Incentives for Interdisciplinary Research*

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Abstract

This paper is a positive analysis of the driving forces in interdisciplinary research. I take the perspective of a research institution that has to decide how to apply its resources among the production of two types of knowledge: specialized or interdisciplinary. Using a prize mechanism of compensation, I show that the choice of interdisciplinarity is compatible with profit maximization when the requirement for the production is sufficiently demanding, and when the new interdisciplinary field is not too neutral. Productive gains due to complementarities of efforts is the main advantage of interdisciplinary organization.

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1 Introduction

Why interdisciplinary research, considered very interesting and important to achieve breakthroughs, is at the same time so neglected among scientific community? There is few understanding of what are the driving forces of the disciplinary pattern in research organizations. In particular, why it is still so scarce the observation of interdisciplinary research, despite all emphasis that it receives from policymakers. The present paper brings some potential useful results in explaining such puzzling situation, by making a positive (rather than normative) comparison between interdisciplinarity and specialization.

In the context of the current paper, specialization refers to the case where scientists work separate and independently on their own fields of expertise. This specialized fields are characterized by well-established and long existent scientific foundations. Foundations of the modern organization of science are based in secular structures of these different and separated disciplines.

Over the years, and especially in the two previous decades, however, it increased the importance attached to an alternative organizational form, interdisciplinarity: the integration of (already existing) separate disciplines on the development of a new scientific area. Interdisciplinarity has been seen as the most suitable way to solve complex questions arising to societies, as illustrated by The National Academies: "Advances in science and engineering increasingly require the collaboration of scholars from various fields. This shift is driven by the urgent need to address complex problems that cut across traditional disciplines, and the capability of new technologies to both transform existing disciplines and generate new ones." (National Academies, 2004). Two world-wide recognized examples of interdisciplinarity illustrate how powerful it can be: the development of genomics, a branch of biotechnology whose roots relate with genetics, molecular biology, analytical chemistry, and informatics; and the development of neurosciences, a new life science evolving from anatomy, physiology, biochemistry, and molecular biology of nerves.

Besides the novelty of the field, interdisciplinarity is also characterized by the requirement of collaboration between different experts. Both defining features of interdisciplinarity are a natural source of difficulties. First, when moving from their scholar background into a new and unexplored discipline, researchers need to adjust to different languages, tools, methodologies, and goals. In the present paper, these differences between disciplines are embody in the concept of scientific distance between fields. A second potential challenge that interdisciplinarity poses to scientists is the need to cooperate with other scientists, but having no common disciplinary background.

The empirical study of Porac et al. (2004) may serve to illustrate my theoretical framework. That paper devotes attention to the scientific performance of two teams of researchers, Astro and Eco. These teams differ in the composition and in the disciplinary expertise of
their members, as well as on their research goal.\(^1\) Scientists in Astro have similar scholarly background and work in the well established field of Astrophysics. Researchers in Eco come from different disciplinary backgrounds and are required to work in modelling ecosystems. Modelling ecosystems is a relatively new science, emerging from previously separated fields related with air, water, and land resources. As in my framework, the paper emphasizes the challenge that Eco team members face to overcome the inherent tensions of the new project. On the one hand, these tensions relate to the need of balancing "between their individual discipline-based paradigms and the joint demands of the Alliance work" (pp. 673). In the language of my framework, this relates with the scientific distance between the background field of scientists and the new interdisciplinary field.\(^2\) On the other hand, the members of the Eco team also identify the need to develop a routine of communication as well a common language among all members.\(^3\)

With such distinction between interdisciplinarity and specialization, interdisciplinary research presents a starting cost disadvantage embodied in its own definition. Nevertheless, when scientists make an extra-investment of adaptation, interdisciplinary difficulties may be reduced (at least partially). Dan Sperber, an anthropologist involved in the interdisciplinary project "Culture and Cognition" at the University of Michigan recognizes the importance of such adaptation concern: "Serious involvement in interdisciplinary research needs a high investment endeavor. To be able to understand each other and conceive of common goals." (Sperber, 2003). In a static framework as mine, I denominate this investment as the acquisition of adaptative-skills for the researchers. It corresponds to an endeavor of learning techniques and tools, allowing the researchers to work on the new discipline, in a less costly way.

Under this framework, I discuss the arguments that lead a research institution to decide between the two types of organization: specialization and interdisciplinarity. For such, I consider researchers to be perfectly coordinated with their employer organization, an university. Then, I consider a simple compensation mechanism for the university, a prize, whose rules are settled by a policymaker.\(^4\)

\(^1\)One main difference between my framework and Porac et al. (2004) is that, in their case, the comparison is between an interdisciplinary and a specialized team. By contrast, I consider that under specialization, researchers work separate and independently. With such separation, I avoid discussing team effects, in order to emphasize the issues of coordination and novelty in interdisciplinarity.

\(^2\)The importance of developing and learning specialized codes in an organization is emphasized in a recent paper of Crémer et al. (2007).

\(^3\)By contrast, among Astro team none of these questions were relevant.

\(^4\)Besides simplicity and linking reward with performance, a mechanism as a prize has two other advantages. First, it gives policymakers have enough flexibility to target preferred areas and methodologies. This fact gives them the possibility to influence research developments so that they become more aligned with social preferences. Second, prizes are mechanisms very similar to grants, which are one of the most common used schemes of rewarding scientific research. The main difference between prizes and grants is their timing. While grants are usually given prior to the realization of the research, prizes are a reward for the achievement of
To emphasize the positive (rather than normative) analysis between interdisciplinarity and specialization, I analyze two alternative informational settings. The first, from the perspective of a policymaker who may allocate funds for one of the two types of organizational structures. For such, it is possible to consider that either it exists perfect and complete information between the policymaker and the research organization, or that a policymaker that has enough flexibility in defining the rules for the prizes so that it can still induce the organization to do the first-best. The second setting takes the point of view of the university that owns the resources. The decision of the university regarding which type of research to implement is influenced by incentives from the policymaker. In this incentives setting, the policymaker defines an unique prize rule for both types of research, and is the university who decides whether it strives for the prize through specialization or through interdisciplinary.

I do consider that the university aims to maximize the net benefit of its projects, financed by the policymaker. The net benefit maximization goal for the university seems a reasonable assumption, given that it is a research institution with limited resources (as it is explicit by the fact that specialization and interdisciplinarity are two disjoint scenarios). Furthermore, by assuming a research institution that receives public funds, its choice of the type of knowledge is linked with the social value that such decision can provide to society.

My results show that when the purpose is to produce a high level of output, interdisciplinarity is more attractive than specialization. The reason is that interdisciplinarity yields complementary gains, which are not possible in specialization. This means that even if interdisciplinarity involves both researchers for a common output, it may be more efficient than to have them working separately for independent areas and, in the end, to sum of values of their separate specialized results. Although interdisciplinarity has a cost disadvantage, comparing with specialization, it is expectable that defining a production goal sufficiently high, favours the choice of interdisciplinarity. These theoretical predictions are aligned with the evidence in Porac et al. (2004). There, when comparing the performance pattern of the two teams, they find that the joint production (sum of publications) of their members increases proportionally more in Eco team, the heterogenous group working in a new area.

The results of this paper may, however, be seen from a broader perspective. Rather than thinking only on scientific research, it is possible to extend some of the findings to firms, both on their internal organization, and on their relation with other firms.

Within the firm, the current paper may be useful to study the relation between a principal (manager) and a group of agents (workers) who can either work separate and independently in their domains of expertise, or can join expertises and work as a team for a project new for all of them. In particular, my results show that when the principal cannot enforce the team a certain performance (ex-post to research). Nevertheless, in the current framework, both timings could be seen as equivalent, since we are assuming: verifiability of levels of output, inexistence of uncertainty during the research process.
creation, it is expectable that the agents actually create it, once the goal that the principal requires is sufficiently difficult to be individually obtained.

Considering inter-firms relations, it is also expectable that firms decide to coordinate efforts in a new project, when the gains from cooperation compensate the costs of coordination and of entrance in a new project.

The theme of interdisciplinarity has some common features with the area of human capital and production organized by teams. In particular, this literature provides empirical evidence on an effect that my framework captures, that a higher productivity is often associated with the heterogeneous composition of the teams (e.g. Van der Vegt & Janssen, 2003; Hamilton et al., 2003).

My results also have common features with the literature on incentives and coordination costs, namely with Dessein et al. (2007). Their work considers the trade-off between the need to standardize and reduce a duplicated activity inside a firm, and the impact that such change in the organizational structure brings in another related task. As in my question, their decision is whether to keep activities working separate and independently and not realize a synergy. Nevertheless, we differ on the focus of the argument. They endogenously condition the optimality of the decision on the distortions that it causes in another related task, namely in terms of incentives to truth revelation of private information. I assume a more general framework (that could be used to encompass their argument) where costs of interdisciplinarity relate with the alternative (rather than sequentially related) scenario of specialization, but also and above all with coordination problems among different parties and with the cost of starting a new, unknown, project.

Despite the above mentioned relation between my paper and the existent literature, up to my knowledge, there are no theoretical developments on interdisciplinarity, and on its relations with the organizational structure of institutions through incentives. The current work is a first step in filling this gap.

In the next section, I formally present the model and the structure of the game between the Government and the research institution, a University. In Section 3, I discuss the equilibrium results under three alternative settings: when the information between the Government and the University is perfect and complete; when the choice of the productive inputs and the acquisition of the adaptative-skills can not be established by contract; and, finally, when the funding rules for the prizes are restricted to be equal among all scientific fields. In this last setting, I explicit analyze what are the main arguments in favour of interdisciplinarity, from the point of view of the research institution. Section 4 concludes. All proofs are in the Appendix.
2 The Model

2.1 Specialization and Interdisciplinarity

Let us consider a research institution, call it University, that employs two researchers, each one with a different expertise field. For simplicity, the researchers are perfectly identified by their own field, that is, researcher $A$ is an expert in scientific field $A$, and researcher $B$ is an expert in scientific field $B$. The two scientific fields are differentiated with respect to their defining characteristics: object of study, language, tools. For the purpose of the current analysis model, I aggregate and reduce those characteristics to a single dimension. Considering such dimension, assume the difference between $A$ and $B$ is measurable and equal to 1, as Figure 1 shows.

![Figure 1: Scientific distance between the specialized fields.](image)

At the University, the research activity may follow one of the two possible disjoint patterns: specialization or interdisciplinarity.

In the specialized scenario, each researcher $i$ ($i = A, B$) works separate and independently on his field of background, producing an amount $Y_i$ of specialized knowledge output, with $C_i$. Explicitly:

\[
Y_i = e_i, \quad C_i = \alpha_i \cdot Y_i, \quad i = A, B,
\]

where $e_i \in \mathbb{R}^+$ is the amount of labor input (effort) spent by researcher $i$, whereas the cost coefficient $\alpha_i \in (0, 1)$. Although only the labor input is explicitly included in the knowledge technology, other factors affecting the production process of $Y_i$ can be reflected in the value of $\alpha_i$.

The current analysis focus on the perspective of the University, considering it as an unified structure in terms of goals and objectives. In that sense, instead of referring to two types of researchers, it would be possible to talk about two different departments or two areas of research, perfectly coordinated with the organization. For the purpose of a simpler exposition, let us keep the reference to two researchers, but not including any informational problem between them and the University. The following two assumptions serve this purpose.

**Assumption 1.1**: The labor inputs $e_i$ are perfectly observable and verifiable between the University and its researchers.
Assumption 1.2: The cost coefficients $\alpha_i$ are publicly known.

As an alternative to have each researcher developing specialized knowledge, the University may combine the work of both experts, $A$ and $B$, for the development of a new scientific discipline, field $I$. In terms of the conjectural dimension for the scientific fields, the new interdisciplinary $I$ lies between the two areas from which it emerges. As Figure 2 represents, I assume that the new $I$ is located at a distance $\rho$ from field $A$ and at a distance $(1 - \rho)$ from field $B$, with $\rho \in (0, 1)$.

![Figure 2: Relative position of the interdisciplinary field $I$.](image)

Using the relative distribution of the three scientific fields and the way it is related to the parameter $\rho$, it is possible to center the analysis on the case where $\rho = \frac{1}{2}$, that is, on the case where researcher $A$ is the closest to $I$. The generalization for the remaining domain of $\rho$ is straightforward. Only punctually, and when opportune, I emphasize the symmetric case of $\rho = \frac{1}{2}$.

Given the relative distribution of the three scientific fields and the way it is related with the parameter $\rho$, it is possible to center the analysis on the case of $\rho \in (0, \frac{1}{2})$, that is, on the case where researcher $A$ is the closest to $I$. The generalization for the remaining domain of $\rho$ is straightforward. Only punctually, and when opportune, I emphasize the symmetric case of $\rho \in (\frac{1}{2}, 1)$, or the even situation of $\rho = \frac{1}{2}$.

For the University, the cost of producing interdisciplinary output reflects, on the one hand, the opportunity cost of the resources employed in production, $e_{AI}$ and $e_{BI}$, and, on the other hand, the difficulties underlying the development of a new and unexplored field. These difficulties may, however, decrease if there is an extra-involvement of the researchers, that is, if the University invests in the adaptative-skills of its workers. Inspired in the concept of individual’s innovative behavior of Van der Vegt & Janssen (2003), I define the adaptative-skills of a researcher as the intentional exercise of intellectual flexibility, in order to decrease the individual marginal cost of working in the interdisciplinary field. The acquisition of these skills describes, on a static framework as the current one, the process of learning the
basic characteristics (language, tools) of the new field. Once such investment is made, the researcher is able to work on the new field in a less costly way. A binary variable \(i\) identifies whether researcher \(i\) (\(i = A, B\)) acquires such adaptative-skills \((\theta_i = 1)\) or not \((\theta_i = 0)\).

Considering these characteristics, the cost of the University to produce interdisciplinary research can, then, be formulated as:

\[
C_I = \left(\frac{\rho + \alpha_A}{1 - \rho}\right) (1 - \rho \theta_A) e_{AI} + \left(\frac{1 - \rho + \alpha_B}{\rho}\right) (1 - (1 - \rho) \theta_B) e_{BI} + \frac{\rho}{1 - \rho} \theta_A + \frac{1 - \rho}{\rho} \theta_B. \tag{4}
\]

This formalization makes explicit that the acquisition of adaptative-skills for researcher \(i\) reduces his marginal cost of effort: for researcher \(A\) from \(\left(\frac{\rho + \alpha_A}{1 - \rho}\right)\) to \((\rho + \alpha_A)\), and for researcher \(B\) from \(\left(\frac{1 - \rho + \alpha_B}{\rho}\right)\) to \((1 - \rho + \alpha_B)\). Let \(m_i\) be the marginal cost of \(e_i\) before the investment in his adaptative-skills, i.e., \(m_A = \frac{\rho + \alpha_A}{1 - \rho}\) and \(m_B = \frac{1 - \rho + \alpha_B}{\rho}\). The function (4) can be re-written as:

\[
C_I = m_A (1 - \rho \theta_A) e_{AI} + m_B (1 - (1 - \rho) \theta_B) e_{BI} + \frac{\rho}{1 - \rho} \theta_A + \frac{1 - \rho}{\rho} \theta_B. \tag{5}
\]

The first two terms in the cost function refer to the cost of the inputs \(e_{II}\). I refer to them as the productive cost. The higher the scientific distance between the original field of researcher \(i\) and the new field \(I\), the higher the productive cost. By \(m_A\) and \(m_B\), the marginal cost of effort for researcher \(A\) is then increasing with \(\rho\), whereas the marginal cost of effort \(B\) is decreasing with \(\rho\). The investment in the adaptative-skills, \(\theta_i = 1\), decreases the marginal cost of \(e_{II}\).

To benefit from adaptative-skills it is necessary to invest on them. The last two terms in function (5) reflect the cost of these investments. I assume the cost of acquiring adaptative-skills is proportional to the distance \(\rho\).

Comparing both benefits and costs of acquiring the adaptative-skills, it is possible to establish the following lemma.

**Lemma 2.1** A University interested in minimizing the cost of producing interdisciplinary research, invests in the adaptative-skills of a researcher only when it employs a sufficiently large amount of his labor input: it invests on adaptative-skills of researcher \(A\) when \(e_{AI} \geq \frac{1}{\rho + \alpha_A}\), and of researcher \(B\) when \(e_{BI} \geq \frac{1}{1 - \rho + \alpha_B}\).
2.2 The Value of Knowledge and the Reward System

Let us consider the existence of a government (the Government), with an endowment of $G$ monetary resources (exogenous in the current setting). The University may receive these resources through a mechanism of prizes. Acting as an advocate for society, the role of the Government is to define the monetary amount of the prize in each field $i$, $g_i$ ($i = A, B, I$), as well as the criterion and respective threshold that the University must fulfill in order to receive that prize. The criterion to receive the prize is unique and defined in terms of a requirement of minimum output $\bar{y}_i$. This means that the University receives $g_i$ monetary units, if its production in field $i$ is at least $\bar{y}_i$. Following what is common in the literature, this required minimum performance may also be denominated as the standard (e.g. Costrell, 1994; or Betts, 1998).

Being aware, not only of a budget constraint of $G$ monetary units, but also that specialization and interdisciplinarity are two disjoint scenarios, the problem of the Government in choosing the funding rules is, then, defined as follows.

G1) In the specialized scenario,

$$\max_{(g_i, \bar{y}_i)_{i=A,B}} V_{AB} [Y_A (g_A, \bar{y}_A), Y_B (g_B, \bar{y}_B)]$$

s.t. \[ \begin{cases} g_A + g_B \leq G, \\
\Pi_{univ} \geq 0, \end{cases} \]

where $Y_i (g_i, \bar{y}_i)$ is the knowledge produced by the University in field $i$ ($i = A, B$), function of the funding rules for field $i$, and $V_{AB}$ measures the social value of the specialized outputs. Since the welfare function $V_{AB}$ captures the benefits that knowledge brings to the society, I assume it is increasing in its arguments, that is, $\frac{\partial V_{AB}}{\partial Y_i} > 0$. For simplicity, I assume additive separability in the social value of the specialized fields:

$$V_{AB} [Y_A (g_A, \bar{y}_A), Y_B (g_B, \bar{y}_B)] = V_A [Y_A (g_A, \bar{y}_A)] + V_B [Y_B (g_B, \bar{y}_B)],$$

where the social value of each specialization is given by an increasing and concave function: $V_i' > 0$, $V_i'' \leq 0$. The generic specification of $V_i$ is compatible with the possibility of $A$ and $B$ being differently important for society.

G2) In the interdisciplinary scenario,

$$\max_{(g_I, \bar{y}_I)} V_I [Y_I (g_I, \bar{y}_I)]$$

s.t. \[ \begin{cases} g_I \leq G, \\
\Pi_{univ} \geq 0, \end{cases} \]
where $Y_I(g_I, \tilde{y}_I)$ is the knowledge produced by the University in field $I$, function of the funding rules for field $I$, and $V_I$ the social value of interdisciplinary research. For the sake of simplicity, I assume $\frac{\partial V_I}{\partial Y_I} > 0$.

After knowing the funding rules defined by the Government, the problem of the University can be seen in two stages:

- first, to decide the type of research to be developed, that is, whether the researchers should produce specialized knowledges $A$ and $B$, collaborate in the interdisciplinary field $I$, or should not produce any research at all (this outside option is assumed to yield zero profit);

- second, to choose the amount of resources to employ in each type of research.

**U1)** In case of specialization, the amount of labor used in each project $(e_A, e_B)$ solves

$$
\max_{(e_A, e_B)} \sum_{i=A, B} g_i - C_i(e_i)
$$

$$
s.t. g_i > 0 \text{ only if } Y_i(e_i) \geq \tilde{y}_i, \ i = A, B;
$$

**U2)** In case of interdisciplinarity, the amount of inputs employed in the new field $I$ $(e_{AI}, e_{BI})$, and the investment in the adaptative-skills of the researchers $(\theta_A, \theta_B)$ in order to

$$
\max_{(e_{AI}, e_{BI}, \theta_A, \theta_B)} g_I - C_I(e_{AI}, e_{BI}, \theta_A, \theta_B)
$$

$$
s.t. g_I > 0 \text{ only if } Y_I(e_{AI}, e_{BI}) \geq \tilde{y}_I.
$$

Without introducing any uncertainty for the outputs and for the rewards, as well as assuming full commitment from both participants, Government and University, the timing of the game is completely described by a two stages sequence: first, the Government announces the funding rules $(g_i, \tilde{y}_i), i = A, B, I$; second, the University decides on the type of research, on the amount of productive resources, and on the acquisition of the adaptative-skills.

The predictions of the model are presented in the following section, where three alternative informational contexts are analyzed. The first, the benchmark situation, deals with complete and perfect information among the Government and the University, as well as no restriction in the funding rules. This means that, when defining the rules for the prizes, the Government is able to delineate all the decisions of the University. In the second scenario, there is the introduction of non-contractibility on the choice of the inputs and on the
acquisition of adaptative-skills. I analyze what is the expected equilibrium in this moral-hazard scenario, and how it is affected by the existence of distinct funding rules per field. In the last setting, imposing the restriction of a unique funding policy for all the three scientific fields, I extend the moral-hazard problem to the choice of the type of research. I then discuss the reasons underlying the preferences of the University between specialization and interdisciplinarity. In all these three contexts, I apply the solution concept of Sub-game Perfect Nash equilibrium.

3 The Equilibrium

3.1 The Benchmark

Consider first that there is complete and perfect information between the Government and the University. With all the research choice variables being contractible, the Government decides: whether it asks the University to undertake the specialized research or interdisciplinarity, what is the amount of the labor inputs that should be employed in each type of research, and in the case of interdisciplinarity whether there is an investment in the adaptative-skills of the researchers. Given the budget restriction of $G$ monetary units, the Government establishes the value of the prize for each field, ensuring that the University is willing to participate in such contract.

Backward induction leads us to the optimal solution. Thus, let us proceed analyzing the result for each type of research. At the end, the comparison of both specialization and interdisciplinarity scenarios, allows to conclude which is socially preferred.

Proposition 3.1 Assuming complete and perfect information between the Government and the University, the social optimal solution for the specialized research satisfies the following conditions:

i) relative marginal benefit equals to relative marginal cost

\[
\frac{\partial v_A}{\partial Y_A} (e_A) = \frac{\alpha_A}{\alpha_B}; \quad \frac{\partial v_B}{\partial Y_B} (e_B) = \frac{\alpha_B}{\alpha_B},
\]

ii) zero-profit for the University

\[
\Pi_i (g_i, e_i) = 0, \quad i = A, B;
\]
iii) exhausting of Governmental budget

\[ g_A + g_B = G. \] (8)

When the decision is for specialization, efficiency drives to the exhausting budget condition (8), since no alternative use is considered for the monetary resources \( G \). With a higher prize, the University is willing to employ more (costly) resources \( e_i \). Nevertheless, due to the symmetry of information, the Government is able to exactly compensate the University for its production costs. Thus, the optimal level of production for the specialized projects yields zero profit for the University, the same as its outside option, according to (7). This result links the monetary value of the prizes with the amount of resources spent in production.

Due to the budget constraint, an increase in the production of one specialized output translates in a reduction of the other specialization. From condition (6), and as expected, the optimal solution for society equates the relative marginal benefit of each knowledge with its relative marginal cost.

Denote by \( e_i^* \) the optimal inputs level of input that is obtained from the previous proposition, and by \( y_i^* \) the associated knowledge production, \( i = A, B \). The maximum social welfare under this choice for the specialized research then comes as \( V_{AB}^* = \sum_{i=A,B} V_i (y_i^*) \).

For the alternative interdisciplinary scenario, it is possible to anticipate that some of the previous results remain valid. In the social optimum solution, both arguments of efficiency and symmetry of information still apply and, hence, both results of budget constraint exhaustion and zero profit are binding.

Differing from the specialized scenario, however, the social welfare function \( V_I \) is increasing in only one argument, \( Y_I \), which enable us to derive explicit functions for the optimal level of the inputs. The best decision concerning the investment on the adaptative-skills of the researchers follows in a straightforward way.

**Proposition 3.2** Assumption complete and perfect information between the Government and the University, the social optimal interdisciplinary solution is defined by:

\[
\begin{align*}
e_{AI}^* &= \frac{g_I - \frac{r}{1-r} \theta_A - \frac{1-r}{r} \theta_B}{m_A \cdot (1 - \rho \theta_A) \cdot \frac{1}{(1-r)}}; \\
e_{BI}^* &= \frac{g_I - \frac{r}{1-r} \theta_A - \frac{1-r}{r} \theta_B}{m_B \cdot [1 - (1 - \rho) \theta_B] \cdot \frac{1}{r}}; \\
\Pi_I (g_I, e_{AI}^*, e_{BI}^*, \theta_A, \theta_B) &= 0; \\
g_I &= G;
\end{align*}
\] (9) (10) (11) (12)
and, therefore, the associated social optimal interdisciplinary output is

\[
y_I^* (\theta_A, \theta_B) = \lambda \left( G - \frac{1}{1-\rho} \theta_A - \frac{1}{1-\rho} \theta_B \right) \frac{m_A^{1-\rho} m_B^{1-\rho}}{\frac{1}{1-\rho} \theta_A + \frac{1}{1-\rho} \theta_B}.
\]

The social optimal investment on the adaptative-skills of the researchers depends positively on the monetary resources \( G \). For \( \rho \in (0, \frac{1}{2}) \), the explicit conditions for optimal \((\theta_A, \theta_B)\) are:

\[
(\theta^*_A, \theta^*_B) = \begin{cases} 
(0, 0) & \text{when } 0 \leq G \leq \frac{1}{1-\rho} \frac{1}{1-\rho}, \\
(1, 0) & \text{when } \frac{1}{1-\rho} \frac{1}{1-\rho} \leq G \leq \frac{1}{1-\rho} + \frac{1}{1-\rho}, \\
(1, 1) & \text{when } G \geq \frac{1}{1-\rho} + \frac{1}{1-\rho}.
\end{cases}
\]

Because the social optimal interdisciplinary solution maximizes the output \( Y_I \), the investment on the adaptative-skills should only be made when it induces an increase on \( Y_I \). For a small budget \( G \), the best option is to spend it only on the productive inputs and not on the adaptative-skills. At an intermediate level of \( G \), it pays to invest in one of the researchers, the closest to \( I \), who is collaborating more in production and has the smallest cost to acquire the adaptative-skills.\(^5\) For a sufficiently high budget, the welfare maximizing decision is to invest in both adaptative-skills. Formally, the conditions for optimal \((\theta_A, \theta_B)\) come from the upper-envelope curve of \( y_I^* (\theta_A, \theta_B) \), considering the different possible combinations of \((\theta_A, \theta_B)\). It is then possible to depict the optimal interdisciplinary production as a function of \( G \) (please refer to Figure 3).

\[ y_I^*(\theta_A, \theta_B) \]

\[ G \]

Figure 3: Social optimal interdisciplinary output, for \( \rho \in (0, \frac{1}{2}) \).

\(^5\)This means that for \( \rho \in (0, \frac{1}{2}) \) researcher \( A \) should have adaptative-skills, for \( \rho \in (\frac{1}{2}, 1) \) researcher \( B \) is the chosen one, and for \( \rho = \frac{1}{2} \) it is indifferent whether the acquisition is for \( A \) or \( B \) (but not both).
Corollary 3.1 The optimal involvement of a researcher $i$ in the common project, $e_{iI}^*$ ($i = A, B$) increases when: i) his cost coefficient $\alpha_i$ decreases, or ii) he acquires adaptative-skills ($\theta_i = 1$). When the other researcher $j$ acquires adaptative-skills ($\theta_j = 1$), $e_{iI}^*$ decreases, $j \neq i$. For $\rho \in (0, \frac{1}{2})$, a marginal increase in $\rho$ : i) has a negative impact on $e_{AI}^*$ when the optimal decision on adaptative-skills is ($\theta_A^*, \theta_B^*$) = (0, 0) or ($\theta_A^*, \theta_B^*$) = (1, 0), and when ($\theta_A^*, \theta_B^*$) = (1, 1) for $G > \frac{1-2\rho^2}{\rho 2^2}$; ii) has a positive impact on $e_{BI}^*$, when the combination ($\theta_A, \theta_B$) is the optimal one.

When the productive marginal cost of a researcher increases through $\alpha_i$, the optimal solution requires that $i$’s marginal benefit also increases. Since $i$’s marginal benefit is decreasing in $e_{iI}$, I obtain that his optimal level of collaboration on interdisciplinarity decreases, $\frac{de_{iI}^*}{d\alpha_i} < 0$.

The acquisition of adaptative-skills has a double effect. On the one hand, the investment on such skills lowers the budget available to compensate the employment of inputs, decreasing $e_{iI}^*$. On the other hand, the researcher acquiring the adaptative-skills lowers his productive marginal cost, and therefore he should work more on the common project. At the optimal combination ($\theta_A^*, \theta_B^*$) : for the researcher acquiring the adaptative-skills, it dominates the decreasing marginal cost effect and, therefore, $\frac{de_{iI}^*}{d\theta_i} > 0$; for the other researcher, however, it only exists the negative effect of a smaller budget, resulting in $\frac{de_{iI}^*}{d\theta_i} < 0$ (or, equivalently, $\frac{de_{iI}^*}{d\theta_j} < 0$).

When $\rho \in (0, \frac{1}{2})$ and it increases marginally (meaning that after the change, $\rho$ is still in the same interval) two effects happen. First, $A$’s effort becomes more costly. Second, it increases the cost of investing in $A$’s adaptative-skills, reducing the budget available to remunerate the productive efforts. Both effects have a negative impact on $e_{AI}^*$ and, therefore, $\frac{de_{AI}^*}{d\rho} (\theta_A^*, \theta_B^*) < 0$. By opposite argument, as $\rho$ increases, $I$ becomes closer to $B$ and, hence, it is optimal to increase $e_{BI}^*$, i.e., $\frac{de_{BI}^*}{d\rho} (\theta_A^*, \theta_B^*) > 0$. When ($\theta_A^*, \theta_B^*$) = (1, 1) the condition of $G > \frac{1-2\rho^2}{\rho 2^2}$ ensures that the increase in the available budget due to a smaller cost of investing in $B$’s adaptative-skills is not sufficiently powerful to invert the sign of $\frac{de_{AI}^*}{d\rho}$.

From the previous proposition, it is also possible to derive some results of comparative statics for the optimal interdisciplinary production.

Corollary 3.2 The optimal interdisciplinary production level, $y_{iI}^*$, decreases with an increase of $i$’s cost coefficient, $\alpha_i$, $i = A, B$. For $\rho \in (0, \frac{1}{2})$ and $\alpha_A = \alpha_B = \alpha$, a marginal increase in $\rho$ decreases $y_{iI}^*$: i) when ($\theta_A^*, \theta_B^*$) = (0, 0), or (1, 1); ii) or when ($\theta_A^*, \theta_B^*$) = (1, 0) and the researcher $B$ has a high relative marginal cost, that is, $\log \left( \frac{m_B}{m_A} \cdot \frac{1}{\rho} \right) > 1 - \frac{1}{m_A} + \frac{1}{m_B} + \frac{1}{\rho(1-\rho)}$. 

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As it follows from the negative relation between $e^*_I$ and $\alpha_i$, whenever the productive marginal cost of at least one of the researchers increases, the maximum possible output decreases, $\frac{\partial y^*_I}{\partial \alpha_i} < 0$.

The response of $y^*_I$ to a change in $\rho$ is ambiguous, since $\rho$ has opposite effects on both $e^*_{AI}$ and $e^*_{BI}$, and also because $\rho$ defines the importance of each researcher in the interdisciplinary production technology (3). Nevertheless, when researchers have equal cost coefficients, $\alpha_A = \alpha_B = \alpha$, and researcher $B$’s relative marginal cost is sufficiently high, I may anticipate that a marginal increase in $\rho$ has a negative impact on $y^*_I$, as Figure 4 shows. Intuitively, when $\rho$ gets closer to $\frac{1}{2}$, and because $A$’s collaboration is still more relevant for the interdisciplinary production, the increase in his cost is not totally compensated by the decrease in $B$’s cost. As a result, the optimal output level decreases.

![Graph](image)

Figure 4: How $y^*_I$ changes when $\rho$ increases, for $\rho \in (0, \frac{1}{2})$.

When $y^*_I(1,0)$, the lack of ambiguity is only solved for a researcher $B$ with sufficiently high marginal cost, that is, for $\log\left(\frac{m_B}{m_A} \rho\right) > 1 - \frac{1}{m_A} + \frac{1}{m_B} + \frac{1}{\rho(1-\rho)}$. This condition guarantees that the negative impact that $\rho$ has on $e^*_{AI}$ dominates over the positive effect that it has on $e^*_{BI}$. To better understand the need for this condition, notice that at the starting situation, $A$ has a relatively higher participation in the production process than $B$. This is due to field $I$ being closer to $A$, $\rho \in \left(0, \frac{1}{2}\right)$, and because $A$ is less costly in production than $B$, by $(\theta_A, \theta_B) = (1,0)$ and $\alpha_A = \alpha_B = \alpha$. When $A$ collaborates more, the interdisciplinary technology (Cobb-Douglas) claims that I are less willing to give up of $B$’s participation. Therefore the ambiguity of $\rho$’s impact over $y^*_I$ only vanishes when $B$’s collaboration is very small, so that the negative impact of $\frac{\partial e^*_{AI}}{\partial \rho}$ dominates over the positive impact of $\frac{\partial e^*_{BI}}{\partial \rho}$. A sufficiently high productive marginal cost for researcher $B$ ensures that small collaboration.
With the social optimal interdisciplinary production defined by the previous proposition, it is possible to represent the interdisciplinary social value by \( V_I(y_I) \). It follows that, for a given \( G \), the social optimal decision is to have interdisciplinarity whenever \( V_I(y_I^*) \geq V_{AB}(y_A^*, y_B^*) \). As modeled, the functions \( V \) reflect the benefits for society from each type of research. Without entering in a normative comparison of such benefits, it is possible to advance that different types of interdisciplinarity (here reflected in the value of \( \rho \)) may lead to different social optimal decisions. In fact, as latter is made explicit, a more central interdisciplinarity may be too costly to undertake.

### 3.2 When resources and adaptative-skills are non-contractible

In a scenario where only the research output, knowledge, is verifiable and contractible through the rules of the prizes, the specialization alternative remains the same as before. In fact, given the technological specification of the specialized scenario in (1), it is equivalent to contract on the research output or on the productive efforts. In terms of interdisciplinarity, however, the non-verifiability of the productive efforts and of the acquisition of adaptative-skills could have an impact on the equilibrium predictions. Nevertheless, if the Government may define rules for the prizes that differ between fields, it benefits from sufficiently high flexibility on choosing between interdisciplinarity and specialization, and is still able to implement its first-best solution. This section shows how the formalization goes in this informational context.

When defining the rules for the prizes, the Government makes the monetary values \( g_i \), \( i = A, B, I \), to be conditional on the production level, \( \tilde{y}_i \). Because \((g_i, \tilde{y}_i)\) may differ between fields, the Government This means that the choice variables of the Government are now: first, the type of research, specialization or interdisciplinarity; second, the funding rules for specialization \((g_A, \tilde{y}_A)\) and \((g_B, \tilde{y}_B)\), and the funding rules for interdisciplinarity \((g_I, \tilde{y}_I)\).

Once accepting the proposal of the Government, and facing a discrete-type of reward (getting or not the prize), the University decides on the resources spent and on the investment in the adaptative-skills. Because there is no extra-benefit of producing above the required standard, a profit maximizer institution seeks the most efficient way of producing, at most, that level.\(^6\)

**Proposition 3.3** Given the funding policy for field \( i \), \((\tilde{y}_i, g_i)\), \( i = A, B, I \), the best choice for the University is:

\[
a) \text{ under specialization } \quad e_{U}^i = \tilde{y}_i , \quad i = A, B; \tag{14}\]

\(^6\)This conclusion is also in line with Result 1 of Betts (1998).
b) under interdisciplinarity

\[
\begin{align*}
\epsilon_{AI} &= \bar{y}_I \cdot \left( \frac{m_A}{m_B} \cdot \rho \left( \frac{1-\rho}{1-\rho \theta_A} \right) \left( \frac{1-(1-\rho) \theta_B}{\rho} \right) \right)^{\rho} \cdot \frac{1}{\lambda}, \\
\epsilon_{BI} &= \bar{y}_I \cdot \left( \frac{m_A}{m_B} \cdot \frac{\rho}{1-(1-\rho) \theta_B} \right)^{1-\rho} \cdot \frac{1}{\lambda}.
\end{align*}
\]

The investment in the adaptative-skills is increasing in both policy variables, \(\bar{y}_I\) and \(g_I\): for smaller values \(g_I\) and \(\bar{y}_I\), the University prefers not to acquire any adaptative-skills; for intermediate \(g_I\) and \(\bar{y}_I\), it invests in the researcher that is closer to field \(I\); and for high values of \(g_I\) and \(\bar{y}_I\), it acquires both adaptative-skills. When \(\rho \in (0, \frac{1}{2})\), the relevant thresholds for acquiring adaptative-skills of \(A\) are \((\bar{y}_I, g_I) = (\bar{y}^{00}_I, g^{00}_I) = (\lambda m_A^{\rho-1} m_B^{-\rho} \left( \frac{\rho}{1-\rho} \right)^{\rho} \left( \frac{1}{1-\rho} \right)^{1-\rho} - 1)^{-1}, (1-\rho) \left( \frac{1}{1-\rho} \right)^{1-\rho}\)) and for the adaptive - skills of \(B\) are \((\bar{y}_I, g_I) = (\bar{y}^{10}_I, g^{10}_I) = (\lambda m_A^{\rho-1} m_B^{-\rho} \left( \frac{1-\rho}{\rho} \right)^{\rho} - 1)^{-1}, \frac{(1-\rho)^2 + \rho^2 (1-\rho^\rho)}{(1-\rho) (1-\rho^\rho)}\).

The outside option of no research is preferred: i) to specialization when the standard \(\bar{y}_i\) is at least \(\frac{g_i}{\alpha_i}, i = A, B\); ii) and to interdisciplinarity when \(\bar{y}_I\) is at least \(\bar{y}^{11}_I = \lambda m_A^{\rho-1} m_B^{-\rho} \left( g_I - \frac{\rho}{1-\rho} - \frac{1-g_I}{\rho} \right)\).

As in the benchmark situation, the acquisition of the adaptative-skills depends positively on the amount of funds available for the interdisciplinary project. Having to choose how to spend the funds, the priority is to remunerate the inputs necessary to produce and, only after, to invest in more efficient ways of producing.

More interesting is to notice that, even if the prize \(g_I\) allows to acquire adaptative-skills, the optimal decision is contingent on \(\bar{y}_I\). As the required production increases, it also increases the effort the researchers must exert to accomplish it. But the higher the effort, the higher the benefits of acquiring adaptative-skills. Thus, only when the policy is sufficiently demanding, the investment is made. This result follows from Lemma 2.1. Formally, I can derive the conditions for University’s best choice \((\theta_A, \theta_B)\) through the upper envelope-curve of the profit curves \(\Pi'_I(\theta_A, \theta_B)\), when considering the different possible combinations of \((\theta_A, \theta_B)\). Figure 5 illustrates the reasoning, plotting the maximum interdisciplinary profit of the University as a function of the standard \(\bar{y}_I\). The figure stands for the case of a prize sufficiently high to allow the acquisition of skills for both researchers, i.e., \(g_I \geq g^{10}_I\).
Figure 5: Maximum interdisciplinary profit, for $\rho \in \left(0, \frac{1}{2}\right)$.

From the solution of the interdisciplinary problem stated in Proposition 3.3, it is possible to derive some comparative statics results.

**Corollary 3.3** At the optimal solution for the University, the effort of researcher $i$ in the interdisciplinary project, $e_{UI}$, increases when: i) his cost coefficient $\alpha_i$ decreases, or the cost coefficient of the other researcher $\alpha_j$ increases, ii) he acquires adaptative-skills ($\theta_i = 1$), or the other expert does not ($\theta_j = 0$), $j \neq i$. A negative relation between $e_{AI}$ and $\rho$ is guaranteed when
\[
\frac{(1-\rho \theta_A)m_A}{(1-(1-\rho)\theta_B)m_B} > \frac{1-\rho}{\rho}.
\]

To produce the required $\tilde{y}_I$ at the most efficient way, whenever the productive marginal cost of one expert decreases, the University should increase his contribution for the common project, so that $\frac{\partial e_{UI}}{\partial \alpha_i} < 0$. By a similar argument, and using the complementary characteristic of interdisciplinarity, $i$ should work more when $j$’s marginal cost increases, $\frac{\partial e_{UI}}{\partial \alpha_j} > 0$. Since the productive marginal cost of the researchers depends negatively on the acquisition of the adaptative-skills, in the optimal solution $\frac{\partial e_{UI}}{\partial \theta_i} > 0$ and $\frac{\partial e_{UI}}{\partial \theta_j} < 0$. Regarding the impact of the distance parameter $\rho$ on the level of efforts chosen by the University, opposite effects emerge. Because $\rho$ determines not only the cost of producing the interdisciplinary output, but also the process of production itself, ambiguity is solved when the relative marginal cost of $e_{AI}$ is higher than the relative importance of $e_{AI}$ for the interdisciplinary production. Under such restriction, a negative relation between $e_{AI}$ and $\rho$ stands out.

**Corollary 3.4** Given the funding policy of the Government for interdisciplinarity $(\tilde{y}_I, g_I)$, the maximum profit that the University may obtain with this type of research, conditional on
the acquisition or not of the adaptative-skills, is given by

\[
\Pi^U_I (\theta_A, \theta_B) = g_I - \frac{\rho}{1 - \rho} \theta_A - \frac{1 - \rho}{\rho} \theta_B - \frac{\tilde{y}_I}{\lambda} \cdot [m_A(1 - \rho \theta_A)]^{1 - \rho} \cdot [m_B(1 - (1 - \rho) \theta_B)]^\rho \cdot \left( \frac{1 - \rho}{\rho} \right) + \left( \frac{\rho}{1 - \rho} \right)^{1 - \rho},
\]

which increases with: i) a higher value of the prize, \(g_I\); and ii) a smaller productive marginal cost for the researchers, \(\alpha_i\).

When \(\rho \in (0, \frac{1}{2})\) and \(\alpha_A = \alpha_B = \alpha\), a marginal increase in \(\rho\) decreases \(\Pi^U_I (\theta_A, \theta_B)\), when i) \((\theta_A, \theta_B) = (0, 0)\); ii) \((\theta_A, \theta_B) = (1, 0)\) and \(\log \left( \frac{m_B}{m_A} \right) > \frac{\rho}{1 - \rho + \alpha}\); or iii) \((\theta_A, \theta_B) = (1, 1)\) and \(\log \left( \frac{m_B}{m_A} \right) > -2 + \frac{1 + \alpha}{1 - \rho + \alpha} - \frac{1 + \alpha}{\rho + \alpha} + \frac{\tilde{y}_I}{\lambda} \cdot \frac{\rho}{\rho} \cdot \frac{1 - \rho^2}{m_B m_A} \).

The achievement of a given standard, at a higher productive marginal costs, necessarily leads to a smaller interdisciplinary profit, meaning that \(\frac{\partial \Pi^U_I}{\partial \alpha_i} (\theta_A, \theta_B) < 0\). Considering the effect of \(\rho\) in the interdisciplinary profit, the conclusion is ambiguous for general values of the parameters. Nevertheless, when \(\alpha_A = \alpha_B = \alpha\) and the researcher B (the researcher further from I) is sufficiently costly, I may conclude that \(\frac{\partial \Pi^U_I}{\partial \rho} (\theta_A, \theta_B) < 0\).

In the particular setting of our analysis, two characteristics have a significative role for the results. First, the lack of incentive of the University to exceed the standard offsets the potential asymmetric information problem. Second, since the funding rules may differ between fields, the Government has enough flexibility to implement the first-best solution.

**Corollary 3.5** Let \((e_A, e_B)\) and \((\theta_A, \theta_B)\) be non-verifiable. Assuming that the Government can choose different funding policies \((\tilde{y}_i, g_i)\) for each field \(i\) \((i = A, B, I)\), it is still possible to achieve the first-best solution. Then, the funding rules for the specialized projects must be \(\tilde{y}_A = y_A^*, \tilde{y}_B = y_B^*, g_A = \alpha_A \cdot y_A^*, \) and \(g_B = \alpha_B \cdot y_B^*,\) whereas for the interdisciplinary research \(\tilde{y}_I (\theta_A, \theta_B) = y_I^* (\theta_A, \theta_B),\) and \(g_I = G.\)^7

The final decision of which type of research must be contracted follows from the comparison of the maximum social welfare on both situations. Interdisciplinarity is the best choice for society when \(V_I (\tilde{y}_I) \geq V_{AB} (\tilde{y}_A, \tilde{y}_B)\).

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7The symbol (*) means that the value of the variables is the same as in first-best (please refer to the benchmark model, Propositions 3.1 and 3.2).
3.3 When the funding policy is restricted

Suppose now that the type of research to implement is now a decision of the University. From the previous setting, let us maintain the assumption of non-contractibility of the amount of resources \( (e_A, e_B) \) and of the investment in the adaptative-skills \( (\theta_A, \theta_B) \). The distinction is that now that the Government decision variables are restricted to a funding rule, common to the three scientific disciplines: \( (\bar{y}_i, g_i) = (\bar{y}, g), i = A, B, I \). As a consequence, the timing of decisions is:

1. the Government chooses the unique funding rule \((\bar{y}, g)\);  
2. once knowing the rule, the University decides,  
   i) whether the researchers work separately on their specialized fields, collaborate with each other on the interdisciplinary field \( I \), or undertake the alternative outside option of no research;  
   ii) the amount of resources to employ in the research, and the acquisition of the adaptative-skills.

At a first glance, it may seem that specialization is being favoured by the funding rule, since it enables the University to receive \( 2g \), whereas interdisciplinarity, at most, yields \( g \). Specialization can, therefore, appear as the obvious choice for the University. As I show, it is not always so. Interdisciplinary research can still be the best option for a profit maximizer organization.

In order to centralize our discussion on the comparison between specialization and interdisciplinarity, rather than in potential asymmetries of costs between the two original fields \( A \) and \( B \), I use the following assumption:

**Assumption 2:** Let the researchers have equal cost coefficients, i.e., \( \alpha_A = \alpha_B = \alpha \).

Our main results remain valid under more general conditions. In the end of this section, I briefly comment the case of \( \alpha_A \neq \alpha_B \).

The comparison between the profitability of the two types of research depends on the value of the parameters, but the two following propositions stand out the main results.

**Proposition 3.4** Let the governmental funding policy be defined per scientific project, and common to all fields, i.e., \( (\bar{y}_i, g_i) = (\bar{y}, g), i = A, B, I \). Then, the University prefers to develop the interdisciplinary research when the required standard is sufficiently high.
I start by illustrating the reasoning using a graph. The relative position of the two profit curves, under specialization and under interdisciplinarity, depends on the value of parameters. As such, Figure 6 considers the case of

$$\frac{2\alpha\lambda}{m_A^r m_B^r} \in (\frac{1-\rho |2-\rho^2| \rho \rho^2 - g(1-\rho)(1-\rho^2)}{(1-\rho)^2 + \rho^2 (1-\rho^2)}) ;$$

![Figure 6: Specialized profit (\(\Pi_{AB}\)) and interdisciplinary profit (\(\Pi_I\)), for \(\rho \in (0, \frac{1}{2})\).](image)

When the requirement of production is small (below \(\tilde{y}_{AB,I}^{10}\)), the cost of joining both researchers in a unique project on a new area, where both have to go native, does not compensate the reward that the organization may receive. Deciding to keep both researchers working in two independent areas, the ones where they are experts, and applying for two separate prizes, turns to be the most rewarding option.

When the funding policy is sufficiently demanding (above \(\tilde{y}_{AB,I}^{11}\)), the conclusion reverses. A larger output level is less costly to produce when the University combines the work of the two different experts. Complementarity generates productivity gains that are not possible to achieve under specialization. This cooperative advantage is reinforced by a potential smaller cost due to the acquisition of adaptative-skills. Together, for a sufficiently high production level, these two characteristics result in such a smaller cost for interdisciplinary research that more than compensates its initial disfavored position on the reward scheme.

Although interdisciplinarity may be a better option than specialization for a standard above \(\tilde{y}_{AB,I}^{10}\), when the requirement is too much demanding (higher than \(\tilde{y}_{AB,I}^{11}\)), the University prefers not to apply to any prize at all.

From the previous analysis, it is possible to infer that the comparison between the profitability of each type of research depends on the value of the parameters. The comparative statistic result then follows.
Proposition 3.5 Let \((e_A, e_B)\) and \((\theta_A, \theta_B)\) be non-contractible and the funding policy \((\tilde{y}, g)\) be unique for all possible fields. Then, under Assumption 2 and \(\rho \in (0, \frac{1}{2})\) :

a) an increase in the researchers’ cost coefficient, \(\alpha\), favors the choice for interdisciplinarity;

b) an increase in \(\rho\) favors the choice for specialization, when the relative marginal cost of researcher \(B\) is sufficiently high, i.e., when: i) \(\log \left( \frac{m_B}{\rho m_A} \right) > \frac{\rho}{1-\rho+\alpha} \) if the optimal \((\theta_A, \theta_B) = (1, 0)\); and ii) \(\log \left( \frac{m_B}{m_A} \right) > -2 + \frac{1+\alpha}{1-\rho+\alpha} - \frac{1+\alpha}{\rho+\alpha} + \frac{1}{\lambda} \cdot \frac{1}{\rho m_B m_A^{\rho}} \) if the optimal \((\theta_A, \theta_B) = (1, 1)\).

Because the coefficient \(\alpha\) stands for the productive marginal cost of the researchers, it affects both interdisciplinarity and specialization. When outputs are separately produced, as in the specialized scenario, an increase in the individual marginal cost affects the cost structures of both fields \(A\) and \(B\), in a direct proportion to researchers’ effort. When both researchers interact, though, the change leads to a reallocation on the individual contributions for the common project \(I\). As a result, interdisciplinary research is less penalized by an increase in \(\alpha\).

The distance parameter \(\rho\) is only relevant for the interdisciplinary option, but it influences in several opposite ways. On the one hand, when \(\rho \in (0, \frac{1}{2})\) and it increases marginally, it becomes more costly for \(A\) to collaborate in the production and to acquire adaptative-skills. For researcher \(B\) the impact is reversed, thus creating an ambiguity on the interdisciplinary profit. When \(B\)’s relative marginal cost is sufficiently high, his collaboration in the common project is small enough to guarantee that \(A\)’s negative result dominates. Intuitively, whenever \(\rho\) gets closer to \(\frac{1}{2}\), but still \(B\)’s higher familiarity with the new discipline does not compensate \(A\)’s higher difficulty, interdisciplinarity becomes less interesting for the University.

3.3.1 Restricted funding policy and \(\alpha_A \neq \alpha_B\)

Let us now discuss how the previous conclusions regarding the choice of the University changes, when the two specialized fields \(A\) and \(B\) have different cost coefficients. Consider the case \(\alpha_A > \alpha_B : \alpha_A = \beta \alpha_B, \beta > 1\).

As far as the specialized projects are concerned, the maximum value of \(\tilde{y}\) that makes the University indifferent between participating or not, is now different for each field. In fact, until \(\tilde{y} = \frac{\alpha_B}{\alpha_B}\) the University is willing to develop specialized output \(B\), but it produces output \(A\) only if \(\tilde{y} \leq \frac{\alpha_A}{\alpha_A}\). For such case of interest, \(\alpha_A > \alpha_B\), this means that when the required standard is sufficiently high, only the specialized field \(B\) is relevant for the University. Figure 7 illustrates the argument.
To compare the maximum profit of the University under specialization and under interdisciplinarity, I have to consider that \( B \) alone may be a relevant choice. Despite this new fact, Proposition 3.4 still holds, since the University still prefers to develop the interdisciplinary research when the required standard \( \tilde{y} \) is sufficiently high.

Regarding Proposition 3.5, the impact that an increase in the marginal cost of one of the specialized fields, \( \alpha_i \) \((i = A, B)\), has on the choice between specialization and interdisciplinarity is now dependent on the distance parameter, \( \rho \).

**Proposition 3.6** Let \((e_A, e_B)\) and \((\theta_A, \theta_B)\) be non-contractible and the funding policy \((\tilde{y}, g)\) be unique for all possible fields. Assume different cost coefficients, such that \( \alpha_A > \alpha_B \). Then,

a) in the case where the specialization in field B, alone, is never an optimal choice for the University, the impact that an increase in \( \alpha_i \) has on the choice between specialization and interdisciplinarity depends on \( \rho \) : i) an increase on \( \alpha_A \) favors interdisciplinarity if \( \frac{1 + \alpha_A}{\alpha_B} > \frac{1}{1-\rho} \); ii) an increase on \( \alpha_B \) favors interdisciplinarity if \( \frac{1 + \alpha_B}{\alpha_A} > \frac{\rho}{1-\rho} \);

b) in the case where the specialization in field B, alone, may be an optimal choice for the University: i) an increase in \( \alpha_A \) enlarges the range of the standard \( \tilde{y} \) where only B is chosen, making interdisciplinarity less interesting for the University; ii) an increase in \( \alpha_B \) has the opposite effect, that is, it favours the choice of interdisciplinarity, when \( \lambda > \frac{1}{\rho^\rho} \left( \frac{m_A}{m_B} \right)^{1-\rho} \).

Changing \( \rho \), and therefore the location of field \( I \), has similar results to the ones described in Proposition 3.5.
4 Concluding remarks

Interdisciplinarity, the development of a new scientific discipline with foundations in well-established disciplines, has recently gain visibility as a promising way of solving complex questions of societies. Despite this renowned importance that is assigned to interdisciplinarity, scholars and scientific institutions do not always share this enthusiasm when deciding the allocation of research resources. This paper shows that under a horizontal differentiation of scientific expertises it is efficient to combine them in a new field, when the resulting complementary gains compensate the entrance and coordination costs. When the goal to achieve is minor, it is better that each researcher works separate and independently in his expertise field. Nevertheless, when the goal is sufficiently audacious, the productive gains from cooperation make interdisciplinarity a more beneficial pattern than specialization.

In this paper I take the perspective of a research organization, an university, whose activities receive a reward from a policymaker, a government, in the form of prizes. In this case, the university seeks to match at most the required standard, since it has no extra-reward from producing above it. The first-best solution is achievable whenever the type of research is contractible and the funding rules differ between fields. The acquisition of adaptative-skills is optimal, not only when the value of the prize is sufficient to pay for such investment, but also when the saving in cost that they allow, compensates the investment. The acquisition of the adaptative-skills is, then, conditional on a high interdisciplinary production.

When the government is restricted to establish a unique funding rule, common to all fields, the type of research is decided by the university. It may seem that specialization is favored, due to its potential higher revenue and less disadvantaged cost structure. Nevertheless, when the required production is sufficiently high to benefit from complementarity gains in production. An excessively high production requirement, however, discourages the development of any type of research.

Besides gains from complementary inputs, the preference of the university for interdisciplinarity is also affected by two other factors. First, the cost of the traditional fields. Higher cost makes both specialized and interdisciplinary research more expensive. Due to the presence of complementarity in interdisciplinary production, it is possible to reallocate the contribution of each research. This is, however, not possible for specialization, which makes interdisciplinarity relatively favored by an increase in these innate costs. In other words, my result shows that institutions with higher costs, thus less efficient in producing the traditional fields of research, may consider interdisciplinarity as a more interesting option. Second, and conversely, when the interdisciplinary field is more central and none researcher is particularly familiar with it, specialization is a better alternative for the research institution. In this case, the increase in the cost of the effort of the closest researcher, and therefore also the one whose involvement is more important, may be too high.
The results in this paper can be applied to a broader range of problems. In particular, it is possible to link them to the organizational structure of firms. From an internal perspective, our results may be applied to a firm that faces the possibility to have two units operating separate and independently in two known domains, or to coordinate them in a new one.

The present work may also be linked with problems in merger of firms, when merging involves the exploration of a new area business, with which none of the partners is familiar.

The aim of this paper is not to discuss organizational or informational issues between the employer organization, the university, and its workers. By definition, interdisciplinarity relies on the collaboration between different researchers, with different scientific backgrounds. Conflict of interests may then arise within the interdisciplinary group, making relevant the design not only of the internal organization among researchers, but also of the relation between the group and the host-research institution. In future work, I plan to develop these issues.

It is worthy emphasizing the pertinent conclusion that interdisciplinarity may be an interesting option for research institutions, once the cost of opening the new scientific path is overcome. Besides the support that policymakers can give to institutions, ensuring the monetary means to face this cost, their role is crucial to guarantee that the performance required is sufficiently high to be worth going through difficulties.

References


5 Appendix

**Proof of Lemma 2.1.** By the definition of the interdisciplinary cost $C_I$ in (5), if there is no acquisition of adaptative-skills of researcher $A$, the cost of the interdisciplinary project is:

$$C_I = m_A e_A + m_B [1 - (1 - \rho) \theta_B] e_B + \frac{1 - \rho}{\rho} \theta_B.$$

When there is an investment in $A$’s adaptative-skills:

$$C_I = (\rho + \alpha_A) e_A + m_B [1 - (1 - \rho) \theta_B] e_B + \frac{\rho}{1 - \rho} + \frac{1 - \rho}{\rho} \theta_B.$$

Comparing both situations, it is cost minimizing to invest in $A$’s adaptative-skills iff:

$$e_A \geq \frac{1}{\rho + \alpha_A}.$$

Similar reasoning can be developed for the adaptative-skills of $B$. ■

**Proof of Proposition 3.1.** In the benchmark situation, when the Government asks for specialized research, the problem may be formalized as:

$$\max_{(e_A, e_B, g_A, g_B)} V_{AB} = \sum_{i=A,B} V_i [Y_i (e_i)]$$

s.t. \( \Pi_i = g_i - C_i (e_i) \geq 0, \ i = A, B, \)

\( g_A + g_B \leq G, \)

where the production functions $Y_i (e_i)$ and the cost functions $C_i (e_i)$ are given by (1) and
The first-order conditions are:

\[
\begin{align*}
&\frac{\partial V_I}{\partial y_I} (e_i) = \mu_I \alpha_i, \\
&\frac{\partial V_I}{\partial \alpha_I} (e_i) = \mu_3, \\
&\mu_I (g_i - \alpha_i e_i) = 0, \quad i = A, B, \\
&\mu_3 (g_A + g_B - G) = 0, \\
&\mu_I \geq 0, \mu_3 \geq 0,
\end{align*}
\]

where \(\mu_i\) is the Lagrangian-multiplier associated with the participation constraint on specialized field \(i\), and \(\mu_3\) is the Lagrangian-multiplier for the budget constraint.

From the assumptions on \(V_i\) and on \(\alpha_i\), it follows that all \(\mu\)-multipliers are strictly positive and, hence, all constraints are binding, in the optimum.

Furthermore, from the assumptions on \(V_i\) and on the linearity of costs, first-order conditions are also sufficient to obtain a maximum. □

**Proof of Proposition 3.2.** In the benchmark situation, when the Government asks for the interdisciplinary research, the optimal contract solves

\[
\begin{align*}
\max_{(e_{AI}, e_{BI}, \theta_A, \theta_B, g_I)} & \quad V_I [Y_I (e_{AI}, e_{BI})] \\
\text{s.t.} & \quad \Pi_I = g_I - C_I (e_{AI}, e_{BI}, \theta_A, \theta_B) \geq 0, \\
& \quad g_I \leq G,
\end{align*}
\]

where the production function \(Y_I\) is presented in (3) and the cost function \(C_I\) in (5).

For simplicity, I divide the resolution of this problem in two steps:

- 1st step: optimal \((e_{AI}, e_{BI}, g_I)\).

The first-order conditions to obtain the optimal level of inputs and the monetary reward are:

\[
\begin{align*}
&\frac{\partial V_I}{\partial y_I} (e_{AI}, e_{BI}) = \mu_1 \cdot \frac{\partial C_I}{\partial e_I} (e_{AI}, e_{BI}, \theta_A, \theta_B), \\
&\mu_1 = \mu_2, \\
&\mu_1 [g_I - C_I (e_{AI}, e_{BI}, \theta_A, \theta_B)] = 0, \quad i = A, B, \\
&\mu_2 (g_I - G) = 0, \\
&\mu_1 \geq 0, \mu_2 \geq 0,
\end{align*}
\]

where \(\mu_1\) is the Lagrangian-multiplier associated with the participation constraint of the University, and \(\mu_2\) the multiplier associated with the budget condition. Solving the system, I obtain results (9), (10), (11), and (12). Assumptions on the function \(V_I\), the linearity of the cost function in the decision variables, and the convexity of the production technology, ensure that the conditions above are necessary and sufficient for a maximum.
- 2nd step: optimal \((\theta_A, \theta_B)\).

From the previous system of conditions, the optimal level of inputs can be written as a function of the adaptative-skills. Therefore, it is also possible to write the interdisciplinary output in terms of \((\theta_A, \theta_B)\), as explicit in (13). Since \(V_I\) is strictly increasing in \(y_I\), the optimal investment in the adaptative-skills must guarantee maximum production.

Take \(\rho \in (0, \frac{1}{2})\). Calculating the upper-envelope curve of \(y_I^* (\theta_A, \theta_B)\) when considering the possible combinations \((\theta_A, \theta_B)\), I verify that the maximum interdisciplinary output is achieved when:

\[
(\theta_A, \theta_B) = \begin{cases} 
(0, 0) & \text{if } 0 \leq G \leq \frac{\rho}{1 - \rho}, \\
(1, 0) & \text{if } \frac{\rho}{1 - \rho} \leq G \leq \frac{1}{\rho} + \frac{\rho}{1 - \rho}, \\
(1, 1) & \text{if } G \geq \frac{1}{\rho} + \frac{\rho}{1 - \rho}.
\end{cases}
\]

Similar and symmetric results can be developed for the remaining possible values of \(\rho\).

**Proof of Corollary 3.1.** From expressions (9) and (10), it is easily verifiable that a decrease in \(\alpha_i\) has a positive effect on \(e_i^*\), \(i = A, B\).

To verify how \(\theta_i\) affects \(e_i^*\) and \(e_j^*\), \(j \neq i\), let us consider the case of \(\rho \in (0, \frac{1}{2})\). In the optimal solution, researcher \(A\) is the first to acquire adaptative-skills and he does it for \(G \geq \frac{\rho}{1 - \rho}\). Comparing the value for \(e_A^*\) when \((\theta_A, \theta_B) = (0, 0)\) with the one when \((\theta_A, \theta_B) = (1, 0)\), I obtain the following condition:

\[
e_A^* (1, 0) \geq e_A^* (0, 0) \Leftrightarrow \frac{G (1 - \rho)^2}{\rho + \alpha_A} \geq \frac{G - \frac{\rho}{1 - \rho}}{\rho + \alpha_A} \Leftrightarrow G \geq \frac{1}{1 - \rho},
\]

which is satisfied for \(G \geq \frac{\rho}{1 - \rho}\). The investment in \(\theta_B\) is interesting when \(G \geq \frac{1}{\rho} + \frac{\rho}{1 - \rho}\). To have a positive relation between \(B\)'s adaptative-skills and his level of input, I need to guarantee that:

\[
e_B^* (1, 0) \geq e_B^* (1, 1) \Leftrightarrow \left( \frac{G - \rho}{1 - \rho} - \frac{1 - \rho}{\rho} \right) \rho \geq \left( \frac{G - \rho}{1 - \rho} \right) \rho^2 \Leftrightarrow G \geq \frac{1}{\rho} + \frac{\rho}{1 - \rho}.
\]

Given the domain where it is optimal to have \(\theta_B = 1\), this condition holds.

For the remaining possible values of \(\rho\), by similar reasoning, I can verify that the participation of each researcher increases when he acquires adaptative-skills. Efficiency arguments support such adjustment.

As far as the effect of \(\rho\) in \(e_j\) is concerned, let us consider again the case of \(\rho \in (0, \frac{1}{2})\) and the optimal decision on the acquisition of the adaptative-skills. Then, the impact that
\( \rho \) has on \( e^*_A \) is:
\[
\frac{\partial e^*_A(0,0)}{\partial \rho} = -\frac{G (1-\rho)(1+\rho+2\alpha_A)}{(\rho+\alpha_A)^2} < 0;
\]
\[
\frac{\partial e^*_A(1,0)}{\partial \rho} = -\frac{G (1+\alpha_A) + \alpha_A}{(\rho+\alpha_A)^2} < 0;
\]
\[
\frac{\partial e^*_A(1,1)}{\partial \rho} = -\frac{\rho (-2 + (2+G)\rho) + \alpha_A [-1 + (2+G)\rho^2]}{\rho^2 (\rho+\alpha_A)^2},
\]
but since \(-2 + (2+G)\rho > 0 \) for \( G \geq \frac{1-\rho}{\rho \cdot (1-\rho^2)} + \frac{\rho}{1-\rho} \),

it is sufficient that \( G > \frac{1-2\rho^2}{\rho^2} \) to ensure that \( \frac{\partial e^*_A(1,1)}{\partial \rho} > 0 \).

The impact that \( \rho \) has on \( e^*_B \) is:
\[
\frac{\partial e^*_B(0,0)}{\partial \rho} = \frac{G \rho(2-\rho+2\alpha_B)}{(1-\rho+\alpha_B)^2} > 0;
\]
\[
\frac{\partial e^*_B(1,0)}{\partial \rho} = \frac{(1-\rho) \rho [G (2-\rho)(1-\rho) - \rho (3-\rho)] + \rho \alpha_B [2G (1-\rho)^2 - \rho (3-2\rho)]}{(1-\rho)^2 (1-\rho + \alpha_B)^2},
\]
but for \( G \geq \frac{(1-\rho) \cdot [1 - (1-\rho)^{1-\rho}]}{(1-\rho)^2 (1-\rho + \alpha_B)^2} \),

always hold \( G (2-\rho)(1-\rho) - \rho (3-\rho) > 0 \)

and \( 2G (1-\rho)^2 - \rho (3-2\rho) > 0 \), so \( \frac{\partial e^*_B(1,0)}{\partial \rho} > 0 \);
\[
\frac{\partial e^*_B(1,1)}{\partial \rho} = \frac{(2+G)(1-\rho) \rho - 1}{(1-\rho) (1-\rho + \alpha_B)} > 0 \text{ because in the domain } G \geq \frac{1-\rho}{\rho \cdot (1-\rho^2)} + \frac{\rho}{1-\rho},
\]
it is true that \( (2+G)(1-\rho) \rho - 1 > 0 \).

---

**Proof of Corollary 3.2.** As stated in Proposition 3.2, in the benchmark model, the optimal solution for the interdisciplinary production is

\[
y^*_I(\theta_A, \theta_B) = \lambda \frac{G - \frac{\rho}{1-\rho} \theta_A - \frac{1-\rho}{\rho} \theta_B}{m_A^\rho m_B^{1-\rho} \left( \frac{1-\rho A_{1-\rho}}{1-\rho} \right)^{1-\rho} \left( \frac{1-(1-\rho) \theta_B}{\rho} \right)^{\rho}},
\]

where \( m_A = \frac{\rho+\alpha_A}{1-\rho} \), and \( m_B = \frac{1-\rho+\alpha_B}{\rho} \). Graphically, it can be represented in terms of \( G \), as the upper envelope curve of the all four lines \( y^*_I(0,0), y^*_I(1,0), y^*_I(0,1), \) and \( y^*_I(1,1) \). As Figure 8 below shows, the relevant thresholds are \( G^{10} = \frac{\rho}{(1-\rho) [1-(1-\rho)^{1-\rho}]} \) and \( G^{11} = \frac{1-\rho}{\rho (1-\rho^2)} + \frac{\rho}{1-\rho} \).
For $\alpha_A = \alpha_B = \alpha$, the change in the slopes of the three upper lines due to a marginal change in $\rho$ is:

$$
\frac{\partial (\text{slope of } y^*_i(0,0))}{\partial \rho} = \frac{\partial \left( \lambda \frac{1}{m_A^{1-\rho} m_B^\rho} \right)}{\partial \rho} = \lambda (1 - \rho)^{1-\rho} \cdot \rho^\rho \cdot m_A^{1-\rho} \cdot \left( \frac{1}{m_B} \right)^\rho \cdot \left\{ \log \left( \frac{\rho m_A}{m_B} \right) - \frac{1}{m_A} + \frac{1}{m_B} \right\},
$$

$$
\frac{\partial (\text{slope of } y^*_i(1,0))}{\partial \rho} = \frac{\partial \left( \lambda \frac{1}{m_A^{1-\rho} m_B^\rho} \right)}{\partial \rho} = \lambda \rho^\rho \cdot \left( \frac{1}{m_A} \right)^{1-\rho} \cdot \left( \frac{1}{m_B} \right)^\rho \cdot \left\{ \log \left( \frac{\rho m_A}{m_B} \right) + 1 - \frac{1}{m_A} + \frac{1}{m_B} \right\},
$$
\[
\frac{\partial (\text{slope of } y^*_I (1, 1))}{\partial \rho} = \frac{\partial \left( \lambda \frac{1}{m_A^{1-\rho} m_B^{\rho}} \right)}{\partial \rho} = \\
= \lambda \left( \frac{1}{m_A} \right)^{1-\rho} \cdot \left( \frac{1}{m_B} \right)^{\rho} \cdot \\
\cdot \left\{ \log \left( \frac{m_A}{m_B} \right) - \frac{1}{m_A} + \frac{1}{m_B} \right\}.
\]

For \( \rho \in (0, \frac{1}{2}) \),

it is true that: \( \log \left[ \left( \frac{m_A}{m_B} \right) \left( \frac{\rho}{1-\rho} \right) \right] - \frac{1}{m_A} + \frac{1}{m_B} < 0, \)
therefore \( \frac{\partial (\text{slope of } y^*_I (0, 0))}{\partial \rho} < 0; \)

it is true that: \( \log \left( \frac{m_A}{m_B} \right) - \frac{1}{m_A} + \frac{1}{m_B} < 0, \)
therefore \( \frac{\partial (\text{slope of } y^*_I (1, 1))}{\partial \rho} < 0; \)

if \( \log \left( \frac{\rho m_A}{m_B} \right) + 1 - \frac{1}{m_A} + \frac{1}{m_B} < 0, \)
then \( \frac{\partial (\text{slope of } y^*_I (1, 0))}{\partial \rho} < 0. \)

As far as the change in the value at the origin is concerned:

\[
\frac{\partial |\text{value at origin of } y^*_I (1, 0)|}{\partial \rho} = \frac{\partial \left( \lambda \frac{\rho}{m_A^{1-\rho} m_B^{\rho} \left( \frac{1}{\rho} \right)^\rho} \right)}{\partial \rho} = \\
= \lambda \frac{1}{\rho + \alpha} \cdot \rho^{1+\rho} \cdot \left( \frac{m_A}{m_B} \right)^{\rho} \cdot \\
\cdot \left\{ \log \left[ \left( \frac{m_A}{m_B} \right)^{\rho} \right] - \frac{1}{m_A} + \frac{1}{m_B} + 1 + \frac{1}{\rho (1-\rho)} \right\},
\]
\[
\frac{\partial \text{ value at origin of } y^*_J (1,1)}{\partial \rho} = \frac{\partial \left( \frac{\rho + \frac{1}{\rho}}{m_A^1 m_B^1} \right)}{\partial \rho} = \\
= \lambda \left( \frac{1}{m_A} \right)^{1-\rho} \cdot \left( \frac{1}{m_B} \right)^{\rho} \cdot \left\{ 1 - \frac{2\rho (1-\rho)}{\rho (1-\rho)} \cdot \log \left( \frac{m_A}{m_B} \right) - \frac{1-2\rho (1-\rho)}{\rho (\rho + \alpha)} + \frac{1}{(1-\rho) (1-\rho + \alpha)} + \frac{1}{(1-\rho)^2} - \frac{1}{\rho^2} \right\}.
\]

For \( \rho \in (0, \frac{1}{2}) \),

it is true that:
\[
\left\{ \frac{1-2\rho (1-\rho)}{\rho (1-\rho)} \cdot \log \left( \frac{m_A}{m_B} \right) - \frac{1-2\rho (1-\rho)}{\rho (\rho + \alpha)} + \frac{1}{(1-\rho)^2} - \frac{1}{\rho^2} \right\} < 0,
\]

therefore
\[
\frac{\partial \text{ value at origin of } y^*_J (1,1)}{\partial \rho} < 0,
\]

if \( \left\{ \log \left( \frac{\rho m_A}{m_B} \right) - \frac{1}{m_A} + \frac{1}{m_B} + 1 + \frac{1}{\rho (1-\rho)} \right\} < 0, \)

then
\[
\frac{\partial \text{ value at origin of } y^*_J (1,0)}{\partial \rho} < 0.
\]

A variation in \( \rho \) also affects the intersection points of the three lines, \( G^{10} \) and \( G^{11} \):

\[
\frac{\partial (G^{10})}{\partial \rho} = \frac{\partial \left( \frac{\rho}{(1-\rho)[1-(1-\rho)^{1-\rho}]} \right)}{\partial \rho} = \\
= \frac{-1 + (1-\rho)^2 + (2-\rho) \rho^2 - (1-\rho)^2 \rho \log (1-\rho)}{(1-\rho)^2 - \rho^2 \rho - 1 + (1-\rho)^{1-\rho}}.
\]

\[
\frac{\partial (G^{11})}{\partial \rho} = \frac{\partial \left( \frac{1-\rho}{\rho (1-\rho^\rho)} + \frac{\rho}{1-\rho} \right)}{\partial \rho} = \\
= \frac{-1 + 2\rho + \rho^{2+2\rho} - \rho^\rho \left[ -1 + \rho (1 + \rho [4 + \rho (\rho - 3)]) + (\rho - 1)^3 \rho \log (\rho) \right]}{(1-\rho)^2 \rho^2 (\rho^\rho - 1)^2}.
\]
For $\rho \in (0, \frac{1}{2})$,

it is true that $\left\{ -1 + (1 - \rho)^2 + (2 - \rho) \rho^2 - (1 - \rho)^2 \rho \log (1 - \rho) > 0 \right\}$,

therefore $\frac{\partial G^{10}}{\partial \rho} > 0$,

it is true that $\left\{ -1 + 2\rho + \rho^{2+2\rho} - \rho^\rho \left[ -1 + \rho (1 + \rho [4 + \rho (\rho - 3)]) + (\rho - 1)^3 \rho \log (\rho) \right] < 0 \right\}$, therefore $\frac{\partial G^{11}}{\partial \rho} < 0$.

**Proof of Proposition 3.3.** When the resources and the adaptative-skills are non-contractible, the problem of the University under specialization is

$$\max_{(e_{AI}, e_{BI})} \Pi_{AB} = \sum_{i=A,B} g_i - C_i (e_i)$$

s.t. $Y_i (e_i) \geq \tilde{y}_i$, $i = A, B$,

where $(\tilde{y}_i, g_i)$ is defined by the Government.

The first-order conditions of this problem are:

$$\left\{ \begin{array}{l}
\mu_i = \alpha_i, \\
\mu_i \cdot [\tilde{y}_i - Y_i(e_i)] = 0, \; i = A, B, \\
\mu_i \geq 0,
\end{array} \right.$$  

where $\mu_i$ is the Lagrangian-multiplier associated with the standard required for field $i$.

By definition, $\alpha_i > 0$, which implies that in the optimum $Y_i(e_i) = \tilde{y}_i$. From the production functions (1) it follows that $e_i^{\tilde{y}_i} = \tilde{y}_i$, $i = A, B$. Linearity of the production functions and of the costs functions ensure the first-order conditions are necessary and sufficient for having a maximum.

When the University decides for the interdisciplinary project, its problem becomes

$$\max_{(e_{AI}, e_{BI}, \theta_A, \theta_B)} \Pi_I = g_I - C_I (e_{AI}, e_{BI}, \theta_A, \theta_B)$$

s.t. $Y_I (e_{AI}, e_{BI}) \geq \tilde{y}_I$.

The first-order conditions are:

$$\left\{ \begin{array}{l}
\mu_I \cdot \frac{\partial Y_I(e_{AI}, e_{BI})}{\partial e_{iI}} = \frac{\partial C_I(e_{AI}, e_{BI}, \theta_A, \theta_B)}{\partial e_{iI}}, \; i = A, B, \\
\mu_I \cdot [\tilde{y}_I - Y_I(e_{AI}, e_{BI})] = 0, \\
\mu_I \geq 0,
\end{array} \right.$$  

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Comparing the value of possible to derive the following profit function for interdisciplinarity:

\[ \Pi_I^U(\theta_A, \theta_B) = g_I - \frac{\rho}{1 - \rho} \theta_A - \frac{1 - \rho}{\rho} \theta_B - \tilde{y}_I \cdot \frac{1}{\lambda} \cdot [m_A (1 - \rho \theta_A)]^{1 - \rho} \cdot [m_B (1 - (1 - \rho) \theta_B)]^{\rho} \cdot \frac{1}{\rho^\rho (1 - \rho)^{1 - \rho}}. \]

Comparing the value of \( \Pi_I^U(\theta_A, \theta_B) \) for the different cases of \((\theta_A, \theta_B) = (0, 0), (1, 0), (0, 1), \) and \((1, 1)\), I obtain the conditions under which the acquisition of the adaptative-skills is profit maximizer. For \( \rho \in (0, \frac{1}{2}) \), this optimal pattern is

\[
(\theta_A, \theta_B) = \begin{cases} 
(0, 0), & \text{if } g_I \leq \frac{\rho}{1 - \rho} \frac{1 - \rho}{1 - (1 - \rho)^{1 - \rho}} \lambda, \\
\text{or if } g_I > \frac{\rho}{1 - \rho} \frac{1 - \rho}{1 - (1 - \rho)^{1 - \rho}} \lambda \\
\tilde{y}_I \leq \frac{m_A^{1 - \rho} m_B^\rho (\frac{1}{\rho}) (\frac{1}{1 - \rho})}{(1 - \rho) [1 - (1 - \rho)^{1 - \rho}]} \lambda; \\
(1, 0), & \text{if } g_I \in \left( \frac{\rho}{1 - \rho} \frac{1 - \rho}{1 - (1 - \rho)^{1 - \rho}} \lambda, \frac{1 - \rho}{1 - \rho} \frac{1 - \rho}{1 - (1 - \rho)^{1 - \rho}} \lambda \right) \\
\text{or if } g_I > \frac{\rho}{1 - \rho} \frac{1 - \rho}{1 - (1 - \rho)^{1 - \rho}} \lambda \\
\tilde{y}_I > \frac{m_A^{1 - \rho} m_B^\rho (\frac{1}{\rho}) (\frac{1}{1 - \rho})}{(1 - \rho) [1 - (1 - \rho)^{1 - \rho}]} \lambda; \\
(1, 1), & \text{if } g_I \geq \frac{(1 - \rho)^2 + \rho^2 (1 - \rho')}{\rho [1 - (1 - \rho)^{1 - \rho}]} \lambda \\
\text{or if } g_I > \frac{(1 - \rho)^2 + \rho^2 (1 - \rho')}{\rho [1 - (1 - \rho)^{1 - \rho}]} \lambda \\
\tilde{y}_I \in \left( \frac{m_A^{1 - \rho} m_B^\rho (\frac{1}{\rho}) (\frac{1}{1 - \rho})}{(1 - \rho) [1 - (1 - \rho)^{1 - \rho}]} \lambda, \frac{m_A^{1 - \rho} m_B^\rho (\frac{1}{\rho}) (\frac{1}{1 - \rho})}{(1 - \rho) [1 - (1 - \rho)^{1 - \rho}]} \lambda \right); 
\end{cases}
\]

For \( \rho \in \left( \frac{1}{2}, 1 \right) \), similar conditions support the choice of \((\theta_A, \theta_B) = (0, 0), (0, 1) \) and \((1, 1)\). For \( \rho = \frac{1}{2} \), the University is indifferent between \((\theta_A, \theta_B) = (1, 0) \) and \((0, 1)\), which means that the choice concerns only the number of researchers with adaptative-skills.

**Proof of Corollary 3.3.** From the solution of the interdisciplinary problem stated in

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Proposition 3.3, I obtain the following results:

\[
\frac{\partial e_{AI}^U}{\partial \alpha_A} = -\frac{\dot{y}_l}{\lambda} \cdot \frac{\rho}{\rho + \alpha_A} \cdot \left[ \frac{m_B}{m_A} \cdot \left( \frac{1}{1 - \rho \theta_A} \right) \cdot \left( \frac{1 - \rho}{\rho} \right) \right]^\rho < 0; \\
\frac{\partial e_{AI}^U}{\partial \alpha_B} = \frac{\dot{y}_l}{\lambda} \cdot \frac{\rho}{1 - \rho + \alpha_B} \cdot \left[ \frac{m_B}{m_A} \cdot \left( \frac{1}{1 - \rho \theta_A} \right) \cdot \left( \frac{1 - \rho}{\rho} \right) \right]^\rho > 0;
\]

\[
\frac{\partial e_{AI}^U}{\partial \theta_A} = \frac{\dot{y}_l}{\lambda} \cdot \frac{\rho^2}{1 - \rho \theta_A} \cdot \left[ \frac{m_B}{m_A} \cdot \left( \frac{1}{1 - \rho \theta_A} \right) \cdot \left( \frac{1 - \rho}{\rho} \right) \right]^\rho > 0; \\
\frac{\partial e_{AI}^U}{\partial \theta_B} = -\frac{\dot{y}_l}{\lambda} \cdot \frac{\rho(1 - \rho)}{1 - (1 - \rho) \theta_B} \cdot \left[ \frac{m_B}{m_A} \cdot \left( \frac{1}{1 - \rho \theta_A} \right) \cdot \left( \frac{1 - \rho}{\rho} \right) \right]^\rho < 0.
\]

The global effect of \( \rho \) on \( A \)'s input is:

\[
\frac{\partial e_{AI}^U}{\partial \rho} = \frac{\dot{y}_l}{\lambda} \cdot \left\{ \log \left[ \frac{m_B}{m_A} \cdot \frac{1}{1 - \rho \theta_A} \cdot \frac{1 - \rho}{\rho} \right] - \frac{3 - \frac{1 - \rho}{1 - \rho \theta_A} - \frac{\rho}{1 - \rho \theta_A} - \frac{1}{m_B}}{1 - \rho} \right\} \cdot \left[ \frac{m_B}{m_A} \cdot \frac{1}{1 - \rho \theta_A} \cdot \frac{1 - \rho}{\rho} \right]^\rho.
\]

A sufficient condition for having a negative relation between \( e_{AI}^U \) and \( \rho \) is:

\[
\frac{m_B}{m_A} \cdot \frac{1}{1 - \rho \theta_A} \cdot \frac{1 - \rho}{\rho} < 1 \Leftrightarrow \frac{(1 - \rho \theta_A)m_A}{[1 - (1 - \rho) \theta_B] m_B} > \frac{1 - \rho}{\rho}.
\]

Similar expressions can be found for comparative statics on \( e_{BI}^U \).

**Proof of Corollary 3.4.** Given the University’s best choice for the interdisciplinary resources \((e_{AI}^U, e_{BI}^U, \theta_A, \theta_B)\) stated in Proposition 3.3, it is straightforward to derive the maximum interdisciplinary profit expression and then to calculate the following derivatives:

\[
\frac{\partial \Pi_{AI}^U}{\partial \alpha_A} (\theta_A, \theta_B) = -\frac{\dot{y}_l}{\lambda} \cdot (m_A)^{1-\rho} \cdot (1 - \rho \theta_A)^{1-\rho} \cdot \left[ m_B (1 - (1 - \rho) \theta_B) \right]^{\rho} \cdot \left\{ \left( \frac{1 - \rho}{\rho} \right)^\rho + \left( \frac{\rho}{1 - \rho} \right)^{1-\rho} \right\} < 0;
\]

\[
\frac{\partial \Pi_{AI}^U}{\partial \alpha_B} (\theta_A, \theta_B) = -\frac{\dot{y}_l}{\lambda} \cdot (m_A (1 - \rho \theta_A))^{1-\rho} \cdot (m_B)^{\rho-1} \cdot \left[ 1 - (1 - \rho) \theta_B \right]^{\rho} \cdot \left\{ \left( \frac{1 - \rho}{\rho} \right)^\rho + \left( \frac{\rho}{1 - \rho} \right)^{1-\rho} \right\} < 0.
\]
For general values of the parameters, the final effect of $\rho$ in the interdisciplinary profit is ambiguous, since:

\[
\frac{\partial \Pi_I}{\partial \rho} (0, 0) = \frac{\tilde{y}_I}{\lambda} \cdot \left( \frac{m_B}{m_A} \right)^{\rho} \cdot \left( \frac{1}{1 - \rho} \right)^{2 - \rho} \cdot \left( \frac{1}{\rho} \right)^{\rho} \cdot \left\{ -1 - \alpha_A + (\rho + \alpha_A) \cdot \left[ \log (m_A) - \log (m_B) + \log \left( \frac{1}{1 - \rho} \right) - \log \left( \frac{1}{\rho} \right) + \frac{1 + \alpha_B}{1 - \rho + \alpha_B} \right] \right\}.
\]

Nevertheless, when $\alpha_A = \alpha_B = \alpha$ and $\rho \in (0, \frac{1}{2})$, it is possible to conclude that:

i) for $(\theta_A, \theta_B) = (0, 0)$: $\frac{\partial \Pi_I}{\partial \rho} (\theta_A, \theta_B) < 0$;

ii) for $(\theta_A, \theta_B) = (1, 0)$:

\[
\frac{\partial \Pi_{MU}^I}{\partial \rho} (1, 0) = -\frac{1}{(1 - \rho)^2} + \frac{\tilde{y}_I}{\lambda} \cdot \left( \frac{m_B}{m_A} \right)^{\rho} \cdot \left( \frac{1}{1 - \rho} \right) \cdot \left( \frac{1}{\rho} \right)^{\rho} \cdot \left\{ -1 - \alpha_A + (\rho + \alpha_A) \cdot \left[ \log (m_A) - \log (m_B) - \log \left( \frac{1}{\rho} \right) + \frac{1}{m_B} \right] \right\}.
\]

This means that if $\log \left( \frac{m_B}{m_A} \right) > \frac{\rho}{1 - \rho + \alpha}$, then $\frac{\partial \Pi_{MU}^I}{\partial \rho} (1, 0) < 0$;

iii) for $(\theta_A, \theta_B) = (1, 1)$:

\[
\frac{\partial \Pi_I}{\partial \rho} (1, 1) = \frac{1}{\rho^2} - \frac{1}{(1 - \rho)^2} + \frac{\tilde{y}_I}{\lambda} \cdot \left( \frac{m_B}{m_A} \right)^{\rho} \cdot \left( \frac{1}{1 - \rho} \right) \cdot \left\{ -1 - \alpha_A + (\rho + \alpha_A) \cdot \left[ \log (m_A) - \log (m_B) - 2 + \frac{1 + \alpha_B}{1 - \rho + \alpha_B} \right] \right\}
\]

from what I conclude that $\log \left( \frac{m_B}{m_A} \right) > -2 + \frac{1 + \alpha}{1 - \rho + \alpha} - \frac{1 + \alpha}{\rho + \alpha} + \frac{1}{m_B} \cdot \frac{1}{m_A} \cdot \frac{1}{\rho} \cdot \frac{1}{1 - \rho + \alpha} \cdot \frac{1}{1 - \rho + \alpha + \beta} \Rightarrow \frac{\partial \Pi_I}{\partial \rho} (1, 1) < 0$.

\[\Box\]

**Proof of Corollary 3.5.** From the profit maximizer behavior of the University stated in Proposition 3.3, I obtain that, once deciding for one type of research, the University exactly matches the required standard. The result of this corollary then follows straightforward. \[\Box\]

**Proof of Proposition 3.4.** To obtain the Sub-game Perfect Nash equilibrium solution, I first concentrate in the problem of the University when choosing the resources
and the investment on the adaptative-skills. Considering the specialized projects, the profit maximizing choice for the resources is given by

$$\max_{(e_A, e_B)} \Pi_{AB} = 2g - \sum_{i=A,B} C_i(e_i)$$

s.t. $Y_i(e_i) \geq \tilde{y}, \ i = A, B,$

from which I obtain the solution $e_i^U = \tilde{y},$ and the profit value $\Pi_i^U = g - \alpha \cdot \tilde{y}, \ i = A, B.$ Under Assumption 2, the University never considers the scenario of developing only one specialized project, since either develops both $A$ and $B,$ or no one of them.

Considering the interdisciplinary field, the amount chosen for the inputs and the acquisition of the adaptative-skills solves

$$\max_{(e_{AI}, e_{BI}, \theta_A, \theta_B)} \Pi_I = g - C_I(e_{AI}, e_{BI}, \theta_A, \theta_B)$$

s.t. $Y_I(e_{AI}, e_{BI}) \geq \tilde{y}.$

The optimal solutions are similar to the ones presented in Proposition 3.3. Again, if choosing interdisciplinarity, the University does not produce above the standard.

For the policymaker, the restriction of having an unique funding rule makes the choice of interdisciplinarity or specialization a non-contractible decision. This means that, if the Government wishes to induce the choice of the interdisciplinary field, the best possible funding rule $(\tilde{y}, g)$ solves:

$$\max_{(\tilde{y}, g)} V_I [Y_I(e_{AI}, e_{BI})]$$

s.t. $\left\{ \begin{array}{l}
\Pi_I(e_{AI}, e_{BI}, \theta_A, \theta_B) \geq 0, \\
\Pi_I(e_{AI}, e_{BI}, \theta_A, \theta_B) \geq \Pi_{AB}(e_A, e_B), \ i = A, B, \\
2g \leq G, \\
e_{ii} = e_{ii}(\tilde{y}), \end{array} \right.$$

where $e_{ii}(\tilde{y})$ is the solution of the profit maximization problem of the University under interdisciplinarity, and $\Pi_{AB}(e_A, e_B)$ is the maximum value of the specialized choice, obtained from the problem of the University for specialization. Given the linearity of the cost function and the strict convexity of the interdisciplinary technology, the necessary and sufficient
conditions to obtain a maximum are:

\[
\begin{align*}
\frac{\partial y}{\partial \beta} + \frac{\partial y}{\partial \beta} (\tilde{y}) + (\mu_1 + \mu_2) \cdot \frac{\partial \Pi}{\partial \beta} (\tilde{y}) - \mu_2 \cdot \frac{\partial \Pi}{\partial \beta} = 0, \\
(\mu_1 + \mu_2) \cdot \frac{\partial \Pi}{\partial g} - \mu_2 \cdot \frac{\partial \Pi}{\partial g} - 2\mu_3 = 0, \\
\mu_1 \cdot \Pi_I (e_{AI}, e_{BI}, \theta_A, \theta_B) = 0, \\
\mu_2 \cdot [\Pi_{AB} (e_A, e_B) - \Pi_I (e_{AI}, e_{BI}, \theta_A, \theta_B)] = 0, \\
\mu_3 \cdot (2g - G) = 0, \\
\mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0.
\end{align*}
\]

From the first condition, I obtain

\[\mu_1 + \mu_2 > 0,\]

and from the second condition,

\[\mu_3 = \frac{\mu_1 - \mu_2}{2}.\]

Considering the fact that all Lagrangian-multipliers \(\mu_i, i = 1, 2, 3,\) are non-negative, I conclude that:

\[\mu_1 > 0,\]
\[\mu_2 \geq 0.\]

Therefore, in the optimal situation,

\[
\begin{align*}
\Pi_I (e_{AI}, e_{BI}, \theta_A, \theta_B) = 0 \Leftrightarrow g = C_I (e_{AI}, e_{BI}, \theta_A, \theta_B), \\
\Pi_I (e_{AI}, e_{BI}, \theta_A, \theta_B) \geq \Pi_{AB} (e_A, e_B). \\
\end{align*}
\]

As far as the budget constraint is concerned, in the optimal I may have

\[\mu_3 > 0 \implies 2g = G,\]
\[\text{or } \mu_3 = 0 \implies 2g \leq G.\]

Nevertheless, when the Governmental budget \(G\) is not exhausted, the other two constraints must be active:

\[2g < G \implies \mu_3 = 0 \implies \mu_1 = \mu_2.\]

Since \(\mu_1 > 0,\) it must be that \(\mu_2 > 0,\) and therefore \(\Pi_I = 0\) and \(\Pi_I = \Pi_{AB}.\)

For the case of \(\rho \in (0, \frac{1}{2})\), depending on the value of the parameters in our model, four alternative situations are possible:

**situation 1**, when \(\frac{(1-\rho)^\delta g (1-\rho) + \rho - g (1-\rho)^2}{(1-\rho)^{1+\delta}} \leq \frac{2\alpha \lambda}{m_A \cdot m_B},\) the comparison between the profit functions under specialization and interdisciplinarity rely on two thresholds, \(\tilde{g}_{AB,I}^{00} = \ldots\)
\[
\begin{align*}
g^\lambda 
\frac{2\alpha\lambda-m_A^{1-\rho} m_B^{\rho}(\frac{1}{\rho})^{1-\rho}}{m_A^{1-\rho} m_B^\rho} 
\text{ and } \tilde{y}^{11} = \frac{(g-\frac{1-\rho}{1-\rho})^\lambda}{m_A^{1-\rho} m_B^\rho}, \text{ as Figure 9 below shows.}
\end{align*}
\]

Figure 9: Specialized profit ($\Pi_{AB}$) and interdisciplinary profit ($\Pi_I$), in situation 1.

For $\tilde{y} \in (0, \tilde{y}_{AB,I}^{00})$, specialization is the best choice of the University. For $\tilde{y} \in (\tilde{y}_{AB,I}^{00}, \tilde{y}^{11})$, it prefers the interdisciplinary project.

Anticipating this behavior, the Government sets $\tilde{y} = \tilde{y}^{11}$ if it wants to induce interdisciplinarity, or $\tilde{y} = \tilde{y}_{AB,I}^{00}$ if it prefers specialization. To know which of the alternative scenarios is actually chosen by the policy-maker, I must compare the value of $V_I(\tilde{y}^{11})$ with $V_{AB} = \sum_{i=A,B} V_i(\tilde{y}_{AB,I}^{00})$;

**situation 2**, when

\[
\frac{1-\rho|2-\rho(2-\rho^\rho)-g(1-\rho)(1-\rho^\rho)|}{\rho^\rho(1-\rho)^2} \leq \frac{2\alpha\lambda}{m_A^{1-\rho} m_B^\rho} \leq \frac{(1-\rho)^{\rho}(1-\rho)^{\rho+\rho}-g(1-\rho)^2}{(1-\rho)^{\rho+\rho}}, \text{ Figure 10}
\]

\[\]
represents the comparison between the two types of profit, \( \Pi_I \) and \( \Pi_{AB} \).

The University prefers the specialized projects when \( \hat{\gamma} \in (0, \hat{\gamma}^{10}_{AB,I}) \), where \( \hat{\gamma}^{10}_{AB,I} = \frac{(g + \frac{1}{\rho})}{2\alpha \lambda - m^A - \rho^p m^B} \). It prefers the interdisciplinary project when \( \hat{\gamma} \in (\hat{\gamma}^{10}_{AB,I}, \hat{\gamma}^{11}) \).

Predicting this optimal reaction, the Government sets \( \hat{\gamma} = \hat{\gamma}^{11} \) or \( \hat{\gamma} = \hat{\gamma}^{10}_{AB,I} \), depending on whether it values more \( V_I (\hat{\gamma}^{11}) \) or \( V_{AB} = \sum_{i=A,B} V_i (\hat{\gamma}^{10}_{AB,I}) \), respectively.

**situation 3**, when \( \frac{2g(1-\rho)}{(2 + g)\rho (1-\rho) - 1} \leq \frac{2\alpha \lambda}{m_A - \rho^p m_B} \leq \frac{1-\rho^\prime(2-\rho^\prime)-g(1-\rho)(1-\rho^\prime)}{\rho^\prime(1-\rho^\prime)} \), Figure 11 shows how to compare the profitability of both types of projects. Here the relevant threshold for the standard is \( \hat{\gamma}^{11}_{AB,I} = \frac{(g + \frac{1}{\rho})}{2\alpha \lambda - m^A - \rho^p m^B} \).
It then follows that the specialized projects are chosen when $\tilde{y} \in (0, \tilde{y}_{AB,I}^{11})$, and the interdisciplinary project is preferred when $\tilde{y} \in (\tilde{y}_{AB,I}^{11}, \tilde{y}^{11})$.

The optimal policy is, therefore, to establish $\tilde{y} = \tilde{y}^{11}$ if the Government prefers interdisciplinarity, and $\tilde{y} = \tilde{y}_{AB,I}^{11}$ if it prefers specialization. This preference is determined by comparing $V_{I}(\tilde{y}^{11})$ with $V_{AB} = \sum_{i=A,B} v_{i}(\tilde{y}_{AB,I}^{11})$.

**situation 4**, when $\frac{2\nu \lambda}{m_{A}^{*} m_{B}^{*}} \leq \frac{2g(1-\rho)}{(2+g)(1-\rho)^{-1}}$, the University never chooses the interdisciplin-
nary project, as I can conclude from Figure 12 below.

![Graph showing specialized profit ($\Pi_{AB}$) and interdisciplinary profit ($\Pi_I$).]

The University prefers the specialized research whenever $\tilde{y} \in (0, \tilde{y}^S)$, and decides for its outside option of no research, otherwise. As a consequence, the Government establishes $\tilde{y} = \tilde{y}^S$.

In all the 4 situations above, whenever interdisciplinarity is a possible best-alternative (situations 1 to 3), I verify that it is actually so, only if the required standard is sufficiently high (above $\tilde{y}_{AB,I}^{10}$, $\tilde{y}_{AB,I}^{10}$, or $\tilde{y}_{AB,I}^{11}$, respectively), but not too much (at most $\tilde{y}^{11}$).

Consider again the previous situation 3. In particular, let $\tilde{y}_{AB,I}^{11} = \tilde{y}^{11}$. This figure enables us to discuss the case where it is not optimal to exhaust the budget $G$ and why it implies
that the incentive-compatibility constraint \( \Pi_I (e_{AI}, e_{BI}, \theta_A, \theta_B) \geq \Pi_{AB} (e_A, e_B) \) is binding.

In this case, if to induce \( \tilde{y} = \tilde{y}^{11} \) the Government still has \( g < \frac{G}{2} \), the budget exhausting is not compatible with the choice for interdisciplinarity. An increase in \( g \) has a higher effect on \( \Pi_{AB} \) than on \( \Pi_I \) (2\( \partial g \) versus \( \partial g \)) and, as a consequence, trying to employ all the monetary resources makes specialization the optimal decision for the University.

**Proof of Proposition 3.5.** From the proof of Proposition 3.4, it is possible to conclude that, when \( \rho \in (0, \frac{1}{2}) \), the comparison between specialized and interdisciplinary profits depends on how the parameters ratio \( \frac{2\alpha \lambda}{m_A^2 m_B^2} \) compares with the 3 thresholds

\[
\frac{(1-\rho)^2(1-\rho)+(1-\rho)^2}{(1-\rho)^2 m_A^4 m_B^4}, \quad \frac{1-\rho(2-\rho)^2}{\rho(1-\rho)^2}, \quad \text{and} \quad \frac{2\rho(1-\rho)}{(2+\rho)(1-\rho)-1}.
\]

An increase in \( \alpha \) affects positively the ratio \( \frac{2\alpha \lambda}{m_A^2 m_B^2} \):

\[
\frac{\partial}{\partial \alpha} \left( \frac{2\alpha \lambda}{m_A^2 m_B^2} \right) = \frac{2\lambda (1 + 2\alpha) m_A^{\rho-2}}{m_B^{1+\rho}} > 0.
\]

Since no threshold depends on \( \alpha \) and because the thresholds have a clear order:

\[
\frac{(1-\rho)^2(1-\rho)+(1-\rho)^2}{(1-\rho)^2 m_A^4 m_B^4} \geq \frac{1-\rho(2-\rho)^2}{\rho(1-\rho)^2} \geq \frac{2\rho(1-\rho)}{(2+\rho)(1-\rho)-1},
\]

an increase in the parameters ratio favors the choice of interdisciplinarity. That is to say, an increase in \( \alpha \) expands the range of \( \tilde{y} \) where interdisciplinarity is more profitable than specialization.

Since \( \rho \) only affects the interdisciplinary profit, I only need the comparative statics results
obtained in Corollary 3.4. Therefore, when $\rho \in (0, \frac{1}{2})$,

for $(\theta_A, \theta_B) = (0, 0) : \frac{\partial \Pi_I}{\partial \rho} (\theta_A, \theta_B) < 0$,

for $(\theta_A, \theta_B) = (1, 0) : \log \left( \frac{m_B}{m_A} \frac{1}{\rho} \right) > \frac{\rho}{1 - \rho + \alpha} \implies \frac{\partial \Pi_I}{\partial \rho} (\theta_A, \theta_B) < 0$,

for $(\theta_A, \theta_B) = (1, 1) : \log \left( \frac{m_B}{m_A} \right) > -2 + \frac{1 + \alpha}{1 - \rho + \alpha} - \frac{1 + \alpha}{\rho + \alpha} + \frac{1}{\rho \frac{m_B}{m_A}} \frac{1}{1 - \rho} \implies \frac{\partial \Pi_I}{\partial \rho} (\theta_A, \theta_B) < 0$.

\[ \square \]

**Proof of Proposition 3.6.** When $\alpha_A = \beta \alpha_B$, $\beta > 1$, each situation described in the proof of Proposition 3.4 gives place to 2 alternative scenarios (so in total I end up with 8 possible situations), depending on whether the kink in the specialized profit curve occurs above or below the interdisciplinary profit curve. In the first case, the relevant alternatives for the University are the specialization in both $A$ and $B$, only in $B$, or the interdisciplinarity. In the second case, the option for $B$ alone is never actually considered and everything remains the same as when $\alpha_A = \alpha_B$. To illustrate, I show two of the eight possible situations:

**situation 2.1**, when \( \frac{1 - \rho \left( 2 - \rho (2 - \rho') \left( 1 - \rho (1 - \rho') \right) \right)}{\rho \rho' (1 - \rho)^2} \leq \frac{m_A}{m_A} \frac{m_B}{m_B} \leq \frac{1 - \rho \rho' + \rho' - g(1 - \rho)}{(1 - \rho)^2 (1 - \rho')^2} \) and the specialization in field $B$ alone is never optimal. The University prefers specialization in $A$ and $B$ if $\tilde{y} \in \left( 0, \tilde{y}_{AB,I}^{10} \right)$, and prefers interdisciplinarity if $\tilde{y} \in \left( \tilde{y}_{AB,I}^{10}, \tilde{y}_{11} \right)$. Figure
situation 2.2, when \[
\frac{1-p}{\rho \cm{a}} \frac{2\frac{m}{a} \frac{m}{b}}{\cm{a} \cm{b}} \geq \left(1-p\right)^{\rho} \frac{g}{\cm{a} \cm{b}} \leq \frac{1-p}{\rho \cm{a}} \frac{2\frac{m}{a} \frac{m}{b}}{\cm{a} \cm{b}} \geq \left(1-p\right)^{\rho} \frac{g}{\cm{a} \cm{b}} \]
and the specialization in field B alone is optimal for \( \tilde{y} \in \left(\frac{g}{\cm{a}} ; \tilde{y}_{B,I}^{10}\right) \), whereas both A and B are chosen for \( \tilde{y} \in \left(\frac{g}{\cm{a}} ; \tilde{y}_{B,I}^{10}, \tilde{y}_{B,I}^{11}\right) \), with \( \tilde{y}_{B,I}^{10} = \frac{1-p}{\cm{a} \cm{b} \cm{a}} \frac{2\frac{m}{a} \frac{m}{b}}{\cm{a} \cm{b}} \left(\tilde{y}^{10}_{B,I} \right)^{\rho} \). Figure 15 represents the situation.
when $\alpha_A > \alpha_B$ and B alone is a possible optimal choice.

The impact of $\alpha_i$ on the choice between specialization and interdisciplinarity can now be analyzed as follows:

a) in the case where B alone is never optimal choice

$$\frac{\partial}{\partial \alpha_A} \left( \frac{(\alpha_A + \alpha_B)\lambda}{m_A^{-\rho} m_B^\rho} \right) = \frac{\lambda (1 - \rho) [\rho (1 + \alpha_A) - (1 - \rho) \alpha_B]}{(\rho + \alpha_A)^2} \left( \frac{m_A}{m_B} \right)^\rho$$

which is $> 0$ when $\frac{1 + \alpha_A}{\alpha_B} > \frac{1 - \rho}{\rho}$.

$$\frac{\partial}{\partial \alpha_B} \left( \frac{(\alpha_A + \alpha_B)\lambda}{m_A^{-\rho} m_B^\rho} \right) = \frac{\lambda (1 - \rho) [(1 - \rho) (1 + \alpha_B) - \rho \alpha_A]}{(\rho + \alpha_A) (1 - \rho + \alpha_B)} \left( \frac{m_A}{m_B} \right)^\rho$$

which is $> 0$ when $\frac{1 + \alpha_B}{\alpha_A} > \frac{\rho}{1 - \rho}$.

With an increase in $\alpha_A$, it may happen that B alone may become a relevant alternative, since it is not affected by that change of inefficiency.

b) in the case where B may be a relevant option for intermediate values of the standard:

$$\frac{\partial \bar{y}_{B,I}^{10}}{\partial \alpha_A} = \frac{\lambda (1 - \rho) \rho^{1+\rho} m_A^\rho m_B^{\rho-1}}{[\lambda (\rho - 1) \rho^\rho m_A^\rho m_B^{\rho-1} + (\rho + \alpha_A) m_B^\rho]^2} > 0,$$

$$\frac{\partial \bar{y}_{B,I}^{10}}{\partial \alpha_B} = -\frac{\lambda \rho \left[ \lambda - \frac{1}{\rho^\rho} m_A^{1-\rho} m_B^{1+\rho} \right]^2}{(1 - \rho) \left[ \lambda \alpha_B - \frac{1}{\rho^\rho} m_A^{1-\rho} m_B^{1+\rho} \right]^2} < 0,$$

for $\lambda > \frac{1}{\rho^\rho} m_A^{1-\rho} m_B^{1+\rho}$. 

$\blacksquare$