

Centre de Referència en Economia Analítica

Barcelona Economics Working Paper Series

Working Paper nº 352

The Effect of Aspirations, Habits, and Social Security on the Distribution of Wealth

Jordi Caballé and Ana I. Moro-Egido

June 18, 2008

The Effect of Aspirations, Habits, and Social Security on the Distribution of Wealth*

Jordi Caballé

Unitat de Fonaments de l'Anàlisi Econòmica and CODE
Universitat Autònoma de Barcelona

Ana I. Moro-Egido

Departamento de Teoría e Historia Económica
Universidad de Granada

June 18, 2008

Abstract

In this paper, we analyze how the introduction of habits and aspirations affects the distribution of wealth when individuals' labor productivity is subject to idiosyncratic shocks and bequests arise from a joy-of-giving motive. In the presence of either bequests or aspirations, labor income shocks are transmitted intergenerationally and this transmission, together with the contemporaneous income shocks, determines the stationary distribution of wealth. We show that the introduction of aspirations increases both the intragenerational variability of wealth and the corresponding degree of intergenerational mobility. The opposite result holds when habits are introduced. Finally, we discuss how aspirations and habits interact with the redistributive features of an unfunded social security system.

JEL classification codes: D31, E21, E62.

Keywords: Aspirations, Habits, Wealth Distribution, Social Security.

* The financial support from the Fundación BBVA to Jordi Caballé and from the Spanish Ministry of Education through grant SEJ2006-11067 and the Junta de Andalucía through grant P07-SEJ-03261 to Ana I. Moro-Egido is gratefully acknowledged.

Correspondence address: Jordi Caballé. Universitat Autònoma de Barcelona. Departament d'Economia i d'Història Econòmica. Edifici B. 08193 Bellaterra (Barcelona). Spain.

Phone: (34)-935.812.367. *Fax:* (34)-935.812.012. *E-mail:* Jordi.Caballe@uab.es

1. Introduction

In this paper, we analyze how the introduction of aspirations and habits affects the distribution of wealth. When aspirations are present the utility of individuals depends on the consumption experience of their parents, while under habits the utility associated with a given amount of current consumption depends on the past experience of consumption of the individual under consideration. In both cases, past consumption is used as a reference with respect to which current consumption is compared and this implies that preferences turn out to be time non-separable.

A large number of empirical studies provide evidence about the effect of the level of past consumption on the satisfaction derived from current consumption. According to this evidence, some authors have used preferences displaying habit formation to improve the predictions made under time separable preferences in different economic scenarios.¹ Moreover, there is also empirical evidence about the existence of aspirations associated with the involuntary transmission of tastes from one generation to the next. For instance, Cox et al. (2004) estimate that parental preferences explain between 5 to 10 percent of the preferences of their children after controlling for their respective incomes.²

Our analysis will be conducted in the framework of an overlapping generations (OLG) economy where preferences of individuals display "joy of giving". This means that individuals' utility will be an increasing function of the amount of bequest left to their children, like in Yaari (1965) and Abel (1986). Several alternative motives leading to intergenerational transfers have been proposed in the literature. Among them, and besides to joy of giving, we could mention strategic behavior (Bernheim et al., 1985), existence of incomplete annuity markets (Abel, 1985), and pure intergenerational altruism (Barro, 1974). However, the empirical evidence is not conclusive about the reasons why individuals make intergenerational transfers and probably a combination of all those motives lies in the core of the mechanism governing the intergenerational transmission of wealth.

When individuals care about the total income of their children, bequests play an equalizing role since then individuals tend to compensate the differences in the random income of their direct descendants. This compensation principle has been used to argue against taxes on inheritances since they could have a disequalizing effect due to the distortion of the optimal risk sharing between two consecutive generations (Becker and Tomes, 1979; and Davies, 1986). In our framework with joy-of-giving preferences, this compensation principle does not come into play since individuals do not seek an optimal allocation of family income between them and their children but an optimal allocation of individual income between own consumption and bequests. Kleiber et al. (2006) have shown that under joy-of-giving preferences, the introduction of bequests results in a reduction of the value of the coefficient of intragenerational variation of

¹See, among others, Abel (1990, 1999), Ljungqvist and Uhlig (2000), Carroll et al. (1997, 2000) and Alonso-Carrera et al. (2004, 2005).

²Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) provide surveys about the evidence on intergenerational transmission of tastes. Among the theoretical studies on the macroeconomic implications of aspirations, we could mention those of de la Croix (1996, 2001), de la Croix and Michel (1999, 2001), and Alonso-Carrera et. al (2007).

wealth. This is so because the average stock of capital becomes larger due to the increase of saving induced by the bequest motive, which offsets the modest increase in the variance of wealth associated with the intergenerational transmission of income shocks through bequests.

Our framework will be also suitable for the study of intergenerational mobility, which is characterized by the correlation between the wealth of parents and their children. Obviously, the introduction of bequests has a negative effect on mobility since they allow the intergenerational transmission of the wealth status.

We will show that habits and aspirations affect both the size of aggregate bequests and the level of the capital stock installed in the economy in a direction similar to the one obtained by de la Croix and Michel (2001), Jellal and Wolff (2002) and Alonso-Carrera et al. (2007), who conducted the analysis under the assumption of altruistic preferences à la Barro (1974). Using the coefficient of variation of wealth as a measure of intragenerational wealth inequality, we also show that the introduction of aspirations increases the intragenerational inequality of wealth since aspirations make the shocks in labor income more persistent. Aspirations make an individual's adult consumption (and his saving) more dependent on his ancestors' income shocks, which results in a transmission of those shocks within the same dynasty. However, the introduction of habits decreases the intragenerational inequality of wealth when aspirations are present. This is so because habits tend to smooth consumption along the individuals' lifetime. Therefore, the old consumption (and saving) of an individual becomes less dependent on the shocks suffered by his parents.

We also evaluate the effects of habits and aspirations on intergenerational mobility. We measure this mobility by the autocorrelation coefficient of asset holdings within a family. We show that, due to the induced reduction in the amount of bequests, aspirations tend to enhance intergenerational mobility. However, habits make saving more correlated with contemporaneous wages and this translates in turn in a larger dependence of inheritances on wages. The final result is that habits reduce the stationary wealth mobility of the economy.

It is also natural to perform some evaluation of the effects of social security in the economy under consideration. Note that social security is a compulsory intergenerational redistributive mechanism that works in a direction opposite to bequests as it forces a transmission of income from young to old individuals. Abel (1985) analyzed the impact of a funded social security system in a setup where the inequality in the distribution of wealth appears as a result of random mortality so that the different mortality histories of families give raise to a non-degenerate distribution of accidental bequests. Cremer and Pestieau (1988) conducted a similar analysis in the presence of fertility shocks. Finally, Caballé and Fuster (2003) analyzed the effects of an unfunded social security system under random altruism where the amount of inheritance received by each individual depends on the realization of an idiosyncratic shock on the altruism factor of parents.

The standard result that social security reduces the aggregate stock of capital also holds in our model. However, we will analyze whether this negative effect becomes stronger or weaker in the presence of habits and aspirations. Our results show that these two phenomena work in opposite directions concerning the efficacy of social security and, moreover, the sign of the effect depends on the dynamic efficiency properties

of the economy. We perform a similar comparative statics analysis concerning the effects of social security on the coefficient of intragenerational variation of wealth to conclude that, while social security has an equalizing effect, this effect becomes smaller when preferences display either aspirations or habits. Finally, we also analyze the interactions among social security, habits and aspirations concerning their implications for intergenerational mobility in wealth.

Our results in a OLG economy with preferences displaying ‘joy of giving’ differ in many respects with the ones of the related paper by Díaz et al. (2003), who considered an economy with infinitely lived agents. In order to make a proper comparison, we should consider the version of their model where the elasticity of intertemporal substitution is not adjusted when habits are introduced. First, our model allows us to introduce the phenomenon of intergenerational transmission of tastes, which cannot be accommodated in non-OLG economies. Furthermore, our economic environment is well suited to conduct an analysis of the effects of social security. Finally, our simple model allows us to obtain closed form expression for the comparative statics exercises when aspirations and habits are marginally introduced. Concerning the results, while Díaz et al. do not obtain a definite sign for the change in aggregate saving when habits are introduced, our life-cycle specification allows to obtain an unambiguous increase in aggregate saving due to the induced shift of income from the adult to the old period of life. Finally, our demographic structure allows a sharp characterization of the effects of habits on intergenerational mobility within a family.

The paper is organized as follows. Section 2 presents the general model with habits, aspirations, and pay-as-you-go social security. Section 3 analyzes some dynamic stability issues of the intragenerational distribution of wealth. In Section 4, we characterize the measure of intergenerational mobility in wealth. In Section 5, we conduct the comparative statics analysis of changes in the intensity of habits and aspirations on the stationary intragenerational distribution of wealth and on intergenerational mobility. In Section 6 we analyze how the size of social security affects the intragenerational distribution of wealth and the stationary mobility under time non-separable preferences. We conclude the paper in Section 7.

2. The Model

Let us consider an OLG economy where a continuum of individuals live for three periods and a new generation is born in each period. Each individual has offspring in the second period of his life and the exogenous number of children per parent is $n \geq 1$. Agents make economic decisions during the last two periods of their lives only.

There is a single commodity, which can be devoted to either consumption or investment. Each agent inelastically supplies one unit of labor in the second period of his life and is retired in the third period. Let us index each generation by the period in which its members work. Therefore, if the size of the generation t is N_t , then $N_{t+1} = nN_t$ for all t . There is also a social security system that collects payroll taxes from the workers (adult individuals) and distributes a benefit to the retired (old) individuals in a pay-as-you-go (PAYG) basis.

An adult individual $i \in [0, N_t]$ of generation t distributes his net labor income and his inheritance between consumption and saving. The budget constraint faced by the

worker i in period t is

$$(1 - \tau) w_t^i + b_t^i = c_t^i + s_t^i, \quad (2.1)$$

where w_t^i is the wage compensation of this worker, c_t^i is the amount of his consumption (adult consumption, henceforth), b_t^i is the amount of inheritance he has received from his parent, s_t^i is the amount of saving and $\tau \in [0, 1]$ is the social security tax rate.

When individuals are old, they receive a social security benefit and a return from their savings, which are distributed between consumption and bequests for their children. Therefore, the budget constraint of an old individual i belonging to generation t will be

$$R_{t+1}s_t^i + p_{t+1}^i = x_{t+1}^i + nb_{t+1}^i, \quad (2.2)$$

where R_{t+1} is the gross rate of return on saving, p_{t+1}^i is the social security benefit he will receive when old, b_{t+1}^i is the amount of bequests he leaves to each of his descendants (who were born in period t), and x_{t+1}^i is the amount of consumption of the old individual i in period $t+1$ (old consumption, henceforth).

We will assume that in each period individuals derive utility from the comparison of their consumption with a consumption reference. Recall that during their first period of life individuals neither work nor consume. However, as in de la Croix (1996), the member i of the generation born in period $t-1$ inherits a certain level of aspirations a_t^i in period t . These aspirations are based on the standard of living achieved by their parents. More precisely, we assume that the inherited aspiration of an individual i of generation t is

$$a_t^i = c_{t-1}^i, \quad (2.3)$$

where c_{t-1}^i is his parent's amount of consumption when the parent was adult (second period of life). We posit the following additive specification for the aspiration adjusted consumption \hat{c}_t^i of an adult individual i belonging to generation t :

$$\hat{c}_t^i = c_t^i - \delta a_t^i. \quad (2.4)$$

This additive formulation for aspiration adjusted consumption makes it possible to preserve the concavity of the objective function with respect to the consumption vector.

Preferences will also exhibit habit formation and, hence, the consumption reference of an old individual i of generation t is given by the consumption level he has reached in the previous period. As we have done for aspirations, we assume that the habit adjusted consumption \hat{x}_{t+1}^i of an old individual i in period $t+1$ is given by the additive function

$$\hat{x}_{t+1}^i = x_{t+1}^i - \gamma c_t^i. \quad (2.5)$$

For tractability we posit the following Cobb-Douglas utility function representing the preferences of the individual i belonging to generation t :

$$U(\hat{c}_t^i, \hat{x}_{t+1}^i, b_{t+1}^i) = \alpha \ln(\hat{c}_t^i) + \beta \ln(\hat{x}_{t+1}^i) + \rho \ln(b_{t+1}^i), \quad (2.6)$$

where α , β and ρ are strictly positive and, without loss of generality, we assume that $\alpha + \beta + \rho = 1$. Note that we are generating positive bequests through a "joy-of-giving" motivation (like in Yaari, 1965; or Abel, 1986) so that the amount of bequests enters directly as an argument in the utility function.

We assume that the good of this economy is produced by means of the aggregate linear production function $F(K_t, L_t) = L_t + rK_t$, where L_t is the amount of efficient units of labor used in period t and K_t is the amount of installed capital. Labor and capital markets are competitive so that the marginal productivity of an efficient unit of labor is equal to one and then the wage w_t^i can be interpreted as the realization of the random number of efficiency units of labor owned by worker i in period t . We assume that the number of efficient units of labor, and thus the wage w_t^i received by the worker i of generation t , is the realization of a random variable that is identically and independently distributed (i.i.d.) with mean w and variance V , for all i and t . Therefore, we are assuming that all workers experience idiosyncratic productivity shocks that are cross-sectionally and intergenerationally independent. These shocks on labor income are assumed to be uninsurable. The equilibrium rental price of one unit of capital is r so that the gross rate of return on savings satisfies $R_{t+1} = 1 + r \equiv R$.³

We will assume that the social security benefit received by an old agent is proportional to his contribution when he was a worker,

$$p_{t+1}^i = \pi \tau w_t^i, \quad (2.7)$$

where π is a positive constant. Moreover, the budget constraint of the PAYG social security system is

$$\int_{[0, N_t]} p_{t+1}^i di = \int_{[0, N_{t+1}]} \tau w_{t+1}^j dj, \quad \text{for all } t,$$

which using (2.7) becomes

$$\int_{[0, N_t]} \pi \tau w_t^i di = \int_{[0, N_{t+1}]} \tau w_{t+1}^j dj.$$

Applying the law of large numbers for a continuum of i.i.d random variables, the previous equation simplifies to $\pi \tau w = n \tau w$ so that $\pi = n$. Therefore, we see from (2.7) that the balanced budget constraint of the social security system can be written as

$$p_{t+1}^i = n \tau w_t^i, \quad \text{for all } t, \quad (2.8)$$

so that retired individuals receive their previous contribution adjusted by the gross rate of population growth.

Individuals maximize (2.6) with respect to $\{c_t^i, x_{t+1}^i, b_{t+1}^i, s_t^i\}$ subject to (2.1), (2.2), (2.4) and (2.5), taking as given p_{t+1}^i , τ , a_t^i , b_t^i , w_t^i and R_{t+1} . If we plug the social security budget constraint (2.8), the aspiration formation equation (2.3), and the competitive gross rate of return $R_{t+1} = R$ into the solution of this individual problem, we find the following amounts of consumptions, bequest, and saving in equilibrium:

$$c_t^i = \left[\frac{((1 - \tau)R + n\tau)\alpha}{R + \gamma} \right] w_t^i + \left[\frac{R\alpha}{R + \gamma} \right] b_t^i + \delta(\beta + \rho) c_{t-1}^i, \quad (2.9)$$

³The assumption of constant rental prices for labor and capital could also be the result of considering a small open economy with perfect capital mobility and no labor mobility. This means that the interest rate is constant and equal to its international level. Under a standard neoclassical production function and competitive input markets, the equilibrium capital-labor ratio turns out to be constant and, thus, the marginal productivity of labor (which is equal to the competitive real wage) is also constant.

$$\begin{aligned} x_{t+1}^i &= \left[\frac{((1-\tau)R + n\tau)(\gamma(1-\rho) + R\beta)}{R+\gamma} \right] w_t^i \\ &+ \left[\frac{R(\gamma(1-\rho) + R\beta)}{R+\gamma} \right] b_t^i - \delta(R\beta - \rho\gamma) c_{t-1}^i, \end{aligned} \quad (2.10)$$

$$b_{t+1}^i = \left[\frac{((1-\tau)R + n\tau)\rho}{n} \right] w_t^i + \left[\frac{\rho R}{n} \right] b_t^i - \left[\frac{(R+\gamma)\delta\rho}{n} \right] c_{t-1}^i, \quad (2.11)$$

and

$$s_t^i = \left[\frac{(1-\tau)(R(\beta+\rho) + \gamma) - n\alpha\tau}{R+\gamma} \right] w_t^i + \left[\frac{R(\beta+\rho) + \gamma}{R+\gamma} \right] b_t^i - \delta(\beta+\rho) c_{t-1}^i. \quad (2.12)$$

Clearly, optimal consumption, bequests and saving of the individual i depend on the realization of his productivity shock w_t^i , on the amount of inheritance b_t^i he has received and, finally, on his aspiration level, which equals the adult consumption of his parents c_{t-1}^i . Note also that all the previous variables depend positively on w_t^i and b_t^i . However, while adult consumption is increasing in the parents consumption c_{t-1}^i due to the effect of aspirations, old consumption, saving, and bequests are all decreasing in c_{t-1}^i .

3. The Dynamics of Consumption, Wealth, and Bequests

The dynamics of the economy is entirely driven by the autonomous system of stochastic difference equations formed by (2.9) and (2.11). To analyze the aggregate behavior of the economy we should obtain the aggregate levels per capita of the endogenous variables. To this end, we can use the law of large numbers to compute the mean values of adult consumption and bequest within a generation by just computing the expectation in both sides of equations (2.9) and (2.11),

$$\bar{c}_t = \left[\frac{R\alpha}{R+\gamma} \right] \bar{b}_t + (\beta + \rho) \delta \bar{c}_{t-1} + \frac{[(1-\tau)R + n\tau]\alpha w}{R+\gamma}, \quad (3.1)$$

and

$$\bar{b}_{t+1} = \left[\frac{\rho R}{n} \right] \bar{b}_t - \left[\frac{(R+\gamma)\delta\rho}{n} \right] \bar{c}_{t-1} + \frac{[(1-\tau)R + n\tau]\rho w}{n}, \quad (3.2)$$

where \bar{c}_t and \bar{b}_t are the average amounts of adult consumption and bequest in period t , respectively.

We could also analyze the dynamics of the second moments of the endogenous variables of our economy. To this end, we compute the variances of equations (2.9) and (2.11) to get

$$\begin{aligned} \text{Var}(c_t^i) &= \left[\frac{R\alpha}{R+\gamma} \right]^2 \text{Var}(b_t^i) + [(\beta + \rho)\delta]^2 \text{Var}(c_{t-1}^i) \\ &+ \left[\frac{2R\alpha(\beta + \rho)\delta}{R+\gamma} \right] \text{Cov}(c_{t-1}^i, b_t^i) + \left[\frac{((1-\tau)R + n\tau)\alpha}{R+\gamma} \right]^2 V, \end{aligned} \quad (3.3)$$

and

$$\text{Var}(b_{t+1}^i) = \left[\frac{\rho R}{n} \right]^2 \text{Var}(b_t^i) + \left[\frac{(R+\gamma)\delta\rho}{n} \right]^2 \text{Var}(c_{t-1}^i)$$

$$- \left[\frac{2R(R + \gamma)\delta\rho^2}{n^2} \right] Cov(c_{t-1}^i, b_t^i) + \left[\frac{((1 - \tau)R + n\tau)\rho}{n} \right]^2 V. \quad (3.4)$$

In order to close the system referred to the dynamics of the second-order moments, we need to compute the covariance $Cov(c_t^i, b_{t+1}^i)$ between the amount of adult consumption and the amount of bequest left to each of his descendants by the generic individual i . This covariance is immediately obtained from the expressions (2.9) and (2.11),

$$Cov(c_t^i, b_{t+1}^i) = \left[\frac{\alpha\rho R^2}{(R + \gamma)n} \right] Var(b_t^i) - \left[\frac{(\beta + \rho)(R + \gamma)\delta^2\rho}{n} \right] Var(c_{t-1}^i) \\ + \left[\frac{(\beta + \rho - \alpha)R\rho\delta}{n} \right] Cov(c_{t-1}^i, b_t^i) + \left[\frac{((1 - \tau)R - n\tau)^2\alpha\rho}{(R + \gamma)n} \right] V. \quad (3.5)$$

It should be noticed that in this large economy $Var(c_t^i)$, $Var(x_t^i)$, $Var(s_t^i)$ and $Var(b_t^i)$ coincide with the empirical intragenerational variances $Var(c_t)$, $Var(x_t)$, $Var(s_t)$ and $Var(b_t)$ of the distribution of adult consumption, old consumption, saving and bequest at date t . Moreover, $Cov(c_t^i, b_{t+1}^i)$ coincides with the empirical covariance $Cov(c_t, b_{t+1})$ between adult consumption and bequest left to each descendant.

The steady state value of average adult consumption and bequests is given by

$$\bar{c} = \frac{\alpha n [(1 - \tau)R + n\tau] w}{[n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta)](R + \gamma)} \quad (3.6)$$

and

$$\bar{b} = \frac{\rho(1 - \delta)[(1 - \tau)R + n\tau]w}{n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta)}, \quad (3.7)$$

where the previous steady state values are obtained by just making $\bar{c}_t = \bar{c}_{t-1} = \bar{c}$ and $\bar{b}_{t+1} = \bar{b}_{t+1} = \bar{b}$ in equations (3.1) and (3.2) and then solving for \bar{c} and \bar{b} . Moreover, taking expectations in both sides of (2.12) and evaluating the resulting equation at the stationary distribution, we obtain the average saving per capita:

$$\bar{s} = \frac{[(\beta(1 - \tau) + \rho)(R + \gamma)(1 - \delta) - \alpha(n\tau - (1 - \tau)\gamma)]nw}{[n(1 - \delta(\beta + \rho)) - \rho R(1 - \delta)](R + \gamma)}. \quad (3.8)$$

Similarly, using equations (3.3), (3.4) and (3.5), we can compute the empirical variances of consumption and bequests, $Var(c)$ and $Var(b)$, and the corresponding stationary covariance $Cov(c, b')$ between adult consumption c and bequest b' left to each descendant in the next period,

$$Var(c) = \frac{n^2[(1 - \tau)R + n\tau]^2\alpha^2(n + R\rho\delta)V}{(n - R\rho\delta)[n(1 - (\beta + \rho)\delta) - \rho R(1 - \delta)][n(1 + (\beta + \rho)\delta) + \rho R(1 + \delta)](R + \gamma)^2}, \quad (3.9)$$

$$Var(b) = \frac{[n(1 - \delta^2(\beta + \rho - \alpha)) - R\rho\delta(1 - \delta^2)][(1 - \tau)R + n\tau]^2\rho^2V}{(n - R\rho\delta)[n(1 - (\beta + \rho)\delta) - \rho R(1 - \delta)][n(1 + (\beta + \rho)\delta) + \rho R(1 + \delta)]}, \quad (3.10)$$

and

$$Cov(c, b') = \frac{\rho n^2[(1 - \tau)R + n\tau]^2\alpha(1 - (\beta + \rho)\delta^2)V}{(n - R\rho\delta)[n(1 - (\beta + \rho)\delta) - \rho R(1 - \delta)][n(1 + (\beta + \rho)\delta) + \rho R(1 + \delta)](R + \gamma)}. \quad (3.11)$$

Moreover, from equation (2.12) we obtain the stationary empirical variance of saving,

$$\begin{aligned}
Var(s) &= \left[\frac{R(\beta + \rho) + \gamma}{R + \gamma} \right]^2 Var(b) + [(\beta + \rho)\delta]^2 Var(c) \\
&\quad - 2 \left[\frac{(R(\beta + \rho) + \gamma)(\beta + \rho)\delta}{R + \gamma} \right] Cov(c, b') \\
&\quad + \left[\frac{(1 - \tau)(R(\beta + \rho) + \gamma) - n\alpha\tau}{R + \gamma} \right]^2 V.
\end{aligned} \tag{3.12}$$

We next proceed to find the conditions under which the first and second moments of the joint distribution of the endogenous variables of our model converge to their steady state values. To analyze the stability of the dynamic system formed by the equations (3.1) and (3.2) determining the evolution of the means of adult consumption and bequest we can rewrite it in matrix form,

$$\begin{pmatrix} \bar{c}_t \\ \bar{b}_{t+1} \end{pmatrix} = P \times \begin{pmatrix} \bar{c}_{t-1} \\ \bar{b}_t \end{pmatrix} + \begin{pmatrix} \frac{[(1-\tau)R+n\tau]\alpha w}{\gamma+R} \\ \frac{[(1-\tau)R+n\tau]\rho w}{n} \end{pmatrix}, \tag{3.13}$$

where the coefficient matrix P is given by

$$P = \begin{pmatrix} \delta(\beta + \rho) & \frac{R\alpha}{R + \gamma} \\ -\frac{\rho\delta(R + \gamma)}{n} & \frac{\rho R}{n} \end{pmatrix}.$$

The following lemma provides a sufficient condition for the dynamic stability of the first moments of the intragenerational distribution of adult consumption and bequest:

Lemma 3.1. *If $\frac{\rho R}{n} < 1$ and the aspirations intensity δ is sufficiently small, then the dynamic system formed by equations (3.1) and (3.2) converges monotonically to the steady state for average adult consumption and average bequest given by (3.6) and (3.7), respectively.*

Proof. The coefficient matrix P appearing in (3.13) has the determinant

$$\text{Det}(P) = \frac{\rho\delta R}{n} > 0,$$

and the trace

$$\text{Tr}(P) = \frac{(\beta + \rho)\delta n + \rho R}{n} > 0.$$

The corresponding characteristic polynomial is

$$Q(\lambda) \equiv \lambda^2 - \left[\frac{(\beta + \rho)\delta n + \rho R}{n} \right] \lambda + \frac{\rho\delta R}{n}, \tag{3.14}$$

so that the values λ_1 and λ_2 solving the equation $Q(\lambda) = 0$ are the eigenvalues of the coefficient matrix P . The discriminant $\Delta(\delta)$ of the quadratic equation $Q(\lambda) = 0$ is

$$\Delta(\delta) = \frac{[(\beta + \rho)\delta n + \rho R]^2 - 4\rho\delta R n}{n^2}.$$

Since $1 - \alpha = \beta + \rho$, it can be checked that $\Delta(\delta) > 0$ if

$$\delta \in \left[0, \frac{\rho R}{n(1 + \sqrt{\alpha})^2} \right]. \quad (3.15)$$

Let us assume for the rest of the proof that δ lies on that interval so that the two eigenvalues are real. We know that $\lambda_1 + \lambda_2 = \text{Tr}(P) > 0$ and $\lambda_1 \lambda_2 = \text{Det}(P) > 0$. Therefore, as the two eigenvalues are real, their sign must be positive. Moreover, if δ tends to zero, the two eigenvalues converge to zero and $\frac{\rho R}{n} \in (0, 1)$, respectively, since the polynomial (3.14) tends to $\lambda^2 - \left(\frac{\rho R}{n}\right)\lambda$. As the eigenvalues are continuous functions of the parameter δ representing the aspiration intensity, we conclude that, for a sufficiently small value of the parameter δ , both eigenvalues are real, positive, and smaller than 1, which proves the desired monotone convergence property. ■

Concerning the stability of the dynamic system driving the evolution of second order moments formed by equations (3.3), (3.4) and (3.5), we can proceed in a similar fashion. Note that we can rewrite that system in matrix form,

$$\begin{pmatrix} \text{Var}(c_t) \\ \text{Var}(b_{t+1}) \\ \text{Cov}(c_t, b_{t+1}) \end{pmatrix} = W \times \begin{pmatrix} \text{Var}(c_{t-1}) \\ \text{Var}(b_t) \\ \text{Cov}(c_{t-1}, b_t) \end{pmatrix} + \begin{pmatrix} \left(\frac{[(1-\tau)R+n\tau]\alpha}{R+\gamma}\right)^2 V \\ \left(\frac{[(1-\tau)R+n\tau]\rho}{n}\right)^2 V \\ \left(\frac{[(1-\tau)R+n\tau]^2\rho\alpha}{(R+\gamma)n}\right) V \end{pmatrix},$$

where the coefficient matrix W is given by

$$W = \begin{pmatrix} (\beta + \rho)^2 \delta^2 & \left(\frac{R\alpha}{R+\gamma}\right)^2 & \frac{2R\alpha(\beta+\rho)\delta}{R+\gamma} \\ \left(\frac{\rho(R+\gamma)\delta}{n}\right)^2 & \left(\frac{\rho\delta}{n}\right)^2 & -\frac{2\rho R(R+\gamma)\delta\rho}{n^2} \\ -\frac{(\beta+\rho)(R+\gamma)\delta^2\rho}{n} & \frac{\alpha\rho R^2}{(R+\gamma)n} & \frac{(\beta+\rho-\alpha)R\rho\delta}{n} \end{pmatrix}.$$

The following lemma provides a sufficient condition for dynamic stability of the second moments of the intragenerational distribution of consumption and bequest:

Lemma 3.2. *If $\frac{\rho R}{n} < 1$ and the aspirations intensity δ is sufficiently small, then the dynamic system formed by equations (3.3), (3.4) and (3.5) converges to the steady state for the variance of adult consumption, variance of bequest, and covariance between adult consumption and the amount of bequest left to each descendant given by (3.9), (3.10), and (3.11), respectively.*

Proof. The characteristic polynomial of the matrix W is

$$T(\hat{\lambda}) \equiv \hat{\lambda}^3 - d\hat{\lambda}^2 + f\hat{\lambda} - g\hat{\lambda}, \quad (3.16)$$

with

$$d = \frac{\rho R (\delta n (1 - 2\alpha) + R\rho) + \delta^2 n^2 (\beta + \rho)^2}{n^2},$$

$$f = \frac{[\rho R (\delta n (1 - 2\alpha) + R\rho) + \delta^2 n^2 (\beta + \rho)^2] R\rho\delta}{n^3} = \left(\frac{R\rho\delta}{n}\right) d,$$

and

$$g = \left(\frac{R\rho\delta}{n}\right)^3.$$

If δ approaches zero, then the three eigenvalues of the coefficient matrix W tend to $\hat{\lambda}_1 = \left(\frac{\rho R}{n}\right)^2 \in (0, 1)$, $\hat{\lambda}_2 = 0$, and $\hat{\lambda}_3 = 0$ since the coefficient d of the characteristic polynomial converges to $\left(\frac{\rho R}{n}\right)^2$, while the coefficients f and g tend to zero. Therefore, the characteristic polynomial (3.16) tends to $\hat{\lambda}^3 - \left(\frac{R\rho}{n}\right)^2 \hat{\lambda}^2$. Finally, since the eigenvalues are continuous functions of the parameter δ , it follows that the three eigenvalues will lie in the interior of the unit circle for a sufficiently small value of δ . ■

For the rest of the paper, we will maintain the assumption $\rho R/n < 1$, which together with a sufficiently small value of the parameter δ , ensures the monotonic convergence of first and second moments of the intragenerational distribution of adult consumption and bequests.

4. Intergenerational Mobility

To perform an analysis of intergenerational mobility in our economy we should analyze the behavior of the correlation coefficient between s_{t+1}^i and s_t^i , $\text{Corr}(s_{t+1}^i, s_t^i)$, i.e., between the amount of assets held by two members of the same family belonging to two consecutive generations. With no bequests and no aspirations this autocorrelation would be equal to zero, so that we would obtain perfect mobility as wages are i.i.d. If we had perfect correlation of asset holdings, i.e. $\text{Corr}(s_{t+1}^i, s_t^i) = 1$, then intergenerational mobility would be null.

It is important to point out that, even if altruism is absent, the presence of aspirations induces a wealth correlation across the members of consecutive generations within the same family. This is so because aspirations induce a correlation between the amount of parents' consumption and the profile of consumption and saving of their descendants.

Note that at a stationary distribution we have

$$\text{Corr}(s_{t+1}^i, s_t^i) = \frac{\text{Cov}(s_{t+1}^i, s_t^i)}{(Var(s_{t+1}^i))^{1/2} \cdot (Var(s_t^i))^{1/2}} = \frac{\text{Cov}(s_{t+1}, s_t)}{Var(s_t)} = \frac{\text{Cov}(s', s)}{Var(s)} = \text{Corr}(s', s),$$

where s' denotes the saving of an individual of the next generation belonging to the same family. The expression (3.12) gives us $Var(s)$ so that we only need to compute the intergenerational covariance of wealth, $\text{Cov}(s', s)$, in the previous formula. To this

end, we first observe that the policy functions (2.9), (2.11), and (2.12) become at the stationary distribution:

$$c_t = Aw_t + Bb_t + Cc_{t-1},$$

$$b_{t+1} = Dw_t + Eb_t + Fc_{t-1},$$

and

$$s_t = Gw_t + Hb_t + Ic_{t-1}, \quad (4.1)$$

where

$$\begin{aligned} A &= \frac{[(1-\tau)R + n\tau]\alpha}{R + \gamma}, & B &= \frac{R\alpha}{R + \gamma}, & C &= \delta(\beta + \rho), \\ D &= \frac{[(1-\tau)R + n\tau]\rho}{n}, & E &= \frac{\rho R}{n}, & F &= -\frac{\rho(R + \gamma)\delta}{n}, \\ G &= \frac{(1-\tau)[R(\beta + \rho) + \gamma] - n\alpha\tau}{R + \gamma}, \\ H &= \frac{\gamma + R(\beta + \rho)}{R + \gamma}, \quad \text{and} \quad I = -\delta(\beta + \rho). \end{aligned}$$

Then, we compute

$$\begin{aligned} s_{t+1} &= Gw_{t+1} + Hb_{t+1} + Ic_t \\ &= Gw_{t+1} + H\underbrace{(Dw_t + Eb_t + Fc_{t-1})}_{b_{t+1}} + I\underbrace{(Aw_t + Bb_t + Cc_{t-1})}_{c_t} \\ &= Gw_{t+1} + (H \cdot D + I \cdot A)w_t + (H \cdot E + I \cdot B)b_t + (H \cdot F + I \cdot C)c_{t-1}^1 \\ &= Gw_{t+1} + Jw_t + Mb_t + Nc_{t-1}^1, \end{aligned} \quad (4.2)$$

where

$$J = H \cdot D + I \cdot A, \quad M = H \cdot E + I \cdot B, \quad \text{and} \quad N = H \cdot F + I \cdot C.$$

Therefore, combining (4.1) and (4.2), we get

$$\begin{aligned} Cov(s_{t+1}, s_t) &= Cov(s', s) = G \cdot J \cdot Var(w) + H \cdot M \cdot Var(b) \\ &\quad + I \cdot M \cdot Cov(c, b') + H \cdot N \cdot Cov(c, b') + I \cdot N \cdot Var(c) \\ &= G \cdot J \cdot V + I \cdot N \cdot Var(c) + H \cdot M \cdot Var(b) + (I \cdot M + H \cdot N) Cov(c, b'). \end{aligned} \quad (4.3)$$

Since the stationary values of $Var(c)$, $Var(b)$, and $Cov(c, b')$ are given by (3.9), (3.10), and (3.11), respectively, we can plug them into (4.3) to find an explicit expressions for $Cov(s', s)$. In the next two sections we will conduct the corresponding comparative statics exercise on the stationary value of the autocorrelation coefficient $Corr(s', s)$ to characterize the effects on the level of intergenerational mobility of changes in the intensity of habits and aspirations and to evaluate also the effects of a PAYG social security system on mobility.

5. Effects of Habits and Aspirations on the Intragenerational Distribution of Wealth and Intergenerational Mobility.

In this section, we will first characterize the effect of habits and aspirations on the stationary intragenerational distribution of wealth. Note that those properties of individual preferences affect the amount of saving since they modify the evaluation of the utility derived from consumption in the two periods of life. Moreover, the distribution of saving is equivalent to the intragenerational distribution of the individuals' asset holdings at the beginning of their last period of life.

By differentiating (3.8) and (3.7) with respect to the parameters δ and γ measuring the intensity of aspirations and habits, respectively, we get the following effects on the average amount of saving and bequests:

$$\begin{aligned}\frac{\partial \bar{s}}{\partial \delta} &= -\frac{((1-\tau)R + n\tau)(n(\beta + \rho) + \rho\gamma)n\alpha w}{[n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta)]^2(R + \gamma)} < 0, \\ \frac{\partial \bar{s}}{\partial \gamma} &= \frac{((1-\tau)R + n\tau)n\alpha w}{[n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta)](R + \gamma)^2} > 0, \\ \frac{\partial \bar{b}}{\partial \delta} &= -\frac{\alpha\rho n((1-\tau)R + n\tau)w}{[n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta)]^2} < 0,\end{aligned}\tag{5.1}$$

and

$$\frac{\partial \bar{b}}{\partial \gamma} = 0.\tag{5.2}$$

The multiplier $\partial \bar{s} / \partial \gamma$ is positive since its denominator is positive. To see this, observe that

$$\begin{aligned}n[1 - \delta(\beta + \rho)] - \rho R(1 - \delta) &= n[1 - \delta(1 - \alpha)] - \rho R(1 - \delta) \\ &> n(1 - \delta) - \rho R(1 - \delta) = (n - \rho R)(1 - \delta) > 0.\end{aligned}\tag{5.3}$$

where the first equality follows from the fact that $\alpha + \beta + \rho = 1$, the first inequality follows as $\alpha \in (0, 1)$ and, finally, the last inequality is a consequence of the dynamic stability assumption, $\rho R/n < 1$.

An increase in the value of the aspiration intensity δ increases the marginal utility of an extra unit of adult consumption since individuals are more sensitive to the level of consumption of their parent when evaluating their own adult consumption. Therefore, individuals optimally increase their adult consumption and, thus, the amounts of both saving and inheritance received by their children should go down. Concerning the effect of an increase in the value of the habit formation parameter γ , we notice that individuals experience an increase in the marginal valuation of their old consumption as they internalize with more intensity their past adult consumption and, thus, they optimally decide to shift consumption from the adult to the old age. On the one hand, the reduction of adult consumption lowers the amount of accumulated habits and, on the other hand, the increase in old consumption is the optimal response to the increase in the marginal utility of old consumption. We also see that the aggregate effects of stronger habits are accommodated along the life cycle of individuals since the aggregate amount of bequests remains unchanged (see equation (5.2)).

We could also analyze how the changes in the values of the parameters δ and γ affect the intragenerational variability of wealth. We will concentrate our analysis on the variability of saving, which fully determines the amount of assets held by individuals at the beginning of the last period of their lives. Since the average amount of saving is also affected by those changes, it seems appropriate to perform our comparative statics exercise on the coefficient of variation, $CV(s) = (Var(s))^{1/2} / \bar{s}$. Using (3.8), (3.9), (3.10), (3.11), and (3.12), and after some painful algebra, we get the following derivative:

$$\frac{\partial CV(s)}{\partial \delta} \bigg|_{\delta=\gamma=\tau=0} = \frac{\alpha n^2 V^{1/2}}{(n + R\rho)^{3/2} (n - R\rho)^{1/2} w} > 0,$$

where the sign of the previous derivative follows immediately under our maintained condition of dynamic stability, $\rho R/n < 1$. Our comparative statics exercise on the coefficient of variation of saving is conducted in a quite restrictive scenario, which allows us to unambiguously sign the effects of stronger aspirations. We do evaluate the derivative of $CV(s)$ with respect to δ at the point $\delta = \gamma = \tau = 0$, that is, we analyze the effect of the marginal introduction of aspirations in an economy when both social security and habits are absent or are present at a small scale. The evaluation of the previous derivative for arbitrary values of δ , γ and τ cannot be explicitly signed. We then see that the marginal introduction of aspirations increases the variability of wealth. As individuals take into account their parents' past consumption experience, there is now a new channel through which the shocks on wages propagate intergenerationally, which results in a large intragenerational variation of wealth.

In order to evaluate the robustness of the sign of the previous derivative, we conduct a numerical analysis. We choose the values of the preference parameters $\alpha = 1/2$, $\beta = 1/4$, and $\rho = 1/4$. Moreover, following Iacoviello (2007), we chose the value of the average wage $w = 2/3$ and make the cross-sectional standard deviation of the log of earnings equal to 0.5173. Therefore, we set the associated variance of wages equal to $V = \left(\frac{2}{3}\right)^2 (\exp(0.5173^2) - 1) = 0.13637$, which amounts to a variation coefficient of wages equal to $V^{1/2}/w = 0.55392$. We assume constant population, $n = 1$. Finally, we choose an interest rate per year of 4% and we consider that each period last for 30 years so that $R = (1.04)^{30} = 3.2434$. We maintain these parameter values for the remaining numerical exercises unless otherwise specified. In Figure 1, we see that the positive effect of aspirations on the coefficient of variation of asset holdings is preserved for all the values of δ and for different combinations of values for the habit parameter ($\gamma = 0$ and $\gamma = 1/4$) and the social security tax ($\tau = 0$ and $\tau = 1/3$). Note that we restrict the domain of the aspiration parameter δ to lie in the interval (3.15) so that $\delta \in [0, 0.27824]$.

[Insert Figure 1]

Concerning the implications for the intragenerational variability of wealth of changes in habit intensity, it can be shown that

$$\frac{\partial CV(s)}{\partial \gamma} \bigg|_{\delta=0} = 0.$$

Therefore, we see that habits affect the level of intragenerational variation of wealth only if aspirations are present. This means that changes in the habit intensity affect

the intergenerational transmission of productivity shocks only through the inherited tastes. Moreover, we can compute the following derivative:

$$\begin{aligned} \frac{\partial CV(s)}{\partial \gamma} \Big|_{\gamma=\tau=0} &= \\ \frac{-\alpha\delta(1+\delta)[n(1-(\beta+\rho)\delta)-R\rho(1-\delta)]^{3/2}V^{1/2}}{(n-R\rho\delta)^{1/2}(1-\delta)^2(\beta+\rho)R[n(1+(\beta+\rho)\delta)+\rho R(1+\delta)]^{1/2}[(n-\delta\rho R)(1-\delta^2)+2n\alpha\delta^2]^{1/2}w} &< 0, \end{aligned} \quad (5.4)$$

where its negative sign follows again under the assumption $\rho R/n < 1$. Note also that the previous derivative (5.4) gives us the effect on the variability of asset holdings of the introduction of habits when the value of the social security tax is around zero. Clearly, when habits are introduced, a shock on wages is more evenly distributed among adult and old consumption since habits enhance consumption smoothing along the life cycle. Therefore, habits make an individual's adult consumption less sensitive to wage shocks and this induces in turn less volatility in the consumption and saving of his descendants. In Figure 2, we see that the negative sign of the derivative (5.4) is preserved for strictly positive values of the habit parameter γ and for different values of the social security tax τ . In the two graphs we set the value of the aspiration parameter at the strictly positive value $\delta = 1/5$. We have also checked the robustness of the sign of the derivative for different values of the aspiration intensity δ in the interval $[0, 0.27824)$.

[Insert Figure 2]

We can proceed now with the analysis of the effects of habit and aspiration intensities γ and δ on the level of intergenerational mobility within a family, which is characterized by the correlation of asset holdings at the stationary distribution. To this end we need to compute the derivatives of the stationary value of the autocorrelation coefficient $Corr(s', s)$ obtained in Section 4 with respect to the parameters representing the aspiration and habit intensities. These partial derivatives are

$$\frac{\partial Corr(s', s)}{\partial \delta} \Big|_{\delta=\gamma=\tau=0} = -\frac{\alpha [n^2 + 2\rho^2 R^2 (R-1)]}{n^2} < 0 \quad (5.5)$$

and

$$\frac{\partial Corr(s', s)}{\partial \gamma} \Big|_{\gamma=\tau=\rho=0} = \frac{\alpha\delta(1-\delta^2)(1-\beta^2\delta^2)}{[1+\delta^2(1-2\beta)]^2\beta R} > 0. \quad (5.6)$$

Again these derivatives characterize the effects of the marginal introduction of either aspirations and habits when the size of the social security system is small. Moreover, the derivative of the correlation coefficient $Corr(s', s)$ with respect to the habits parameter γ can only be explicitly signed when the degree of altruism, which is parametrized by ρ , is sufficiently low. Note that, if we evaluate the derivative (5.6) when there are no aspirations, we clearly obtain

$$\frac{\partial Corr(s', s)}{\partial \gamma} \Big|_{\delta=\gamma=\tau=0} = 0$$

so that, again, changes in habits only affect the level of intragenerational mobility through the transmission of tastes across the generations within the same family. We see thus that the introduction of aspirations and habits have opposite effects on intergenerational mobility. On the one hand, the introduction of aspirations raises the degree of mobility. As the marginal utility of adult consumption increases when aspirations are introduced *ceteris paribus*, workers tend to increase their consumption by both reducing their saving and, thus, by reducing the amount of bequests they plan to leave to their children. Obviously, this results in a smaller correlation between the assets of the members of the same family belonging to two consecutive generations. On the other hand, when habits are introduced, workers wish to smooth more their consumption along the life cycle. Hence, a positive shock in their labor income results in a larger increase in their saving aimed at shifting adult consumption towards old consumption. In the presence of aspirations, consumption and saving levels of individuals belonging to consecutive generations become correlated and this correlation becomes indeed larger as the saving of each individual becomes more sensitive to productivity shocks. We see then that the introduction of habits results in an increase in the correlation of wealth between members of consecutive generations within the same dynasty.

In Figures 3 and 4 we conduct a numerical analysis to check the robustness of the signs of the derivatives (5.5) and (5.6). When aspirations are present in an economy with no habits, no altruism, and no social security, the autocorrelation coefficient of asset holdings is negative. Obviously, since adult individuals want to mimic the consumption level of their parents and labor income is uncorrelated across generations, the saving of two consecutive generations of the same family becomes negatively correlated. When the aspiration intensity increases, individuals raise their adult consumption in response to the larger parental consumption by reducing their saving. This negative effect of aspirations on asset autocorrelation is preserved through the numerical examples of Figure 3 with $\delta \in [0, 0.27824]$ and $\rho = 1/4$. Note that, since we have introduced altruism in these examples, bequests are positive and, hence, the autocorrelation $\text{Corr}(s, s')$ becomes positive as a consequence of the intergenerational transmission of wealth.

[Insert Figure 3]

In Figure 4 we make again the value of the aspiration parameter equal to $\delta = 1/5$. Moreover, in that figure we consider two values for the altruism parameter, $\rho = 0$ and $\rho = 1/4$. When there is no altruism, $\rho = 0$, we see in the first two panels of Figure 4 that the autocorrelation of asset holding is negative due to the presence of aspirations. Both for $\tau = 0$ and $\tau = 1/3$ the relationship between the autocorrelation $\text{Corr}(s, s')$ and the value of the habit parameter γ is positive, which agrees with (5.6). In the last two panels of Figure 4 we consider the case with $\rho = 1/4$ and, then, the wealth autocorrelation becomes positive due to the introduction of altruism and the corresponding positive bequests. Here, the positive relationship between $\text{Corr}(s, s')$ and the intensity of habits is not preserved since the corresponding graph is inverted U-shaped. We see from (2.11) that when $\rho > 0$ the amount of bequests decreases with the habit intensity γ . This is so because individuals want to shift more resources towards old consumption and they achieve this goal by reducing the amounts of adult consumption (see (2.9)) and bequests. Therefore, this reduction in the level of intergenerational transfers ends up

offsetting the initial positive effect on $\text{Corr}(s, s')$ of habits.

[Insert Figure 4]

6. The Effects of Social Security

In this economy the effect of raising the social security tax, and thus the social security benefits, results in a reduction of aggregate saving. This is a standard result accruing from the transfer of income from the second period of life to the third one brought about by the social security system. Individuals offset this compulsory transfer by lowering the amount of saving. We confirm this effect by differentiating (3.8),

$$\frac{\partial \bar{s}}{\partial \tau} = -\frac{[(1-\delta)\beta(R+\gamma) + \alpha(n+\gamma)]nw}{[n(1-\delta(\beta+\rho)) - \rho R(1-\delta)](R+\gamma)} < 0.$$

The effect of social security on bequests depends on whether the economy is dynamically efficient (i.e., $R > n$) or inefficient (i.e., $R < n$) in the steady state according to the dynamically optimality criterion of Cass (1979). Since the return of the PAYG social security system is equal to the gross rate of population growth n , if $R > n$ then an increase in the social security tax shifts resources from the productive investment earning the gross rate R to the social security system, which yields a lower return n . This implies that the present value of individuals' lifetime income decreases and, therefore, these poorer individuals end up leaving a lower amount of bequest to their descendants. The converse argument applies when $R < n$. The previous discussion is confirmed after taking the derivative of (3.7) with respect to τ ,

$$\frac{\partial \bar{b}}{\partial \tau} = \frac{\rho(1-\delta)(n-R)w}{n[1-\delta(\beta+\rho)] - \rho R(1-\delta)},$$

which is positive (negative) when $R < (>)n$ as follows from (5.3).

Concerning the effects of social security on the variability of wealth, we obtain the following derivative of the coefficient of variation of saving with respect to the social security tax:

$$\frac{\partial \text{CV}(s)}{\partial \tau} \bigg|_{\delta=\gamma=\tau=0} = -\frac{\rho(n-R\rho)^{3/2}V^{1/2}}{n(\beta+\rho)(n+R\rho)^{1/2}w} < 0.$$

Note that the previous derivative is evaluated at the point $\delta = \gamma = \tau = 0$ so that we are characterizing the effect on wealth variability of introducing a PAYG social security system when habits and aspirations are not very important. The coefficient of variation of asset holdings decreases when the social security tax is introduced. This is so because the equalizing impact of the introduction of the proportional payroll tax on adult income is translated into a lower variability of adult consumption and saving. Figure 5 shows how the negative sign of the previous derivative is maintained for different configurations of the parameter values of aspiration and habit intensity when the social security tax τ is not too large. However, when the tax rate is large (above 60% in our simulations), the relationship between variability of wealth and the tax rate becomes negative. When the social tax rate is so large the average amount of saving becomes negative since individuals want to transfer income from their third to

their second period of life. The variability of this negative saving will be now driven by the variability of the income that individuals receive when they are old. Obviously, the variability of the third period income is linked to the variability of social security benefits, which rises with the tax rate τ .

[Insert Figure 5]

Since aspirations and habits affect the intragenerational distribution of wealth, we could also analyze how these two phenomena affect the efficacy of social security. This analysis will allow us to see whether habits and expectations exacerbate or not the effects of social security on the intragenerational distribution of wealth. Therefore, we can compute the following cross derivatives of saving:

$$\frac{\partial^2 \bar{s}}{\partial \tau \partial \delta} = \frac{(n(\beta + \rho) + \rho\gamma) \alpha (R - n) nw}{(R + \gamma) [n(1 - (\beta + \rho)\delta) - \rho R(1 - \delta)]^2},$$

which is positive (negative) when $R > (<)n$, and

$$\frac{\partial^2 \bar{s}}{\partial \tau \partial \gamma} = \frac{(n - R) n \alpha w}{[n(1 - (\beta + \rho)\delta) - \rho R(1 - \delta)] (R + \gamma)^2},$$

which is positive (negative) when $R < (>)n$. We see thus that the negative effect of social security on saving becomes even more negative or less negative depending on whether are the intensity of aspirations or of habits that become stronger and also on the dynamic efficiency properties of the economy.

Just to gain some intuition about the effects involved in the analysis, let us consider the effect of an increase in the value of the habit intensity parameter γ on the multiplier $\partial \bar{s} / \partial \tau$. When the economy is dynamically inefficient ($R < n$) and an increase in the social security tax takes place, the present value of lifetime income of the individuals increases. As individuals exhibit more habits they want to rebalance their consumption profile and shift adult consumption to the old age. This is achieved through larger saving and, thus, the negative effect of social security on saving becomes now more moderate. Similar arguments can be applied to dynamically efficient economies and to changes in the intensity of aspirations.

We could also extend our analysis to see whether the negative effect of social security on the coefficient of variation of saving is exacerbated or not by the presence of aspirations and habits. To see this, we compute the following cross derivatives:

$$\left. \frac{\partial^2 CV(s)}{\partial \tau \partial \delta} \right|_{\delta=\gamma=\tau=\rho=0} = \frac{\alpha n V^{1/2}}{\beta R w} > 0$$

and

$$\left. \frac{\partial^2 CV(s)}{\partial \tau \partial \gamma} \right|_{\delta=\gamma=\tau=0} = \frac{\alpha \rho (n - R\rho)^{3/2} V^{1/2}}{n R (\beta + \rho)^2 (n + R\rho)^{1/2} w} > 0.$$

The crossed derivative $\partial^2 CV(s) / \partial \tau \partial \delta$ has an ambiguous sign if it is evaluated at $\delta = \gamma = \tau = 0$. However, that derivative can be signed if the strength of the bequest motive is small. This is why we add the parameter restriction $\rho = 0$ when computing

the crossed derivative. We do not need to impose this additional restriction to sign the derivative $\partial^2 CV(s) / \partial \tau \partial \gamma$.

We see thus that the negative effect of the introduction of social security on the dispersion of asset holdings of old individuals becomes weaker when either habits or aspirations come into play (under a low value of the altruism factor ρ). Note that in both cases the endogenous variables will exhibit more inertia as they depend from the realization of past consumption. This dependence from the initial conditions puts a limit on the strength of the effects of social security on cross-sectional wealth inequality.

We can analyze now the effects of the introduction of a PAYG social security system on intergenerational mobility. To this end we compute the derivative of the stationary value of the autocorrelation coefficient $Corr(s', s)$ obtained in Section 4 with respect to the social security tax to obtain

$$\frac{\partial Corr(s', s)}{\partial \tau} \Big|_{\delta=\gamma=\tau=0} = \frac{(n^2 - R^2 \rho^2) \rho}{(\beta + \rho) n^2} > 0.$$

The positive sign of this derivative tells us that the introduction of social security reduces mobility. This is so because the individuals enjoying higher wages are the ones that get a larger social security benefit when old. Obviously, those individuals will be the ones that will end up leaving even larger bequests. The converse argument applies to the individuals earning lower wages. The final result is that intergenerational mobility in wealth decreases when the social security system is introduced. The first two panels of Figure 6 show how the negative effect of social security on intergenerational mobility is preserved under different configurations of the parameter values of aspiration and habit intensity under small social security tax rates. However, in these first two panels we see that for large values of the tax rate τ the autocorrelation between assets becomes negative. This is so because in this case the richer individuals end up saving a negative amount in spite of leaving a larger amount of bequests, which raises the amount of saving of their direct descendants. Again, the converse argument applies for the poor individuals. In the last two panels of Figure 6, aspirations are present and this pushes adult consumption up and the average amount of saving becomes negative for all values of τ . This results in a negative effect of social security on the autocorrelation coefficient of wealth for all the values of the social security tax.

[Insert Figure 6]

Finally, we can compute

$$\frac{\partial^2 Corr(s', s)}{\partial \tau \partial \delta} \Big|_{\delta=\gamma=\tau=\rho=0} = -\frac{(1-\beta) n}{\beta R} < 0$$

and

$$\frac{\partial^2 Corr(s', s)}{\partial \tau \partial \gamma} \Big|_{\delta=\gamma=\tau=0} = -\frac{\alpha (n^2 - R^2 \rho^2) \rho}{(\beta + \rho)^2 R n^2} < 0,$$

to conclude that habits and aspirations tend to decrease the negative effect of social security on mobility.⁴ As for the case of intragenerational inequality, the

⁴We also need to impose $\rho = 0$, when computing the crossed derivative $\partial^2 \rho(s', s) / \partial \tau \partial \delta$ in order to get a definite sign.

introduction of habits and aspirations reduce the efficacy of social security concerning the intergenerational transmission of wealth since this transmission is relatively more dependent on the preference parameters δ and γ and less on the policy parameter τ .

7. Conclusion

We have developed a simple model that enable us to study the effect of the introduction of habits and aspirations on the intragenerational distribution of wealth. Moreover, we have discussed the interaction of these two phenomena with the redistributive features of an unfunded social security system. Our results show that the introduction of habits and aspirations has opposite effects on both the average amount of asset accumulated by individuals and the level of wealth inequality measured by the coefficient of variation of wealth. Moreover, the efficacy of the PAYG social security in reducing wealth inequality becomes weaker when habits and aspirations are present.

Concerning mobility in wealth within the same family, we see that the introduction of aspirations increases intergenerational mobility as the amount of bequests tend to be lower. However, the introduction of either habits in preferences or a social security system result in an increase of the correlation between the amount of assets held by two members of the same family belonging to consecutive generation since the amount of bequests left by individuals becomes more correlated with their wealth.

Our model is simple enough to obtain explicit characterizations of other policy experiments, like the introduction of taxes on capital income or on consumption. The latter tax is specially relevant since it would affect directly the reference that individuals take into account when they evaluate their current consumption. Another potential extension of our model would be the introduction of either idiosyncratic or aggregate risks affecting the return on saving. This will create a source of volatility in the income of old individuals which will give raise to precautionary saving. How this saving will be affected by the presence of habits and aspirations is a topic for future research.

References

Abel, A. (1985). 'Precautionary Saving and Accidental Bequests', *American Economic Review*, vol. 75, pp. 777-91.

Abel, A. (1986). 'Capital Accumulation and Uncertain Lifetimes with Adverse Selection', *Econometrica*, vol. 54, pp. 1079-97.

Abel, A. (1990). 'Asset Prices under Habit Formation and Catching up with the Joneses', *American Economic Review*, vol. 80, pp. 38-42.

Abel, A. (1999). 'Risk Premia and Term Premia in General Equilibrium', *Journal of Monetary Economics*, vol. 43, pp. 3-33.

Alonso-Carrera, J., Caballé, J., and Raurich X. (2004). 'Consumption Externalities, Habit Formation and Equilibrium Efficiency', *Scandinavian Journal of Economics*, vol. 106, pp. 231-51.

Alonso-Carrera, J., Caballé, J., and Raurich X. (2005). 'Growth, Habit Formation, and Catching Up with the Joneses', *European Economic Review*, vol. 49, pp. 1665-91.

Alonso-Carrera, J., Caballé, J., and Raurich X. (2007). 'Aspirations, Habit Formation, and Bequest Motive', *Economic Journal*, vol. 117, pp. 813-36.

Barro, R.J. (1974). 'Are Government Bonds Net Wealth?', *Journal of Political Economy*, vol. 82, pp. 1095-117.

Boyd, R. and Richerson, P. (1985). *Culture and the Evolutionary Process*, Chicago: University of Chicago Press.

Becker, G. and Tomes, N. (1979). 'An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility', *Journal of Political Economy*, vol. 87, pp. 1153-89.

Bernheim, B., Schleifer, A., and Summers, L. (1985). 'The Strategic Bequest Motive', *Journal of Political Economy*, vol 93, pp. 1045-75.

Caballé, J. and Fuster, L. (2003). 'Pay-as-you-go Social Security and the Distribution of Altruistic Transfers', *Review of Economic Studies*, vol. 70, pp. 541-67.

Carroll, C., Overland, J., and Weil, D. (1997). 'Comparison Utility in a Growth Model', *Journal of Economic Growth*, vol. 2, pp. 339-67.

Carroll, C., Overland, J., and Weil, D. (2000). 'Saving and Growth with Habit Formation', *American Economic Review*, vol. 90, pp. 341-55.

Cass, D., 1979, 'On Capital Overaccumulation in the Aggregate, Neoclassical Model of Economic Growth: a Complete Characterization', *Journal of Economic Theory*, vol. 4, pp. 200-23.

Cavalli-Sforza, L. and Feldman, M. (1981). *Cultural Transmission and Evolution: A Quantitative Approach*, Princeton: Princeton University Press.

Cox, D., Ng, S., and Waldkirch, A. (2004). Intergenerational Linkages in Consumption Behavior', *Journal of Human Resources*, vol. 32, pp. 355-81.

Cremer, H. and Pestieau, P. (1988). 'The Joint Impact of Fertility Differentials and Social Security on the Accumulation and Distribution of Wealth', in: D. Kessler and A. Masson (eds.) *Modelling the Accumulation and Distribution of Personal Wealth*, Oxford University Press, pp. 169-85.

Davies, J. (1986). 'Does Redistribution Reduce Inequality?', *Journal of Labour Economics*, vol. 4, pp. 538-59.

de la Croix, D. (1996). 'The Dynamics of Bequeathed Tastes', *Economics Letters*, vol. 53, pp. 89-96.

de la Croix, D. (2001). 'Growth Dynamics and Education Spending: The Role of Inherited Tastes and Abilities', *European Economic Review*, vol. 45, pp. 1415-38.

de la Croix, D. and Michel P. (1999). 'Optimal Growth when Tastes Are Inherited', *Journal of Economic Dynamics and Control*, vol. 23, pp. 519-37.

de la Croix, D. and Michel P. (2001). 'Altruism and Self-Restraint', *Annales d'Economie et de Statistique*, vol. 63-64, pp. 233-60.

Díaz, A., Pijoan-Mas, J., and Rios-Rull, J.V. (2003). 'Precautionary Savings and Wealth Distribution under Habit Formation Preferences', *Journal of Monetary Economics*, vol. 50, pp. 1257-91.

Iacoviello, M. (2007). 'Household Debt and Income Inequality, 1963-2003', *Journal of Money, Credit and Banking* (forthcoming).

Jellal, M. and Wolff, F.C. (2002). 'Altruistic Bequests with Inherited Tastes', *International Journal of Business and Economics*, vol. 1, pp. 95-113.

Kleiber, C., Sexauer, M., and Waelde, K. (2006). 'Bequests, Taxation and the Distribution of Wealth in a General Equilibrium Model', *CESifo Working Paper* 1723.

Ljungqvist, L. and Uhlig, H. (2000). 'Tax Policy and Aggregate Demand Management under Catching Up with the Joneses', *American Economic Review*, vol. 90, pp. 356-66.

Yaari, M. (1965). 'Uncertain Lifetime, Life Insurance, and the Theory of the Consumer', *Review of Economic Studies*, vol. 32, pp. 137-50.

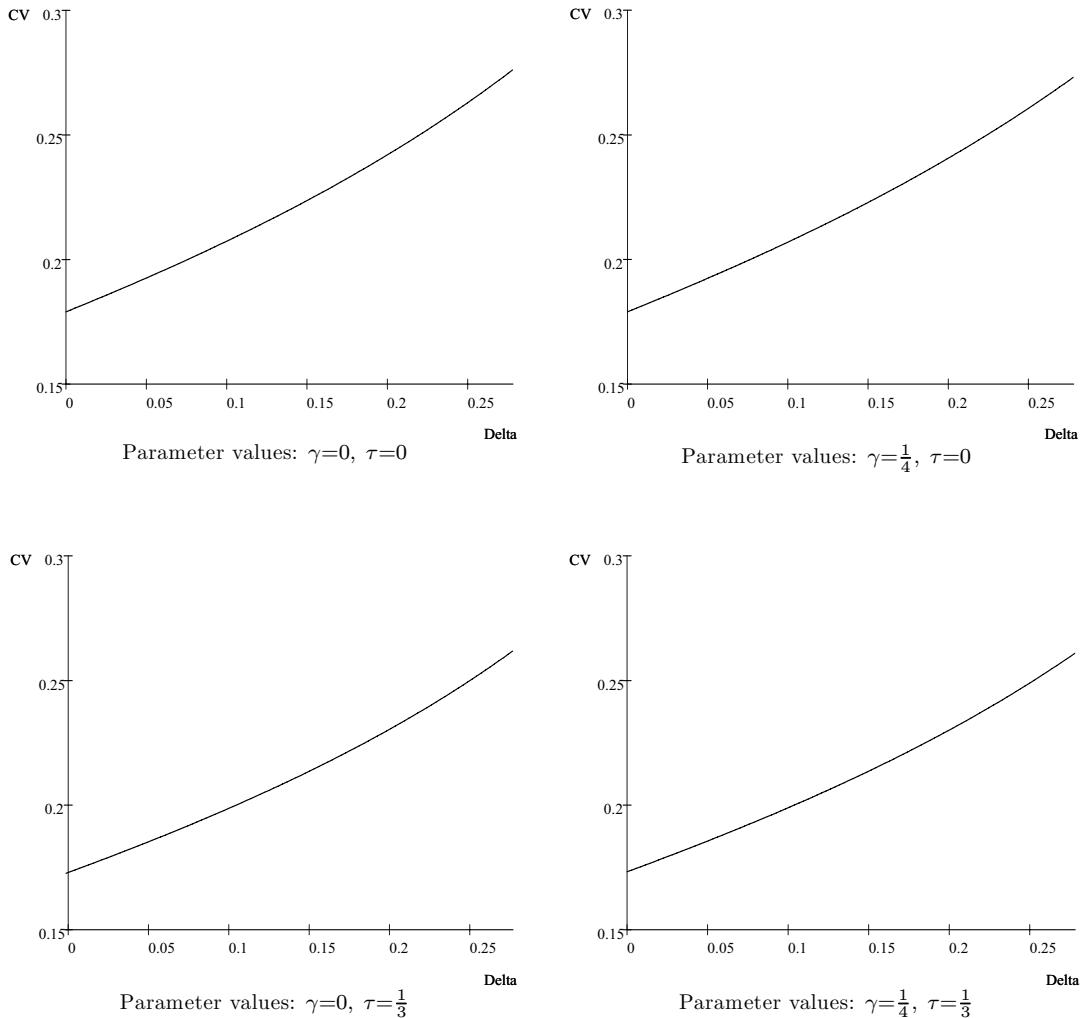


Figure 1: The effects of δ on the coefficient of variation of wealth $CV(s)$.

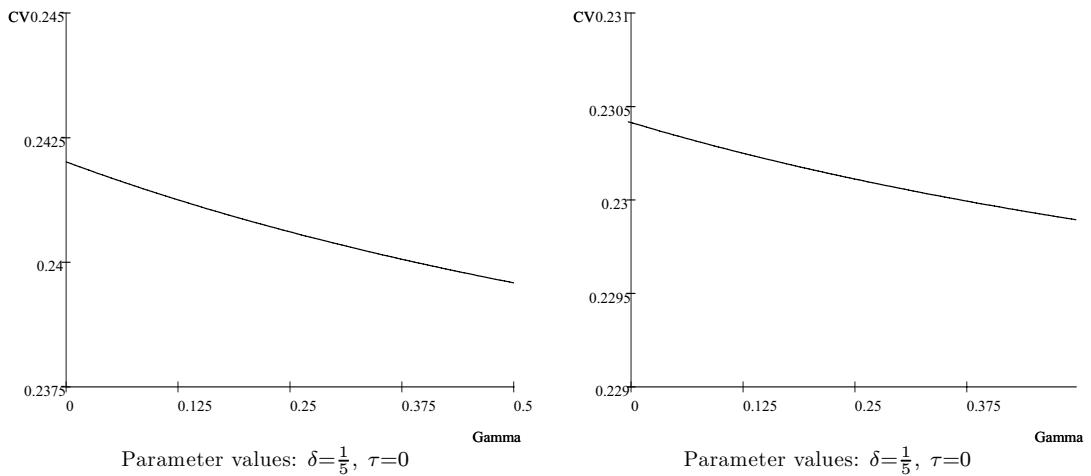


Figure 2: The effects of γ on the coefficient of variation of wealth $CV(s)$.

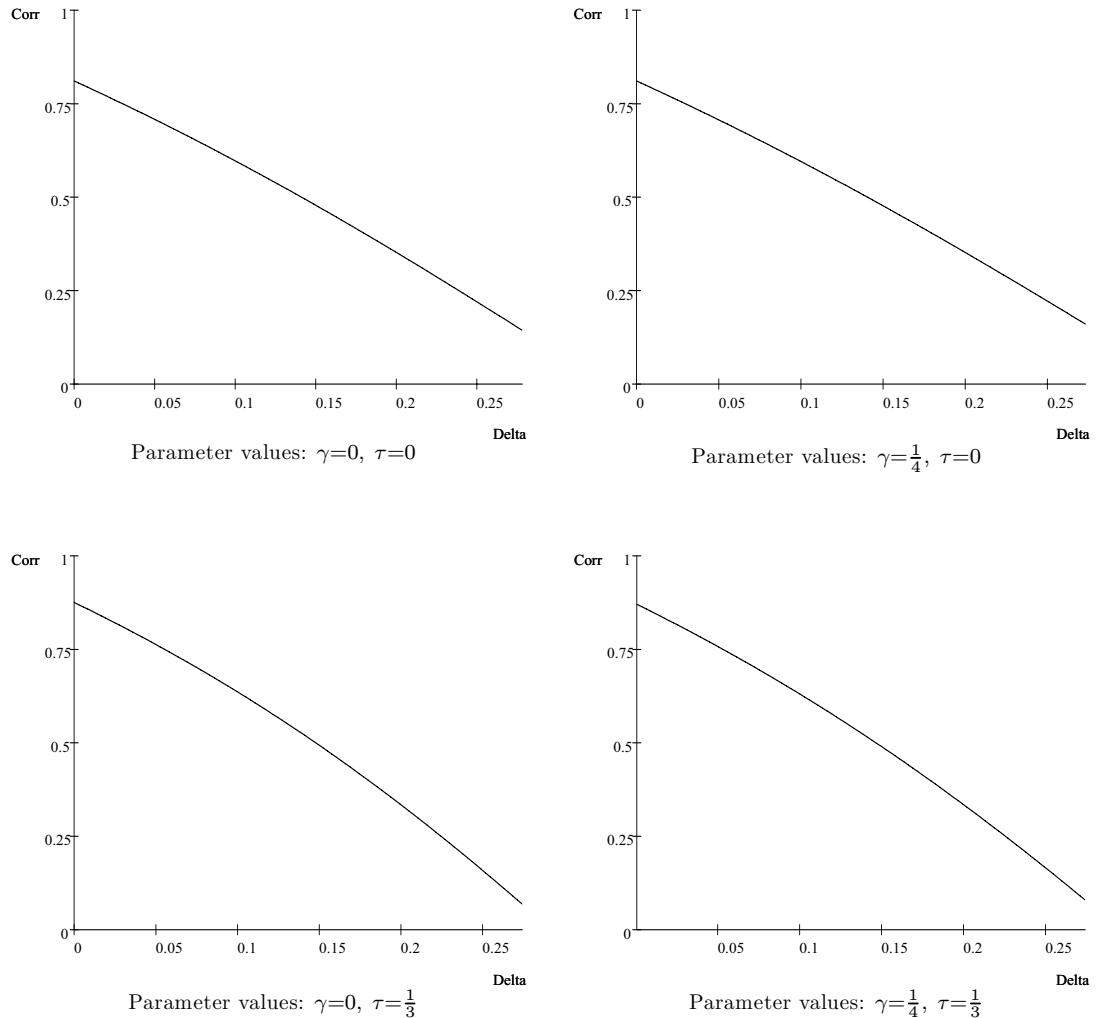


Figure 3: The effects of δ on the autocorrelation coefficient of wealth $\text{Corr}(s', s)$.

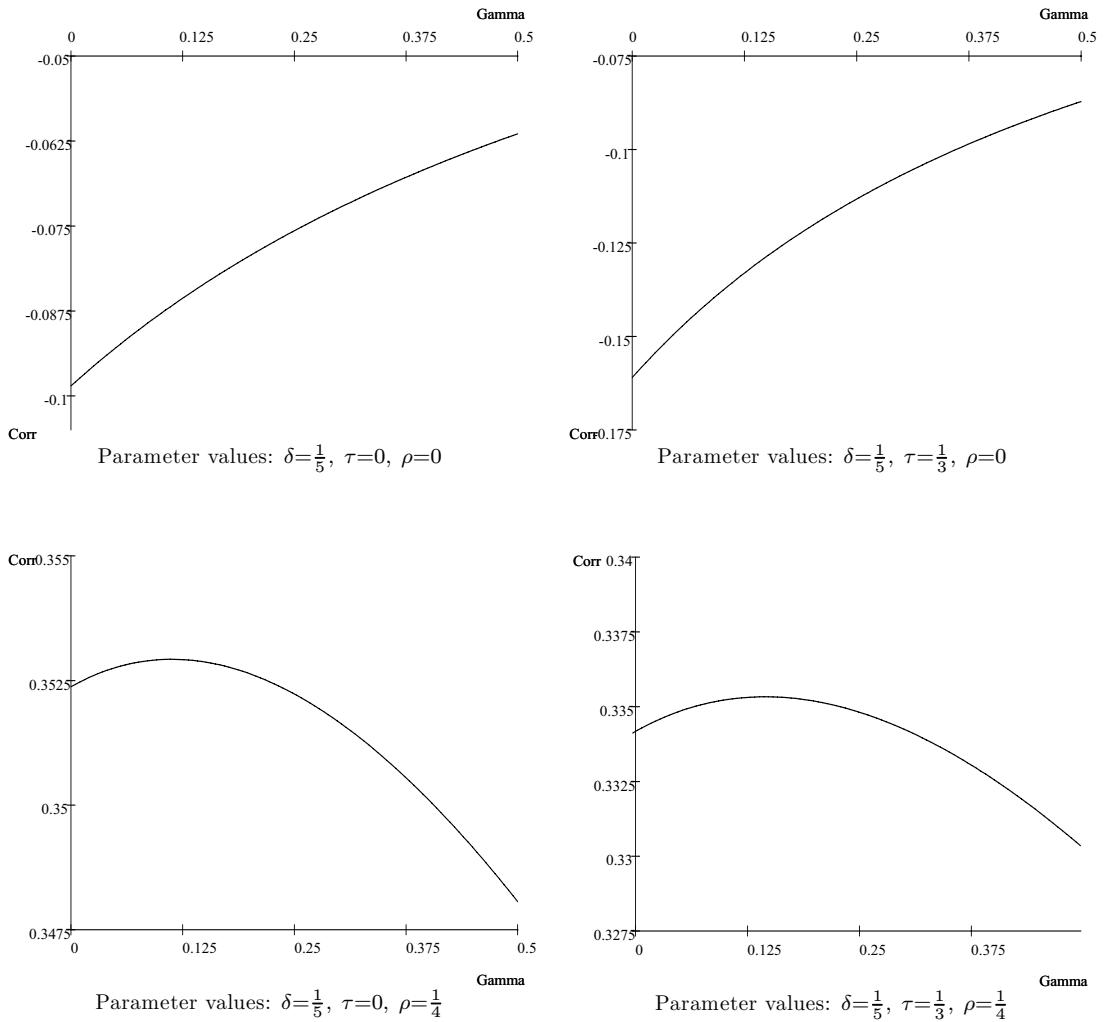


Figure 4: The effects of γ on the autocorrelation coefficient of wealth $\text{Corr}(s', s)$.

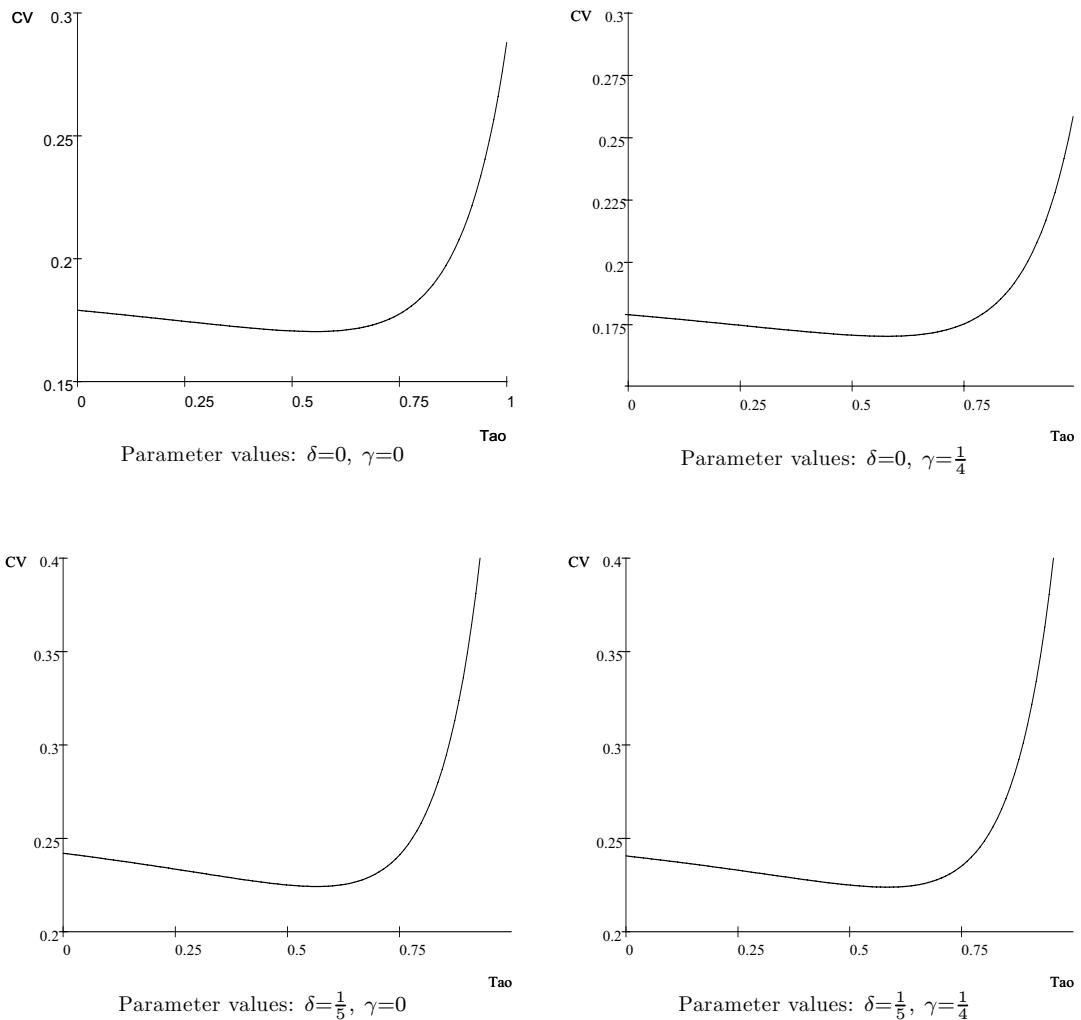


Figure 5: The effects of τ on on the coefficient of variation of wealth $CV(s)$.

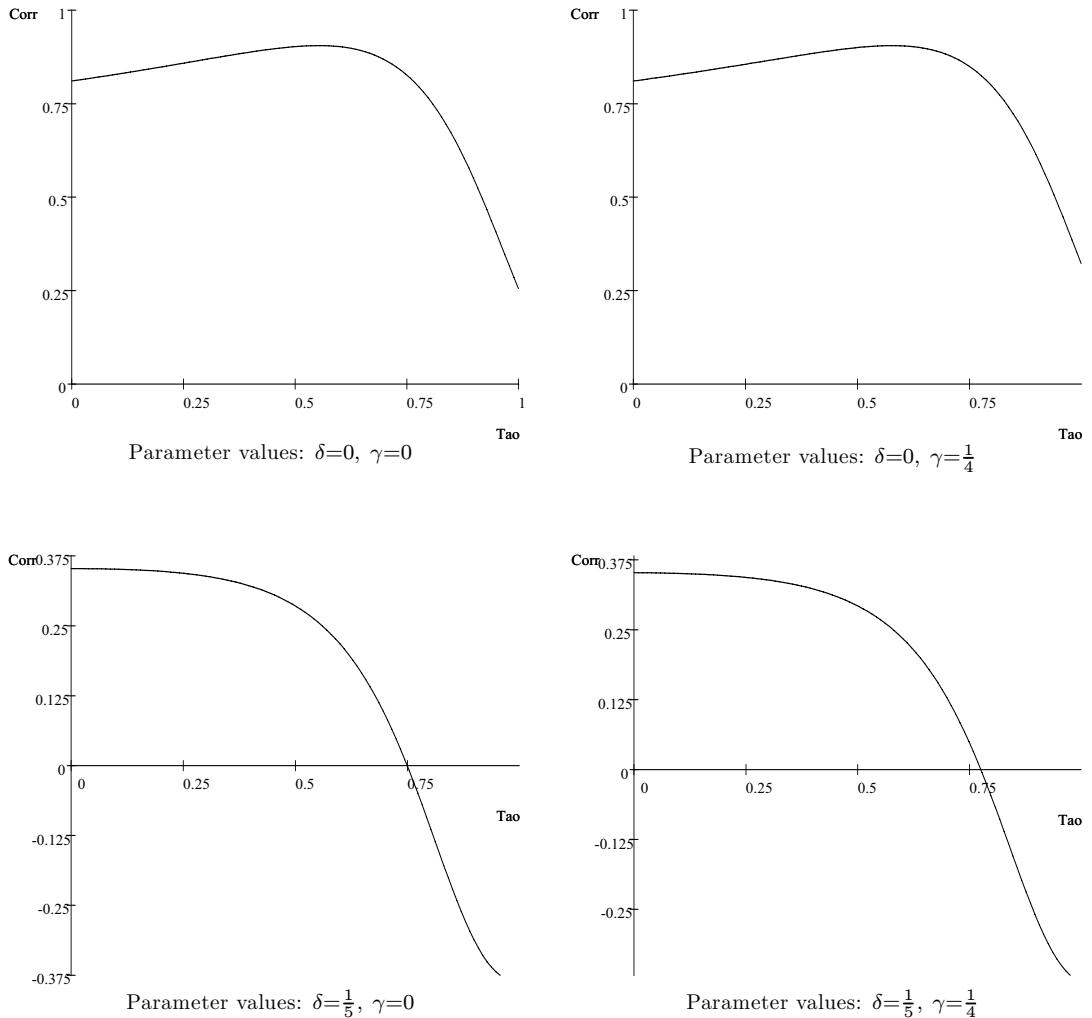


Figure 6: The effects of τ on the autocorrelation coefficient of wealth $\text{Corr}(s', s)$.