'The Child is Father of the Man:'
Implications for the Demographic Transition

David de la Croix

Department of Economics and CORE, Université catholique de Louvain

Omar Licandro

European University Institute and CEPR

March 10, 2009

Abstract

We propose a new theory of the demographic transition based on the evidence that body development during childhood is an important predictor of adult life expectancy. This theory is embodied in an OLG framework where fertility, longevity and education all result from individual decisions. The model displays different regimes, allowing the economy to move slowly from an initial Malthusian regime towards the Modern era. The dynamics reproduces the key features of the demographic transition, including the permanent increase in life expectancy, resulting from improvements in body development, the hump in both population growth and fertility, and a late increase in secondary educational attainments.

JEL Classification Numbers: J11, I12, N30, I20, J24.
Keywords: Life Expectancy, Height, Education, Fertility, Mortality.

1 Introduction

Providing children with appropriate hygienic conditions and good nutrition, as well as a good attitude towards health during childhood, are very effective ways of affording them a longer life. Starting with Kermack, McKendrick, and McKinlay (1934), who showed that the first fifteen years of life were central in determining the longevity of the adult, the relationship between early development and late mortality has been well-established. Fogel (1994) emphases that nutrition and physiological status are at the basis of the link between childhood development and longevity. Another important mechanism stressed by epidemiologists links infections and related inflammations during childhood to the appearance of specific diseases in old age (Crimmins and Finch 2006). In the same direction, Barker and Osmond (1986) relates lower childhood health status to higher incidence of heart disease in later life. It is in this sense that, following Harris (2001), we echo Wordsworth (1802)'s aphorism "The Child is Father of the Man," meaning that the way a child is brought up determines what she or he will become in the future.[1]

It is well documented that the timing of fertility, mortality and education differ during the demographic transition. We illustrate it for Sweden in Section 2. First, body development and adult life expectancy have both permanently improved from the beginning of the 19th century. Second, the time profile of net fertility rates, net of infant mortality, is hump shaped, declining from the dawn of the 20th century. As it is well known, the combination of these two forces made population growth rates also hump shaped. Finally, years of schooling that start increasing around 1850, accelerate at the beginning of the 20th century with the development of secondary education.

In this paper, we propose a new theory of the demographic transition based on the cited evidence that body development during childhood favors adult longevity. We claim the demographic transition have been shaped by a slow but permanent improvement in childhood development, promoting adult life expectancy. As pointed out by (Galor 2005a) "The reversal in the fertility patterns in England ... suggests therefore that the

---

[1]The state of the debate between nature and nurture may be synthesized by the following statement "The effects of genes depend on the environment," (Pinker 2004) where genes are associated with nature, and the environment, as a short-hand for the effects of human behavior, with nurture. In this paper, we are mainly interested in understanding how changes in human behavior, for a given distribution of genes, affect childhood development and then demographics. By restricting the analysis to a representative cohort member, a standard assumption in OLG models, we do not pay much attention to within cohort differences in genes, but concentrate on how changes in nurture over time affect childhood development on average. The implicit assumption is that the complex interaction between nature and nurture shaping the distribution of childhood development across cohort members in society has no significant effect on the mean over a period of time covering some few centuries.

demographic transition was prompted by a different universal force than the decline in infant and child mortality." In our view, contrary to reductions in *infant and child* mortality, deliberate improvements in childhood development promoting *adult* survival have prompted the demographic transition, in particular the observed fertility patterns.

When today's developed world was in the Malthusian era, most individuals were living at the subsistence level. Small changes in the environment inducing improvements in adult life expectancy slowly increased individual's lifetime surplus, allowing people to afford more and healthy descendants, which reinforced the initial effect of an improved environment on survival. As a consequence, net fertility rates were increasing as predicted by the Malthusian theory. At some point at the beginning of the 19th century, the Western world escaped the subsistence trap and the desire of having more children became dominated by the search of an individual better life. Net fertility rates remained high but stable for some decades while life expectancy kept growing. Finally, with the arrival of the 20th Century, for the first time in human history adults started finding it attractive to invest in their own human capital, which reduced the incentives to have children and, consequently, net fertility rates. More educated, richer cohorts have additional incentives to invest in childhood development, keeping life expectancy permanently growing up to our days.

The model in this paper is a continuous time overlapping generations economy as in Boucekkine, de la Croix, and Licandro (2002), where the key demographic variables, fertility, mortality and education, are all decided by individuals. Parents like to have children, but they also care about child longevity. By ensuring an appropriate physical development for their children, parents provide them with a longer life. Such provision is costly though, and its cost is increasing with the number of children. As a consequence, having many children prevents parents of spending much on their body development, which makes longevity and fertility negatively related in the modern era.[2] We assume adults decide about their own education, and we take basic education, even if provided by parents, as being exogenously given. Adults face a trade-off between having children

---

[2]We are aware that longevity does not depend solely on childhood development. Adults' investment in health and government spending on the elderly also contribute considerably to reductions in mortality. However, adding these mechanisms into our setup would not alter the trade-off we want to put forward, and, therefore, we abstract from them in order to streamline the argument. We are also aware that there are at least two different types of health capital, as pointed out by Murphy and Topel (2006). One extends life expectancy so that individuals can enjoy consumption and leisure for longer; the other increases the quality of life, raising utility from a given quantity of consumption and leisure. In this paper, since we are mainly interested in longevity, we restrict the analysis to the first type of health capital.

and improving their own education, which makes the number of children and schooling negatively related. This is similar to the trade-off faced by parents in a Beckerian world, where they care about the quantity and quality (education) of their offsprings. However, observed data on primary school education anticipate in many decades the decline in fertility predicted by the Beckerian theory, but not secondary school, as shown by Figure 1. Finally, the mechanism relating demographics and education is the Ben-Porath hypothesis that longevity positively affects education, by extending individual active life.[3]

The model dynamics displays the key features of the demographic transition, including the observed permanent rise in adult life expectancy, the hump in net fertility and the late increase in secondary educational attainments. For this reason, we claim that body development and adult life expectancy played a fundamental role explaining the demographic transformations faced by Western economies. In particular, it has promoted an initial increase in fertility, with improvements on secondary education and reductions in fertility arriving late with the modern era at the beginning of the 20th century.[4]

This article differs from the previous attempts in the literature to endogenize fertility and longevity. Many papers have the standard education/fertility trade-off with exogenous longevity, for example Doepke (2004) and Soares (2005). Other papers model health investment either by households (Chakraborty and Das (2005) and Sanso and Aisa (2006)) or by the government (Chakraborty (2004) and Aisa and Pueyo (2006)) but have exogenous fertility. A few treat both fertility and longevity as endogenous variables, but the mechanism leading to longer lives always relies on an externality: more aggregate human capital or more aggregate income leads to higher life expectancy (Blackburn and Cipriani (2002), Kalemli-Ozcan (2002), Lagerloef (2003), Cervellati and Sunde (2005) and (2007) and Hazan and Zoabi (2006)).

Two recent papers have modeled the trade-off between the number and survival of children exclusively in the context of pre-modern societies. In Galor and Moav (2005)'s paper, there is an evolutionary trade-off (i.e. not faced by individuals but by nature), between the survival to adulthood of each offspring and the number of offspring that can be supported. Lagerloef (2007) suggested that agents chose how aggressively to behave, given that less aggressive agents stand a better chance of surviving long enough to have

---

[3]Conditions for this mechanism to hold in the presence of endogenous fertility are derived in Hazan and Zoabi (2006). The main argument in our paper would also hold if, as in Hazan and Zoabi, childhood development affects education directly because healthier children perform better.

[4]The role of life expectancy on raising income well before the industrial revolution has been stressed by Boucekkine, de la Croix, and Licandro (2003).

children, but gain less resources, so more of their children die early from starvation. In both cases, the trade-off is between the number and survival of children, while in our paper it is between the number of children and adult longevity.

This paper is organized as follows. Section 2 presents some evidence on the link between the improvement in childhood development and the demographic transition. In Section 3.1, we present and solve the problem for individuals. Section 3.2 is devoted to the study of the dynamics of dynasties. The demographic transition is analyzed in Section 4. Implications for growth are studied in Section 5. Section 6 presents our conclusions.

# 2 Childhood Development and Demographic Transition

Height at age 18 is a good measure of childhood development, since both better nutrition and lower exposure to infections leads to increased height, and a good predictor of adult life expectancy.[5] According to Waaler (1984), the trend towards greater height found in the data means that younger cohorts, which have grown up under better conditions, develop more and live longer as adults. Height is also frequently used as a health indicator in microeconomic studies. Weil (2007) finds that the effect on wages of an additional centimeter of height ranges between 3.3% and 9.4%. In a second step, he exploits the correlation between height and direct measures of health such as the adult survival rate to evaluate health's role in accounting for income differences among countries; he finds that eliminating health variations would reduce world income variance by a third.

The height of conscripts has been systematically recorded by the Swedish army since 1820, which provides a unique source of time-series information on changes in body development throughout the demographic transition. Figure 1 presents data for the cohorts born between 1760 and 1960. The left panel shows that the height of soldiers (measured at approximately age 20) is highly correlated with life expectancy at age 10.[6] The right panel of Figure 1 reports net fertility rates, as well as years of schooling. We

---

[5]According to Silventoinen (2003), height is a good indicator of childhood living conditions (mostly family background), not only in developing countries but also in modern Western societies. In poor societies, the proportion of cross-sectional variation in body height explained by living conditions is larger than in developed countries, with lower heritability of height as well as larger socioeconomic differences in height.

[6]Notice that this strong correlation over time can also be established in a cross section of countries: Baten and Komlos (1998) regressed life expectancy at birth on adult height and explained 68% of the variance for a sample of 17 countries in 1860.

Figure 1: Height, Life Expectancy, Fertility and Education in Sweden
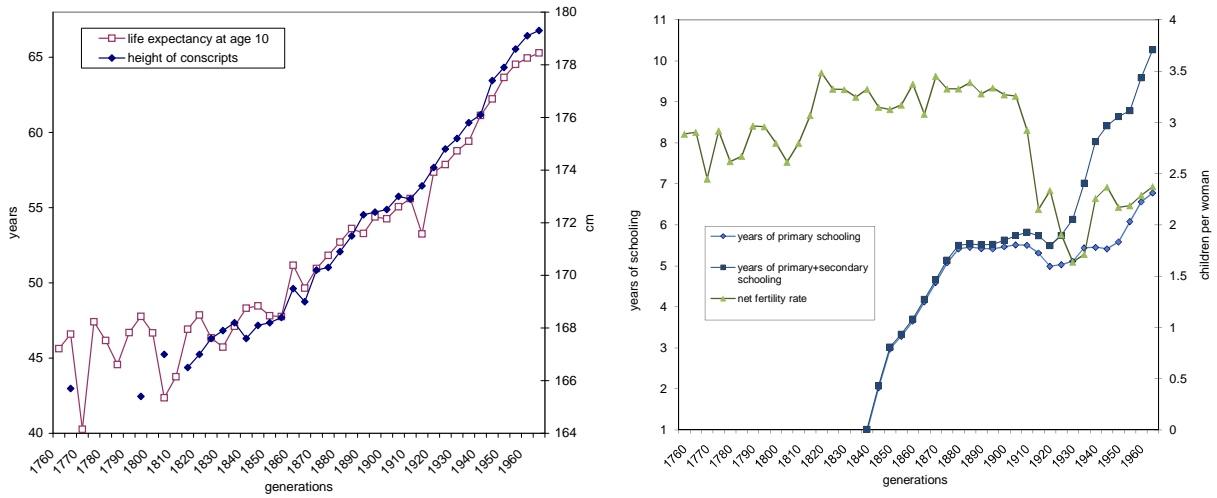
Sources: Sandberg and Steckel (1997) for height data from 1820; Floud (1984) for height data before 1820 from Denmark); The "Human Mortality Database" for life expectancy data; and de la Croix, Lindh, and Malmberg (2007) for education data. Net fertility is computed as the product of the fertility rate (Statistics Sweden) with the probability of survival until age 15 (Human Mortality Database).

Figure 2: Fertility Rates in Germany
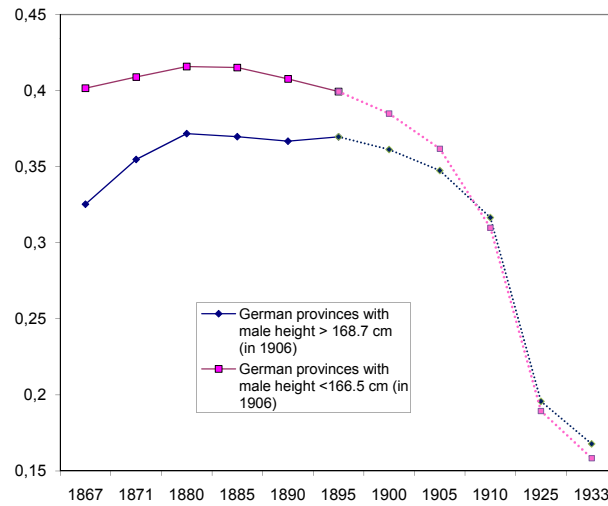
Sources: Baten (2003) for height data; Knodel (1974) for fertility data. Fertility is the ratio of total fertility rate to a benchmark.

observe that fertility raises for cohorts 1800-1830, then becomes relatively stable and reduces sharply since the beginning of the 20th century. Years of elementary schooling were growing well before fertility declined, but secondary schooling started moving up when net fertility started to decline. Permanent improvements in survival rates over the nineteenth and twentieth centuries and the hump shaped profile of fertility rates during the demographic transition are well established facts for most developed countries, as reported for example by Galor (2005a).

Further insights into the links between childhood development and fertility during the demographic transition can be gained by combining two data sets. Baten (2003) classified the former provinces of the German empire into six categories according to conscripts' height in 1906, i.e., for men born in the 1880s. In Figure 2 we retain the two extreme categories: provinces with the tallest (168.70 cm and more) and the shortest (166.50 cm and less) soldiers. The Princeton European Fertility Project provides information on fertility in these provinces for the years 1867-1933 (see Knodel (1974)). In the period 1870-1890, which is when the soldiers of 1906 were born, we can see that fertility rates were systematically higher in the provinces with shorter soldiers, which is consistent with the idea of a trade-off between the number of children and childhood development (as measured by adult height). Later on, fertility rates dropped and converged.

3   The Model

The model is a continuous-time overlapping generations economy with endogenous fertility and mortality inspired by de la Croix and Licandro (1999) and Boucekkine, de la Croix, and Licandro (2002), who modeled the link between longevity and education in a framework where all the demographic variables are exogenous. The interest of using continuous time rather than discrete is to be able to model *length* of schooling and *length* of life in a meaningful way.

There is a continuum of dynasties defined in the time domain. However, a dynasty is an ordered sequence of individuals born at different points in time. In the following, the individual problems is solved first, then the dynasty problem and finally the aggregates.

3.1 Individuals

Let us denote by B the age of puberty, i.e., the age at which individuals acquire regular fertility. B is assumed to be strictly positive and constant. Individuals reaching puberty at time t are said to belong to cohort t, whose size is denoted by $P(t)$. Life expectancy at age B is denoted by $A(t)$, which is referred to as life expectancy below. We abstract from infant mortality, and assume that the survival law is rectangular, with mortality rates equal to zero for ages below $B + A$, and individuals dying with probability one at this age. Consequently, $B + A$ is life expectancy at birth. Consistently with the thesis developed by Aries (1962) according to which people become adults much earlier in old times and start making decisions themselves, we assume that choices are made by individuals reaching puberty.[7] Preferences are represented by (we drop the cohort index to ease the exposition)

$$\int_0^A c(t) \ dt + \left(\beta \ln \hat{n} + \delta \ln \hat{A}\right). \tag{1}$$

We assume that individuals do not consume until they reach age B. $c(t)$ represents consumption at age $B+t$. Preferences in consumption are linear and the time preference parameter is assumed to be zero. Under this assumption, the equilibrium interest rate is zero and the marginal value of the intertemporal budget constraint, the associated Lagrange multiplier, is unity. In addition to their own consumption flow, individuals value the number of children, denoted by $\hat{n}$, as well as the life expectancy of their children, denoted by $\hat{A}$. Parameters β, $\beta > 0$, weight the marginal utility of the number of children relative to adult consumption. The marginal utility of the children's life expectancy is weighted by δ, $\delta > 0$.

Our assumption that utility is linear in consumption and logarithmic in children is highly consistent with the objective pursued by humans as a living specie. The first thing that matters is survival. Imposing the constraint

$$C = \int_0^A c(t) \ dt \geq 0$$

amounts to require that consumption is above the survival threshold; here zero. Once subsistence is achieved, priority moves to reproduction. This is what quasi-linear pref-

---

[7]Modeling family behavior is not a simple issue. As children grow, parents take child preferences into account more and more, but parents still have something so say as long as they support children financially until they find a job, leave home and become fully independent. Since modeling this complex process is beyond the objective of this paper, we assume that children become fully independent at one stroke.

erences deliver. Reproduction covers the two relevant dimensions in the transmission of genes: the quantity and survival of descendants.[8]

The technology producing human capital depends on the time allocated to education, denoted by T:

$$h = \mu \left(\theta + T \right)^{\alpha}.$$

The productivity parameter μ and the parameter θ, which relates to schooling before puberty, are strictly positive, and $\alpha \in (0, 1)$. θ ensures that human capital, and hence income, are positive even if individuals choose not to go to school after age B.

The budget constraint takes the form

$$\int_{0}^{A} c\left(t \right) \ \mathrm{d}t + \hat{n} \ \Psi(\hat{A}) = \mu \left(\theta + T \right)^{\alpha} \left(A - T - \phi \hat{n} \right). \tag{2}$$

The right hand side is the total flow of labor income under the assumption that one unit of human capital produces one unit of good. For simplicity, we assume that people have and raise their children immediately after finishing their studies and before becoming active in the labor market. This greatly simplifies the dynastic structure of the model. Raising a child takes a time interval of length $\phi > 0$, implying that individuals work for a period of length $A - T - \phi \hat{n}$. Parental expenditure on each child's development is

$$\Psi(\hat{A}) = \kappa \, \frac{1}{2} \frac{1}{A} \hat{A}^{2}, \tag{3}$$

which implies that the expenditure is quadratic in $\hat{A}$ and inversely related to A.[9] This formulation is consistent with the complex interaction between nurture and nature observed by biologists and psychologists. It stresses the difficulty of raising life expectancy above that of the parents, reflecting how the genes interact with human behavior (the environment) in building up a child body. The parameter $\kappa > 0$ measures the costs of developing children in a broad sense.

Figure 3 summarizes the life cycle of a representative individual of generation z, born at $z - B$, becomes independent at z, going to school until $z + T$ and entering the labor market at $z + T + \phi \hat{n}$. Her children belong to generation $z + T + B$, since T is the

---

[8]In nature, different species develop different fertility-mortality patterns. See for example Anderson et al. (2008).

[9]An example of a technology of childhood development is given by Dalgaard and Strulik (2006), where the metabolic energy to create a new cell is an exponential function of the body mass the individual wants to reach.

Figure 3: The life cycle

$z - B$	z	$z + T(z) + \phi\hat{n}(z)$	$z + A(z)$
birth	puberty	work starts	death
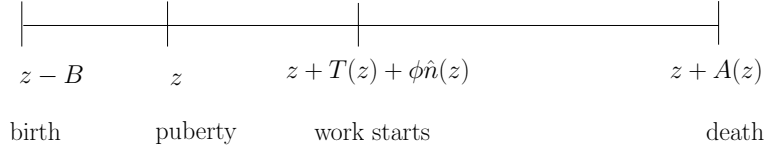
age at which individuals have children, and children reach puberty after a period of length B. Individuals chose their own education T, the number of children $\hat{n}$ and their life expectancy $\hat{A}$. Their choice depends on three types of parameters. First, those related to preferences, β and δ. Second, the parameters associated with child rearing and childhood development, ϕ, B and κ. Finally, the educational technology parameters θ, μ and α.

The maximization of utility (1), subject to the budget constraint (2) and to the positivity constraints $T \geq 0$ and $C \geq 0$, can be interior or corner. Note that the integral in Equation (2) may be substituted in Equation (1). The resulting objective function, depending on $\hat{A}$, $\hat{n}$ and T, is concave. We make the following assumption about preferences:

Assumption 1 *Preferences satisfy $\delta < 2\beta$.*

Assumption 1 states that the preference weight attached to childhood development, δ, cannot exceed twice the weight attached to the number of children, β. The trade-off between number of children and their survival depends on the ratio of marginal utilities to marginal costs, which crucially depends on the factor two because of the quadratic form of the childhood development costs. A similar condition can be found in Moav (2005) and de la Croix and Doepke (2006), when parents face the standard fertility/education trade-off.

We make the following assumptions about education technology μ:

Assumption 2 *The productivity of education technology satisfies:*

$$\mu > \max \left[\frac{(\beta - \delta/2)\alpha^2}{\theta^{1+\alpha}(1 + \alpha)}, \frac{\delta\alpha}{2\theta^{1+\alpha}} \right] \equiv \underline{\mu}.$$

This assumption requires the productivity coefficient μ to be large enough. Let us establish the main proposition on individual behavior.

Proposition 1 *Under Assumptions 1 and 2, there exist two thresholds $\underline{A}$ and $\overline{A}$, $0 <$ $\underline{A} < \overline{A}$, such that:*
If $A \geq \overline{A}$, there is a unique interior solution satisfying

$$\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A, \tag{4}$$

$$T = \frac{\alpha}{1+\alpha}(A - \phi \hat{n}) - \frac{\theta}{1+\alpha}, \tag{5}$$

$$\hat{n} = \frac{\beta - \delta/2}{\mu \phi}(\theta + T)^{-\alpha}. \tag{6}$$

If $\underline{A} \leq A < \overline{A}$, there is a unique corner solution with positive consumption satisfying

$$\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A, \tag{7}$$

$$T = 0, \tag{8}$$

$$\hat{n} = \frac{\beta - \delta/2}{\mu \phi \theta^\alpha}. \tag{9}$$

If $0 < A < \underline{A}$, there is a unique corner solution with zero consumption satisfying

$$\hat{A}^2 = \frac{\delta}{\kappa} \frac{\mu \theta^\alpha \phi}{\beta - \delta/2} A, \tag{10}$$

$$T = 0, \tag{11}$$

$$\hat{n} = \frac{\beta - \delta/2}{\beta \phi} A. \tag{12}$$

Proof. Using the Kuhn-Tucker conditions for constrained optimization, we can identify the two thresholds $\underline{A}$ and $\overline{A}$ and characterize the different regimes. See Appendix A. ∎

Restriction $A \geq \overline{A}$ in Proposition 1 states that parental life expectancy has to be large enough for schooling to be positive. At the interior solution, Equation (4) shows the trade-off faced by parents between the number and the life expectancy of their children. The relation is negative, since the total cost of providing children with a good body development increases as their number increases. Equation (5) is the standard Ben-Porath (1967) result, as described by de la Croix and Licandro (1999), where life expectancy positively affects the time allocated to education since it allows people to work for a longer time. The term $\phi \hat{n}$ in Equation (5) shows an additional trade-off of having children: parents expecting to have many children will postpone their entry into

the labor market, reducing the incentives to take additional education. This trade-off also shows up in Equation (6).

When $\underline{A} \leq A < \overline{A}$, parental life expectancy A is not long enough to render optimal a positive investment in education. For lower levels of life expectancy, i.e. when $A < \underline{A}$, both education and consumption are zero.[10] Expected life time earnings are so low that parents use all their resources in bearing a limited number of children.

From now on, the interior solution, (4)-(6), and the corner solutions, (7)-(9) and (10)-(12), are referred to as $\hat{A} = f_A(A)$, $T = f_T(A)$ and $\hat{n} = f_n(A)$. The effect of an increase in A is given by Corollary 1:

Corollary 1 $f_A'(A) > 0;$

$f_n'(A) < 0$, for $A \geq \overline{A}$, $f_n'(A) = 0$, for $\underline{A} \leq A < \overline{A}$, and $f_n'(A) > 0$ otherwise

$f_T'(A) > 0$, for $A \geq \overline{A}$, and $f_T'(A) = 0$ otherwise

Proof. See Appendix A. ∎

In the interior solution, increased life expectancy raises optimal schooling and human capital levels via the Ben-Porath effect. This increases the opportunity cost (time cost) of raising children. Hence, the optimal number of children drops as life expectancy increases.

In the corner solutions, since $T = 0$, a change in parental life expectancy does not affect education, canceling the Ben-Porath effect. In the corner regime (7)-(9) the number of children remains constant whatever the life expectancy, but childhood development is still positively affected by life expectancy, since the efficiency of body development activities depends positively on parental life expectancy. In the corner regime (10)-(12), when consumption C is zero, however, the effect of life expectancy on the number and survival of children reverses, since the number of children is directly determined by the $C = 0$ constraint, which allows for more children as life expectancy, and hence life-cycle income, increases. This is a pure income effect as in the Malthusian "positive check" hypothesis.

---

[10]Imposing a strictly positive minimum consumption level with $C \geq \bar{C}$ for the sake of realism would not change the results. The only difference is that $\underline{A}$ would be larger and increasing in $\bar{C}$.

3.2 Dynasties

Since individual decisions do not depend on aggregate variables, we can study the dynamics of life expectancy, fertility and education within dynasties separately, before the analysis of the aggregates.

Let us consider the dynamics of life expectancy first. At any point t, individuals reaching puberty belong to a representative dynasty with life expectancy $A(t)$. Let us denote as A_1 the life expectancy of the first generation of this dynasty. The operator f_A defined above consists in a difference equation governing the evolution of the dynasty's life expectancy since

$$A_{i+1} = \hat{A}_i = f_A(A_i),$$

where the index $i = 1, 2, 3, ...$, is associated with generations. From Proposition 1, for any initial value A_1 there exists a sequence of solutions $T_i = f_T^i(A_1)$, $\hat{A}_i = f_A^i(A_1)$ and $\hat{n}_i = f_n^i(A_1)$ for $i = 1, 2, 3, ...$, where X^i is the ith consecutive application of operator X. In the following, we characterize the dynamics of life expectancy. Once this has been done, the sequence $\{A_1, A_2, ...\}$ determines through the operators f_T and f_n the date and size of the following generations.

Proposition 2 *Under Assumptions 1 and 2, a stationary solution $A = f_A(A)$ exists, is unique and globally stable.*

Proof. See Appendix A. ■

Proposition 2 states that life expectancy converges to a constant value in the long run. Consequently, from Proposition 1, fertility and education also converge to a constant value. Demographic variables are then stationary, meaning that the demographic transition only occurs, as the name itself indicates, as a transitional phenomenon.

The results obtained so far allow us to assess some theoretical characteristics of the demographic transition in our model. Consider Figure 4. The lower panel plots the function $f_A(A)$. It describes a situation where the globally stable steady state is in the modern regime. The top panel shows fertility as a function of life expectancy. Suppose now that initial life expectancy is very low, below $\underline{A}$. The dynamics of life expectancy will be monotonic and converge to the steady state. A rise in life expectancy will firstly drive fertility up (as long as the economy is in the Malthusian regime $A < \underline{A}$), then fertility will peak in the zone where $\underline{A} \leq A < \overline{A}$ (i.e. where $T = 0$ but $C > 0$), and then decrease in the modern era. (Secondary) schooling will be zero until we reach the

13

Figure 4: A Steady State in the Interior Regime
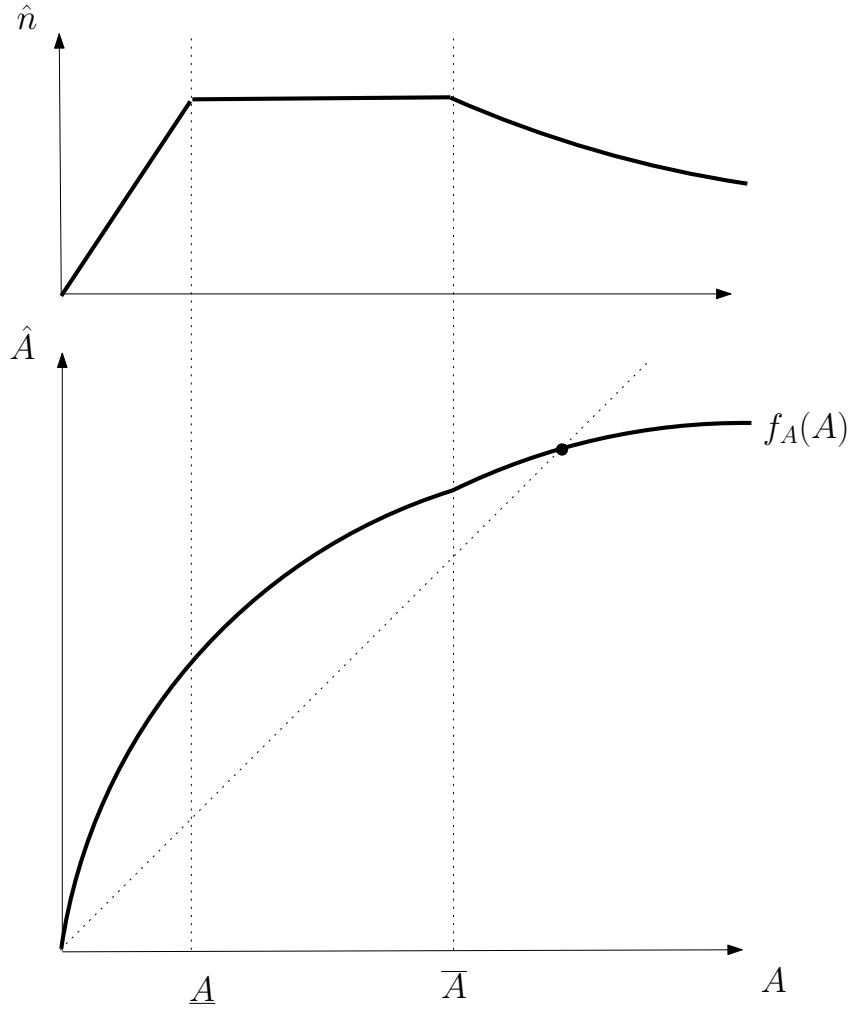
modern regime and will then increase monotonically at the time fertility is decreasing. This sharp characterization is very much in line with the stylized facts of the demographic transition as reported in Figure 1.

In this description of the theoretical dynamics, we have assumed a steady state in the interior regime. A condition for such a situation to occur is given by Remark 1.

Remark 1 *The steady state is in the modern regime if*

$$\kappa < \frac{4\alpha\delta\theta^{2\alpha}\mu^2\phi}{(2\beta - \delta)\,(\alpha[2\beta - \delta] + 2\theta^{1+\alpha}\mu)} \equiv \overline{\kappa} \tag{13}$$

Proof. See Appendix A. ∎

Condition (13) states that if the childhood development technology is cheap enough, there is an interior steady state with positive education. Changing the value of κ is a natural way of generating a demographic transition: if κ is initially high, the economy may be in one of the corner regimes. Once κ decreases, body development becomes more affordable, and the new steady state may move to the modern regime, with life expectancy converging monotonically.

4   Simulating the Demographic Transition

In this section, we investigate to what extent changes in the cost of childhood development helps reproducing the key facts of the demographic transition. The transition is studied as the reaction to a change in the environment occurring slowly from 1700 and leading the economy to a new balanced growth path in the 21st century characterized by longer lives. For this purpose, we implement a change in cost of childhood development as measured by parameter κ and analyze how the economy adjusts to this change.[11]

4.1   The Change in Parameter κ

Before analyzing the demographic transition as a response to a reduction in the cost of childhood development, we need to discuss briefly the interpretation to be given to such a change. The quantitative exercise requires, as explained in the following section, a slow and long reduction in κ starting in the 18th century and lasting to our days, with more than 80% of this reduction occurring between 1840 and 1960. Reducing the cost of body development may be due to improvements in hygienic habits, medical treatments and nutrition. Nobody would object to such progresses in the late nineteenth and twentieth century. The references below claim that improvements in these dimensions started in fact before the 19th century.

Johansson (1999) argues against the therapeutic nihilism denying that medicine had any effectiveness before the end of the nineteenth century. Firstly, in the period 1500-1800, medicine showed an increasingly experimental attitude: no improvement was effected on the grounds of the disease theory (which was still mainly based on traditional ideas),

---

[11]A more sophisticated version of the model, in line with Galor and Weil (2000), would allow the demographic transition to be associated with an endogenously generated industrial revolution. This could be achieved by letting the body development technology, as measure by the inverse of κ, depend upon population size or density, for example.

but significant advances were made based on practice and empirical observations. For example, although the theoretical understanding of how drugs work only came progressively in the nineteenth century with the development of chemistry (Weatherall 1996), the effectiveness of the treatment of some important diseases was improved thanks to the practical use of new drugs coming from the New World. Second, the number of books containing lifestyle advice increased significantly over the period 1750-1800, which provides some indirect evidence on the fact that personal and domestic cleanness, for example, became popular before the 19the century. Third, as early as 1829, Dr.F.B. Hawkins wrote a book entitled *Elements of Medical Statistics*, in which he described what could be called an early modern epidemiological transition. He describes a set of diseases which were leading causes of death but could at the time be treated effectively: leprosy, plague, sweating sickness, ague, typhus, smallpox, syphilis and scurvy. The cumulative effects of these improvements could have produced a net increase in the efficacy of medicine as early as in the eighteenth century (see de la Croix and Sommacal (2008) for further arguments).

However, medicine did not play a major role during the 19th century, as claimed by Omran (1971). "The reduction of mortality in Europe and most western countries during the nineteenth century ... was determined primarily by ecobiologic and socioeconomic factors. The influence of medical factors was largely inadvertent until the twentieth century, by which time pandemics of infection had already receded significantly." In the same direction, refereing to the first half of the 19th century, Landes (1999) argues that "Much of the increased life expectancy of these years has come from gains in prevention, cleaner living rather than better medicine." He relates it to reductions in the price of washable cotton, along with the mass production of soap. In a recent survey conducted by the British Medical Journal, medical professionals consider the sanitary revolution of the nineteen century as the major medical milestones since 1840, leading medical innovations as antibiotics, anaesthesia and vaccines.

The fundamental role of nutrition improvements on the reduction of mortality during and before the Industrial Revolution has been stressed by McKeown and Record (1962), and recently restated by Harris (2004).

4.2 The Demographic Transition

4.2.1 Calibration

Some parameters are set a priori. The age of puberty, B, is assumed equal to 13.5 (average between women, 12, and men, 15). The rearing cost per child, ϕ, is set to one year. The elasticity of human capital to schooling, α, is equal $1/6$, which is a conservative value. Finally, the length of basic skills, θ, is equivalent to six years. We next calibrate the remaining parameters to reproduce a steady state having the following properties in the pre-1700 balanced path: low life expectancy at age B ($A = 27$), no education after puberty ($T = 0$) and a population growth rate of 0.5% per year. We also set the parameters to obtain the thresholds $\underline{A} = 28$, which ensure that the economy is initially in the Malthusian regime and ends in the modern regime. Parameters μ, κ, δ and β have been computed to match the properties given above. This leads to the following results: $\mu = 0.7418$, $\kappa = 1.7954 \equiv \kappa_0$, $\delta = 53.7811$, and $\beta = 28$. For these values, the life expectancy threshold leading to the modern regime, $\overline{A}$, is 37.1095, and Assumption 1 and 2 hold.

The demographic transition is generated by a change in the cost of childhood development increasing life expectancy to 76.5 years at the new steady state (implying $B + A = 90$ in 2100 from Li and Lee (2005)), which requires $\kappa = 0.7077 \equiv \kappa_1$. This change is assumed to take place smoothly, following a logistic curve:

$$\kappa(t) = \kappa_0 + \frac{\kappa_1 - \kappa_0}{1 + e^{1890-t}/30}.$$

Under these assumptions, most of the change takes place after 1840.[12] We also assume that $\kappa(t)$ is specific to generation t. Hence any change only affects new generations, leaving past decisions unaffected.

4.2.2 Simulation

Figure 5 depicts the simulation results which are in line with the observed Swedish data as shown in Section 2. We first observe that following the drop in childhood development

---

[12]If, instead, the change in κ were discrete, we would observe intervals of times with no births, corresponding to periods where everybody increases their length of schooling in a discrete way, giving rise to permanent replacement echoes which are typical of models with delays (Boucekkine, Germain, and Licandro 1997). In this case, the economy keeps fluctuating forever, moving from baby booms to baby busts. Non-monotonic convergence also occurs in the Galor and Weil model - see Lagerloef (2006).

Figure 5: Example of dynamics - drop in κ
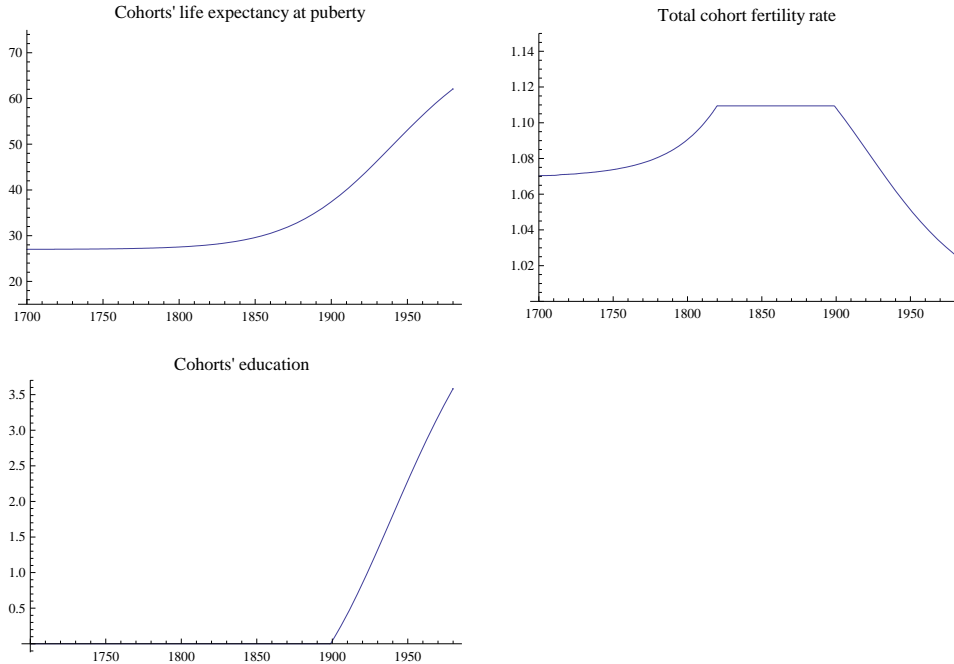
costs cohorts' life expectancy increases monotonically over time. Note that Figure 5 stops well before life expectancy has converged to steady state. Cohorts' secondary education remains nil up to the beginning of the 20th century when the economy enters the modern regime, and it increases from then monotonically. Notice that the magnitude of the increase is about right, with secondary schooling reaching 3 years around 1970. Cohorts' fertility (per individual, to be multiplied by 2 to get fertility per women) first increases as long as the economy is in the Malthusian regime, then peaks in the intermediary regime, to monotonically drop as a consequence of the trade-off between education and the number of children in the modern era.

4.3   Regional Variations

We conclude from the above simulation exercise that our model is able to reproduce the main features of the demographic transition. Another question is whether we can also shed some light on regional variations in the demographic transition. Considering the German data presented in Figure 2, we have seen that adult height (a proxy for

childhood development) and fertility were negatively associated across provinces on the eve of the twentieth century.

At that time, the prevailing regime is the one where consumption is above subsistence but it is not yet optimal to invest in (secondary) education. One reason for different places exhibiting different fertility and height levels during this phase of demographic transition is that the productivity parameter μ could vary in different places. The high μ regions will have lower fertility and taller citizens as it is clear from Equations (7) and (9). A similar argument can be made by letting the other parameters vary across regions. For example, regions with a higher θ or ϕ will also have tall parents with few children.

In Figure 2 we also observe that differences across regions seem to vanish as soon as the interior regime is reached in the beginning of the twentieth century. This convergence could reflect a reduction in the variance of the distribution of μ across regions, which would be the case for example if the education system is more and more framed by a central authority.

5   From Malthus to Modern Growth

The transition from a world of low economic growth with high mortality and high fertility to one with low mortality and fertility but sustained growth has been the subject of intensive research in recent years.[13] In this literature, the relation between growth and fertility results from the quantity/quality trade-off faced by parents between the number of children and their education. Indeed, the gradual increase in the observed level of human capital during the nineteenth century 'has led researchers to argue that the increasing role of human capital in the production process induced households to increase investment in the human capital of their offspring, ultimately leading to the onset of the demographic transition' (Galor 2005b).

If the rise in education was indeed driven by a stronger demand for skills from the industrial sector, one should have observed a rise in the skill premium during and following the industrial revolution. Looking for such evidence, Clark (2005) computes a skill premium over the period 1220-1990 in two different ways. First by measuring the relative

---

[13]Rostow (1960) presents an early attempt to understand the transition from stagnation to growth. The first modern treatment of the issue is in the seminal paper by Galor and Weil (2000). See Doepke (2006) for a recent survey.

wage of all skilled building workers [14] relative to all laborers and, second, by using only those observations in which there is a matched pair for the same place and year of wages for craftsmen and laborers. The two methods lead to the same conclusion: the skill premium did not rise during the Industrial Revolution. And Clark concludes that 'The market premium for skills, does not explain the increased investment in human skills evident after 1600.' Hence, we might wonder whether the human capital interpretations of the Industrial Revolution are based on the right trade-off. The new mechanism we develop in this paper could be seen as an alternative to the usual one.

5.1 A Growth Model

We introduce endogenous growth to the model in Section 3 by adding a human capital externality in the education technology:

$$h = \mu \left(\theta + T\right)^{\alpha} \bar{H},$$

where $\bar{H}$ is human capital per worker. It may reflect, for example, the quality of education or the cultural ambience in the society.

To keep utility balanced in a growing economy, we assume that preferences are

$$\int_0^A c\left(z\right) \, \mathrm{d}z + \bar{H} \left(\beta \ln \hat{n} + \delta \ln \hat{A}\right), \tag{14}$$

where $\bar{H}$ now multiplies the term associated with children, implying that the value of children also grows with cultural ambience in the society. Finally, for similar reasons, childhood development costs are also indexed on average human capital per worker:

$$\Psi(\hat{A}) = \kappa \, \frac{1}{2} \frac{\bar{H}}{A} \hat{A}^2. \tag{15}$$

These assumptions do not affect the household decision problem and all the results in the previous sections can be applied directly.

Figure 6: Life cycle (from left to right) and living cohorts (from right to left)
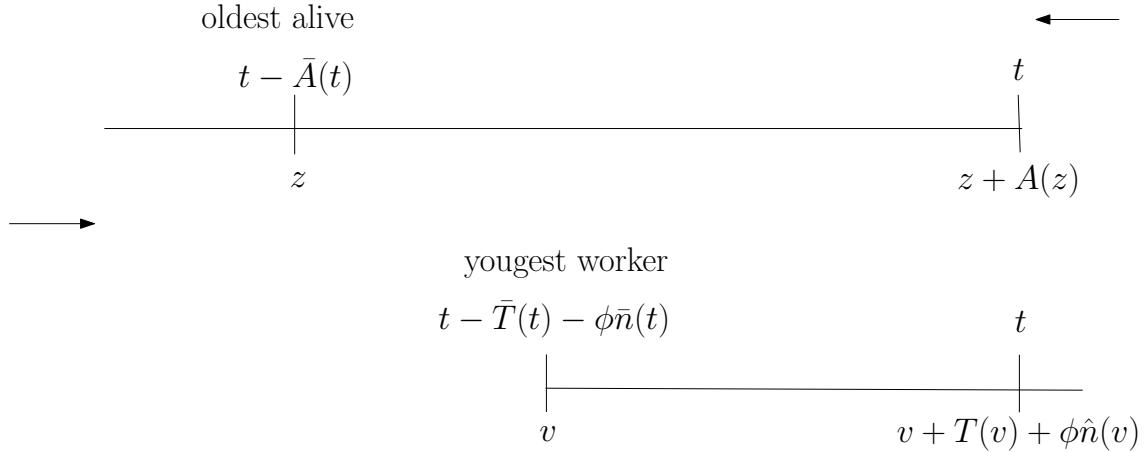
5.2 Aggregates

Some definitions are useful to study the dynamics of population growth and output growth. In Figure 6, t and z represent time and cohort, respectively. Let us define $\tilde{A}(t)$ as the age of the oldest cohort still alive at time t, which then represents the life expectancy at time t of cohort $t - \tilde{A}(t)$. By definition, $A(z)$ is the life expectancy of cohort z. Then, given that generations z and $t - \tilde{A}(t)$ are the same, $A(z)$ has to be equal to $\tilde{A}(t)$, implying that

$$\tilde{A}(t) = A\left(t - \tilde{A}(t)\right).$$

A similar argument applies to functions $T(.)$ and $\hat{n}(.)$. Let us define $\tilde{T}(t)$ and $\tilde{n}(t)$ as the schooling time and the number of children of the youngest cohort entering the labor market at time t, i.e., cohort $v = t - \tilde{T}(t) - \phi\tilde{n}(t)$ in Figure 6. Since $\tilde{T}(t) = T(v)$ and $\tilde{n}(t) = \hat{n}(v)$,

$$\tilde{T}(t) = T(t - \tilde{T}(t) - \phi\tilde{n}(t)),$$

and

$$\tilde{n}(t) = \hat{n}(t - \tilde{T}(t) - \phi\tilde{n}(t)).$$

---

[14]According to Clark, Skilled building workers typically acquired those skills by apprenticing themselves to a craftsman, with the traditional apprenticeship lasting up to seven years.

Total population is computed by integrating over all the living cohorts:

$$N(t) = \int_{t-\tilde{A}(t)}^{t+B} P(z)\ \mathrm{d}z, \tag{16}$$

from the oldest $t - \tilde{A}(t)$ to the youngest $t + B$. The cohort size $P(z)$ is given by

$$P(z + T(z) + B) = \hat{n}(z) P(z), \tag{17}$$

since members of cohort z have $\hat{n}(z)$ children at time $z + T(z)$, who belong to cohort $z + T(z) + B$.

Aggregate human capital is defined by the human capital of active cohorts

$$H(t) = \int_{t-\tilde{A}(t)}^{t-\tilde{T}(t)-\phi\tilde{n}(t)} P(z) \underbrace{\mu(\theta + T(z))^\alpha \bar{H}(z)}_{h(z)}\ \mathrm{d}z \tag{18}$$

where average human capital per worker is given by

$$\bar{H}(t) = \frac{H(t)}{E(t)},$$

and total employment $E(t)$ is

$$E(t) = \int_{t-\tilde{A}(t)}^{t-\tilde{T}(t)-\phi\tilde{n}(t)} P(z)\ \mathrm{d}z.$$

The technology producing the consumption good, the only final good in this economy, is linear in aggregate human capital with productivity one, implying that the real wage per unit of human capital is unity. Output per capita is then $H(t)/N(t)$.

5.3   Balanced Growth Path

A balanced growth path is an equilibrium path where population grows at rate η, human capital at rate γ, and, the demographic variable T, n and A are all constant. From Equation (17), the grow rate of cohorts' size is such that $e^{\eta(T+B)} = n$, i.e.

$$\eta = \frac{\ln(n)}{T + B},$$

with $P(t) = P^\star e^{\eta t}$, $P^\star > 0$. The population growth rate depends on the fertility rate n and on the age at child's birth $B + T$. At a given fertility rate, the smaller the age at birth, the larger the frequency of births and thus the population growth rate.

Total population, as defined in Equation (16), evolves along a balanced growth path following

$$N(t) = N^\star \, e^{\eta t} = P^\star \, \frac{e^{\eta B} - e^{-\eta A}}{\eta} \, e^{\eta t},$$

with $N^\star > 0$. Population also grows at rate η and its size depends positively, as expected, on life expectancy. When η approaches zero, i.e., when population is constant, its size is given by $N(t) = P^\star(B + A)$, which is the product of the cohort size and life expectancy at birth. Along a balanced growth path, the active population is given by

$$E\,(t) = E^\star \, e^{\eta t} = P^\star \, \frac{e^{-\eta(T+\phi n)} - e^{-\eta A}}{\eta} \, e^{\eta t}.$$

Similarly as for total population, when η approaches zero $E(t)$ converges to $P^\star(A - T - \phi n)$, where the term in brackets is the length of active life.

Finally, the growth rate of human capital γ satisfies at the balanced growth path

$$\gamma = \frac{P^\star}{E^\star} \, \mu(\theta + T)^\alpha \left(e^{-\gamma(T+\phi n)} - e^{-\gamma A} \right).$$

To understand this result better, let us differentiate, at the balanced growth path, the definition of $H(t)$ in Equation (18) with respect to time:

$$H'(t) = P(t - T - \phi n)h(t - T - \phi n) - P(t - A)h(t - A).$$

The change in aggregate human capital is the difference between the human capital of the youngest workers and that of the oldest. From the human capital technology, and using the balanced growth path assumption

$$\gamma = \frac{H'(t)}{H(t)} = \frac{P^\star}{E^\star} \, \mu(\theta + T)^\alpha \left(e^{-\gamma(T+\phi n)} - e^{-\gamma A} \right).$$

The first term on the r.h.s, $P^\star/E^\star$, derives directly from the assumption that per worker human capital affects the human capital of the current cohort. If, instead of normalizing total human capital by E, we normalized it by P, this term would vanish. It basically corresponds to the length of active life. The second term reflects the fact that both

Figure 7: Example of dynamics - drop in κ - human capital
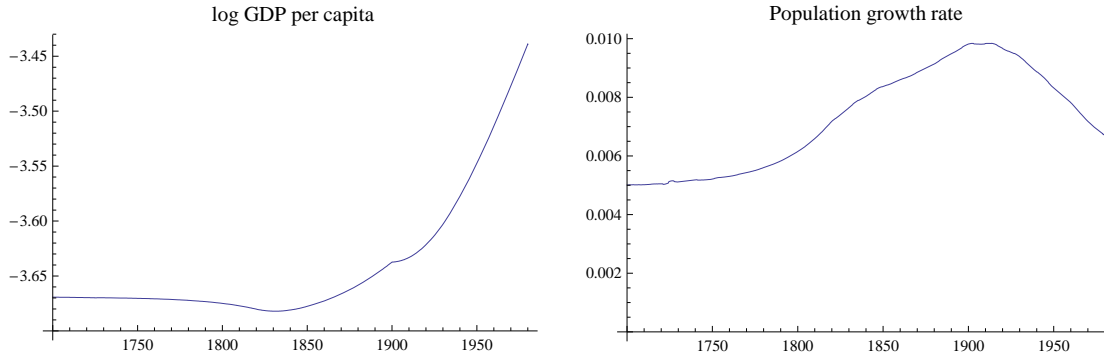
the oldest and the youngest cohort share the same human capital technology, with a common length of education. For this reason, the term $\mu(\theta + T)^\alpha$ is common. Finally, the last term in brackets reflects the fact that aggregate human capital was not the same at the time the two cohorts were at school, the difference depending on the growth rate itself and the age difference between the cohorts.

5.4 Simulating the Transition to Modern Growth

No theorem is available to assess the asymptotic behavior of the solutions of our dynamic system directly, and in particular, whether income per capita converges to its balanced growth path.[15] In the simulation below though, the solution converges asymptotically to the balanced growth path.[16]

Figure 7 illustrates the complex relationship between the demographic transition and the transition from Malthus to Modern growth. As long as the economy is in the Malthusian regime, an increase in life expectancy induces an increase of the population growth rate. Since fertility goes up, the associated raise in the dependency ratio reduces income per capita. In the intermediary regime, with consumption above subsistence but still no education, increases in life expectancy promote growth, because it raises adult longevity and the number of workers per dependent children. Finally, in the modern era, the take-off of education generates an acceleration in growth and a switch to a balanced growth

[15]No direct stability theorem is available for delay differential systems with more than one delay since the stability outcomes depend on the particular values of the delays. See Mahaffy, Joiner, and Zak (1995).

[16]The simulation was performed using the method in Boucekkine, Licandro, and Paul (1997).

24

path with positive income growth.[17] Notice that the model does not generate enough growth compared to observations, as it relies only on population and human capital as the engine of growth.

6   Conclusion

The epidemiology literature stresses that life expectancy depends greatly on body development during childhood. Both better nutrition and lower exposure to infections leads to increased body height and a longer life. We have proposed a theory of the demographic transition based on this fact. The novel mechanism of the model is that parents face a trade-off between the quantity of children they have and the amount they can afford to spend on childhood development of each of them. Parents like to have many children, but they also care about their longevity. Having many children prevents parents spending much on their body development. If its cost decreases, parents will increase their investment in their children's longevity. The number of children will first increase in the Malthusian regime as a consequence of higher lifetime income. As longevity rises, fertility starts falling as a result of the trade-off faced by parents between investing in their own human capital and spending time rearing children. Following the trade-off between the number of children and childhood development, adult longevity keeps increasing.

The model we have developed reproduces the characteristics of the demographic transition well, displaying the appealing features that longer education delays birth and reduces fertility. Our theory can be seen as an alternative to the one based on a rise in the return to human capital investment induced by economic progress, leading parents to substitute quality for quantity. A distinctive implication of our theory is that improvements in childhood development should precede the increase in education. Taking height as a proxy for childhood development, we have observed just such a pattern in Swedish historical data.

Our theory can also provide an explanation for the puzzling fact that height at age 18 is a strong predictor of education attained later in life (Magnusson, Rasmussen, and Gyllensten (2006) showed that Swedish men taller than 194 cm were two to three times more likely to obtain a higher education than men shorter than 165 cm), even

---

[17]The slowdown around 1900 is related to the fact that the first educated generations postpone their entry on the labor market.

after controlling for parental socioeconomic position, other shared family factors, and cognitive ability. A further test of our theory would consist of checking whether family size is related to childhood development as measured by average height on historical micro-data.

References

Aisa, Rosa, and Fernando Pueyo. 2006. "Government health spending and growth in a model of endogenous longevity." *Economics Letters* 90 (2): 249–253 (February).

Anderson, Holly B., Melissa Emery Thompson, Cheryl D. Knott, and Lori Perkins. 2008. "Fertility and mortality patterns of captive Bornean and Sumatran orangutans: is there a species difference in life history?" *Journal of Human Evolution* 54 (February): 34–42.

Aries, Philippe. 1962. *Centuries of Childhood.* New York: Alfred A. Knopf.

Barker, David, and Clive Osmond. 1986. "Infant mortality, childhood nutrition, and ischaemic heart disease in England and Wales." *Lancet* i:1077–1088.

Baten, Joerg. 2003. "Anthropometrics, consumption, and leisure: the standard of living." In *Germany: A New Social and Economic History, Vol III: 1800-1989*, edited by Sheilagh Ogilvie and Richard Overy. London: Edward Arnold Press.

Baten, Jorg, and John Komlos. 1998. "Review: Height and the Standard of Living." *The Journal of Economic History* 58 (3): 866–870.

Ben-Porath, Yoram. 1967. "The production of human capital and the life-cycle of earnings." *Journal of Political Economy* 75 (4): 352–365.

Blackburn, Keith, and Giam Petro Cipriani. 2002. "A model of longevity, fertility and growth." *Journal of Economic Dynamics and Control* 26 (1): 187–204.

Boucekkine, Raouf, David de la Croix, and Omar Licandro. 2002. "Vintage human capital, demographic trends and endogenous growth." *Journal of Economic Theory* 104:340–375.

———. 2003. "Early mortality declines at the dawn of modern growth." *Scandinavian Journal of Economics* 105:401–418.

Boucekkine, Raouf, Marc Germain, and Omar Licandro. 1997. "Replacement echoes in the vintage capital growth model." *Journal of Economic Theory* 74 (2): 333–348.

Boucekkine, Raouf, Omar Licandro, and Christopher Paul. 1997. "Differential-difference equations in economics: On the numerical solution of vintage capital growth models." *Journal of Economic Dynamics and Control* 21:347–362.

Cervellati, Matteo, and Uwe Sunde. 2005. "Human capital formation, life expectancy and the process of development." *American Economic Review* 95 (5): 1653–1672.

―――. 2007. "Fertility, Education, and Mortality: A Unified Theory of the Economic and Demographic Transition." mimeo, IZA, Bonn.

Chakraborty, Shankha. 2004. "Endogenous lifetime and economic growth." *Journal of Economic Theory* 116 (1): 119–137.

Chakraborty, Shankha, and Mausumi Das. 2005. "Mortality, human capital and persistent inequality." *Journal of Economic Growth* 10 (2): 159–192.

Clark, Gregory. 2005. "The Condition of the Working Class in England, 1209-2004." *Journal of Political Economy* 113 (6): 1307–1340.

Crimmins, Eileen, and Caleb Finch. 2006. "Infection, inflammation, height, and longevity." *Proceedings of the National Academy of Sciences of the United States of America* 103:498–503.

Dalgaard, Carl-Johan, and Holger Strulik. 2006, September. "Subsistence - A Bio-economic Foundation of the Malthusian Equilibrium." Discussion papers 06–17, University of Copenhagen. Department of Economics (formerly Institute of Economics).

de la Croix, David, and Matthias Doepke. 2006. "To Segregate or to Integrate: Education Politics and Democracy." Discussion paper 5799, C.E.P.R.

de la Croix, David, and Omar Licandro. 1999. "Life expectancy and endogenous growth." *Economics Letters* 65 (2): 255–263.

de la Croix, David, Thomas Lindh, and Bo Malmberg. 2007. "Economic Growth and Education Since 1800: How Much does Longer Life Explain?" unpublished.

de la Croix, David, and Alessandro Sommacal. 2008. "A Theory of Medical Effectiveness, Differential Mortality, Income Inequality and Growth for Pre-Industrial England." *Mathematical Population Studies*, p. forthcoming.

Doepke, Matthias. 2004. "Accounting for Fertility Decline During the Transition to Growth." *Journal of Economic Growth* 9 (3): 347–383.

———. 2006. "Growth Takeoffs." Working paper 847, UCLA Department of Economics. forthcoming in the New Palgrave Dictionary of Economics, 2nd Edition.

Floud, Roderick. 1984, April. "The Heights of Europeans Since 1750: A New Source For European Economic History." Working paper 1318, National Bureau of Economic Research, Inc.

Fogel, Robert. 1994. "Economic growth, population theory and physiology: the bearing of long-term processes on the making of economic policy." *American Economic Review* 84 (3): 369–395.

Galor, Oded. 2005a. "The Demographic Transition and the Emergence of Sustained Economic Growth." *Journal of the European Economic Association* 3 (23): 494–504.

———. 2005b. "From Stagnation to Growth: Unified Growth Theory." Chapter 4 of *Handbook of Economic Growth*, edited by Philippe Aghion and Steven Durlauf, Volume 1 of *Handbook of Economic Growth*, 171–293. Elsevier.

Galor, Oded, and Omer Moav. 2005. "Natural Selection and the Evolution of Life Expectancy." Discussion papers 5373, C.E.P.R.

Galor, Oded, and David Weil. 2000. "Population, technology, and growth: from the Malthusian regime to the demographic transition and beyond." *American Economic Review* 90 (4): 806–828.

Harris, Bernard. 2001. "'The child is father of man.' The relationship between child health and adult mortality in the 19th and 20th centuries." *International Journal of Epidemiology* 30:688–696.

———. 2004. "Public health, nutrition and the decline of mortality: The McKeown thesis revised." *Social History of Medicine* 17:379–407.

Hazan, Moshe, and Hosny Zoabi. 2006. "Does longevity cause growth? A theoretical critique." *Journal of Economic Growth* 11:363–376.

Johansson, Ryan. 1999. "Death and the doctors: medicine and elite mortality in Britain from 1500 to 1800." *Cambridge Group for the History of Population and Social Structure Working Paper Series.* No. 7.

Kalemli-Ozcan, Sebnem. 2002. "Does the Mortality Decline Promote Economic Growth?" *Journal of Economic Growth* 7:411–439.

Kermack, William, Anderson McKendrick, and P McKinlay. 1934. "Death Rates in Great Britain and Sweden: some general regularities and their significance." *Lancet* i:698–703.

Knodel, John. 1974. *The decline of fertility in Germany, 1871-1939.* Princeton: Princeton University Press.

Lagerloef, Nils-Petter. 2003. "From Malthus to modern growth: can epidemics explain the three regimes ?" *International Economic Review* 44 (2): 755–777.

———. 2006. "The Galor-Weil Model Revisited: A Quantitative Exercise." *Review of Economic Dynamics* 9:116–142.

———. 2007. "Violence." mimeo, York University, Canada.

Landes, David. 1999. *The Wealth and Poverty of Nations.* New York: W.W. Norton & Company.

Li, Nan, and Ronald Lee. 2005. "Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method." *Demography* 42 (3): 575594.

Magnusson, Patrik, Finn Rasmussen, and Ulf Gyllensten. 2006. "Height at age 18 years is a strong predictor of attained education later in life: cohort study of over 950,000 Swedish men." *International Journal of Epidemiology* 35 (3): 658–663.

Mahaffy, Joseph, Kathryn Joiner, and Paul Zak. 1995. "A Geometric Analysis of Stability Regions for a Linear Differential Equation with two delays." *International Journal of Bifurcation and Chaos* 5 (3): 779–796.

McKeown, Thomas, and R. G. Record. 1962. "Reasons for the Decline of Mortality in England and Wales during the Nineteenth Century." *Population Studies* 16 (3): 94–122.

Moav, Omer. 2005. "Cheap Children and the Persistence of Poverty." *Economic Journal* 115 (500): 88–110.

Murphy, Kevin, and Robert Topel. 2006. "The Value of Health and Longevity." *Journal of Political Economy* 114:871–904.

Omran, Abdel. 1971. "The Epidemiologic Transition: A Theory of the Epidemiology of Population Change." *Milbank Memorial Fund Quarterly* 49:509–538.

Pinker, Steven. 2004. "Why nature & nurture won't go away." *Daedalus* 133 (4): 5–17.

Rostow, Walt. 1960. *The Stages of Economic Growth.* New York: Cambridge University Press.

Sandberg, Lars, and Richard Steckel. 1997. "Was Industrialization Hazardous to your Health? Not in Sweden." In *Health and Welfare during Industrialization*, edited by Richard Steckel and Roderick Floud. University of Chicago Press.

Sanso, Marcos, and Rosa M. Aisa. 2006. "Endogenous longevity, biological deterioration and economic growth." *Journal of Health Economics* 25 (3): 555–578.

Silventoinen, Karri. 2003. "Determinants of Variation in Adult Body Height." *Journal of Biosocial Science* 35:263–285.

Soares, Rodrigo. 2005. "Mortality Reductions, Educational Attainment, and Fertility Choice." *American Economic Review* 95(3):580–601.

Waaler, Hans. 1984. "Height, weight and mortality: the Norwegian experience." *Acta medica Scandinavica. Supplementum* 679:1–56.

Weatherall, Miles. 1996. "Drug treatment and the rise of pharmacology." In *Cambridge Illustrated History: Medicine*. Cambridge: Cambridge University Press.

Weil, David. 2007. "Accounting for the Effect of Health on Economic Growth." *Quarterly Journal of Economics* 122:1265–1306.

Wordsworth, William. 1802. My heart leaps up when I behold. Poem.

A   Proofs of Propositions

Proof of Proposition 1

After substituting the integral in (2) into (1), the objective becomes

$$\left(\beta \ln \hat{n} + \delta \ln \hat{A}\right) + \underbrace{\mu \left(\theta + T\right)^{\alpha} \left(A - T - \phi \hat{n}\right) - \hat{n} \left(\frac{\kappa}{2} \frac{\hat{A}^2}{A}\right)}_{C}$$

which is maximized under the restrictions $T \geq 0$ and $C \geq 0$.

First order conditions to this problem are (omitting the Kuhn-Tucker conditions):

$$(1 + \eta)\hat{A}^2 = \frac{\delta}{\kappa \hat{n}} A \tag{A.1}$$

$$(1 + \eta)\alpha\mu \left(\theta + T\right)^{\alpha - 1} \left(A - T - \phi \hat{n}\right) = (1 + \eta)\mu \left(\theta + T\right)^{\alpha} - \lambda \tag{A.2}$$

$$\frac{1}{\hat{n}} \left(\beta - \frac{\delta}{2}\right) = (1 + \eta)\mu \left(\theta + T\right)^{\alpha} \phi \tag{A.3}$$

where λ and η are the Kuhn-Tucker multipliers associated with the constraints $T \geq 0$ and $C \geq 0$, respectively. The interior solution (4)-(6) is (A.1)-(A.3) under $\eta = \lambda = 0$. The corner solution (7)-(9) results from the same system under $\eta = T = 0$, and finally, the corner solution (10)-(12) results from the first order conditions under $T = C = 0$. Under Assumption 2, $\eta = C = 0$ is not optimal.

Interior Regime. The solution to the first order conditions (4)-(6) exists and is unique iff the loci in (5) and (6) cut once and only once for positive n and T, and $C \geq 0$ at the solution. The locus in (5) is a straight line with negative slope and cuts the $\hat{n}$ axes at $\frac{A - \theta/\alpha}{\phi} \equiv n_0$, see Figure A.1. The locus in (6) has a negative slope, is convex, and is such that $\hat{n}$ goes to zero when T goes to infinity and cuts the $\hat{n}$ axes at $\frac{\beta - \delta/2}{\mu \phi \theta^{\alpha}} \equiv n_1$. Comparing these two points and imposing $n_0 \geq n_1$ leads to the condition $A \geq \overline{A}$, where

$$\overline{A} = \frac{\beta - \frac{\delta}{2}}{\mu \theta^{\alpha}} + \frac{\theta}{\alpha}.$$

Substituting (4) and (5) in the definition of C gives

$$C = \frac{\mu}{\alpha}(\theta + T)^{1+\alpha} - \frac{\delta}{2},$$

Figure A.1: The interior solution
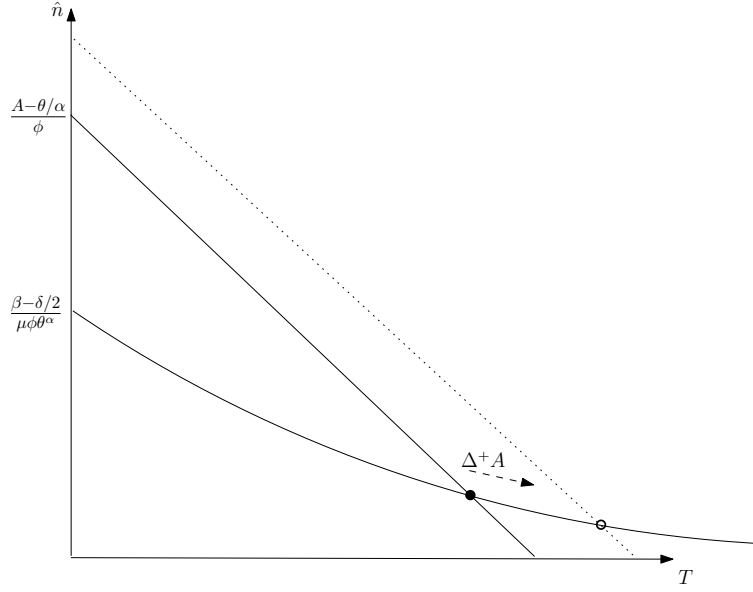
which is positive under Assumption 2 for all $T \geq 0$.

Corner regime $\underline{A} \leq A < \overline{A}$. If $A < \overline{A}$, the straight line is above the convex curve at $T = 0$ (see Figure A.1). A sufficient condition for these two curves not to intersect in the positive plane is that the straight line is steeper than the convex curve at zero. This is guaranteed by Assumption 2. In that case, there is no interior solution, since negative values for T are not feasible. Consequently, the solution must be corner with $T = 0$. From equations (7)-(9), at this corner solution

$$C = \mu \theta^\alpha \left(A - \frac{\beta - \delta/2}{\mu \theta^\alpha} \right) - \delta/2,$$

which is positive for $\underline{A} \leq A < \overline{A}$,with

$$\underline{A} \equiv \frac{\beta}{\mu \theta^\alpha}.$$

From Assumption 2, $\underline{A} < \overline{A}$. It is easy to see that the solution is unique.

Corner regime $0 < A < \underline{A}$. Finally, when $0 < A < \underline{A}$, the optimal solution is (10)-(12), with both inequality constraints being binding. Uniqueness is trivial.

32

Proof of Corollary 1

For the interior solution, we apply the implicit function theorem to (4)-(6), which leads to

$$f'_A = \mathrm{d}\hat{A}/\mathrm{d}A = \sqrt{\frac{A\delta}{\hat{n}\kappa}}\,\frac{(T(\alpha+1)+\theta+\alpha((A-\hat{n}\phi)\alpha+\theta))}{2A((\alpha+1)(T+\theta)-\hat{n}\alpha^2\phi)}.$$

The numerator is positive. Under Assumption 2, the denominator is also positive. The results for f'_n and f'_T can be proved using the same arguments. For the corner solutions, the result is straightforward.

Proof of Proposition 2

Let us denote the function $f_A(.)$ by $f_{A1}(.)$ when $A \geq \overline{A}$, $f_{A2}(.)$ when $\underline{A} \leq A < \overline{A}$, and $f_{A3}(.)$ when $0 < A < \underline{A}$. The dynamics of life expectancy following $A_{i+1} = f_A(A_i)$ are monotonic because f_A is continuous and non-decreasing.

Let us first prove the existence of a solution. From corollary 1, $f'_A(A) > 0$. It is easy to see that $\lim_{A\to 0} f_A(A) = \lim_{A\to 0} f_{A3}(A) > 0$. To prove the existence it is enough to show

$$\lim_{A\to\infty} \frac{f_A(A)}{A} = \lim_{A\to\infty} \frac{f_{A1}(A)}{A} = 0.$$

From (5) and (6)

$$\hat{n} = \mathrm{cste}(A - \phi\hat{n})^{-\alpha}.$$

Substituting in (4), and dividing by A^2 gives

$$\left(\frac{\hat{A}}{A}\right)^2 = \mathrm{cste}\frac{(A-\phi\hat{n})^\alpha}{A}.$$

Since $\lim_{A\to\infty} f_n(A) = 0$, it's now easy to see that $\lim_{A\to\infty} \frac{f_{A1}(A)}{A} = 0$.

Let us now prove its unicity. For $0 < A < \underline{A}$, the function $f_{A3}(.)$ is increasing and concave, with $f_{A3}(0) = 0$ and $f'_{A3}(0) = \infty$, implying that if it crosses the diagonal on the interval $(0, \underline{A})$, it crosses it only once.

Function $f_{A2}(.)$ is increasing and concave, with $f_{A2}(0) = 0$ and $f'_{A2}(0) = \infty$, implying that if it crosses the diagonal on the interval $[\underline{A}, \overline{A})$, it crosses it only once.

33

Finally, let us prove that $f'_{A1}(A) < 1$ for any fixed point of $f_{A1}(.)$ in $A \geq \overline{A}$. From the implicit function theorem applied to (4)-(6),

$$\frac{\mathrm{d}\hat{A}}{\hat{A}} \Big/ \frac{\mathrm{d}A}{A} = \frac{1}{2}\frac{(1+\alpha)(A - \phi\hat{n})}{A - (1+\alpha)\hat{n}}.$$

At a fixed point of f_{A1}, since Corollary 1 shows that $f'_A(A) > 0$ in this interval, the denominator must be strictly positive. It is then easy to see that $f'_A(A) < 1$ iff $A > \frac{1+\alpha}{1-\alpha}\phi\hat{n}$. Since, from Corollary 1, $f'_n(.) < 0$ in this interval, $f'_A(A) < 1$ for all $A \geq \overline{A}$ iff $\overline{A} > \frac{1+\alpha}{1-\alpha}\phi\hat{n}$, which holds under Assumption 2.

Global stability is then trivial, since f_A is above the diagonal before the unique steady state equilibrium and below it afterwards.

Proof of Remark 1

A steady state for A in the interior regime exists iff there is a solution to the system (4)-(6) evaluated at the steady state. Eliminating A and n from Equation (5) using Equations (4) and (6) we find that the steady state T should satisfy:

$$T(1+\alpha) + \theta = \alpha \left(\frac{2\delta\phi\mu(T+\theta)^\alpha}{\kappa(2\beta - \delta)} - \frac{(2\beta - \delta)(T+\theta)^{-\alpha}}{2\mu} \right)$$

The left hand side is a linear increasing function of T. The right hand side is a concave function of T, with a slope going to zero as T goes to infinity. A sufficient condition for existence and uniqueness of a stationary solution is that the right hand side is larger than the left hand side at $T = 0$. This leads to Condition (13).

34