to impact education. So far, we have assumed that the median voter is the median of the natives, the only ones who can vote. But suppose that immigrants earn the right of vote sufficiently early after arrival to the destination country. Then, poorly selected immigrants would shift the median voter toward an individual who cares less about education and hence lowers the level of resources even further. Obviously, the vicious cycle of selection becomes virtuous in case of positive selection. There is a higher level of \( c_2 \), a higher level of resources \( r \) and the immigrant effect may be improved by enfranchising the immigrants.

Another important observation is that our assumption on funding resources implies that immigrants are legal, so they pay taxes. If they are illegal (non-tax paying) but exogenous in number, we would effectively have a higher level of \( \rho \), which would entail a lower level of resources. If they were illegal and also their number were endogenous, an increase in resources would bring more of
them, and the effect is less easy to compute but similar to having a technology with more rapidly decreasing returns to extra resources.

7 Concluding Discussion

In this paper, we propose a model of endogenous migration and human capital production. The model allows us to understand for example the differential selection, and hence performance, of immigrant from the same country into different destinations. It can also explain why students from different origins exhibit so widely different performances in the same host country, even after controlling for observables. The model also informs about the effects of different policies in terms of the selection of immigrants. Finally, we can study endogenous reactions of the school system to the presence of immigrants, and through that the impact on natives and immigrants alike.

The focus of this paper is on the school effects of immigration in the host country. However, it is straightforward applying our model to understand the effect on the educational system in the source country. For example, if immigrants were positively selected and, thus, the most motivated parents leave their countries, this would imply negative effects on their compatriots. In particular, this can lead to lower school incentives in the source country, and hence to smaller learning efforts of non-emigrant children under plausible conditions. Refocusing the analysis to the home country is an obvious follow up of this paper.

We restrict our analysis to the effects of immigration on the school system. Clearly, immigration involves effects beyond schools; in the health sector, in the labor market and in many other socially important phenomena. Hence, we do not provide any specific prediction about the optimal policy mix regarding the number of immigrants. Nevertheless, our model uncovers important side and feedback effects, which are generally overlooked in the design and implementation of immigration policy. Many labor market regulations fall into this category. Consider the case of interventions affecting wages of immigrants, like non-discriminatory policies that ease skill transferability of immigrants. This may be achieved, for example, by recognizing qualifications obtained abroad. Recall now that proposition 2 established that selecting the most motivated immigrants required not only a skill premium that is higher in the host country than in the origin country, but also imposed restrictions on parental wages. In particular, the condition for positive selection is easier to satisfy for lower

\[ \text{For example, if assumption (4) holds in the home country. The same would is also true if assumption (4) fails but condition (31) holds and all emigrants are high-skilled.} \]
base rate of wages in the destination country. Insofar skill transferability of immigrants determines the chances for skilled immigrants to get a skilled job abroad, it becomes an important factor determining expected parental wage. While it is hard to think of a situation where impeding skill transferability would make sense in general, our model predicts that skill transferability might worsen immigrant selection in parental skills.\(^{24}\) Moreover, any labor market intervention that affects wages of immigrant parents would condition immigrant selection in parental skills by modifying the opportunity cost for parents to be involved in enhancing their children’s learning effort. It is easy to show that imposing a maximum on the amount of hours will improve selection by forcing parents to dedicate more time on inducing their children to devote more learning effort; a more valuable measure for parents characterized by higher motivation. Similarly, as proved in appendix A.8, higher taxation improves immigrant selection provided the host country preserves the skill premium advantage. Notwithstanding the importance of these side effects, a rigorous evaluation of immigration policies requires a model able to capture their general equilibrium implications; an avenue we leave for future research.

Another important extension concerns the interactions between the political economy of the host country and education; immigrants, or at least their children, often eventually achieve political rights and could importantly, and perhaps unexpectedly, affect political outcomes.\(^{25}\)

References


\(^{24}\) A formal analysis of this claim is in Appendix A.7.

\(^{25}\) See Levy (2005) for an example of the subtle interaction between different types of groups and education provision in a political economy context.
ALBORNOZ, F., S. BERLINSKI, AND A. CABRALES (2011): “Incentives, resources and the organization of the school system,”.


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A Appendices

A.1 Proof of Proposition

For $\Delta < 1$, we have that

$$c_1 = (\phi^S - \phi^U) \frac{\theta \Delta}{\phi'} - \frac{1}{2} c_2,$$

where $c_2$ is still

$$c_2 = \frac{\theta^T (\phi^S - \phi^U)}{\Psi^T} - \frac{1}{2} c_1.$$

Thus,

$$c_1 = \frac{4}{3} (\phi^S - \phi^U) \left( \frac{\theta \Delta}{\phi'} - \frac{1}{2} \frac{\theta^T}{\Psi^T} \right),$$

$$c_2 = \frac{4}{3} (\phi^S - \phi^U) \left( \frac{\theta^T}{\Psi^T} - \frac{1}{2} \theta \Delta \frac{\phi'}{w'} \right),$$

$$e = \frac{2}{3} (\phi^S - \phi^U) \left( \frac{\theta \Delta}{\phi'} + \frac{\theta^T}{\Psi^T} \right),$$

and

$$U^P = T \phi' + \Delta \theta \phi^U + \frac{2}{9} (\phi^S - \phi^U)^2 \left( \frac{\theta^2 \Delta^2}{\phi'} + 2 \theta \Delta \frac{\theta^T}{\Psi^T} + \phi' \left( \frac{\theta^T}{\Psi^T} \right)^2 \right).$$

Remember we assumed that $\Delta_H = 1$ and $\Delta_A \leq 1$. Bearing this in mind, and drawing on the analysis in section 3, we obtain that immigration occurs whenever

$$T \phi' (\phi_A' - \phi_H') + \theta (\Delta \phi_A^U - \phi_H^U) + \frac{2}{9} \theta^2 \left( \frac{\Delta^2 (\phi_A^S - \phi_A^U)^2}{\phi_A'} - \frac{(\phi_H^S - \phi_H^U)^2}{\phi_H'} \right)$$

$$+ \frac{4}{9} \theta \frac{\theta^T}{\Psi^T} \left( \Delta (\phi_A^S - \phi_A^U)^2 - (\phi_H^S - \phi_H^U)^2 \right) + \frac{2}{9} \left( \frac{\theta^T}{\Psi^T} \right)^2 \left( (\phi_A^S - \phi_A^U)^2 (\phi_A' - (\phi_H^S - \phi_H^U)^2) \phi_H' \right)$$

$$> F.$$
The derivative with respect to parental motivation is

\[ v(\Delta \phi_A^U - \phi_H^U) + \frac{4}{9} \left( \theta \left( \frac{\Delta^2 (\phi_A^S - \phi_A^U)^2}{\phi_A'} - \frac{(\phi_H^S - \phi_H^U)^2}{\phi_H'} \right) + \frac{\theta T}{\Psi T} \left( \Delta \phi_A^S - \phi_A^U \right)^2 - \left( \phi_H^S - \phi_H^U \right)^2 \right) \]  

(43)

A similar argument as the one used in the proof of proposition 2 now establishes the result. ■

### A.2 Proof of Lemma 1

Using the optimal children’s effort decision, as expressed in (2), the utility of the headmaster becomes:

\[ U_{HM}^A = \theta_A^T \left( \sum_{k=1}^{N_N} \left( (\phi^S - \phi^U) (c_{1k}^N + c_{2k}^A) + \phi^U \right) + \sum_{l=1}^{N_A} \left( (\phi^S - \phi^U) (c_{1l}^A + c_{2l}^A) + \phi^U \right) \right) \]

\[ + \left( T - \frac{1}{2} c_{2A}^{c_{2A}} \left( \sum_{k=1}^{N_N} \left( c_{1k}^N + c_{2k}^A \right) + \sum_{l=1}^{N_A} \left( c_{1l}^A + c_{2l}^A \right) \right) \right) \gamma_{HM}^A, \]

which implies:

\[ c_{2A} = \frac{\theta_A^T}{\gamma_{HM}^A} \left( \phi^S - \phi^U \right) - \frac{N_N \overline{c_1^N} + N_A \overline{c_1^A}}{2N_j}, \]  

(44)

where

\[ \overline{c_1^N} = \frac{1}{N_N} \sum_{k=1}^{N_N} c_{1k}^N, \]

\[ \overline{c_1^A} = \frac{1}{N_A} \sum_{l=1}^{N_A} c_{1l}^A. \]

It is clear now that the incentives set by schools depend on the average involvement of both natives and immigrants, to which we turn now. The utility of a native parent in country \( A \) is given by:

\[ U_{NP_i}^N = \theta_i \left( (c_{1i}^N + c_{2i}^A) \left( \phi^S - \phi^U \right) + \phi^U \right) + \left( T - \frac{1}{2} c_{1i}^{c_{2i}} \left( c_{1i}^N + c_{2i}^A \right) \right) w_i', \]

leading to the following first order condition

\[ \frac{\partial U_{NP_i}^N}{\partial c_{1i}^N} = \theta_i \left( \phi^S - \phi^U \right) - c_{1i}^N w_i' - \frac{1}{2} c_{2i}^A w_i' = 0, \]  

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so that

\[ c_{1i}^N = \theta_i \frac{v'_i}{w'_i} (\phi^S - \phi^U) - \frac{1}{2} c_A^2. \]  

(45)

Similarly, we obtain

\[ c_{1i}^A = \theta_i \frac{v'_i}{w'_i} (\phi^S - \phi^U) - \frac{1}{2} c_A^2. \]  

(46)

Thus, the optimal parental incentives are expressed by:

\[ c_{1i}^N = \psi_i N (\phi^S - \phi^U) - \frac{1}{2} \theta_A^T \left( \frac{\theta_A^T}{\gamma_{HM}} (\phi^S - \phi^U) - \frac{N_N c_{1i}^N + N_A c_{1i}^A}{2N_j} \right). \]  

(47)

\[ c_{1i}^A = \psi_i A (\phi^S - \phi^U) - \frac{1}{2} \theta_A^T \left( \frac{\theta_A^T}{\gamma_{HM}} (\phi^S - \phi^U) - \frac{N_N c_{1i}^N + N_A c_{1i}^A}{2N_j} \right). \]  

(48)

Notice as well that \( c_A^1 = (\Omega_A - \Omega_N) (\phi^S - \phi^U) + c_N^1 \). Using this, \( c_N^1 \) and \( c_A^1 \) become:

\[ \overline{c}_1^N = (\phi^S - \phi^U) \left( \frac{4N_j - N_A \Omega_N}{3N_j} \Omega_A + \frac{N_A \Omega_A}{3N_j} \Omega_N - \frac{2}{3} \theta_A^T \right). \]  

(51)

\[ \overline{c}_1^A = (\phi^S - \phi^U) \left( \frac{3N_j + N_A \Omega_A}{3N_j} \Omega_N + \frac{N_j - N_A \Omega_N}{3N_j} \Omega_A - \frac{2}{3} \theta_A^T \right). \]  

(52)

and therefore:

\[ N_N c_{1i}^N + N_A c_{1i}^A = \frac{2}{3} (\phi^S - \phi^U) \left( 2 (N_A \Omega_A + N_N \Omega_N) - \frac{(N_N + N_A) \theta_A^T}{\gamma_{HM}} \right). \]  

(53)

Plugging (53) into (47), (48) and (44) we then get the desired result.■
A.3 Proof of Lemma 2

The utility of a parent who stays in $H$ is given by

$$U^H_{P_i} = \theta_i (c_i^H (\phi^S_H - \phi^U_H) + \phi^U_H) - \frac{1}{2} c_i^H w_i^H + Tw_i^H$$  \hspace{1cm} (54)

while the utility of a parent that emigrates from $H$ to $A$ is given by

$$U^A_{P_i} = \theta_i (c_i^A (\phi^S_A - \phi^U_A) + \phi^U_A) - \frac{1}{2} c_i^A w_i^A + Tw_i^A - F_i$$  \hspace{1cm} (55)

Emigration if

$$U^A_{P_i} - U^H_{P_i} > 0$$

or equivalently

$$\theta_i (c_i^A (\phi^S_A - \phi^U_A) + \phi^U_A) - \frac{1}{2} c_i^A w_i^A + Tw_i^A - F_i > 0$$  \hspace{1cm} (56)

Hence from the first order conditions on 54 we get

$$c_i^H = \frac{\theta_i v_i'}{w_i'} (\phi^S_H - \phi^U_H) - \frac{1}{2} c_i^H.$$

Clearly, (23) can be used to calculated the incentives for schools at home by using zero foreign students and replacing all the parameters for the country abroad by the home country’s parameters

$$c_i^H = \frac{2}{3} (\phi^S_H - \phi^U_H) (\frac{2\theta_i^T}{\gamma_{HM}} - \frac{\Omega_i}{H}).$$  \hspace{1cm} (57)

So parental incentives are

$$c_i^H = \frac{\theta_i v_i'}{w_i'} (\phi^S_H - \phi^U_H) - \frac{2}{3} \frac{\theta_i^T}{\gamma_{HM}} (\phi^S_H - \phi^U_H) + \frac{1}{3} (\phi^S_H - \phi^U_H) \frac{\Omega_i}{H}$$

$$= (\phi^S_H - \phi^U_H) \left( \frac{\theta_i}{\phi_i'} \right) - \frac{2}{3} \frac{\theta_i^T}{\gamma_{HM}} + \frac{1}{3} N_H \sum_{i \in N_H} \frac{\theta_i}{\phi_i'}.$$  \hspace{1cm} (58)

and the total effort of the child is given by

$$e_i^H = (\phi^S_H - \phi^U_H) \left( \frac{\theta_i}{\phi_i'} + \frac{2}{3} \frac{\theta_i^T}{\gamma_{HM}} - \frac{1}{3} \frac{\Omega_i}{H} \right).$$  \hspace{1cm} (59)
Using the optimal child’s effort at home defined by (59) and abroad given by (24) and parental incentives at home (58) and abroad (21) and defining
\[ \Omega_{NA} = \frac{N_A \Omega_A + N_N \Omega_N}{N_N + N_A} \]
we can calculate
\[ e_i^A (\phi_A^S - \phi_A^U) + e_i^U (\phi_H^S - \phi_H^U) - e_i^A (\phi_A^H - \phi_A^U) - e_i^H (\phi_H^S - \phi_H^U) = \phi_U^i - \phi_H^i + (\phi_A^S - \phi_A^U)^2 \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) \]
\[ - (\phi_H^S - \phi_H^U)^2 \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \]
and
\[ c_1^A e_i^A w_i^A - c_1^H e_i^H w_i^H = (\phi_A^S - \phi_A^U)^2 \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) - (\phi_H^S - \phi_H^U)^2 \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \]
Hence the condition that people emigrate (56) can be rewritten as
\[ T \nu' \left( \phi_A^i - \phi_H^i \right) + \theta_i \left( \phi_A^i - \phi_H^i \right) + \theta_i \left( \phi_A^i - \phi_H^i \right)^2 \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) - (\phi_H^S - \phi_H^U)^2 \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \]
\[ \gamma \left( \phi_A^S - \phi_A^U)^2 \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) - (\phi_H^S - \phi_H^U)^2 \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \]
\[ > F_i \]
or equivalently
\[ T \nu' \left( \phi_A^i - \phi_H^i \right) + \theta_i \left( \phi_A^i - \phi_H^i \right) + (\phi_A^S - \phi_A^U)^2 \left( \frac{\theta_i}{\phi_A^i} + \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \gamma_A \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_{NA} \right) \phi_A^i \right) \]
\[ - (\phi_H^S - \phi_H^U)^2 \left( \frac{\theta_i}{\phi_H^i} + \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \gamma_H \frac{\theta^T}{H_M} - \frac{1}{3} \Omega_H \right) \phi_H^i \right) \]
\[ > F_i \]
A.4 Proof of Proposition 5

Let \( I_+ (F_\xi) = \{ i \in N | \xi_i = \xi, F_i < F_\xi \} \), and \( I_- (F_\xi) = \{ i \in N | \xi_i = \xi, F_i \geq F_\xi \} \). Denote by \( N_+ (F_\xi) \) the cardinality of \( I_+ (F_\xi) \) and by \( N_- (F_\xi) \) the cardinality of \( I_- (F_\xi) \).

Then, under a threshold equilibrium, we can write for any vector of thresholds \( F = (F_\xi)_{\xi \in \Xi} \):

\[
\overline{\Omega}_A (F) = \frac{\sum_{i \in I_+ (F_\xi)} \theta_i \phi_i}{\sum_{\xi \in \Xi} N_+ (F_\xi)}, \quad \overline{\Omega}_H (F) = \frac{\sum_{i \in I_- (F_\xi)} \theta_i \phi_i}{\sum_{\xi \in \Xi} N_- (F_\xi)}.
\]

Clearly

\[
\overline{\Omega}_{NA} (F) = \frac{N_+ (F) \overline{\Omega}_A (F) + N_N \overline{\Omega}_N}{(N_N + N_+ (F))}.
\]

Let for any \( i \) with \( \xi_i = \xi \in \Xi \)

\[
G_\xi (F) \equiv Tv' (\phi^A_i - \phi^H_i) + \theta_i (\phi^U_i - \phi^U_H)
\]

\[
+ (\phi^S_i - \phi^U_i)^2 \left( \frac{\theta_i}{\phi_i} + \frac{2}{3} \frac{\theta^T_A}{\gamma_{HM}} - \frac{1}{3} \overline{\Omega}_{NA} (F) \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \frac{\theta^T_A}{\gamma_{HM}} - \frac{1}{3} \overline{\Omega}_{NA} (F) \right) \phi^A_i \right)
\]

\[
- (\phi^S_H - \phi^U_H)^2 \left( \frac{\theta_i}{\phi_i} + \frac{2}{3} \frac{\theta^T_H}{\gamma_{HM}} - \frac{1}{3} \overline{\Omega}_H (F) \right) \left( \frac{1}{2} \theta_i + \frac{1}{2} \left( \frac{2}{3} \frac{\theta^T_H}{\gamma_{HM}} - \frac{1}{3} \overline{\Omega}_H (F) \right) \phi^H_i \right).
\]

Under these conditions existence is guaranteed by a straightforward application of Brouwer's fixed point theorem, since \( G(.) \) is a continuous function and we have defined \( F \) to belong to the convex, compact set \([0,A]\).

\[\blacksquare\]
A.5 Proof of Proposition 6

Under the assumption that \( \frac{\partial^2 \Phi}{\partial \bar{N}^2} = \frac{\partial^2 \Phi}{\partial \bar{H}^2} = 0 \) and individuals are homogeneous in base line wages we can write

\[
G_\xi (F) - G_{\xi'} (F) \equiv (\theta - \theta') (\phi_A - \phi_H') \\
+ (\phi_A^S - \phi_A^U)^2 \left( \frac{((\theta - \theta')^2)}{2\phi} \right) \\
+ (\phi_A^S - \phi_A^U)^2 \frac{\phi_A^A}{2} \left( \frac{\theta - \theta'}{2\phi} \right) \left( \frac{2}{3} \frac{\theta^T_r}{\gamma_{HM}} - \frac{1}{3} \frac{\Omega_{NA}}{F} \right) \\
+ (\phi_A^S - \phi_A^U)^2 \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} \right) \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} - \frac{1}{3} \frac{\Omega_{H}}{F} \right) \\
- (\phi_H^S - \phi_H^U)^2 \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} \right) \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} - \frac{1}{3} \frac{\Omega_{H}}{F} \right)
\]

Hence we have that

\[
G_\xi (F) - G_{\xi'} (F) \\
\equiv (\theta - \theta') (\phi_A - \phi_H') \\
+ \left( \frac{(\phi_A^S - \phi_A^U)^2}{2\phi} - \frac{(\phi_H^S - \phi_H^U)^2}{2\phi} \right) ((\theta - \theta')^2) \\
+ (\phi_A^S - \phi_A^U)^2 (\theta - \theta') \left( \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} \right) - \left( \frac{2}{3} \frac{\theta^T_{r_f}}{\gamma_{HM}} - \frac{1}{3} \frac{\Omega_{H}}{F} \right) \right),
\]

or equivalently

\[
G_\xi (F) - G_{\xi'} (F) \\
\equiv (\theta - \theta') (\phi_A - \phi_H') \\
+ \left( \frac{(\phi_A^S - \phi_A^U)^2}{2\phi} - \frac{(\phi_H^S - \phi_H^U)^2}{2\phi} \right) ((\theta - \theta')^2) \\
+ (\phi_A^S - \phi_A^U)^2 (\theta - \theta') \frac{1}{3} \left( \left( \frac{1}{N_{A+1}} \sum_{i=1}^{N_{A+1}} \left( \frac{2k_A}{\gamma_{HM}} - \frac{1}{\phi_i} \right) \theta_i \right) \right) - \left( \frac{1}{N_H} \sum_{i=1}^{N_H} \left( \frac{2k_H}{\gamma_{HM}} - \frac{1}{\phi_i} \right) \theta_i \right).
\]
Let $\theta_\xi > \theta_{\xi'}$. By our assumption that baseline wages abroad are at least as high as wages at home $\phi_A \geq \phi_H$, the first line is nonnegative. The second line is nonnegative if $(\phi_S - \phi_U)^2 / 2\phi_A \geq (\phi_S - \phi_U)^2 / 2\phi_H$ and the third line is nonnegative if $(2k_A / \gamma_{HM}) \theta_A - (2k_H / \gamma_{HM}) \theta_H \geq \Omega_{NA} - \Omega_{H}$. ■

A.6 Proof of Lemma 3

Using the first order conditions for children’s effort decision (2) we get:

$$U_A^{HM} = \theta_A^T \left( \sum_{k=1}^{N} ((\phi_S - \phi_U) (c_{ik} + c_A^2) + \phi_U) + \sum_{l=1}^{N_A} ((\phi_S - \phi_U) (c_A^1 + c_A^2) + \phi_U) \right) + \left( T - \frac{1}{2r_j} c_A^2 \left( \sum_{k=1}^{N} (c_{ik} + c_A^2) + \sum_{l=1}^{N_A} (c_A^1 + c_A^2) \right) \right) \gamma_{HM}^A.$$

Hence

$$\frac{\partial U_A^{HM}}{\partial c_A^2} = \theta_A^T \left( \sum_{k=1}^{N} (\phi_S - \phi_U) + \sum_{l=1}^{N_A} (\phi_S - \phi_U) \right) - \left( \frac{1}{2} \left( \sum_{k=1}^{N} c_{ik} + \sum_{l=1}^{N_A} c_A^1 \right) + N_j c_A^2 \right) \frac{\gamma_{HM}^A}{r_j} = 0.$$

So

$$c_A^2 = \frac{r_j \theta_A^T (\phi_S - \phi_U) - N_N c_{1i}^N + N_A c_A^1}{2N_j}, \quad (60)$$

For parents the only change now is that school resources cost money which they will have to pay from general taxation, but given the quasi-linearity in income of utility and that taxation is already decided at the time parents choose their effort, the amount of those taxes do not affect the parental effort decision. Hence for a native parent in country $A$

$$c_{1i}^N = \theta_i \left( \frac{v_i'}{w_i'} (\phi_S - \phi_U) - \frac{1}{2} c_A^2 \right). \quad (61)$$

And for a parent who emigrates from country $H$ to country $A$ is given by

$$c_{1i}^A = \theta_i \left( \frac{v_i'}{w_i'} (\phi_S - \phi_U) - \frac{1}{2} c_A^2 \right). \quad (62)$$

Similar calculations as in Lemma 1 yield the desired result. ■
A.7 Imperfect skill transferability

Assume now that people who have a high skill job at home do not necessarily get a high skill job abroad but their job expectations abroad are \( \phi'_{A} = \left( e_{A} \phi_{A}^{S} + (1 - e_{A}) \phi_{A}^{U} \right) \geq \phi_{H}^{S} \). In this case the derivative with respect to work ethic for high-skill workers (12) becomes

\[
\left( \phi_{A}^{U} - \phi_{H}^{U} \right) + \frac{4}{9} \left( \theta \left( \frac{\left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2}}{e_{A} \phi_{A}^{S} + (1 - e_{A}) \phi_{A}^{U}} - \frac{\left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2}}{\phi_{H}^{S}} \right) \right) + \frac{\theta^T}{\Psi^T} \left( \left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2} - \left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2} \right)
\]

and we would get positive selection in work ethic if the skill differential is higher abroad than at home and if \( \left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2} / \left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2} \geq \left( e_{A} \phi_{A}^{S} + (1 - e_{A}) \phi_{A}^{U} \right) / \phi_{H}^{S} \).

The right hand side is smallest for \( e_{A} = 0 \) and the constraint is most difficult to satisfy for \( e_{A} = 1 \). This indicates that reducing skill transferability might improve immigrant selection. While all high-skill parents have more incentives to emigrate the higher the skill transferability, this effect is biggest for parents with a lower work ethic, due to their smaller parental involvement. Cutting the wages improves parental involvement and also the selection of immigrants.

A.8 Taxation

Here we will show that taxation improves immigrant selection provided the host country preserves the skill premium advantage (condition (13) holds).

Assume proportional taxes in the host country \( (t_{A}) \), so people keep a \( (1 - t_{A}) \) fraction of their income. In this case, immigration requires

\[
T \left( (1 - t_{A}) w'_{A} - w'_{H} \right) + \theta \left( (1 - t_{A}) \phi_{A}^{U} - \phi_{H}^{U} \right) + \frac{2}{9} \theta^2 \left( (1 - t_{A}) \left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2} \frac{v'}{w'} - \left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2} \frac{v'}{w'} \right) + \frac{4}{9} \frac{\theta^T}{\Psi^T} \left( (1 - t_{A})^{2} \left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2} - \left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2} \right) + \frac{2}{9} \left( \frac{\theta^T}{\Psi^T} \right)^{2} \left( (1 - t_{A})^{3} \left( \phi_{A}^{S} - \phi_{A}^{U} \right)^{2} \frac{w'_{A}}{v'} - \left( \phi_{H}^{S} - \phi_{H}^{U} \right)^{2} \frac{w'_{H}}{v'} \right) > \bar{F}.
\]

Let us call the left-hand-side of this expression \( G(\theta, \phi) \). Its derivative with respect to \( \theta \) is:
\[
\frac{\partial G}{\partial \theta} = (1 - t_A) \phi_A^U - \phi_H^U + \frac{4}{9} \theta \left( (1 - t_A) \left( \phi_A^S - \phi_A^U \right)^2 \frac{v'}{w_A} - \left( \phi_H^S - \phi_H^U \right)^2 \frac{v'}{w_H} \right) + \frac{4}{9} \frac{\theta^T}{\Psi^T} \left( (1 - t_A)^2 \left( \phi_A^S - \phi_A^U \right)^2 - \left( \phi_H^S - \phi_H^U \right)^2 \right),
\]

which, assuming that \( \phi_A' > \phi_H' \), is always positive if \( (1 - t_A) \left( \phi_A^S - \phi_A^U \right)^2 > \left( \phi_H^S - \phi_H^U \right)^2 \phi_A' / \phi_H' \). In other words,

\[
t_A < \overline{t_A} = 1 - \frac{\left( \phi_H^S - \phi_H^U \right)^2 \phi_A'}{\left( \phi_A^S - \phi_A^U \right)^2 \phi_H'}.
\]

Now

\[
\frac{\partial \theta^*}{\partial t_A} = -\frac{\frac{\partial G}{\partial t_A}}{\frac{\partial G}{\partial \theta}},
\]

so that if \( \partial G / \partial t_A < 0 \) for \( t_A < \overline{t_A} \), then an increase in taxation induces a better selection of immigrants in terms of \( \theta \). Indeed,

\[
\frac{\partial G}{\partial t_A} = -T w_A' - \theta \phi_A^U - \frac{2}{9} \theta^2 \left( \phi_A^S - \phi_A^U \right)^2 \frac{v'}{w_A'} - \frac{8}{9} \frac{\theta^T}{\Psi^T} \left( 1 - t_A \right) \left( \phi_A^S - \phi_A^U \right)^2
\]

\[
- \frac{2}{3} \left( \frac{\theta^T}{\Psi^T} \right)^2 \left( 1 - t_A \right)^2 \left( \phi_A^S - \phi_A^U \right)^2 \frac{w_A'}{v'} < 0.
\]