Firm Dynamics Support the Importance of the Embodied Question*

Alain Gabler† Omar Licandro‡

September 23, 2009

Abstract

This paper contributes to the literature on both embodied technical progress and firm dynamics, by formulating an endogenous growth model where selection and imitation play a fundamental role in helping capital good producers to learn about the productivity of technologies embodied in new plants. By calibrating the model to some key aggregates particularly relevant for the embodied capital literature, among them the growth rate of the relative investment price, the model quantitatively replicates the main facts associated to firm dynamics, such as the entry rate and the tail index of the establishment size distribution. In line with the previous literature, it also predicts a contribution to productivity growth of embodied technical progress and selection of around 60%.

*We thank Paolo Giordani, Bart Hobijn, Thanh Le, Pietro Peretto, Morten Ravn, Roberto Samaniego, Paul Segerstrom, and seminar participants at the European University Institute, the Swiss Federal Institute of Technology Zurich, IZA Bonn, and the SED, EEA, ASSET, and ISS conferences for helpful comments and suggestions. A previous version of this paper has circulated under the name "Growth through Selection under Rational Expectations." The authors acknowledge the financial support of the EC, contract HPRN-CT-2002-00237. Omar Licandro also acknowledges support from the Spanish Ministry of Education under projects SEJ2004-0459/ECON and SEJ2007-65552, and the Catedra Sabadell-FEDEA.

†Université Laval.

‡Instituto de Análisis Económico.

JEL codes: B52, O3, O41. *Keywords*: endogenous growth; investment-specific technological change; selection and imitation; firm entry and exit.

1 Introduction

Gordon (1990)'s estimations of quality adjusted price indexes for durable goods reopened the embodiment controversy during the nineties. In a highly influential paper, Greenwood, Hercowitz, and Krusell (1997) calibrate a two-sector growth model using Gordon's estimates as a measure of investment-specific technical change to conclude that about 58% of per capita output growth can be attributed to productivity improvements specific to equipment investment.[1] The production technology in Greenwood, Hercowitz, and Krusell (1997) derives from Solow (1957), who postulates general assumptions under which a vintage capital structure aggregates at equilibrium into a two-sector (nondurable consumption and investment) technology with investment-specific technical change. In Solow's theory, technology differs across vintages, but there is within-vintage homogeneity; all capital units belonging to the same vintage are identical.

In recent years, following the seminal papers by Jovanovic (1982) and Hopenhayn (1992) and benefiting from increasing micro evidence, a developing literature studies different dimensions of the dynamics of heterogeneous firms. Using a similar endogenous growth engine as the one in this paper, Luttmer (2007) finds that around 60% of productivity growth is due to a selection-imitation mechanism emerging from the dynamics of firm behavior. As we show in this paper, the observed similarity of the contribution to productivity growth of both embodied technical progress and the selection-imitation mechanism is not a fortunate coincidence.

This paper contributes to the literatures on both embodied technical progress and firm dynamics, by formulating an endogenous growth model

[1] The literature following Greenwood, Hercowitz, and Krusell (1997)'s publication, see Cummins and Violante (2002) and Gort, Greenwood, and Rupert (1999), pointed in the same direction by providing similar estimations. Endogenous growth versions of Greenwood, Hercowitz, and Krusell (1997) are in Krusell (1998), Hsieh (2001) and Boucekkine, del Río, and Licandro (2003).

where selection and imitation play a fundamental role in helping capital good producers to learn about the productivity of technologies embodied in new plants. By calibrating the model to some key aggregates particularly relevant for the embodied capital literature, among them the growth rate of the relative investment price, the model quantitatively replicates the main facts associated to firm dynamics, such as the entry rate and the tail index of the firm distribution. In line with Greenwood, Hercowitz, and Krusell (1997), it also predicts a contribution to productivity growth of embodied technical progress and selection-imitation of around 60%.

As in Greenwood, Hercowitz, and Krusell (1997), the paper assumes there are two sectors. A nondurable sector is composed of plants using labor to produce both consumption goods and inputs for the investment sector. The investment sector produces 'plants'. In the nondurable sector, there is within and between vintage plant heterogeneity; the productivity of new plants is stochastic, and existing plants are continuously hit by idiosyncratic productivity shocks. The heterogeneous productivity of new plants is initially unknown and subject to stochastic learning-by-doing. This process is assumed to have a common component, meaning that average learning is common to all plants. But, the random component is plant-specific, so that some plants regress, while others improve when compared to the mean. A second important implication of plant specific learning is associated to the selection process typical from models of firm dynamics. Plants cumulating negative shocks realize they are particularly inefficient and exit the market.

Concerning new plants, the model assumes that their productivity is drawn from a known distribution, whose mean depends on the mean productivity of incumbent plants. We read this assumption in terms of learning in the investment sector. The learning-by-doing process taking place in the nondurable sector partially informs capital producers about the characteristics of existing plants. The outcome of this learning process is then used by them to build new plants, which are expected to be on average more efficient than those previously produced. By learning about the efficiency of existing technologies, capital producers are able to design better and better plants. Let us call this process imitation, as in Luttmer (2007), which we model by assuming that the mean of new plants' productivity depends on the mean of incumbents' productivity. Note that signals coming from the selection process are highly informative, since exiting plants' productivity does not affect the expected productivity of new plants.

Behind the stochastic productivity process just described, and its learning interpretation, there are two sources of growth: disembodied technical progress represented by the common component of the incumbents' productivity process; and embodied technical progress, associated to the selection-imitation mechanism. The result is a model of investment-specific technological change in which the relative price of investment is endogenous. Average learning-by-doing then corresponds to what Greenwood, Hercowitz, and Krusell (1997) call neutral technological progress, while the gains in average productivity due to learning in the investment sector correspond to investment-specific technological progress.

In learning-by-doing models of economic growth, as in Romer (1990), technical progress is disembodied and benefits all plants; individual learning occurs at the final production technology and immediately becomes common knowledge. It does not matter 'who learns what', since any technological news is common knowledge. In our framework, nondurable plants are heterogeneous and their learning is plant specific. There is a common component as in Romer and an idiosyncratic component spilling information over capital producers to learn about the ways of moving up the frontier technology by producing better and better plants.

The growth engine in this paper is selection and imitation, which has much in common with the Schumpeterian idea of creative destruction. The stochastic evolution of individual technologies makes some plants obsolete, opening market opportunities to new entrants. Moreover, selection transmits information to the investment sector about the quality of technology favoring the development of more productive plants. Growing through selection reverses the role of destruction and creation, when compared with the existing literature. For example, in Aghion and Howitt (1992), growth is generated by a random sequence of quality-improving, sector-specific innovations; better products or technologies render previous ones obsolete, and this occurs through the replacement of the incumbent local monopolist by a new firm. Something similar occurs in the Solow vintage capital model, where new capital vintages require labor to produce, pushing up wages and reallocating resources out of old vintages. These two are theories of destructive creation more than creative destruction.

We evaluate the plausibility of our assumptions by doing a quantitative exercise. Because of the aggregate formulation of our model, we can calibrate it without recurring to establishment-level data. Instead, we use US data from the National Income and Product Accounts (NIPA), as well as estimates on the speed of embodied technological progress from Cummins and Violante (2002) and Gort, Greenwood, and Rupert (1999), and find that the model implies a rate of firm entry which is only slightly larger, as well as a distribution of production units whose tail index is only slightly smaller, than in the data. From this, we conclude that our assumption that only entrants profit from embodied technological change is not overly restrictive. Therefore, it should not be surprising that Greenwood, Hercowitz, and Krusell (1997) measuring the contribution to productivity growth of embodied technological progress and Luttmer (2007) that of selection and imitation yield very similar results.

The remainder of the paper is organized as follows: section 2 describes the model; section 3 deals with its calibration; section 4 looks at the results, and section 5 concludes.

2 The Model

2.1 Production Technology

There are two sectors, one producing a non-durable good and the other producing capital goods or *machines*. The price of the non-durable good is normalized to one every period. The non-durable good is homogeneous and produced by the means of a continuum of machines of measure K_t and labor. Its output is assigned to consumption and is the sole input in the capital goods sector. As in Campbell (1998), it is assumed that an operative plant in the non-durable sector uses one and only one machine.

Machines are heterogeneous with idiosyncratic productivities Z_t at time t, $Z_t \in R^+$. A machine with productivity Z uses $L(Z)$ of the homogeneous labor input to produce $Y(Z)$ of the non-durable good by the means of technology

$$Y_t(Z_t) = A_t Z_t^{\alpha} L_t(Z_t)^{1-\alpha}. \tag{1}$$

Returns to labor are assumed to be decreasing, meaning that parameter $\alpha \in (0,1)$. Note that a firm's productivity has two components. The first, A_t, is assumed to follow a learning-by-doing process $A_t = K_t^{\xi}$, $\xi \in (0, 1-\alpha)$, which depends on the aggregate capital stock K_t.[2]

The second productivity component, Z_t, is machine-specific and its random growth rate is assumed to be independently and identically distributed over time and across machines. Let us denote by $\varphi(.)$ the log-normal density of Z_t/Z_{t-1}, which is assumed to have unit mean and variance σ^2. By the previous assumption, existing machines have no expected idiosyncratic gain in productivity. As is shown below, however, the dispersion of productivity gains will make *selection* to operate: Machines exposed to a sequence of negative idiosyncratic shocks are optimally scrapped, implying that the average productivity of remaining machines is growing. The cumulative distribution of plants across productivity levels is endogenous and denoted by $\Phi_t(Z)$, with the associated density denoted by $\phi_t(Z)$; the average machine-specific productivity is

$$\bar{Z}_t = \int_0^{\infty} Z d\Phi_t(Z).$$

In the investment sector, a continuum of firms of unit measure are operative. They transform one unit of the non-durable good into a machine. As in the one sector growth model, machines produced at period $t-1$ become operative at t. The initial productivity of a new machine Z_t, relative to the average productivity of operative machines $\bar{Z}_{t-1}$, is assumed to follow a log-normal distribution $\varphi_e(.)$ with mean ψ, $\psi > 0$, and variance σ_e^2. This is the

[2]Notice that capital is measured in units of machines, without any quality adjustment. This important issue is discussed below. The upper bound restriction on ξ avoids unbounded growth.

simplest way of modeling *imitation*: The expected initial productivity of new machines is proportional to the average productivity of existing machines, ψ being the proportionality factor. Consequently, the investment sector will be producing better and better machines if the average productivity of existing machines were permanently growing.[3] Imitation is restricted to the capital goods sector.

Let us give some economic interpretation to the previous assumptions. In the non-durable sector a random *learning-by-doing* process occurs at the plant level. Average learning as represented by A_t is common, but the random component Z_t is machine-specific, meaning that some plants improve, but others regress when compared to the mean. This means that some machines are more adapted to the new technological environment, as described by A_t, than others. This learning process may be interpreted as learning about the quality of machines, which improves randomly over time as the economy develops. *Imitation* means that by observing the distribution of output across machines, capital good producers learn about the quality of existing machines, and then they try to replicate the best machines as closely as possible. Imitation is random. The output of the imitation process is represented by the density $\varphi_e(.)$, with an expected initial productivity close to the mean productivity of existing machines, the distance being represented by ψ.[4]

Total output in the non-durable sector is given by

$$Y_t^N = K_t \int_0^\infty Y_t\left(Z\right) d\Phi_t\left(Z\right), \tag{2}$$

while an efficient allocation of labor satisfies

$$K_t \int_0^\infty L_t\left(Z\right) d\Phi_t\left(Z\right) = L_t, \tag{3}$$

[3] In fact, the calibrated value of ψ will be smaller than unity, replicating the observed evidence that the average productivity of entering plants is smaller than the average productivity of incumbent plants.

[4] Luttmer assumes that the productivity distribution of entrants is a scaled-down version of the productivity distribution of incumbents.

where L_t is the aggregate labor supply. Non-durable output has to be efficiently allocated to consumption C_t and as an input in the investment sector I_t^N:

$$Y_t^N = C_t + I_t^N. \tag{4}$$

2.2 Consumers and Producers Behavior

There is a continuum of individuals of measure L. Individuals have a labor endowment of one unit every period, meaning that L also measures the aggregate labor supply. Preferences are represented by

$$U = \sum_{t=0}^{\infty} \beta^t \ln\left(C_t\right), \tag{5}$$

where C_t denotes consumption, and the discount factor is β, with $\beta \in (0,1)$. The representative household maximizes its life-time utility (5) subject to a standard budget constraint. Given that plants are atomistic, and all shocks are independently and identically distributed across plants, households may diversify any individual risk by owning a positive measure of plants. The first order condition for consumption yields the usual Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta\left(1 + r_t\right), \tag{6}$$

where r is the interest rate.

At any period t, a plant with productivity Z solves the following static problem

$$\Pi_t(Z) = \max_{L_t(Z)} K_t^{\xi} Z^{\alpha} L_t(Z)^{1-\alpha} - w_t L_t(Z),$$

taking the wage rate w_t and the mass of plants K_t as given. The optimal labor demand is

$$L_t(Z) = \left(\frac{(1-\alpha)K_t^{\xi}}{w_t}\right)^{\frac{1}{\alpha}} Z.$$

The real wage rate is obtained by aggregation, using (3),

$$w_t = (1-\alpha)\left(\frac{\bar{Z}_t}{L_t}\right)^{\alpha} K_t^{\xi+\alpha}$$

implying that

$$L_t(Z) = \frac{L_t}{K_t}\frac{Z}{\bar{Z}_t}. \tag{7}$$

Average per plant labor $\frac{L_t}{K_t}$ is allocated across plants depending on their relative productivities. Consequently, a plant's profits can be written as a linear function of the plant's productivity Z

$$\Pi_t(Z) = \alpha K_t^{\xi}\left(\frac{L_t}{\bar{Z}_t K_t}\right)^{1-\alpha} Z. \tag{8}$$

As will be shown below, at a balanced growth path a plant's profits are stationary, but the mass of plants grows at the same rate as the economy, meaning that total profits follow output.

Note that non-durable technology (2), after substitution of (1) and (7), may be written as a Cobb-Douglas technology on capital and labor

$$Y_t^N = \left(K_t^{\xi}\bar{Z}_t^{\alpha}\right) K_t^{\alpha} L_t^{1-\alpha}. \tag{9}$$

Total factor productivity has two components, a *disembodied* component related to the learning-by-doing term K_t^{ξ} and an *embodied* component related to the average productivity of incumbents $\bar{Z}_t$. Since $\xi + \alpha < 1$ by assumption, learning-by-doing is not strong enough to generate growth endogenously. However, the selection-imitation mechanism described in the following section makes $\bar{Z}$ grow at the positive rate g_Z, which will have as implication that non-durable production will be growing at the growth factor

$$g_N = g_Z^{\frac{\alpha}{1-\alpha-\xi}},$$

at the stationary solution.

Finally, it can be easily proved that total profits are αY^N. Therefore, consistently with aggregate technology (9), α is the capital share in value added.

2.3 Entry, Exit and Productivity Distribution

Given that capital is a fixed factor and since its relative productivity may decline over time, less productive plants definitively cease production. In this case plant's capital is scrapped, being transformed into new capital at the rate θ, $\theta \in (0,1)$, which represents the scrapping value of capital.

At period $t-1$, knowing its productivity Z_{t-1}, an incumbent plant has to decide whether to exit the market at period t. Let $V_t\left(Z_{t-1}\right)$ be the expected value at period t of a plant with observed productivity Z_{t-1}. If the plant chooses to stay, it will draw a new productivity Z_t, produce and get profits Π_t and then the discounted expected value V_{t+1}. Otherwise, it recovers the scrap value θ at time $t-1$, and transfers it to period t getting the corresponding return r_t. The optimal policy then involves choosing a *reservation productivity* Z^*_{t-1} at which plants are indifferent between staying and exiting:

$$\int_0^\infty \left[\Pi_t\left(Z'\right) + \frac{1}{1+r_{t+1}} V_{t+1}\left(Z'\right)\right] \varphi\left(Z'/Z^*_{t-1}\right) dZ' = \theta(1+r_t), \qquad (10)$$

with $\Pi_t\left(Z\right)$ as defined above. Since the decision is taken at period $t-1$, the productivity cutoff Z^*_{t-1} corresponds to the $t-1$ productivity distribution. The plant's value is then given by

$$V_t\left(Z_{t-1}\right) = \begin{cases} \int_0^\infty \left[\Pi_t\left(Z'\right) + \frac{1}{1+r_{t+1}} V_{t+1}\left(Z'\right)\right] \varphi\left(Z'/Z_{t-1}\right) dZ' & \text{if } Z_{t-1} \geq Z^*_{t-1}, \\ \theta(1+r_t) & \text{otherwise.} \end{cases} \qquad (11)$$

To create a plant requires a machine, which costs one unit of the non-durable good. Under free entry, expected profits have to cover the investment cost:

$$\int_0^\infty \left[\Pi_t\left(Z'\right) + \frac{1}{1+r_{t+1}} V_{t+1}\left(Z'\right)\right] \varphi_e\left(Z'/\bar{Z}_{t-1}\right) dZ' = 1 + r_t. \qquad (12)$$

Since the machine has to be bought one period in advance, the investment cost, on the right hand side, includes the user cost of capital r_t.

At period $t-1$, after observing Z_{t-1}, an operative plant faces the following alternatives: to keep producing with the same machine or to exit, which opens the option of buying a new machine by creating a new plant. This trade-off may be better understood by comparing (10) and (12). The right hand sides reflects the investment costs of these two alternatives, being both faced at $t-1$ but evaluated at t [this explains the $1+r$ factor]. The cost of a new machine is unity and the opportunity cost of an old machine is the scrapping value θ. At equilibrium, the distance between Z^* and $\bar{Z}$ depends on ψ and the relative variance of both random process. By direct observation of these two equations, we may expect that parameters affecting the value function would not have a strong effect on the cutoff point, since the value function is the same in both conditions.

In order to map the time t productivity distribution $\Phi_t(Z)$ into the next period's distribution, one has to take into account (i) idiosyncratic shocks hitting plants, (ii) the disappearance of those plants which choose to shut down, and (iii) the entrance of new plants. Since there is a continuum of plants in the economy, the evolution of the distribution of plants across productivity levels is deterministic even though each particular firm experiences random shocks. The transition function for the distribution of plants across productivity levels is

$$K_{t+1}\phi_{t+1}(Z) = \underbrace{K_t \int_{Z_t^*}^{\infty} \varphi(Z/Z')\, d\Phi_t(Z')}_{\text{remaining plants}} + \underbrace{(I_t + \theta X_t)\, \varphi_e\left(Z/\bar{Z}_t\right)}_{\text{entering plants}}, \tag{13}$$

where $X_t = K_t \Phi_t(Z_t^*)$ is the measure of exiting plants and the total amount of scrapped capital recovered from exiting plants is equal to θX_t.

New plants in $t+1$ are created using new machines, I_t^N, and machines recovered from scrapping, θX_t, so that the evolution law of the stock of machines follows:

$$K_{t+1} = K_t - X_t + I_t^N + \theta X_t = (1-\delta_t) K_t + I_t^N, \tag{14}$$

where $\delta_t = (1-\theta)\frac{X_t}{K_t}$. The above equation can be easily derived from (13) by integrating it over $Z \in (0, \infty)$. The number of operative machines endogenously depreciates at the *obsolescence* rate δ.

2.4 Balanced Growth

Let us define the relative productivity z, $z_t = Z_t/\bar{Z}_{t-1}$, and the average production per firm y, $y_t = K_t^{\xi+\alpha-1} L_t^{1-\alpha} \bar{Z}_t^{\alpha}$. In a balanced growth path, the distribution of relative productivities, the profit and the value functions are all three stationary, and y is constant. Using the proposed variable changes, the profit function (8) becomes

$$\pi(z) = \alpha z y \tag{15}$$

The value function (11), after substituting r from the Euler equation (6), the exit condition (10), and the entry condition (12) become

$$v(z) = \begin{cases} \int_0^\infty \left[\pi(z') + \frac{\beta}{g_N} v(z')\right] \varphi(z' g_Z / z)\, dz' & \text{if } z \geq z^* \\ \theta g_N/\beta & \text{otherwise} \end{cases} \tag{16}$$

$$\int_0^\infty \left[\pi(z') + \frac{\beta}{g_N} v(z')\right] \varphi(z' g_Z / z^*)\, dz' = \frac{\theta g_N}{\beta} \tag{17}$$

$$\int_0^\infty \left[\pi(z') + \frac{\beta}{g_N} v(z')\right] \varphi_e(z') dz' = \frac{g_N}{\beta}. \tag{18}$$

Finally, the evolution law of the cumulative probability distribution of relative productivities across plants $\Lambda(z)$ derives from (13), after dividing both sides by K_t, substituting $I/K = g_N + (\theta - 1)\Lambda(z^*)$ coming from (14), and using the definition of δ, and $X/K = \Lambda(z^*)$:

$$g_N \lambda(z) = \int_{z^*}^\infty \varphi(z g_Z / z')\, d\Lambda(z') + (g_N + \Lambda(z^*))\, \varphi_e(z), \tag{19}$$

where λ is the density associated to Λ.

A balanced growth path equilibrium is a growth rate g_Z, an average production y, a cutoff point z^*, a value function $v(z)$ and a density function $\lambda(z)$, both for $z \in R^+$. They solve (16) to (19), with π defined by (15) and

$g_N = g_Z^{\frac{\alpha}{1-\alpha-\xi}}$, and $\int_{z^*}^{\infty} z\lambda(z)\,dz = g_Z$. The last condition comes directly from the definition of variable z, which has a mean of g_Z by construction. An algorithm designed to solve for it is described in the appendix.

The model distinguishes itself from the evolutionary economics literature in the line of Nelson and Winter (1982) through equations (17) and (18), which state that exit and entry follow rational, instead of adaptive, expectations. It distinguishes itself from the industrial evolution literature, which notably includes papers by Jovanovic (1982) and Hopenhayn (1992), through equation (19), which states that the average productivity of entering plants is not exogenous but instead depends on the average productivity of existing plants. It differs from Luttmer (2007) through the existence of an aggregate capital stock, and the fact that part of an exiting firm's capital can be reused by entrants (equation 14). This allows us to define a quality-adjusted measure for aggregate capital, which we then use in order to calibrate the model.

2.5 Embodied Technological Progress

In this section, we show that the aggregate relations in the above setup can be rewritten as a model of embodied technical change, in the same line as Greenwood, Hercowitz, and Krusell (1997), with the novelty that the relative price of investment and the depreciation rate are both endogenous.

To compute the aggregate technology, we follow Solow (1957) in defining the quality-adjusted capital stock

$$J_t = K_t \bar{Z}_t.$$

After substituting it in (9), we get

$$Y_t^N = A_t J_t^{\alpha} L_t^{1-\alpha}. \tag{20}$$

From (14), the law of motion of quality-adjusted capital can be written as

$$J_{t+1} = \left(1 - \hat{\delta}_t\right) J_t + q_t I_t^N, \tag{21}$$

where $\hat{\delta}_t = \left(\int_0^\infty Z d\Phi_t(Z) - \psi\theta\right) \frac{X_t}{K_t}$ and $q_t = \psi \bar{Z}_{t-1}$.

Equivalent equations to (20) and (21) are in Greenwood, Hercowitz, and Krusell (1997). Disembodied technical progress is represented by A_t and embodied technical progress by q_t. Quality adjusted output of the investment sector, $q_t I_t^N$, grows at the rate

$$g_I = (g_Z)^{\frac{1-\xi}{1-\alpha-\xi}} > g_N$$

at the stationary solution, while the price of investment goods relative to the price of non-durable goods permanently decreases at the rate g_Z.

Disembodied technical progress is a direct result of learning-by-doing. The embodied nature of technical progress is due to the assumption that imitation takes place at the investment sector. Capital good producers offer more and more productive investment goods since they are imitating surviving, successful machines.

3 Calibration

The aim of this section is to study the behavior of a parameterized version of the model economy, in order to assess the quantitative impact of selection and imitation on U.S. productivity growth, in analogy to the contribution of embodied technological change as estimated by Greenwood, Hercowitz, and Krusell (1997). The length of a period is set to a quarter. The parameters which need to be calibrated are the preference parameter β, the technology parameters α and ξ, the scrapping value of capital θ, the variance of idiosyncratic shocks to all plants σ, under the assumption $\sigma_e = \sigma$, and the average relative productivity of entering plants ψ.

Gomme and Rupert (2007) find an average capital income share α for the non-housing and non-government sector of 28.3%. The discount rate β is set to .99.

In order to impose some rigor on the quantitative analysis, the procedure advanced by Kydland and Prescott (1982) is followed. Parameters are set such that the balanced growth path is consistent with some average observations from U.S. National Income and Product Accounts (NIPA) for the period 1948-2000. The Government and housing sectors are netted out of GDP, given that the selection mechanism which is at work in the model is specific to a competitive business sector. Also, to avoid the issue of accounting for quality improvements in consumer durables, consumption in the model is matched up with nondurable goods and non-housing services, as in Greenwood, Hercowitz, and Krusell (1997).

In order to compute the remaining parameters, the following four moments of the data were used: the growth rate of per capita consumption, $g_N - 1$ in the model, the investment share, I^N/Y^N, the depreciation rate, δ, and the decline rate of investment prices relative to consumption prices $g_Z - 1$. The average real growth rate of quarterly non-durable consumption per capita was 0.48% and the investment share was 18.5%.

Given that current NIPA methodology does not fully take into account quality improvements when computing investment goods prices, we use estimates from Cummins and Violante (2002) for fixed investment in equipment and software and Gort, Greenwood, and Rupert (1999) for fixed investment in structures. Cummins and Violante (2002) find that in post-war US data, the relative price of equipment in terms of nondurable consumption goods has decreased by an average of 0.98% per quarter, while the physical depreciation rate of equipment was 2.6%. Gort, Greenwood, and Rupert (1999) find an average decrease in the relative price of structures of 0.25% per quarter, and a physical depreciation rate of around 0.48%. When combined, weighted by their nominal shares, this results in a quarterly decline rate in the relative price of aggregate investment of 0.72% and a depreciation rate for aggregate capital of 1.85%.

Parameter ξ is calibrated from $g_N = g_Z^{\overline{\frac{\alpha}{1-\alpha-\xi}}}$, using the observed $\alpha = 0.283$, $g_N = 1.0048$ and $g_Z = 1.0072$, which imply $\xi = 0.29$. The remaining three parameters $\{\theta, \sigma, \psi\}$ are calibrated jointly following the algorithm described in the Appendix. Table 3 contains a summary of the calibrated parameters.

Table 1: Parameter Values

Parameter	Value	Parameter	Value
α	.283	σ	.413
β	.99	θ	.215
ξ	.29	ψ	.554

4 Results

Differently from the exogenous growth framework in Greenwood, Hercowitz, and Krusell (1997), the economy in this paper faces endogenous growth due to selection. In case the selection mechanism were shut down, the economy would stop growing. To estimate the sole contribution of embodied technical change, we isolate the effect of selection from the aggregate externality by assuming $\xi = 0$. In this case, the growth of nondurable consumption would be reduced by about 40%, meaning that embodied technical change (as well as the selection mechanism) contributes the remaining 60%. This calculation is in accordance with the contributions estimated by Greenwood, Hercowitz, and Krusell (1997) and Luttmer (2007).

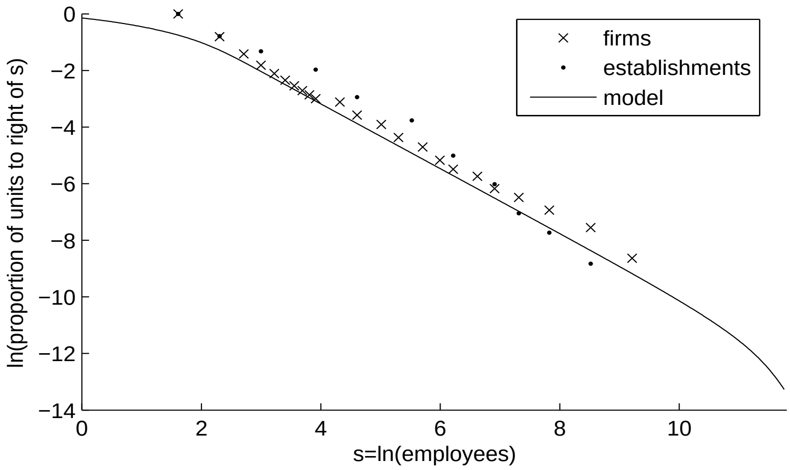
Despite within-vintage heterogeneity, the model is calibrated in such a way that aggregate productivity evolves at the same rate as in Greenwood, Hercowitz, and Krusell (1997). As an implication, both models have to generate similar contributions of embodied technical change. How is it the case that the contribution of selection is similar to the one in Luttmer (2007)? It is due to the fact that the model has predictions about firm dynamics that are very close to those in the data.

Concerning firm dynamics, the model implies an entry rate of 10.9% per year, which is slightly more than the average entry rate for firms observed for the years 1989 - 1995, which is 9.76%.[5] Figure 1 shows the observed productivity distributions of firms and establishments,[6] along with the distribution

[5] Own calculations, based on data from the OECD Firm-Level Project, which are originally from the U.S. Census Bureau's Longitudinal Business Database (LBD).

[6] The data for the distribution of firms according to the number of employees is for the year 2005 and was obtained from the Small Business Administration internet site, while the distribution of establishments is for 2004 and was obtained from the County Business Patterns database.

Figure 1: The Distribution of Production Units



implied by the model. The slope of the log right tail probability of the size distribution, with size measured by the log of employment, is -1.15. The implied model distribution is more compressed than the observed firm distribution, which has a slope of -1.06, but less compressed than the distribution of establishments, which has a slope of -1.37; the definition of production units in the data which corresponds most closely to the one in our model is that of establishments. Consequently, the model implies a rate of firm entry which is only slightly larger, as well as a distribution of production units whose tail index is only slightly smaller, than in the data.

5 Conclusion

This paper sets up a simple model of endogenous growth incorporating both embodied technical progress and a selection mechanism across heterogeneous plants. We show that under the assumption that only new plants can use the latest technology, those sources of growth are in fact one and the same.

The model is calibrated using US NIPA aggregates and the usual measure of investment-specific technical progress. Surprisingly, the model predicts very well some key moments of establishment-level data. We conclude that assuming that new technologies spread through new plants is not overly restrictive, and that the literatures on embodied technical change and on firm selection are measuring similar concepts. One should then not be surprised that they yield similar answers as to what proportion of productivity growth is due to the mechanisms they examine.

A Algorithm

1. Set α, β, δ, θ, g_Z and g_N as described in section 3. Average output per machine y is fixed to its estimated value over the balanced growth path, which is $(g_N - 1 + \delta)$ divided by the investment share.

2. Guess θ and σ.

3. Choose a vector

$$\vec{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

of productivity levels; construct the transition matrix

$$T = \begin{bmatrix} P(z_1 \mid z_1) & \cdots & P(z_n \mid z_1) \\ \vdots & \ddots & \vdots \\ P(z_1 \mid z_n) & \cdots & P(z_n \mid z_n) \end{bmatrix},$$

where $P(z_n \mid z_1)$ is the probability, given a productivity level of z_1 today, to have productivity level z_n tomorrow.

4. Make an initial guess for the value function

$$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}.$$

From this guess, iterate on $v = max\left[\alpha \cdot \vec{z} \cdot y/g_Z + \frac{\beta}{g_N} \cdot T \cdot \vec{v}, \theta \cdot g_N/\beta\right]$ until convergence. This equation is the discretized equivalent of (16).

Also, $E(\alpha \cdot z \cdot y) = \alpha \cdot z \cdot y / g_N$, since profits are linear in z and expected relative productivity growth is $1/g_Z$. So now we have the (discretized) value vector $\vec{v}$ for the values of σ and θ which we guessed. The cutoff point $z^\star$ is equal to the s^{th} element of $\vec{z}$, where s is the index of the topmost element of $\vec{v}$ which is larger than $\theta \cdot g_N/\beta$.

5. Guess ψ, and get the corresponding distribution of entrants $\vec{e}$, where

$$\vec{e} = \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}.$$

Use the discretized version of the zero profit condition in equation (18) to update ψ_e:

$$\left[\alpha \cdot \vec{z} \cdot y / g_Z + \frac{\beta}{g_N} T \cdot \vec{v}\right]' \cdot \vec{e} - g_N/\beta = 0. \tag{22}$$

If the left-hand side of (22) is greater than zero, then ψ should be revised downwards, and vice-versa.

6. Guess a distribution of machines across productivity levels

$$\vec{k} = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}.$$

The discretized equivalent of the evolution law for the relative productivity distribution in equation (19) is

$$\vec{k} = \frac{1}{g^N}\left[P \cdot \vec{k} + \left(g^N - \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \cdot P \cdot \vec{k}\right) \cdot \vec{e}\right], \tag{23}$$

where

$$P = \begin{bmatrix} 0 & \cdots & 0 & P(z_1 \mid z_s) & \cdots & P(z_1 \mid z_n) \\ 0 & \cdots & 0 & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & P(z_n \mid z_s) & \cdots & P(z_n \mid z_n) \end{bmatrix}$$

is the transpose of T, but for which all entries to the left of the cut-off point $z_s = z^\star$ have been replaced by zero (because the transition function in (19) is censored below $z^\star$). Iterate on equation (23) to get $\vec{k}$.

7. Check the normalization condition

$$\vec{z}' \cdot \vec{k} = 1 \tag{24}$$

If the left-hand side is larger than one, then revise σ upwards, and vice-versa. Restart from point 3.

8. Once equation 24 holds, we still need to match the physical depreciation rate, which in the model is equal to

$$\delta = (1-\theta)\left(1 - \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \cdot P \cdot \vec{k}\right), \tag{25}$$

One then needs to revise the guess for θ, and restart from point 3 until equation (25) holds.

References

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60(2), 323–351.

Boucekkine, R., F. del Río, and O. Licandro (2003): "Embodied Technological Change, Learning-by-doing and the Productivity Slowdown," *The Scandinavian Journal of Economics*, 105(1), 87–97.

Campbell, J. R. (1998): "Entry, Exit, Embodied Technology, and Business Cycles," *Review of Economic Dynamics*, 1(2), 371–408.

Cummins, J., and G. L. Violante (2002): "Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences," *Review of Economic Dynamics*, 5(2), 243–284.

Gomme, P., and P. Rupert (2007): "Theory, Measurement and Calibration of Macroeconomic Models," *Journal of Monetary Economics*, 54(2), 460–497.

Gordon, R. J. (1990): *The Measurement of Durable Goods Prices*. Chicago: University of Chicago Press.

Gort, M., J. Greenwood, and P. Rupert (1999): "Measuring the Rate of Technological Progress in Structures," *Review of Economic Dynamics*, 2(1), 207–230.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87(3), 342–62.

Hopenhayn, H. (1992): "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127–1150.

Hsieh, C. (2001): "Endogenous Growth and Obsolescence," *Journal of Development Economics*, 66(1), 153–171.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50(3), 649–670.

Krusell, P. (1998): "Investment-Specific R&D and the Decline in the Relative Price of Capital," *Journal of Economic Growth*, 3(2), 131–141.

Kydland, F., and E. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

Luttmer, E. G. J. (2007): "Selection, Growth, and the Size Distribution of Firms," *Quarterly Journal of Economics*, 122(3), 1103–1144.

Nelson, R. R., and S. G. Winter (1982): *An Evolutionary Theory of Economic Change*. Belknap Press of Harvard University Press, Cambridge (MA), U.S.A.

Romer, P. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98(S5), 71.

Solow, R. M. (1957): "Investment and Technical Progress," *Mathematical Methods in the Social Sciences*, pp. 89–104.