Fiscal Insurance and Debt Management in OECD Economies*

Elisa Faraglia[†], Albert Marcet[‡] and Andrew Scott[§]

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Abstract

Assuming the role of debt management is to provide hedging against fiscal shocks we consider three questions: i) what indicators can be used to assess the performance of debt management? ii) how well have historical debt management policies performed? and iii) how is that performance affected by variations in debt issuance? We consider these questions using OECD data on the market value of government debt between 1970 and 2000.

Motivated by both the optimal taxation literature and broad considerations of debt stability we propose a range of performance indicators for debt management. We evaluate these using Monte Carlo analysis and find that those based on the relative persistence of debt perform best. Calculating these measures for OECD data provides only limited evidence that debt management has helped insulate policy against unexpected fiscal shocks. We also find that the degree of fiscal insurance achieved is not well connected to cross country variations in debt issuance patterns. Given the limited volatility observed in the yield curve the relatively small dispersion of debt management practices across countries makes little difference to the realised degree of fiscal insurance.

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[†]Department of Economics, London Business School. efaraglia@london.edu

[‡]Institut d'Anàlisi Econòmica, CSIC and CEPR albert.marcet@iae.csic.es

[§]Department of Economics, London Business School and CEPR. ascott@london.edu

1 Introduction

This paper considers from an empirical perspective the performance of OECD debt management between 1970 and 2000. In doing so we seek to provide insights into three key questions i) what indicators should be used to assess the performance of debt management? ii) how does performance so measured vary across the OECD? and iii) do these variations have any systematic link to the structure of outstanding government debt and can conclusions be drawn concerning the optimal portfolio?

In order to monitor the performance of debt management it is necessary to evaluate outcomes against some criteria. In practice countries have set a number of different goals - for instance, to help support monetary policy in influencing short term interest rates, to provide a liquid market in risk free assets, to minimize borrowing costs, etc.[1]. In this paper we focus on one specific goal - namely the role of debt management in providing insurance against budget shocks so as to support optimal taxation or stabilize the debt-to-GDP ratio (or to minimise variations in the tax rate or in the debt-to-GDP ratio).

In focusing on this concept of fiscal insurance and the connection it implies between debt management and fiscal policy we are not asserting that this has been the main aim of government debt management over our period. In practice the majority of government debt managers make no explicit reference to fiscal policy and instead focus on aims broadly based around the notion of "minimizing cost subject to risk". Despite this policy lacunae we firmly believe debt management has an important role in supporting fiscal policy and hence focus our paper accordingly[2]. We also believe that even if other considerations are paramount in the operation of debt management the information contained in our measures of fiscal insurance are informative.

Our motivation for focusing on the interaction between debt management and fiscal policy comes from the optimal taxation literature. For instance, Lucas and Stokey (1983) show how, through issuing a complete range of Arrow Debreu contingent securities, the government can stabilise the

[1] See Missale (1999) for a survey.

[2] Obviously policy perspectives do change over time. Sargent and Wallace (1975) was clearly influential in emphasising the links between monetary and fiscal policy. The current literature on optimal fiscal policy and debt management may have the analagous potential.

excess burden of taxation and so minimise the distortionary costs of taxation. Angeletos (2002) and Buera and Nicolini (2004) show how this outcome can be achieved even in the absence of a complete set of contingent securities, by exploiting variations in the yield curve across different maturities of risk free securities. Critical to this approach is the idea that debt management can exploit a negative covariance between adverse fiscal shocks and bond prices. When governments receive news of adverse future fiscal trends a fall in bond prices helps maintain the intertemporal budget constraint - the market value of government debt equals the net present value of future primary surpluses with minimal need for tax rates to change. We term the ability of governments to insulate tax rates (and the excess burden of taxation) in this manner *fiscal insurance*. The aim of the paper is to evaluate the potential of a wide range of possible indicators of fiscal insurance, construct estimates of fiscal insurance using OECD data and then see whether there is any relationship between realized degrees of fiscal insurance and the composition of government debt.

Our notion of fiscal insurance is also relevant to the case where the aim of debt management is not to stabilize the excess burden of taxation but rather to stabilize or target the level of debt, possibly so as to maximize the probability of governments achieving stated fiscal rules (see Giavazzi and Missale (2004), Borenztein and Mauro (2004), Goldfajn (1998), Lloyd Ellis and Zhu (2001) for a similar motivation to debt management). We therefore use both optimal tax smooth and debt stabilization as possible motivations for why governments will be interested in the fiscal insurance potential of debt management and the various tests we propose.

A distinctive feature of our analysis is an empirical evaluation of observed practices rather than a normative analysis leading to a recommended optimal portfolio. A substantial literature exists using the optimal taxation paradigm to draw normative recommendations regarding the optimal portfolio structure for government debt (see, *inter alia,* Bohn (1990), Barro (1997), Missale (1999), Angeletos (2002), Lustig, Sleet and Yeltekin (2005), Nosbusch (2008)). While offering many insights this approach provides few implications for analysing existing portfolios. Further, as Faraglia, Marcet and Scott (2007) show the predictions of these analyses are often dramatically different from observed debt portfolio structures. In the absence of performance indicators for debt management there is no way of knowing whether these sharp differences in portfolio structure actually lead to a substantial difference in performance. By proposing indicators of fiscal insurance the aim of this paper is to try and remedy this gap and provide a less model specific approach to analysing debt management.

The simplicity of our performance indicators also contrasts with the simulation approach used in a number of recent studies by debt management organisations (see Bergstrom et al (2002), Bolder (2003), Pick and Anthony (2006)). The approach of these papers is to make a number of assumptions regarding the underlying structure of the economy, specify a policy rule for the setting of interest rates as well as a model for how these fluctuations impact on the yield curve and then perform a detailed analysis of alternative debt portfolios, normally with a focus on the average funding cost. By contrast our approach to ranking debt management strategies requires fewer assumptions and is based on purely empirical information.

The structure of the paper is as follows. In Section 2 we outline a number of potential measures of fiscal insurance, motivated both by theory and fiscal accounting identities. In Section 3 we consider the relationship between two different motivations for our focus on fiscal insurance - tax smoothing and debt stability. Section 4 then uses simulations of a stochastic dynamic general equilibrium model to investigate the performance of these fiscal insurance measures, their ability to discriminate between different debt management strategies and draws recommendations as to the most reliable measures. A key focus of our analysis is the importance of working with the market value of debt rather than the more readily available outstanding value of debt. Therefore Section 5 calculates the market value of debt for a range of OECD economies. Section 6 uses this data to calculate fiscal insurance measures for a range of OECD economies since 1970 and finds little evidence that debt management has provided much support to fiscal policy. This section also performs cross section regressions to estimate whether these differences in performance are connected with differences in debt composition but finds little evidence of any relationship. A final section concludes.

2 Measures of Fiscal Insurance

We propose two distinct classes of indicators for the performance of debt management. The first are motivated by implications from the tax smoothing literature while the second are based around the period by period budget constraint, namely :

$$\begin{aligned} MV_{t+1} &= \omega_{t+1} + R_{t+1} MV_t \\ &\text{or} \\ MV^*_{t+1} &= \omega^*_{t+1} + R^*_{t+1} MV^*_t \end{aligned} \tag{1}$$

where MV_{t+1} denotes the market value of government debt, ω_{t+1} denotes the primary deficit and R_{t+1} is the gross one period holding return on government debt i.e. including both coupon payments and capital gains. MV^* and ω^* denote the ratio of debt and deficits to GDP respectively and R^*_{t+1} is the growth adjusted interest rate $(R_{t+1}Y_t/Y_{t+1})$.

2.1 Tax Smoothing Perspective

A key insight of Lucas and Stokey (1983) is that time variation in the returns on assets held by government can reduce the need for changes in tax rates. This suggests debt management can be used to reduce the deadweight loss arising from distortionary taxation. Building on this insight, Bohn (1990) uses the tax smoothing framework of Barro (1979) and derives first order conditions that the returns on debt instruments have to meet to support optimal fiscal policy. In a similar vein Farhi (2005) outlines a government CAPM model based on a securities mean return and its covariance with the excess burden of taxation and the marginal utility of consumption. While these approaches have the advantage of focusing on individual funding instruments they also require joint assumptions concerning optimal taxation, functional forms and the unobserved excess burden of taxation. For these reasons we take a different approach to assessing the quality of debt management and use the results of Marcet and Scott (2004) and derive measures depending on the aggregate measure of debt.

Marcet and Scott (2004) analyze the case of a Ramsey planner who maximizes consumer welfare under two different bond market scenarios. In one case governments have access to complete markets and a full set of contingent Arrow Debreu securities while in the second case markets are incomplete with the government having access to only a one period risk free bond. The key differences induced by these different bond market structures are :

- Under incomplete markets, the market value of government debt shows *greater* persistence than other endogenous variables including taxes, government expenditure, output and the primary fiscal deficit. Under complete markets, the market value of debt shows *less* persistence than these other variables.

- Under complete markets and an optimising government the market value of government debt *declines* in response to *increases* in the primary fiscal deficit. By contrast, under incomplete markets, the market value of debt *rises* when the primary deficit increases.

  Complete markets is the case where the government fully exploits the risk characteristics of the securities available and so minimizes the distortionary costs of taxation. In assessing the statistical performance of debt management across the OECD we will therefore interpret better debt management as outcomes closer to the complete market case and so interpret findings of less peristence in debt relative to deficits as better debt management[3].

2.1.1 Persistence Tests

According to the above result we can use measures of the relative persistence of debt, compared to other variables, to capture the quality of debt management.

Let

$$P_y^k = \frac{Var(y_t - y_{t-k})}{kVar(y_t - y_{t-1})}$$

If $\{y_t\}$ is stationary and ergodic then $P_y^k \to 0$,as $k \to \infty$; if $\{y_t\}$ is i.i.d then $P_y^k = 1/k$ while if $\{y_t\}$ has a unit root then $P_y^k \to 1$. We propose as measures of debt management the following :

$$\Psi_{1k} = P_{MV}^k - P_\omega^k$$

$$\Psi_{2k} = (P_{MV}^k - P_\omega^k)/P_\omega^k$$

[3]The Marcet-Scott result hinges on the assumption of optimal fiscal policy. It is possible that despite poor debt management the government adjusts the fiscal deficit to reduce the persistence of debt. In this case the persistence of debt would be uninformative regarding the quality of debt management. However, our analysis mitigates this by examining the *relative* persistence of debt compared to the deficit.

where MV denotes the market value of government debt and ω the primary deficit. The only difference between these two measures is that Ψ_{2k} is normalized by the degree of persistence in the primary deficit. The greater our estimate of $\Psi_{ik}, i = 1, 2$ the worse the performance of debt management and negative values of Ψ_{ik} are indicative of complete market outcomes.

2.1.2 Impact Measures

The second discriminating feature between complete and incomplete markets is that under complete markets a persistent and unanticipated increase in the fiscal deficit leads to a *fall* in the market value of debt whereas under incomplete markets the market value increases. The intuition is straightforward. The market value of debt equals the expected present value of future primary surpluses. Under complete markets a persistent increase in government expenditure is *not* matched by an equivalent rise in taxes as debt management provides fiscal insurance. Instead bond prices fall so that the market value of debt matches the expected present value of the now reduced level of future fiscal surpluses. By contrast under incomplete markets governments have to raise taxes in response to adverse expenditure shocks and so raise the NPV of future primary surpluses leading to an increase in the level of debt. Denoting orthogonal shocks to the fiscal deficit by u_t^ω this suggests the following measure of the impact of fiscal deficits on debt

$$\mathrm{Im} = \frac{\frac{\partial MV_t}{\partial u_t^\omega}}{\frac{\partial \omega_t}{\partial u_t^\omega}}$$

i.e. the ratio of the impact of deficit shocks on the market value of debt compared to the impact of deficit shocks on the primary deficit. If this measure is negative (or 0) then debt management supports the complete market outcome. The more positive the number the less fiscal insurance debt management provides. Note that Im measures how the market value of debt *after the current deficit has been financed* responds to innovations in the current periods primary deficit. In other words, we examine how end of period debt responds to the within period deficit.

2.2 Debt Stabilization Perspective

The previous section focused on the role of debt management in supporting optimal fiscal policy as defined by tax smoothing. In this section we focus on issues of debt stability instead. A number of countries have recently used debt targets or ceilings as a guide to fiscal policy. Viewed from this perspective "debt stability" can be interpreted as whether the debt process is mean reverting (towards the target debt level) and if so how quickly it reverts. However the concept of debt stability is an ambiguous one and also covers the notion of a government that seeks to minimise fluctuations around a target value e.g $Min\ E_t(MV_{t+1} - T)^2$ where T is the target level for debt. It is also possible that the government may wish to "debt smooth" by, for instance, minimising the conditional variance of the market value of debt $-MinE_t(MV_{t+1} - E_tMV_{t+1})^2$.

To minimise the conditional variance of debt the government should choose its debt portfolio so that $Cov_t(\omega_{t+1}, R_{t+1})/\sigma_t(R_{t+1})\sigma_t(\omega_{t+1}) = -1$ where σ_t denotes the conditional standard deviation. In the case of a government seeking to minimise fluctuations around a target value of debt the optimal portfolio has to satisfy the condition[4] $Cov_t(\omega_{t+1}, R_{t+1}) + Var_t(R_{t+1})MV_t = -(E_tR_{t+1} - E_tr^*_{t+1})E_t(\omega_{t+1} + R_{t+1}MV_t - T)$ where r^*_{t+1} denotes the risk free rate of return. Minimising debt fluctuations therefore involves exploiting a negative covariance between the rates of return on debt and the primary deficit. The precise condition to be exploited however hinges on the exact aim of the government.

In what follows we pursue a weak implication of using debt management to stabilise debt. Because debt stability is capable of several different interpretations and because estimating conditional variances involves considerable time series structure[5] we focus instead on the unconditional variance of debt e.g $Var(MV^*_{t+1}) = Var(\omega^*_{t+1}) + Var(R^*_{t+1}MV^*_t) + 2Cov(R^*_{t+1}MV^*_t, \omega^*_{t+1})$. Our concept of fiscal insurance is the notion that debt management can help offset the impact of the primary deficit on the market value of debt. At a mimimum this involves creating a negative covariance between R^*_{t+1} and MV^*_{t+1} As we have shown above specific goals of debt stability imply achieving precise values for this covariance. However the essence of fiscal insurance is that the holding period returns on debt offset the impact of budget shocks on the level of debt. We therefore interpret this covariance as showing better debt management the more negative the correlation coefficient

[4]We are grateful to a referee for pointing out the relevance of these expressions for our analysis.

[5]In the expressions above, for instance, we require the conditional covariance of holding rates of return at the time of the deficit shock.

between :

$$\rho_{\omega^*,R^*MV^*} = \frac{Cov(\omega_t^*, R_t^* MV_t^*)}{\sigma_{\omega^*}\sigma_{R^*MV^*}}$$

where σ_x denotes the standard deviation of x.

Another way of capturing the ability of debt management to insulate debt levels from deficit fluctuations is to examine the relative volatility of the total deficit to the primary deficit. The more successful debt management is at exploiting a negative covariance between interest/coupon payments and the primary deficit then the less volatile the total deficit (including interest payments but excluding capital gains - ω_T^*) is relative to the primary deficit. Further the better the government is at exploiting a negative covariance between holding period returns and the primary deficit then the less volatile will be changes in the end of period market value of debt relative to the primary deficit. This suggests the following unconditional variances as additional measures of fiscal insurance:

$$\sigma_{\omega_T^*}/\sigma_{\omega^*}$$

$$\sigma_{\Delta MV^*}/\sigma_{\omega^*}$$

Our final measure focuses on the dynamic response of debt to shocks to the primary fiscal deficit. Using the period by period budget constraint (1) recursively we arrive at :

$$\begin{aligned} MV_{t+j}^* &= MV_t^* \prod_{k=1}^{j} R_{t+k} \frac{Y_t}{Y_{t+k}} + \sum_{n=1}^{j} \omega_{t+n}^* (\prod_{l=n}^{j-1} R_{t+1-l} \frac{Y_t}{Y_{t+1-l}}) \\ &= MV_t^* \phi_{0t+j} + \phi_{1t+j}(L)\omega_{t+j}^* \end{aligned} \tag{2}$$

where $\phi_{0t+j} = \prod_{k=1}^{j} R_{t+k} Y_t / Y_{t+k}$ and $\phi_{1t+j}(L)\omega_{t+j}^* = \sum_{n=1}^{j} \omega_{t+n}^* (\prod_{l=n}^{j-1} R_{t+1-l} Y_t / Y_{t+1-l})$. This equation defines the $t+j$ level of debt/GDP as equal to i) the level of debt at time t multiplied by the relevant compound rate of return ii) plus the sum of all intervening primary deficits grossed up by the relevant compound interest rate. We therefore have :

$$\frac{\partial MV_{t+j}^*}{\partial \varepsilon_{t+1}^{\omega^*}} = MV_t^* \frac{\partial \phi_{0t+j}}{\partial \varepsilon_{t+1}^{\omega^*}} + \frac{\partial \phi_{1t+1}(L)}{\partial \varepsilon_{t+1}^{\omega^*}} \omega_{t+j}^* + \phi_{1t+j}(L) \frac{\partial \omega_{t+j}^*}{\partial \varepsilon_{t+1}^{\omega^*}}. \tag{3}$$

In other words, the impact of deficit shocks ($\varepsilon_{t+1}^{\omega^*}$) on the level of debt can be decomposed into : (a) a term reflecting how interest rate charges on the original level of debt alter in response to such a shock $MV_t^*\frac{\partial\phi_{0t+j}}{\partial\varepsilon_{t+1}^{\omega^*}}$ (b) a term reflecting how the financing costs of future deficits alter because of the deficit shock ($\frac{\partial\phi_{1t+1}(L)}{\partial\varepsilon_{+1t}^{\omega^*}}\omega_{t+j}^*$) and (c) a term reflecting the impact of the shock on future fiscal deficits $\phi_{1t+j}(L)\frac{\partial\omega_{t+j}^*}{\partial\varepsilon_{t+1}^{\omega^*}}$. In the case where the return on government debt is independent of shocks to the economy (as in the case where the government issues only one period risk free debt and shocks are i.i.d, as in Aiyagari et al (2001))[6], then (3) reduces to

$$\frac{\partial MV_{t+j}^*}{\partial\varepsilon_{t+1}^{\omega^*}} = \sum_{k=1}^{j}\phi_{1t+j}^{j+1-k}\frac{\partial\omega_{t+k}^*}{\partial\varepsilon_{t+1}^{\omega^*}}. \tag{4}$$

where ϕ_{1t+j}^{j+1-k} is the $j+1-k$th coefficient in the lag polynomial $\phi_{1t+j}(L)$. In other words debt would only rise by the accumulated impact of the shock on future deficits with no offsetting effects through changes in bond prices. Comparing (3) and (4) we have a natural measure of fiscal insurance, namely

$$\Phi_j = \frac{\frac{\partial MV_{t+j}^*}{\partial\varepsilon_t^{\omega^*}}}{\sum\limits_{k=1}^{j}\phi_{1t+j}^{j+1-k}\frac{\partial\omega_{t+k}^*}{\partial\varepsilon_t^{\omega^*}}} \tag{5}$$

This statistic[7] is the ratio between the estimated change in the market value of debt in response to a deficit shock (as estimated from a VAR) and the increase in debt that would have occurred from cumulating the total effect on current and future deficits in response to the current deficit shock. If $\Phi_j^* = 1$ then debt levels rise by exactly the amount of the primary fiscal deficits accumulated over time in which case debt management offers no fiscal insurance. In the case where $\Phi_j^* = 0$ then debt shows no response to increases in the deficit because bond prices fall to offset the higher deficits and debt is stabilized. Note that in the case of complete markets and persistent shocks we know that debt levels *fall* on impact so that $\Phi_0^* < 0$ but then increases thereafter. A measure of Φ_j^* between 0 and 1 implies that the government achieves some degree of fiscal insurance - debt levels increase by less than the accumulated impact on the fiscal deficit as changes in interest rates offset the impact of shocks. Notice that it is also possible for $\Phi_j^* > 1$. In this case the debt level rises by more than the accumulated impact of fiscal deficits as interest rates move *adversely* in response to the shock.

[6] The market value of debt will vary with fiscal shocks both because the government issues explicitly state contingent debt or fixed coupon bonds whose market price varies endogenously with the economy.

[7] Because the coefficients in the lag polynomial $\phi_{1t+j}(L)$ depend on the growth adjusted interest rate and are time varying this measure of fiscal insurance is also time varying. To avoid this complication we estimate (5) evaluating $R_{t+j}Y_t/Y_{t+j}$ at its sample averages and denote this measure Φ_j^*.

This situation might occur for economies where as debt rises investors demand a higher risk premia so that funding costs actually rise when the deficit increases. Although in our analysis we focus on calculating Φ_j^* in terms of the response of debt and deficit to deficit shocks the approach can easily be extended to consider the degree of fiscal insurance provided against shocks to GDP, inflation, etc.

3 Tax Smoothing or Debt Stabilization?

The previous sections outlined a range of potential performance indicators for debt management drawn from two different motivations - tax smoothing, where the role of debt management is to minimise fluctuations in the excess burden of taxation, and debt stabilization, where debt management exploits negative covariances between bond prices and primary deficits to offset the impact of deficit fluctuations on the level of debt. Only if fluctuations in debt are correlated with fluctuations in the excess burden of taxation should we expect these motivations to produce the same assessment of debt management performance. In this section we perform simulations to consider the relationship between debt stabilization and optimal taxation and the implications for our two sets of indicators. Our aim is to investigate whether the indicators motivated by debt stability issues would also perform well in a tax smoothing context and more generally to understand the relationship between tax smoothing and debt stability and the role of debt management in linking the two.

In order to do this we take a canonical Real Business Cycle model where a Ramsey planner sets taxes to minimise their distortionary costs and does so in both a complete and incomplete market scenario. Later we will use these simulations to record the efficacy of our performance indicators but in this section we are focusing on the link between debt stability and debt management.

Our model economy consists of a consumer with utility function :

$$u(c_t, l_t) = \frac{c_t^{1-\gamma_1}}{1-\gamma_1} + B\frac{l_t^{1-\gamma_2}}{1-\gamma_2}$$

where c_t, l_t denote consumption and leisure respectively. We set B so that the share of leisure in the time endowment equals 30% on average, use a discount factor of 0.98 and set γ_1=1, γ_2=2. Output is given by $y_t = \theta_t(1 - l_t)$ and θ_t is a stochastic productivity shock. The resource constraint is $y_t = c_t + g_t$ with g_t denoting exogenous government expenditure. Model 1 is the case of no capital accumulation and Model 2 allows for capital. In this latter case we have $y_t = \theta_t k_{t-1}^{\alpha}(1 - l_t)^{1-\alpha}$ (where $\alpha = 0.4$) and $y_t = c_t + g_t + k_t - (1 - \delta)k_{t-1}$ where δ is the depreciation rate which we set equal to 0.05. The stochastic process $\{g_t, \theta_t\}$ is exogenous and Markov. We assume consumers get paid a competitive wage (in equilibrium equal to θ_t) and pay labour taxes to the government which are levied at a proportional rate τ_t on labor income.

Under complete markets there exists a full range of Arrow Debreu securities such that at time t there exists a spot market for claims contingent on all possible values of $\{g_t, \theta_t\}$. We denote by $b_t(\overline{g_t}, \overline{\theta_t})$ the quantity of bonds issued at $t-1$ and which pay out at time t contingent on the simultaneous occurrence of $\overline{g}$ and $\overline{\theta}$ and the price of such a bond is $p_t^b(\overline{g}, \overline{\theta})$. The market value of debt is therefore given by $vb_t \equiv \int b_{t+1}(\overline{g}, \overline{\theta})\ p_t^b(\overline{g}, \overline{\theta})\ d\overline{g}d\overline{\theta}$. Under these assumptions the government faces the budget constraint

$$b_t(g_t, \theta_t) + g_t - \tau_t\theta_t(1 - l_t) = \int b_{t+1}((\overline{g}, \overline{\theta})\ p_t^b(\overline{g}, \overline{\theta})d\overline{g}d\overline{\theta}$$

Under incomplete markets we consider the extreme case where the government can only issue one period risk free bonds[8] so that

$$b_t + g_t - \tau_t\theta_t(1 - l_t) = p_t^b b_{t+1}.$$

To solve our models for the optimal policy we make the standard Ramsey assumptions that governments have a fixed initial level of bonds and choose tax rates and government bonds so as to maximize consumer welfare. Both the consumer and the government observe all shocks up to the current period.

For the stochastic shocks we assume g follows a truncated AR(1), and θ_t a log AR(1) process e.g.

$$g_t = \begin{array}{ll} \overline{g} & \text{if } (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g > \overline{g} \\ \underline{g} & \text{if } (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g < \underline{g} \\ (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g & \text{otherwise} \end{array}$$

[8] We therefore assume exogenously incomplete asset markets. See Sleet and Yeltekin (2004) for an endogenous explanation founded on asymmetric information.

$$\log \theta_t = \rho^\theta \log \theta_{t-1} + \varepsilon_t^\theta$$

for $\varepsilon_t^g, \varepsilon_t^\theta$ i.i.d., mean zero and mutually independent. We assume $\varepsilon_t^\theta \sim N(0, 0.007^2)$, $\varepsilon_t^g \sim N(0, 1.44^2)$, $g^* = 25$, with an upper bound $\overline{g}$ equal to 35% and a lower bound $\underline{g} = 15\%$ of average GDP. We consider two sets of cases. In our first model both shocks are $i.i.d$ e.g. $\rho^g = \rho^\theta = 0$ but we also consider the case of highly persistent shocks when $\rho^g = \rho^\theta = 0.95$. We solve the models using the Parameterized Expectations Algorithm described in den Haan and Marcet (1990). We solve each model 1000 times and simulate for 200 observations but discard the first 150 observations and compute our statistics on the remaining data.

One possible and simple way of testing for debt stability is to estimate an AR(1) process for debt of the form :

$$MV_t^* = a + bMV_{t-1}^* + \varepsilon_t.$$

This autoregressive representation for debt can be thought of as consistent with a fiscal rule with a target level $a/(1-b)$. If $b \geq 1$ then debt is unstable and has no well defined average and will show explosive dynamics (if b>1). In the case of $b = 0$ then debt shows no autoregressive behaviour and aside from temporary shocks ε_t is stable with a value a.

Figures 1a-c shows the Monte Carlo distribution of estimates of b obtained by OLS regression on simulated data of 40 periods (similar to the length of sample with our OECD data). For all three models (without capital and i.i.d shocks, no capital and persistent shocks, capital and persistent shocks) we find that incomplete markets leads to substantially higher estimates of persistence in debt e.g higher b. For instance, in the case of no capital and persistent shocks we have for complete markets $E(b^{OLS}) = 0.814$ whereas under incomplete markets $E(b^{OLS}) = 0.988$. For i.i.d shocks the difference is even more stark - -0.008 for complete markets and 0.826 for incomplete markets. These results clearly illustrate that under incomplete markets, regardless of the persistence of shocks, it is optimal for debt to show large and substantial persistent fluctuations. When shocks are persistent then even under complete markets it is optimal for debt to show persistent fluctuations - maintaining a constant value of debt is not a feature of optimal policy under complete markets. However, while debt fluctuations are part of optimal policy it is also clear that under complete markets these fluctuations tend to be stable whereas under incomplete markets there is often evidence of apparent instability. In the case of Model 1 and persistent shocks in only 0.4% of cases with complete markets do we find evidence that debt is explosive or "unsustainable" e.g estimates of $b > 1$. By contrast under incomplete markets this occurs in 36.4% of cases. When we add capital accumulation to the model then the relevant proportions are 0.8% for complete markets and 38.7% for incomplete markets. It should be stressed that these findings that debt is on an "unsustainable" path under incomplete markets are misleading. By construction debt is sustainable and is indeed following an optimal path. It is simply that under incomplete markets debt is used as a buffer and during these periods displays such persistent and long lasting swings that simple AR models suggest data is following a non-stationary path[9].

From these simulation results we draw the following conclusions:

- Tax smoothing does not produce complete debt stability even in the case of complete markets. Therefore evidence of substantial fluctuations in debt does not rule out the existence of complete markets.

- Complete markets produces less volatility in debt compared to incomplete markets *for a given economic structure.* Therefore changes in debt issuance practice that lead to lower debt fluctuations reflect better fiscal insurance and a move towards complete markets.

- Debt management can have an important role in helping to achieve fiscal rules involving debt targets (specified levels for $a/(1-b)$). The better is debt management e.g. the closer policy is to complete markets, then the quicker debt is mean reverting and the less persistent are swings in debt.

- Under incomplete markets debt displays large and long term fluctuations which may appear "unsustainable" and debt based fiscal rules are likely to prevent optimal policies being followed.

  These results suggest that regardless of whether the aim of debt management is to smooth taxes or stabilize the level of debt our performance indicators will rank different portfolios in

[9] It should also be stressed that Bohn (2005) shows how the non-stationarity of debt alone is not sufficient to infer that debt is unsustainable. The intertemporal budget constraint places a restriction on the *relative* orders of integration of debt and the primary deficit rather than imposes that debt is stationary.

the same relative order. However for many of the measures inspired by debt stability there is a normalization issue such that the *absolute* size of a statistic may not be indicative of the presence or otherwise of high quality debt management. However, for a given economic structure we can interpret lower measures for our performance indicator as reflecting better quality debt management from both a tax smoothing and debt stability perspective.

4 Evaluating Performance Indicators

In this section we use simulations of our models to assess the ability of our 7 different indicators of debt management performance (3 motivated by tax smoothing - two relative persistence measures and the impact test; 4 motivated by debt stabilization - correlation coefficient between deficits and bond prices; two measures of relative standard deviations and the dynamic measure of the extent to which debt is insulated against deficit shocks). We do so by solving under both complete and incomplete markets and then examine the ability of these indicators to discriminate successfully between the two cases. We know from the previous section that for a given economy complete markets produce debt that is more swiftly mean reverting than under incomplete markets. Therefore, indicators that successfully detect the difference between complete and incomplete markets will also discriminate debt management policies that produce more stable levels of debt.

In performing these simulations we abstract from two key issues. Firstly, we consider only real and not nominal denominated debt and secondly we consider the case where the government is able to implement the Ramsey outcome due to full commitment. In practice governments issue nominal debt (and in our later empirical results we use real data for debt, deficits and GDP). As suggested by Benigno and Woodford (2003) this raises the possibility that even if governments issue fixed rate nominal securities through variations in inflation the complete market outcome can be achieved. In our empirical analysis we allow for this possibility by examining the role of inflation and inflation volatility in influencing the degree of fiscal insurance achieved. Introducing nominal debt also opens the possibility that the existence of nominal imperfections will require governments to trade off the tax smoothing possibilities of inflation against the need to reduce other distortions. If this were the case then we might well find in the data that governments achieve limited fiscal insurance through their debt management practices - other aims conflict with the ability to deliver fiscal insurance. By focusing on the case of full commitment we also abstract from the issues analysed in Lucas and Stokey (1983) of how debt management may be needed to resolve time inconsistency issues. Further reputational issues may mean that governments face a trade off between tax smoothing considerations and cost minimisation. Once again this may lead to governments achieving limited fiscal insurance due to the presence of other aims - in this case cost minimisation. A similar problem arises from our choice of a standard representative agent Real Business cycle model in our simulations. If in reality the price of risky assets does not reflect agents risk aversion then debt management will once again require a trade off between cost minimisation and tax smoothing - an issue our simulations abstract from.

The results of our simulations are shown in Tables 1a-c - the first two tables are for the case of no capital accumulation and i.i.d and persistent shocks respectively and Table 1c is for the case with capital and persistent shocks. We show results for both complete and incomplete markets and quote the average value of each test statistic as well as the 25th and 75th quartile values from the distribution of the simulated test statistics.

i) The persistence measures Ψ_{1k} and Ψ_{2k} show across all three simulations an excellent ability to discriminate between complete and incomplete markets. Not only are the mean value of the statistics distinct between complete and incomplete markets but the quartile values also suggest that sampling error is not an issue. Ψ_{1k} takes low values across all three complete market simulations with values of around zero. However the value of Ψ_{1k} in the incomplete market simulations depends on the details of the model. Understandably the persistence of debt is a function of the persistence of shocks in the economy and the persistence of the transmission mechanisms at work. The normalization that is involved in Ψ_{2k} reduces this issue substantially and leads to less variation in estimates across the incomplete market scenarios.

ii) Implementing the impact measure, Im, requires identifying fiscal shocks. We do so using a Cholesky decomposition on a trivariate VAR ordered as the primary deficit/GDP, GDP growth and debt/GDP. Although we report results for this ordering and VAR we also tested our results were robust to the ordering and the inclusion as a fourth variable the holding period return. The impact measure also performs well in discriminating between the complete and incomplete market case. As shown in Marcet and Scott (2004) in the case of persistent shocks the market value of debt falls in response to adverse shocks and so Im < 0 whereas under incomplete markets debt rises so Im > 0. This is confirmed in our simulation results in Tables 1b and c. Table 1a shows the case of i.i.d shocks but although the impact effect is not negative it is essentially zero and still offers a tool to discriminate between complete and incomplete markets.

iii) The correlation statistic ρ_{ω^*,R_t^*} does discriminate successfully between complete and incomplete markets within the context of each model but the values of the correlation coefficients depends strongly on the underlying features of the model. This variation across the model simulations reduces its value as a measure of the absolute quality of debt management although its ability to discriminate between complete and incomplete markets shows its use as a relative measure. There are also signs that the distribution of the correlation coefficient is quite diffuse under incomplete markets which again makes inference difficult.

v) The standard deviation measures $\sigma_{\omega_T^*}/\sigma_{\omega^*}$ and $\sigma_{\Delta MV^*}/\sigma_{\omega^*}$ show a mixed performance. Overall the indicator $\sigma_{\omega_T^*}/\sigma_{\omega^*}$ does manage within each simulation to distinguish complete markets from the incomplete market outcome. However, the differences are often small between complete and incomplete markets and once again the absolute value of the test statistic varies considerably across the three models meaning it is unlikely to be a useful statistic when applied to OECD rather than simulated data. The indicator based on the volatility of debt to deficits, $\sigma_{\Delta MV^*}/\sigma_{\omega^*}$, also performs unreliably. In the case of complete markets and persistent shocks the statistic clearly discriminates between the market settings - however in this case the volatility of debt is huge under complete markets as debt falls sharply in value in response to an adverse shock. However in the case of i.i.d shocks the market value of debt is more volatile under incomplete markets. Therefore without knowing the persistence of the shocks $\sigma_{\Delta MV^*}/\sigma_{\omega^*}$ cannot be used to distinguish between complete and incomplete markets.

vi) Our final proposed indicator of debt management was Φ_j - based on the estimated IRF of debt to the cumulative IRF for the primary deficit using the same VAR as for Im. Across all simulations this statistic provides a reliable test for complete versus incomplete markets. This indicator essentially combines information in Ψ_{2k} and Im and so not surprisingly performs well. Notice that in the case of persistent shocks under incomplete markets Φ_j is initially extremely negative and then increases slowly but for all lags and also in the case of i.i.d shocks the statistic discriminates clearly between complete and incomplete markets, as can be seen by looking at the quartile values.

Summarizing across these measures we have that $\sigma_{\Delta MV^*}/\sigma_{\omega^*}$ is a poor indicator of fiscal insurance; ρ_{ω^*,R_t^*} and $\sigma_{\omega_T^*}/\sigma_{\omega^*}$ perform better in the sense that for a given simulation they can discriminate between complete and incomplete markets but comparing across all three simulations suggests they too are unreliable measures. By contrast Ψ_{2k}, Im and Φ_j prove themselves to be reliable indicators in an absolute sense of the difference between complete and incomplete markets. As a consequence it is these measures we will focus on in our empirical analysis.

5 Data

Key to our concept of fiscal insurance are fluctuations in the market value of debt. This creates an empirical problem as official published data on government debt record the outstanding value of debt to be repaid. To construct market value data we use the approximation suggested by Butkiewicz (1983), namely :

$$MV_t \approx B_t \frac{1 + n_t c_t}{1 + n_t y_t}$$

where B_t is the outstanding value of government debt, n is the average maturity of debt, c the average coupon rate and y the average redemption yield on government debt. A data appendix lists how we use this approximation across OECD countries but the majority of data is taken from the OECD's *Central Government Debt Statistical Yearbook* and Missale (1999) with the yield data taken from Global Financial Data. For Australia, Germany, Italy, Netherlands, UK and US we are able to construct a series for 1970-2000, for Canada and Belgium from 1976, for Ireland from 1977, Austria from 1981 and Norway from 1984. All variables are deflated using the GDP deflator.

Table 2 reports a number of statistics to help gauge the differences between our estimates of the market value of debt and the official outstanding value series and gives some crude indication of the extent of fiscal insurance achieved. Given that $\frac{1+n_t c_t}{1+n_t y_t}$ should be stationary it is not surprising that the levels of the two different debt measures are strongly correlated and that they share the same order of integration (see the Unit root tests). The two series are also very strongly correlated in differences which suggests that fluctuations in bond prices offer limited fiscal insurance. Both debt series show a strong negative correlation in first differences between debt and GDP e.g. debt changes behave counter cyclically. The correlation between changes in the debt/GDP ratio and the primary deficit are invariably positive. That the outstanding value of debt should increase with the primary deficit is to be expected but the fact that the market value does as well suggests limited fiscal insurance. However, in the majority of cases the market value of debt shows a weaker correlation with the primary deficit than the outstanding value of debt suggesting at least some fiscal insurance has been achieved.

6 Determinants of OECD Debt Management Performance

Table 3 shows estimates of our preferred measures[10] of fiscal insurance. While there is clearly variation across countries the general message of Table 3 confirms the findings of Marcet and Scott (2004) based on US data - the behaviour of OECD debt and deficits is not consistent with the complete market outcome. Examining the relative persistence of debt compared to deficits and the impact effect of deficit shocks shows no evidence at all in favour of complete market outcomes - debt displays substantially more persistence than deficits. However, the main point of Table 3 is not to adjudicate between complete and incomplete markets but to assess the relative performance of debt management across countries. Focusing on the relative persistence measures Ψ_{1k} and Ψ_{2k} we find that at all horizons Norway performs well whereas Belgium and Netherlands tend to be the worst performers. At horizons of a year we also find evidence of relatively good performance in Germany, Ireland, Australia and Austria although performance in these countries rapidly deteriorates. From the impact measure there is once more evidence of good performance in Norway and Australia and also interestingly Netherlands. Belgium once more shows relative poor performance as do Austria and UK. The relative dynamic measure Φ^* again confirms the good performance of Norway and the bad performance of Belgium and suggests that at longer lags the

[10] Our results are unaffected if we also include all 7 performance measures.

performance of France and particularly Australia improves while that of Netherlands and Ireland deteriorates. To provide greater insight into the differences in debt management across countries Table 4 shows summary statistics. Norway tends to score well across our performance indicators but in terms of the proportion of fixed rate debt issued and how much was short term and how much longer term its portfolio structure is almost exactly the average across all countries. The two areas where Norway is distinctive is the amount of foreign currency debt it issues and the average maturity of its debt, where only the UK issues longer average maturity. In addition to these features of its debt structure Norway also stands out because throughout the period it tends to run fiscal surpluses and reduces it level of debt. Belgium and the Netherlands tend to be assessed poorly by our indicators and from Table 4 their debt structure stands out in having above average issuance of fixed rate debt and more long term debt but despite this the overall debt structure has below average maturity - suggesting larger issuance of medium term debt.

While Table 4 offers some insights into differences in debt structure and how this may account for the differences in Table 3 we now consider more formally whether variations in achieved fiscal insurance is linked to cross country differences in debt composition and macroeconomic performance. We do this by estimating a relationship of the form :

$$P_i = \alpha' X_i + \beta' Z_i + u_i \tag{6}$$

where P_i denotes one of our performance indicators for debt management, X_i denotes a vector of macroeconomic variables (e.g. the average level of debt or deficit in a country, inflation or GDP growth) and Z_i a vector of variables describing the portfolio composition of country i (e.g. proportion of indexed debt, proportion of variable rate debt etc.). Finding significant variables in Z_i is critical if we are to make recommendations for debt management. In estimating (6) we face severe data limitations. For reliable inference our performance measures need to be identified over our whole sample and so we cannot use a time series approach in estimating (6). This problem is reinforced by the fact that many of our key variables.such as maturity of debt, do not change much over time. As a consequence we are reliant on cross sectional variation to identify the determinants of debt management. We therefore estimate the equation in cross sectional form. Further, we only have data for 12 OECD countries so we cannot estimate this equation whilst simultaneously including a broad range of macroeconomic variables as well as the debt composition indicators at our disposal. Therefore we follow a sequential approach in Tables 4 to 6 and show results from regressing P_i first on macro variables, then separately on a set of variables capturing broad characteristics of the debt portfolio and then finally on detailed statistics on the debt composition[11].

Table 5 shows the results of regressing our performance indicators on the average maturity of debt, the average debt and deficit to GDP ratio, real GDP growth and inflation. The point estimates suggest that issuing longer maturity debt helps improve fiscal insurance although the effect is nowhere significant. Similarly large values of debt tend to worsen fiscal insurance (although again not at any conventional significant levels) whereas surprisingly a large primary deficit leads to better performance and this latter effect sometimes is of borderline significance. The results regarding inflation are mixed in terms of the sign of the impact but always statistically insignificant which is the same as for GDP growth although here the point estimates suggest faster growth helps improve fiscal insurance.

Table 6 shows results of regressing our performance indicators on average maturity and the proportion of fixed rate, foreign currency denominated and indexed debt. Once again the regressions reveal few statistically significant effects. The point estimates suggest that foreign currency debt may contribute to fiscal insurance but the only statistically significant finding is that higher levels of indexed debt improve fiscal insurance. Table 7 adds the proportion of short and long run debt and continues to find few results of significance, there is some very weak evidence that longer term debt and indexed debt helps achieve better fiscal insurance but the explanatory power of the equations are very poor. Similar results were found irrespective of the combination of variables used in the regression

Summarizing these results it seems there is very little relationship between observed cross country differences in debt structure and the degree of fiscal insurance achieved. Only occasionally are any variables significant and even then not in a consistent manner and the R^2 from all regressions is invariably very poor. Whether the government issues short, medium or long; fixed, variable or

[11] Some governments have made increasing use of derivatives in manging the debt. The consequence is that the reported debt statistics may not capture the effective maturity structure of debt. This may contribute to the weak statistical evidence we find although in the absence of reliable data it is impossible to comment on the importance of this bias.

indexed there seems to be little systematic connection with the degree of fiscal insurance achieved. It is of course possible that our inability to find a relationship between our measures of fiscal insurance and debt structure is due to the poor power of the former, although our simulation results in Section 4 suggest otherwise. With such limited data multicollinearity may be another reason for weak significance levels although not for the overall explanatory power of the regressions.

Our empirical results point to two facts - the first is that governments have achieved limited fiscal insurance and the second is that there is little link between variations in acheived fiscal insurance and the structure of debt issued. The first point could be due to many reasons - for instance that governments wish to smooth taxes but the existence of incomplete markets and the lack of contingent debt available means fiscal insurance is hard to achieve. However other possibilities also exist. For instance, that governments may focus on cost minimisation rather than tax smoothing or that concerns over time inconsistency and inflation control pin down the optimal debt composition. Our empirical analysis is unable to determine the relevance of these other aims - all we can conclude is that potentially pursuing these aims comes at a cost in terms of fiscal insurance.

To better understand our second finding - a weak relationship between debt structure and achieved fiscal insurance - it is useful to reconsider the results of Angeletos (2002) and Buera and Nicolini (2004). These authors show how even in the case where governments issue risk free bonds of different maturities it is still possible to achieve the level of fiscal insurance attained by the complete market outcome by exploiting shifts in the term structure of interest rates. Assume that fiscal policy follows a two state Markov process and that the government can issue a short and a long bond. Let the net present value of future primary surpluses be denoted z_H when the Markov process for the deficit is in a high state and z_L when the low state is realized. Bond prices are denoted p_i^j for $j = L, H$ and i=1,2 where 1 denotes a short bond and 2 a long bond. The complete market outcome is achieved by setting

$$\begin{aligned} z^H &= p_1^H b_1 + p_2^H b_2 \\ z^L &= p_1^L b_1 + p_2^L b_2. \end{aligned}$$

Without loss of generality let N denote the total number of bonds issued by the government i.e. $b_1 + b_2$ and let $b_1 = \phi b_2$ where ϕ can be either negative or positive. Using these assumptions and the fact that each Markov state is equally likely we can use these equations to derive

$$\frac{\mu_z}{\mu_{p_1} N} = 2 + \frac{1}{1+\phi}\{\frac{p_2^L - p_1^L}{p_1^L} + \frac{p_2^H - p_1^H}{p_1^H}\}$$

where μ_x denotes the mean of x. The usual approach is to calibrate with actual data the terms reflecting the slope of the yield curve $\frac{p_2^L - p_1^L}{p_1^L}, \frac{p_2^H - p_1^H}{p_1^H}$ and the terms μ_z and μ_{P_1} and then derive the relative portfolio share ϕ. As noted by Buera and Nicolini (2004) and Faraglia, Marcet and Scott (2007) the limited volatility in the slope of observed term structures requires the government to set $\phi \approx -1$ and N large and so the optimal portfolio is to issue a large amount of long term debt and hold a large negative short position in order to magnify the limited shifts in the yield curve to match $\frac{\mu_z}{\mu_{p_1} N}$. The magnitude of these positions are large - frequently requiring very large multiples of GDP to be held as assets or issued as debt. However, amongst our sample of OECD economies b_1 and b_2 are both positive (and on average $\phi \approx 0.4$). Issuing positive amounts of short and long term debt can only be the optimal issuance policy if the term structure displays fluctuations an order of magnitude greater than that observed. Given the limited volatility in observed term structures even substantial differences in b_1 and b_2 around these positive values will make little contribution towards achieving the complete market outcome of $\frac{\mu_z}{\mu_{p_1} N}$. It is for this reason that despite the fact countries differ in both their debt structure and the level of fiscal insurance achieved we are unable to find a reliable relationship between the two. The flatness of the yield curve and its limited volatility mean that the narrow range of different debt issuance that is witnessed across the OECD countries is not sufficient to deliver substantial differences in fiscal insurance.

This in turn raises a further substantive research issue - why do governments tend to avoid the "extreme" portfolios that a normative optimal taxation analysis proposes? Instead, as shown in Missale (1999) and Faraglia, Marcet and Scott (2007) governments tend to issue positive amounts of debt at most maturities and the maturity structure of debt changes smoothly as maturity changes. This suggests once more either the presence of non-tax smoothing motives by governments or the existence of numerous other constraints that affect the issuance of debt and that need to be accounted for. A better understanding of these factors would help shed light on why governments pursue other objectives than fiscal insurance. If small variations in debt structure affect fiscal insurance insignficantly and if other constraints restrict governments issuing more extreme portfolios then it is perhaps understandable that cost minimisation or other objectives may dominate.

7 Conclusion

A growing literature investigates the role of debt management in supporting fiscal policy, what we term "fiscal insurance". We propose a battery of tests with which to assess the quality of debt management and using Monte Carlo simulations suggest that performance indicators based on the relative persistence of debt are most reliable. Using data on the market value of government debt between 1970 and 2000 we find substantial evidence to support Marcet and Scott (2004) in their finding that bond markets are incomplete. Although we do detect differences in the actual quality of debt management across nations (with Norway performing the best and Belgium and Netherlands the worst) we find little evidence to link these variations in fiscal insurance to differences in debt structure. We argue that this is because relative yields show little variation and so minor variations in debt composition have little effect on fiscal insurance. Given these findings achieving better fiscal insurance will require either holding more extreme portfolio positions, issuance of new forms of contingent securities or magnifying existing positions through the use of derivatives. Our lack of significant empirical results also provides a possible justification for why traditionally debt management offices have focused on objectives other than fiscal policy in deciding their debt issuance policy.

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Data Appendix

In order to construct market value data a number of different data sources were used and approximations required. The following summarises these calculations.

Australia 1980-2000. The outstanding debt series used was provided by the Reserve Bank of Australia and is non-official holdings of marketable commonwealth government securities. The maturity series was also provided by the RBA. Yield data was from the Global Financial Database. Coupons were estimated as gross general government interest payments (SourceOECD) divided by gross general government debt.

Austria 1980-2000. The outstanding value of government debt is from the OECD as is the maturity data (*Central Government Statistical Yearbook).* The yield is taken from Global Financial Database and the coupon is estimated as gross interest payments (OECD) divided by gross outstanding debt.

Belgium 1980-2000. The data is constructed as above and was also calculated using Missale's (1999) maturity data.

Canada 1980-2000 As above.

Germany 1980-2000. As above.

Ireland 1980-2000 As above but using Missale's (1999) maturity data updated with information supplied by Central Bank of Ireland.

Italy 1980-2000 As for Belgium.

Netherlands 1980-2000 As above.

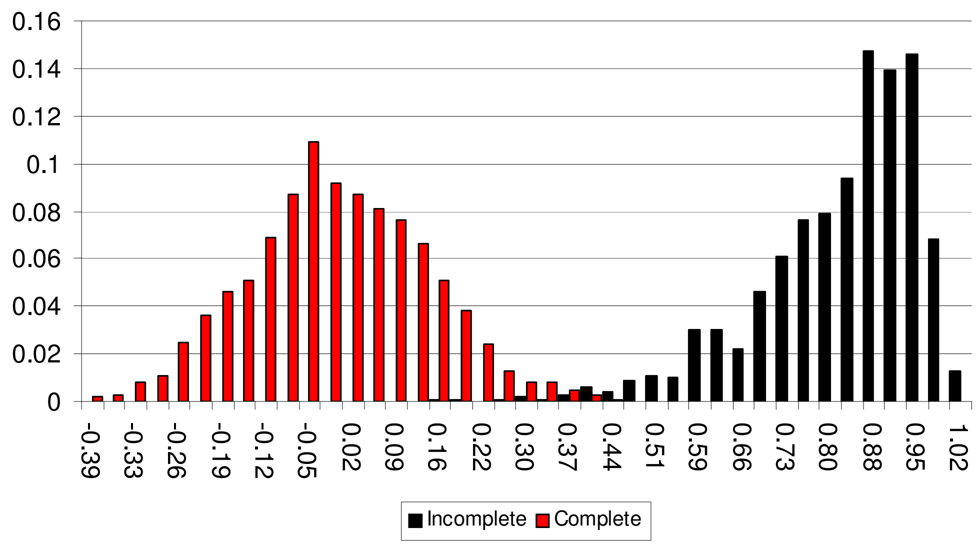
Norway 1982-2000 As above.

UK 1980-2000 As above but where the outstanding value of debt is supplied by the Debt Management Office.

US 1980-2000 As for Belgium.

All macroeconomic data was taken from the OECD's *SourceOECD* database and all data series are converted into real terms by dividing by the GDP deflator. The data is net central government debt and includes foreign currency debt, fixed and variable rate debt as well as nominal and indexed but excludes non-marketable debt. A spreadsheet containing full documentation and details for how each series was constructed is available on request.

Figure 1a: Distribution of OLS Estimator for b, Model 1, i.i.d Shocks



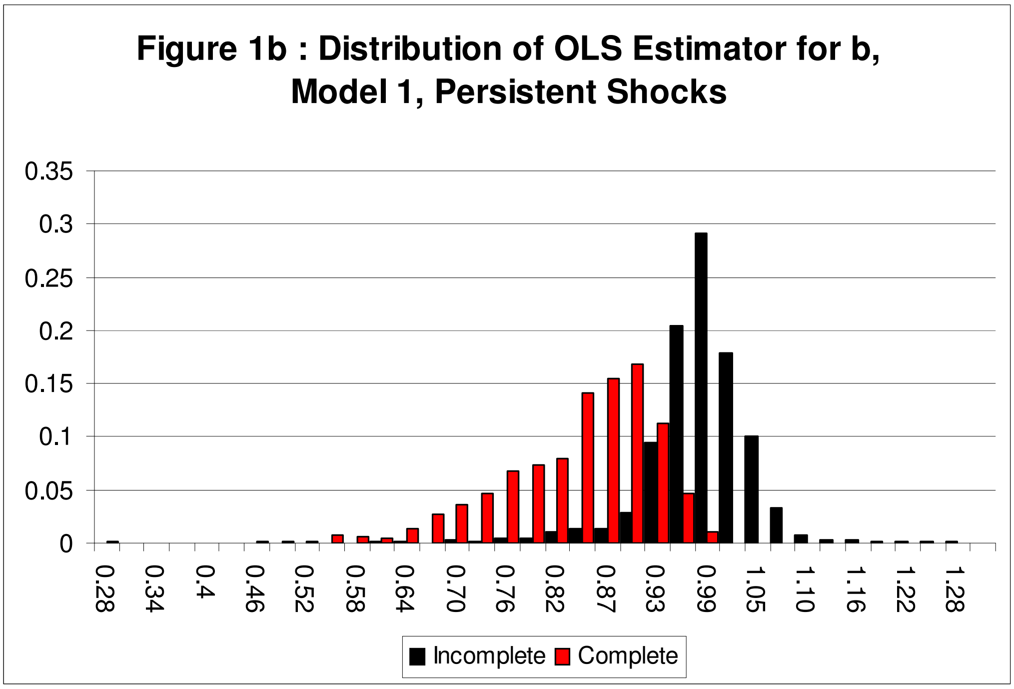


Figure 1b : Distribution of OLS Estimator for b,
Model 1, Persistent Shocks
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.28
0.34
0.4
0.46
0.52
0.58
0.64
0.70
0.76
0.82
0.87
0.93
0.99
1.05
1.10
1.16
1.22
1.28
Incomplete
Complete

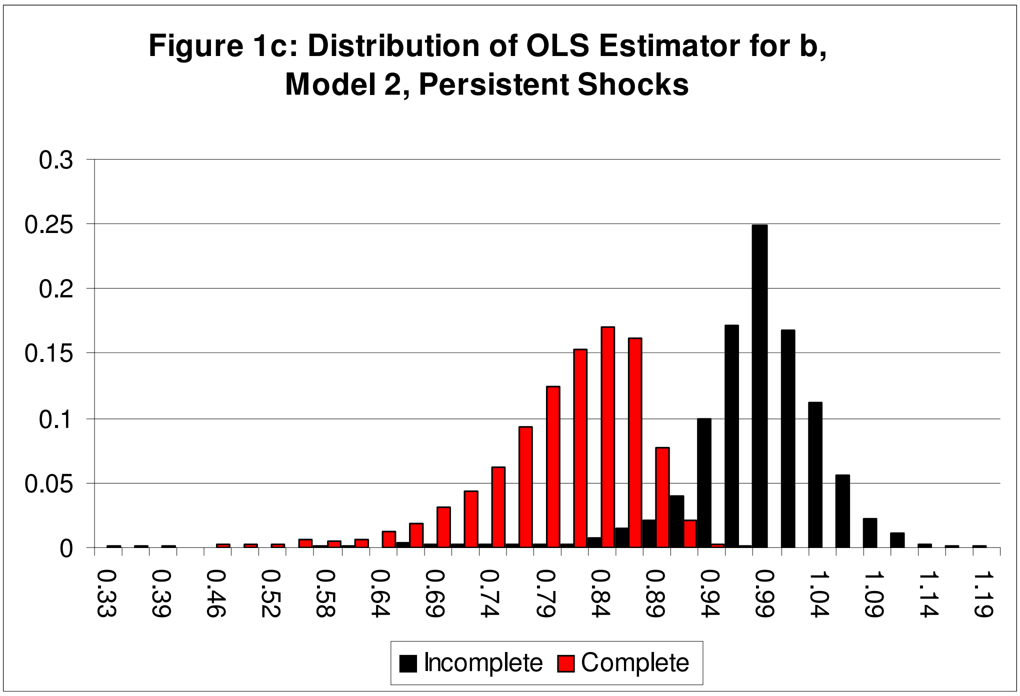


Figure 1c: Distribution of OLS Estimator for b,
Model 2, Persistent Shocks
0.3
0.25
0.2
0.15
0.1
0.05
0
0.33
0.39
0.46
0.52
0.58
0.64
0.69
0.74
0.79
0.84
0.89
0.94
0.99
1.04
1.09
1.14
1.19
Incomplete
Complete

.

	COMPLETE			INCOMPLETE		
	aver	**25%**	**75%**	**aver**	**25%**	**75%**
$\Psi_{1,3}$	0.004	-0.042	0.051	0.389	0.209	0.519
$\Psi_{1,5}$	0.002	-0.026	0.028	0.446	0.223	0.608
$\Psi_{2,3}$	0.035	-0.116	0.167	1.161	0.636	1.524
$\Psi_{2,5}$	0.028	-0.123	0.154	2.200	1.171	2.877
Im	0.008	0.006	0.009	1.033	1.010	1.057
$\rho_{\omega,R}$	-0.215	-0.423	-0.022	-0.042	-0.144	0.059
$\sigma(\omega_T)/\sigma(\omega)$	0.993	0.986	1.000	1.041	0.996	1.070
$\sigma(\Delta MV)/\sigma(\omega)$	0.018	0.017	0.020	1.041	0.996	1.070
Φ^*_1	0.000	-0.002	0.002	0.752	0.590	0.911
Φ^*_3	0.000	-0.002	0.001	0.721	0.558	0.907
Φ^*_5	0.000	-0.001	0.001	0.707	0.523	0.901
Φ^*_{10}	0.000	0.000	0.000	0.793	0.467	0.932

Table 1a - Assessing Debt Management Performance Statistics - i.i.d shocks and no capital accumulation

Results show outcomes from 1000 simulations of 200 periods where first 150 periods are discarded and statistics calculated on final 50 observations. "Average" denotes the average of each statistic over all 1000 simulations, 25% shows the bottom quartile outcome and 75% the third quartile. Ψ_{ij} $i = 1, 2$ is the persistence test described in the text evaluated at lag j. Im is the impact measure comparing the effect of a shock on debt and the deficit. $\rho_{\omega,R}$ is the correlation coefficient between primary deficit and interest payments. $\sigma(\omega_T)/\sigma(\omega)$ the ratio of the variance of the total to the primary deficit, $\sigma(MV)/\sigma(\omega)$ the ratio of the variance changes to the market value of debt to the variance of the primary deficit. Φ^*_j is the statistic based on the dynamic impact of deficit shocks on debt evaluated for j periods.

	COMPLETE			INCOMPLETE		
	aver	25%	75%	aver	25%	75%
$\Psi_{1,3}$	0.001	-0.005	0.006	1.596	1.396	1.850
$\Psi_{1,5}$	0.000	-0.007	0.008	2.842	2.381	3.367
$\Psi_{2,3}$	0.001	-0.006	0.007	2.103	1.437	2.629
$\Psi_{2,5}$	0.000	-0.009	0.009	4.548	2.841	5.691
Im	-10.061	-10.125	-10.000	1.026	0.930	1.102
$\rho_{\omega,R}$	-0.970	-0.985	-0.961	-0.370	-0.667	-0.123
$\sigma(\omega_T)/\sigma(\omega)$	0.746	0.715	0.792	0.944	0.879	1.000
$\sigma(\Delta MV)/\sigma(\omega)$	5.482	4.326	6.498	0.944	0.879	1.000
Φ^*_1	-4.450	-4.831	-4.144	0.990	0.949	1.043
Φ^*_3	-1.715	-2.141	-1.391	0.969	0.905	1.045
Φ^*_5	-0.788	-1.195	-0.523	0.980	0.880	1.039
Φ^*_{10}	-0.145	-0.450	0.042	1.201	0.798	1.045

Table 1b - Assessing Debt Management Performance Statistics - Persistent shocks and No Capital Accumulation

Results show outcomes from 1000 simulations of 200 periods where first 150 periods are discarded and statistics calculated on final 50 observations. "Average" denotes the average of each statistic over all 1000 simulations, 25% shows the bottom quartile outcome and 75% the third quartile. Ψ_{ij} $i = 1, 2$ is the persistence test described in the text evaluated at lag j. Im is the impact measure comparing the effect of a shock on debt and the deficit. $\rho_{\omega,R}$ is the correlation coefficient between primary deficit and interest payments. $\sigma(\omega_T)/\sigma(\omega)$ the ratio of the variance of the total to the primary deficit, $\sigma(MV)/\sigma(\omega)$ the ratio of the variance changes to the market value of debt to the variance of the primary deficit. Φ_j^* is the statistic based on the dynamic impact of deficit shocks on debt evaluated for j periods.

	COMPLETE			INCOMPLETE		
	aver	25%	75%	aver	25%	75%
$\Psi_{1,3}$	-0.006	-0.029	0.018	1.650	1.467	1.867
$\Psi_{1,5}$	-0.011	-0.041	0.021	3.057	2.563	3.557
$\Psi_{2,3}$	-0.006	-0.031	0.021	1.838	1.385	2.180
$\Psi_{2,5}$	-0.011	-0.051	0.027	3.699	2.543	4.471
Im	-13.712	-14.321	-13.022	0.495	0.150	0.958
$\rho_{\omega,R}$	-0.975	-0.993	-0.972	-0.438	-0.730	-0.231
$\sigma(\omega_T)/ \sigma(\omega)$	0.627	0.595	0.661	0.727	0.625	0.839
$\sigma(\Delta MV)/ \sigma(\omega)$	6.757	5.199	8.036	0.727	0.625	0.839
Φ^*_1	-6.106	-6.750	-5.541	0.712	0.565	1.002
Φ^*_3	-2.403	-3.010	-1.907	0.794	0.757	1.019
Φ^*_5	-1.180	-1.741	-0.750	0.920	0.818	1.022
Φ^*_{10}	-0.312	-0.662	0.007	0.978	0.779	1.027

Table 1c - Assessing Debt Management Performance Statistics - Persistent shocks and Capital Accumulation

Results show outcomes from 1000 simulations of 200 periods where first 150 periods are discarded and statistics calculated on final 50 observations. "Average" denotes the average of each statistic over all 1000 simulations, 25% shows the bottom quartile outcome and 75% the third quartile. Ψ_{ij} $i = 1, 2$ is the persistence test described in the text evaluated at lag j. Im is the impact measure comparing the effect of a shock on debt and the deficit. $\rho_{\omega,R}$ is the correlation coefficient between primary deficit and interest payments. $\sigma(\omega_T)/\sigma(\omega)$ the ratio of the variance of the total to the primary deficit, $\sigma(MV)/\sigma(\omega)$ the ratio of the variance changes to the market value of debt to the variance of the primary deficit. Φ^*_j is the statistic based on the dynamic impact of deficit shocks on debt evaluated for j periods.

	$\rho_{(MV,B)}$	$\rho_{(\Delta MV,\Delta B)}$	$\rho_{(\Delta MV,\Delta Y)}$	$\rho_{(\Delta Y,\Delta B)}$	$\rho_{(\Delta MV,\omega)}$	$\rho_{(\Delta B,\omega)}$	UR MV	UR B
Australia	0.897	0.653	-0.183	0.048	0.549	0.408	-2.47	-2.19
Austria	0.798	0.925	-0.228	-0.367	0.131	0.304	-1.69	-1.61
Belgium	0.997	0.883	-0.172	-0.117	-0.117	-0.154	-0.17	-0.34
Canada	0.994	0.858	-0.262	-0.321	0.303	0.602	-0.03	-0.8
France								
Germany	0.996	0.794	-0.347	-0.397	-0.213	-0.315	-1.62	-1.26
Ireland	0.903	0.63	-0.491	-0.799	0.469	0.77	-1.23	-0.85
Italy	0.993	0.604	-0.547	-0.337	-0.117	0.087	-0.46	-0.59
NL	0.871	0.904	-0.517	-0.684	0.317	0.477	-0.43	-1.81
Norway	0.804	0.867	-0.132	0.11	0.36	0.232	-1.95	-2.31
UK	0.81	0.445	0.039	-0.299	0.369	0.539	-2.12	-1.11
US	0.987	0.715	-0.505	-0.45	0.564	0.871	-0.69	-1.52

Table 2 - Comparing Debt Measures

ρ_{XY} is the correlation coefficient between X and Y where MV is the market value of debt, B is the outstanding value of debt, Y is GDP, ω denotes the primary deficit. The statistic quoted in the UR column is the Augmented Dickey Fuller test for each variable including four lags and a time trend.

	$\Psi_{1,2}$	$\Psi_{1,3}$	$\Psi_{1,5}$	$\Psi_{2,1}$	$\Psi_{2,3}$	$\Psi_{2,5}$	Im	Φ^*_1	Φ^*_3	Φ^*_5
Australia	0.335	0.53	0.5	0.238	0.369	0.592	0.403	0.441	0.383	0.243
Austria	0.34	0.878	0.588	0.299	1.039	1.185	1.393	1.03	0.726	0.519
Belgium	0.915	2.14	2.507	1.039	2.418	2.498	1.399	1.412	1.522	1.868
Canada	0.556	1.09	1.183	0.477	0.79	1.097	0.813	0.691	0.526	0.458
France	0.416	1.236	1.394	0.321	0.987	1.478	0.907	0.949	0.691	0.172
Germany	0.172	0.691	0.623	0.183	1.211	1.83	0.745	0.429	0.851	0.545
Ireland	0.19	1.293	2.15	0.13	0.942	1.879	0.458	0.784	0.835	0.849
Italy	0.586	1.394	1.526	0.771	2.902	2.446	0.774	0.418	0.305	0.306
NL	0.813	2.306	2.993	0.868	3.671	4.822	0.492	0.738	0.973	0.939
Norway	0.028	0.038	0.172	0.022	0.033	0.166	0.551	0.139	0.103	0.197
UK	0.342	0.653	0.761	0.328	0.816	2.667	1.427	1.456	0.597	0.643
US	0.572	1.376	1.651	0.559	1.789	3.021	1.056	0.775	0.535	0.27

Table 3 - Relative Persistence Measures

	Fixed	Fixed Short	Fixed Long	Indexed	Foreign Currency	Variable	Maturity
Australia	**0.707**	**0.348**	**0.359**	**0.036**	**0.255**	**0.052**	**5.610**
Austria	**0.860**	**0.860**		**0.000**	**0.228**	**0.031**	**5.733**
Belgium	**0.876**	**0.417**	**0.460**	**0.000**	**0.064**	**0.037**	**4.240**
Canada	**0.603**	**0.291**	**0.313**	**0.014**	**0.036**	**0.007**	**5.820**
Germany	**0.967**	**0.396**	**0.571**	**0.000**	**0.000**	**0.015**	**5.000**
Italy	**0.296**	**0.150**	**0.146**	**0.002**	**0.049**	**0.349**	**3.360**
Netherlands	**0.910**	**0.158**	**0.752**	**0.000**	**0.000**	**0.000**	**5.210**
Norway	**0.779**	**0.419**	**0.360**	**0.000**	**0.172**	**0.009**	**8.790**
UK	**0.763**	**0.615**	**0.148**	**0.150**	**0.000**	**0.019**	**11.070**
US	**0.745**	**0.485**	**0.259**	**0.023**	**0.007**	**0.000**	**4.500**

Table 4 - Summary Statistics

First 6 columns show proportion of debt issued in different formats. “Maturity” shows average length of bond issued

	Constant	Maturity	MV/Y	ω/Y	Δy	Inflation	R^2
$\Psi_{1,2}$	0.77 (0.07)	-0.04 (0.32)	0.12 (0.79)	-19.23 (0.14)	-0.474 (0.96)	-5.364 (0.480)	0.24
$\Psi_{1,3}$	1.64 (0.10)	-0.113 (0.26)	0.552 (0.61)	-60.148 (0.07)	-15.878 (0.50)	1.109 (0.95)	0.29
$\Psi_{1,5}$	1.72 (0.20)	-0.142 (0.32)	0.639 (0.69)	-80.01 (0.09)	-36.94 (0.30)	18.17 (0.49)	0.18
$\Psi_{2,2}$	0.75 (0.11)	-0.035 (0.44)	0.369 (0.484)	-22.767 (0.13)	-0.483 (0.96)	-8.611 (0.33)	0.33
$\Psi_{2,3}$	1.56 (0.34)	-0.064 (0.71)	0.806 (0.68)	-95.730 (0.11)	-7.164 (0.87)	-10.680 (0.74)	0.15
$\Psi_{2,5}$	1.63 (0.45)	-0.132 (0.58)	-0.064 (0.98)	-103.511 (0.18)	-1.325 (0.98)	12.191 (0.78)	-0.262
Im	1.54 (0.027)	-0.033 (0.57)	0.873 (0.23)	6.546 (0.71)	5.869 (0.69)	-22.43 (0.09)	0.19
Φ^*_1	1.56 (0.04)	-0.112 (0.13)	0.327 (0.67)	-11.531 (0.56)	-0.861 (0.96)	-6.285 (0.62)	0.09
Φ^*_3	0.48 (0.23)	-0.074 (0.12)	0.768 (0.15)	-31.887 (0.04)	3.323 (0.75)	1.342 (0.86)	0.57
Φ^*_5	0.31 (0.50)	-0.070 (0.21)	1.279 (0.07)	-38.199 (0.05)	-4.463 (0.72)	2.327 (0.81)	0.61

Table 5 - Macroeconomic Determinants of Fiscal Insurance

Table shows regression results of $P_i = a + bxMaturity_i + cMV_i/Y_i + d\omega_i/Y_i + e\Delta y_i + f\pi_i$ where P_i is the variable listed in the first column, $Maturity_i$ is the average maturity length of bonds for country i in our sample, MV_i/Y_i denotes the average debt/GDP ratio, ω_i/Y_i denotes country i's average deficit/GDP ratio, Δy_i denotes average GDP growth and π_i average inflation. Number in each cell is the estimated coefficient and in parantheses the p-value of significance for each variable.

	Constant	Maturity	Fixed	FX	Indexed	R^2
$\Psi_{1,2}$	0.00 (0.99)	0.062 (0.33)	0.325 (0.49)	-0.334 (0.71)	-5.956 (-0.09)	0.27
$\Psi_{1,3}$	0.61 (0.48)	0.222 (0.15)	-0.270 (0.79)	-1.664 (0.42)	-19.39 (0.03)	0.43
$\Psi_{1,5}$	1.41 (0.29)	0.324 (0.16)	-1.511 (0.35)	-3.099 (0.33)	-25.25 (0.05)	0.29
$\Psi_{2,2}$	-0.15 (0.76)	0.075 (0.36)	0.479 (0.44)	-0.663 (0.58)	-6.853 (-1.840)	0.17
$\Psi_{2,3}$	-0.83 (0.58)	0.478 (0.09)	0.467 (0.79)	-2.921 (0.42)	-32.503 (0.03)	0.41
$\Psi_{2,5}$	0.50 (0.78)	0.542 (0.11)	-0.933 (0.67)	-3.800 (0.39)	-37.134 (0.05)	0.29
Im	0.59 (0.39)	-0.115 (0.31)	1.240 (0.16)	-0.159 (0.92)	1.691 (0.75)	0.01
Φ^*_1	1.03 (0.17)	-0.094 (0.41)	0.509 (0.54)	-0.201 (0.90)	-1.680 (0.76)	0
Φ^*_3	0.74 (0.31)	0.005 (0.97)	0.006 (0.97)	0.097 (0.95)	-5.166 (0.37)	-0.34
Φ^*_5	0.90 (0.40)	-0.003 (0.99)	-0.212 (0.87)	-0.126 (0.96)	-3.997 (0.63)	-0.55

Table 6 - Portfolio Determinants of Fiscal Insurance

Table shows regression results of $P_i = a + bxMaturity_i + cFixed_i + dForeignCurrency_i + eIndexed_i$ where P_i is the variable listed in the first column, $Maturity_i$ is the average maturity length of bonds for country i in our sample, $Fixed_i$ the proportion of government debt that is fixed rate, $ForeignCurrency$ the proportion of debt that is issued in foreign currency terms and $Indexed_i$ the proportion of indexed bonds issued. Number in each cell is the estimated coefficient and in parantheses the p-value of significance for each variable.

	Constant	Variable	Fixed Long	Fixed Short	FX	Indexed	R^2
$\Psi_{1,2}$	0.36 (0.58)	-0.646 (0.69)	0.227 (0.80)	1.388 (0.60)	0.767 (0.57)	-6.147 (0.24)	0.19
$\Psi_{1,3}$	0.96 (0.63)	0.626 (0.90)	0.539 (0.85)	2.467 (0.76)	1.143 (0.77)	-14.369 (0.34)	-0.15
$\Psi_{1,5}$	1.20 (0.66)	2.415 (0.72)	0.326 (0.93)	3.337 (0.76)	1.311 (0.81)	-17.866 (0.39)	-0.32
$\Psi_{2,2}$	0.06 (0.94)	-0.183 (0.93)	0.729 (0.57)	2.414 (0.52)	0.881 (0.63)	-7.291 (0.29)	-0.01
$\Psi_{2,3}$	-0.94 (0.79)	2.995 (0.73)	3.842 (0.46)	10.245 (0.49)	4.077 (0.58)	-24.819 (0.36)	-0.25
$\Psi_{2,5}$	2.67 (0.54)	-2.226 (0.83)	-1.300 (0.83)	4.785 (0.78)	1.498 (0.86)	-29.169 (0.37)	-0.5
Im	1.514 (0.11)	-0.773 (0.68)	-1.135 (0.33)	-0.946 (0.76)	-5.687 (0.03)	4.107 (0.46)	0.74
Φ^*_1	0.654 (0.27)	0.484 (0.72)	-0.207 (0.78)	1.281 (0.57)	-0.393 (0.72)	-9.104 (0.08)	0.62
Φ^*_3	2.438 (0.10)	-4.743 (0.17)	-2.461 (0.20)	-2.087 (0.66)	-1.148 (0.63)	-9.003 (0.32)	0.27
Φ^*_5	1.600 (0.55)	-0.252 (0.97)	-0.152 (0.97)	5.312 (0.63)	-1.446 (0.79)	-19.343 (0.34)	-0.23

Table 7 - Portfolio Determinants of Fiscal Insurance II

Table shows regression results of $P_i = a + bxVariable_i + cFixed_i Longterm_i + dFixedshort + eForeignCurrency$ $+ fIndexed_i$ where P_i is the variable listed in the first column, $Maturity_i$ is the average maturity length of bonds for country i in our sample, $Variable_i$ denotes the proportion of variable rate debt, $FixedLongterm$ the proportion of fixed long term debt issued Number in each cell is the estimated coefficient and in parantheses the p-value of significance for each variable.