Does Affirmative Action Reduce Effort Incentives? A Contest Game Analysis[‡]

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Abstract

In this paper a contest game with heterogeneous players is analyzed in which heterogeneity could be the consequence of past discrimination. Based on the normative perception of the heterogeneity there are two policy options to tackle this heterogeneity: either it is ignored and the contestants are treated equally, or affirmative action is implemented which compensates discriminated players. The consequences of these two policy options are analyzed for a simple two-person contest game and it is shown that the frequently criticized trade-off between affirmative action and total effort does not exist: Instead, affirmative action fosters effort incentives. A generalization to the n-person case and to a case with a partially informed contest designer yields the same result if the participation level is similar under each policy.

Keywords: Asymmetric contest; affirmative action; discrimination

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1 Introduction

Affirmative action can be described as a policy instrument that should ameliorate the adverse effects of discrimination on affected groups of individuals.[1] Affirmative action programs are a frequently observed policy instrument which usually gives rise to intense public discussions in countries where those policies are in fact implemented. One of the reasons for this controversy seems to be the fact that its implementation goes beyond formal equal treatment considerations by addressing discriminated groups directly which is, for example, reflected by phrases like 'positive discrimination', or 'preferential treatment' as synonyms for affirmative action. However, even in contemporary societies in which formal equality is legally guaranteed and executed, there exists empirical evidence of ongoing discrimination with respect to specific minority groups. Hence, although open discrimination is prohibited, some minority groups may be disadvantaged out of reasons for which they cannot be held ethically responsible.[2] In such cases in which formal 'equal treatment of equals'-legislation is ineffective because individuals are not ex-ante equal, the implementation of affirmative action policies could be justified on ethical grounds; see Loury (1981) and Loury (2002).

However, opponents of affirmative action do not only criticize the, from their perspective, formal violation of the equal treatment principle but also they refer to potential adverse consequences with respect to effort incentives. The following statement by Thomas Sowell from his book "Affirmative Action Around the World" reflects the concern of those opponents that there could exist a trade-off between affirmative action (i.e. preferential treatment) and social efficiency due to potential disincentive effects with respect to effort provision:

> Both preferred and non-preferred groups can slacken their efforts - the former because working to their fullest capacity is unnecessary and the latter because working to their fullest capacity can prove to be futile. [...] While affirmative action policies are often thought of, by advocates and critics alike, as a transfer of benefits from one group to another, there can also be net losses of benefits

[1] Discrimination is interpreted here as a disadvantage of a group of individuals in different social contexts that is based on some kind of exogenous marker, e. g. race, gender, or nationality, that is (at least initially) not related to these contexts and for which the members of these groups are personally not responsible. Alternatively, more shortly and less technical, discrimination can be described as "allowing racial identification [or gender, nationality etc.] to have a place in an individual´s life chances"; see Arrow (1998), p. 91.

[2] This persistence of discrimination could, for instance, be interpreted as the consequence of historical discrimination that affects negatively the contemporaneous generation, e.g. if investment in human capital depends on the historical segregation of work and living places along races; see Lundberg (1998).

> when both groups do less than their best. What might otherwise be a zero-sum game can thus become a negative-sum game. (Sowell (2004), p. 14)

This line of critique is also addressed in Fryer and Loury (2005a), where it is coined 'Myth No. 3' because "confident a priori assertions about how affirmative action affects incentives are unfounded. Indeed, economic theory provides little guidance" (ibid., p.). The simple contest game in the style of Tullock (1980), introduced in the next section, is an attempt to fill this gap in theoretical analysis by addressing the question whether the criticized trade-off does exist in this kind of stylized model. An affirmative answer to this question would then imply that optimizing players reduce their respective effort levels if they face affirmative action policies which creates the mentioned trade-off.

The implementation of affirmative action is modeled as a biased contest rule[3] where weak contestants are favored because ethical perception interprets their weakness as being the consequence of past discrimination. The alternative perception, i. e. holding the contestants ethically responsible for their heterogeneity, requires instead the implementation of an unbiased contest rule. Both policies are defined formally as restrictions on the contest rule which imply different incentives for the individuals depending on the implemented policy option. The key question is therefore how individuals react to the distortion of incentives that is induced by the two policies.

There exists a limited number of articles with a similar focus. Fu (2006) models college admission as a two-player all-pay auction under complete information and shows that favoring the discriminated player to some extent induces the maximal expected academic effort (interpreted as the expected test score) by both candidates. A similar conclusion is derived in Schotter and Weigelt (1992) that analyze, also experimentally, a two-player tournament with unobservable effort. However, none of the models mentioned so far specifies the normative objective of affirmative action, i.e. in these papers affirmative action is considered simply as a deviation from an unbiased 'equal treatment'-policy. This is a crucial difference to the contest model presented below because here the normative objective of affirmative action is explicitly defined and integrated into the model.[4] Kranich (1994) formalizes a similar idea for a two-

[3]The underlying game theoretic model is an asymmetric contest game with n heterogeneous players. Asymmetric contest games are applied in different frameworks, for example, to analyze legal presumption in trials; see Bernardo, Talley, and Welch (2000), with the interpretation of prior probabilities; see Corchón (2000), or in a two-stage rent-seeking contest; see Leininger (1993).

[4]In Fryer and Loury (2005b) a model with incomplete information is introduced where a continuum

player production economy where the jointly produced output is shared according to a 'division rule'. He introduces a normative requirement for the class of feasible division rules which is phrased equal-division-for-equal-work principle[5] and shows that there exist feasible division rules that satisfy this principle in combination with Pareto-efficiency. In contest games, however, the notion of Pareto-efficiency is meaningless because there is no production, i.e. the contested prize does not depend on the exerted effort of the contestants. Therefore, a Pareto-ranking of the outcomes of the two normatively defined policies is pointless in a contest game framework. Instead, the two policy alternatives are evaluated with respect to the total equilibrium effort that they generate.[6]

Contrary to Sowell´s prediction, it is shown that in the two-player contest game the optimal individual response to the implementation of affirmative action would be to increase individual effort level in comparison to the unbiased contest game (irrespective of the fact whether the individual is discriminated or not). However, relaxing the restriction on the number of players is not innocuous: the result for the two-player case can only be sustained in the n-player contest game if the underlying heterogeneity is not too severe because otherwise participation effects dominate incentive effects. As the model is kept sufficiently simple to facilitate analytical tractability, it is too stylized to give any sort of direct policy implications. Nevertheless it shows that a trade-off between affirmative action and aggregated effort, as stated by Sowell, may not exist, especially if participation effects are not an important issue.

The contest model is formulated in general terms to reflect in a stylized way a variety of situations in which the implementation of affirmative action can have consequences on the incentive structure of effort provision. Possible real world examples of contest-like

of contestants compete for positions in simultaneous pair-wise tournaments. There, the analysis is focused on the comparison of group-sighted and group-blind affirmative action policies without addressing explicitly the incentive effects of affirmative action versus unbiased tournament rules.

[5]The subsequently stated definitions of the two policies are modified versions of this principle in the sense that they now capture the two different normative perceptions of the heterogeneity of the contestants. Therefore, the interpretation of 'equal work' has a different meaning depending on the perceived underlying reason for the heterogeneity.

[6]This interpretation of exerted effort as being socially valuable is the crucial difference to the extensive literature on rent-seeking contests (comp. the literature survey in Nitzan (1994) and a collection of related articles in Lockard and Tullock (2001)). There, exerted effort is usually interpreted as pure social waste, while in situations where affirmative action is applied it is more appropriately characterized as socially valuable (which is also suggested by the quotation of Sowell). In the recent literature on sport contests effort, i.e. the performance of the athletes, has a similar interpretation; see Szymanski (2003).

environments in which affirmative action is implemented are:[7] university admission, in which applicants compete for places in a university program by means of their high school grade point average and minority students get some kind of bonus; fixed bonus-payment tournaments within a firm where the bonus is payed to the employee with the highest sales performance and discriminated employees might get some limited advantage; and even sport contests, for example horse riding, in which jockeys that weigh less than their competitors are forced to carry additional weight.

2 The Model

Affirmative action instruments are usually applied in situations of competitive social interaction. The competitive structure of these situations can be captured by a contest game in which contestants compete for an indivisible prize. The contestants can increase their respective probability of winning the contested prize by exerting more effort. This feature seems to be appropriate to model the basic structure of the above mentioned examples because there exists a relatively high grade of discretion on the side of the organizer of the competition. This is reflected in a contest game in which contestants face a probabilistic outcome. To guarantee analytical tractability and closed form solutions, the model is formulated under complete information, i. e. the only element of uncertainty is the final winner of the contest.

2.1 The Contestants

Let $N = \{1, 2, \dots, n\}$ denote the set of individuals that compete against each other in a contest game. Each contestant $i \in N$ exerts an effort level $e_i \in \Re^n_+$ and takes the effort level $e_{-i} = (e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n) \in \Re^{n-1}_+$ of its rivals as given. Additionally, it is assumed that all contestants are risk-neutral and have the same positive valuation V for the contested prize. The only element of heterogeneity among the contestants is the respective 'cost function' that captures the disutility of exerting effort e_i which depends additionally on parameter β_i that (potentially) reflects the degree of discrimination of contestant i. It is assumed that this cost function is linear in e_i and multiplicative in β_i for all $i \in N$, with β_i normalized in such a way that for the most able contestant

[7]For empirical results with respect to the consequences of different affirmative action policies compare the survey in Holzer and Neumark (2000).

$\hat{\beta} = 1$ and for less able contestants $\beta \in \left(\hat{\beta}, \infty\right)$:

$$c_i(e_i) = \beta_i e_i \text{ for all } i \in N. \tag{1}$$

The contestants perceive the outcome of the contest game as probabilistic. However, they can influence the probability of winning by exerting effort, i.e. the outcome depends on the vector of effort levels exerted by all individuals. The following Contest Success Function (CSF), axiomatized in Clark and Riis (1998), that will be applied in the model allows also an asymmetric treatment of the contestants that can be interpreted as affirmative action policy:

$$p_i(e) = \frac{\alpha_i^P e_i^r}{\sum_{j \in N} \alpha_j^P e_j^r} \tag{2}$$

with $\alpha_i^P > 0$ for all $i \in N$ and $r \in (0, 1]$. This function maps the vector of effort levels $e = (e_1, \ldots, e_n)$ into win probabilities for each contestant: $p_i(e) : \Re_+^n \to [0, 1]$. The parameter r measures the sensitivity of the outcome of the contest game with respect to differences in effort.[8] Additionally, each individual effort level is weighted by a positive parameter α_i^P that depends on the policy P, formally defined in the next subsection. If no contestant exerts positive effort it is assumed that none of the individuals receives the prize, i.e. $p_i(0, \ldots, 0) = 0$ for all $i \in N$.

The specification of the cost function in eq. (1) and the contest mechanism in eq. (2) are already the necessary elements to state the following expected (additive separable) utility function of contestant i:

$$u_i(e_i, e_{-i}) = p_i(e)V - c_i(e_i) \text{ for all } i \in N. \tag{3}$$

This contest game can therefore be interpreted as a standard non-cooperative game: $\Gamma[N, P, (A_i)_{i \in N}, (u_i)_{i \in N}]$, where $e_i \in A_i$ and P is an additional policy parameter.

2.2 The Policy Options

It is assumed that the choice of the policy P is based on the ethical perception of the heterogeneity of the contestants (i.e. the different marginal cost functions)[9] which directly implies the normative objective of the respective policy option and therefore

[8] The upper bound $r \leq 1$ is imposed because otherwise the existence of pure strategy equilibria cannot be guaranteed.

[9] As the model is formulated under complete information, the individual marginal cost functions are common knowledge.

also governs the specification of the individual effort weights $(\alpha_1^P, \ldots, \alpha_n^P)$. With respect to the ethical perception of the difference in cost functions, there are two potential interpretations for the source of this heterogeneity.

The first interpretation holds the contestants ethically responsible for their respective cost function in which case the probability to win the contest game (i.e. the CSF) should only depend on the vector of exerted effort. In other words, if a contestant i exerts the same effort level as a contestant j then both contestants should win the contest game with the same probability. This policy option would therefore treat the contestants equally with respect to their exerted effort level.

Definition 1 *A policy is called* ***equal treatment approach (ET)*** *if:*

$$e_i = e_j \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j.$$

For the class of contest games as defined by the CSF in eq. (2) equal treatment implies that the policy weights $(\alpha_1^{ET}, \ldots, \alpha_n^{ET})$ must be identical for all players:

$$\alpha_i^{ET} = \alpha^{ET} \text{ for all } i \in N.$$

The last line is derived by observing that for all $e_i = e_j$ it has to be the case that $p_i(e) = p_j(e)$. Solving this expression according to eq. (2) for all possible values of $e_i = e_j$ yields the above specification for the weight α_i^{ET} for all $i \in N$.

This policy could also be interpreted as an anonymity principle because it postulates that the contest success function neither depends on the specific names nor on the exogenous characteristics of the players.[10] However, the outcome, i.e. expected equilibrium utility, of the contest game will indirectly depend on the characteristics of the players because weaker players will exert less effort in equilibrium.

The second interpretation is based on the perception that the contestants cannot be held ethically responsible for their heterogeneity, for instance, if it is the consequence of past discrimination. As heterogeneity affects the cost function for each contestant, fairness would require that two contestants that face equal disutility induced by the chosen effort level (that could be different) should have the same probability to win the contest game. The normative justification for this interpretation is the "moral

[10] In Skaperdas (1996), theorem 2, this CSF (specified by eq. (2) and the relevant ET weights) is axiomatized based on a conventional anonymity axiom, comp. footnote 12.

intuition that two people incurring equal disutility deserve equal rewards" (Kranich 1994, p. 178).[11]

Definition 2 *A policy is called **affirmative action (AA)** if:*

$$c_i(e_i) = c_j(e_j) \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j.$$

For the class of contest games as defined by the CSF in eq. (2) the following relation with respect to the policy weights $(\alpha_1^{AA}, \ldots, \alpha_n^{AA})$ satisfies the definition of affirmative action:

$$\frac{\alpha_i^{AA}}{\beta_i^r} = \frac{\alpha_j^{AA}}{\beta_j^r} \text{ for all } i \neq j. \tag{4}$$

This relation is derived by using the following transformation of variables: $z_i = c_i(e_i)$ for all $i \in N$. As $c_i(e_i)$ is linear it can be inverted: $e_i = z_i/\beta_i$. For the so transformed model the condition in Definition 2 then states that if $z_i = z_j$ then $p_i(z/\beta) = p_j(z/\beta)$, where $z/\beta = (z_1/\beta_1, \ldots z_n/\beta_n)$ denotes the vector of transformed individual effort. Solving $p_i(z/\beta) = p_j(z/\beta)$ for $z_i = z_j$ implies that: $\alpha_i^{AA}(z_i/\beta_i)^r = \alpha_j^{AA}(z_i/\beta_j)^r$ which has to hold for all values $z_i = z_j$. This condition is satisfied if the above mentioned relation holds. The following normalization simplifies the subsequent analysis. As the CSF is homogeneous of degree zero, there is no loss in generality if the weights are normalized such that:

$$\alpha_i^{AA} = \beta_i^r \text{ for all } i \in N. \tag{5}$$

The policy AA therefore generates a bias[12] of the contest success function in favor of discriminated contestants in such a way that both contestants have the same probability of winning the contest whenever they face the same disutility of effort. Note that this definition requires that the affirmative action bias is implemented multiplicatively through α_i^{AA} which increases the marginal efficiency of exerted effort for contestant i and therefore changes the incentives for effort distribution. This type of affirmative

[11] This quotation from Kranich (1994) justifies a related 'equal-division-for-equal-work' principle in his model. The difference to the model presented here is that 'equal work' should be interpreted here as equal disutility of effort as this is the relevant normative standard of comparison if contestants are not responsible for the differences in marginal costs.

[12] In Clark and Riis (1998) it is argued that the anonymity axiom of Skaperdas (1996) should be relaxed because "in many situations, however, contestants are treated differently (due to affirmative action programs for instance)" (Clark and Riis 1998, p. 201). The resulting CSF is asymmetric as in eq. (2) but without any further specification of this asymmetry. Definition 2 can therefore also be interpreted as a substitute of the anonymity axiom that entails now a specific normative restriction with respect to the asymmetry of the CSF.

action also has an interesting normative interpretation as shown in a more general framework in Calsamiglia (2004). There, it is justified normatively because it equalizes 'reward to effort' which guarantees a notion of 'global equality of opportunity'.[13]

Both policies, ET and AA, can be considered as principles that guarantee a notion of procedural fairness because they are defined with respect to the outcome (in the sense of winning probabilities) of the contest game. An alternative (welfaristic) approach would be to equalize the expected equilibrium utilities as objective of affirmative action. This alternative interpretation is discussed in Section 6 where it is also shown that the resulting affirmative action bias is identical under both normative interpretations.

In the introduction it was already remarked that the difference between the contest game presented here and the literature on rent-seeking is based on the assumption that exerting effort is perceived as socially valuable (and not as social loss). This is also implicitly reflected by the citation of Sowell in which less effort of all participants is interpreted as socially inferior. Therefore, the positive analysis of the two normative policy options is carried out by simply comparing the sum of equilibrium effort that each policy induces (interpreted as a measure of 'social efficiency'). Using total equilibrium effort as the standard of comparison seems to be appropriate because in situations in which affirmative action is potentially implemented, this assumption captures the notion of social loss (or gain).[14] The equilibrium effort level of each contestant will depend on the ex-ante announced policy parameter P and the standard of comparison will therefore be expressed and denoted in the following way: $E_P^* = \sum_{i \in N} e_i^*(P)$ for $P \in \{ET, AA\}$.

[13]Global equality of opportunity in a contest framework is defined as equality of welfare achieved for individuals that compete in several contests simultaneously and where the respective contest organizers implement affirmative action that is based on local information which is limited to the respective contest.

[14]In the college admission example, the education authorities are interested in high effort levels, i.e. grades, by all students that are possibly affected by affirmative action irrespective of the fact that they are admitted. Also in the bonus tournaments the employer is obviously interested in high effort levels by all employees, irrespective of the identity of the final winner (the interpretation of this kind of tournament as an incentive device is obvious here). And even in sport competitions it can be argued that spectators are interested in the overall performance of all athletes because ex-ante predictable sport competitions are usually perceived as boring. Note that also the quotation by Sowell suggests that the effort of the individuals can be simply summed up to evaluate affirmative action.

As the objectives and the entailed bias of the two policy options are clarified now, the timing of the complete contest game can be summarized in the following way: The heterogeneity of the contestants (i.e. different marginal cost parameters) is observed. Based on the ethical perception of this observation a policy option $P \in \{ET, AA\}$ is selected that determines the weighting parameters $(\alpha_1^P, \ldots, \alpha_n^P)$ for the respective policy. The contestants exert the optimal (with respect to their expected utility) equilibrium effort level $e_i^*(P)$ for each $i \in N$, taking as given the effort levels of their rivals and the relevant weights induced by policy P. In the last step the exerted efforts are observed and the winner of the contest game is determined according to the announced policy option. After that the total equilibrium effort that is generated by each policy can be compared which directly answers the question whether a trade-off between affirmative action and total effort does in fact exist or not.

3 The Two-Player Contest Game

Restricting the number of contestants in the two player case yields the key result of the comparative policy analysis: in equilibrium both contestants will exert more effort under AA than under ET. Contrary to the n-player contest game this result holds without any extra assumption and the derivation of equilibrium in the two-player contest is based on simple first order conditions.[15] Therefore, the two-player contest game is analyzed separately.

Applying the CSF as specified in eq. (2), the expected utility function for policy $P \in \{ET, AA\}$ can be expressed as:

$$u_i(e_i, e_j) = \frac{\alpha_i^P e_i^r}{\alpha_1^P e_1^r + \alpha_2^P e_2^r} V - \beta_i e_i \text{ for } i = 1, 2;$$

where contestant 1 is assumed to be the one with the lowest marginal cost parameter such that $\beta_1 = 1$ and $\beta_2 > 1$. By Definition 1 and 2 the bias for contestant 1 is normalized to $\alpha_1^P = 1$ for $P \in \{ET, AA\}$. Solving first order conditions for a given policy parameter P yields the equilibrium effort candidate for $i = 1, 2$:

$$e_i^*(P) = \frac{\alpha_1^P \alpha_2^P \beta_i^{r-1} \beta_j^r}{(\alpha_1^P \beta_2^r + \alpha_2^P \beta_1^r)^2} rV \qquad \text{for } i \neq j, \tag{6}$$

that would imply positive expected equilibrium utility by the assumption on r:

$$u_i(e_i^*(P), e_j^*(P)) = \frac{(\alpha_i^P \beta_j^r)^2 + \alpha_1^P \alpha_2^P (\beta_1 \beta_2)^r (1 - r)}{(\alpha_1^P \beta_2 + \alpha_2^P \beta_1)^2} V > 0. \tag{7}$$

[15] Nti (1999) is based on a similar set-up with a non-biased CSF in a rent-seeking framework where heterogeneity does affect individual valuations instead of marginal costs.

A non-interior equilibrium in which a contestant exerts zero effort cannot exist because there always exists a profitable deviation for one of the contestants.[16] The second order conditions can be expressed in the following way:

$$\frac{\partial^2 u_i(e_i,e_j)}{\partial e_i^2} = \frac{\alpha_1^P \alpha_2^P r V e_i^{2r-2} e_j^r}{(\alpha_1^P e_1^r + \alpha_2^P e_2^r)^3} \left[\alpha_j^P (r-1)\left(\frac{e_j}{e_i}\right)^r - \alpha_i^P (r+1)\right] < 0,$$

which proves concavity by the assumption on r. Hence, the equilibrium is interior and unique. From eq. (6) it can also be noted that the relative equilibrium effort levels are independent of the implemented policy because:

$$\frac{e_1^*(P)}{e_2^*(P)} = \beta \quad \text{for } P \in \{ET, AA\}\,. \tag{8}$$

The two policy alternatives ET and AA can now be evaluated with respect to the sum of equilibrium effort $E_P^* = \sum_{i=1,2} e_i^*(P)$ that each policy generates. The following Proposition states the result for the two-player contest game: The affirmative action policy as specified in Definition 2 will induce higher individual and also higher aggregated effort than the equal treatment policy. This result refutes the above mentioned critique of affirmative action policy because in the contest game as specified here a trade-off between affirmative action and aggregate effort does not exist.

Proposition 1 *In the two-player contest game (i) the sum of equilibrium effort, and (ii) each individual equilibrium effort level is higher under policy AA than under the policy ET.*

Proof: Using eq. (6) and Definition 1 and 2, the inequality $E_{AA}^* > E_{ET}^*$ can be reduced to $\frac{rV}{4}\frac{\beta_2+1}{\beta_2} > \frac{rV\beta_2^r}{(1+\beta_2^r)^2}\frac{\beta_2+1}{\beta_2}$, which is always satisfied because it can be simplified to $(1-\beta_2^r)^2 > 0$. This establishes part (i) of the Proposition.

Using the fact that the relation between the equilibrium effort levels remains constant, as stated in eq. (8), proves part (ii).□

The reason for this at first sight surprising result lies in the fact that the implementation of the AA policy yields a contest game that is more balanced with respect to the characteristics of the contestants (the heterogeneity of the contestants is reduced

[16]If both contestants would exert zero effort a deviating player i will always win the contest with certainty by exerting a slightly positive effort level ϵ: $u_i(\epsilon,0) > u_i(0,0) = 0$. If only one contestant j would exert zero effort player i can deviate profitably by decreasing his chosen effort level by a small amount ϵ because then he still wins the contest game with certainty: $u_i(e_i - \epsilon, 0) > u_i(e_i, 0)$ as long as $e_i - \epsilon > 0$.

by the biased CSF). As the contestants are more similar under AA, the competitive pressure is higher which implies higher equilibrium effort by both contestants.[17]

In fact, the bias that is induced by AA for the two-person contest game yields a level playing field, i.e. the contestants are as similar as possible under this set-up. Therefore, the policy AA also generates the maximal aggregated effort even for a contest game that is not restricted by any normative constraint. In other words, if the objective would solely be the maximization of total equilibrium effort by implementing an appropriate weight $\hat{\alpha_2}$ then this weight would coincide with the bias that is required by the AA policy.[18]

Proposition 2 *The policy option AA generates the maximal sum of equilibrium effort in the two-player contest game.*

Proof: Consider the sum of equilibrium effort for an arbitrary parameter α_2 that favors the discriminated contestant: $E^* = \frac{\alpha_2 \beta_2^r}{(\alpha_2+\beta_2^r)^2} \frac{\beta_2+1}{\beta_2} rV$. This expression is maximized for $\hat{\alpha_2} = \beta_2^r$ which coincides with $\alpha_i^{AA} = \beta_i^r$ for $i = 1, 2.\square$

Opponents of affirmative action policies claim that those policies could result in less aggregated effort level. The last two propositions reveal that in the above specified two-player contest this concern is not justified. Instead, both contestants will exert higher effort levels in equilibrium if they face affirmative action. In fact, as it was shown in proposition 3, the affirmative action bias even leads to the highest possible level of total equilibrium effort. In the next section it is analyzed if these results are also valid for contest games with more than two players.

4 The n-Player Contest Game

Contrary to the two-player case the derivation of the equilibrium and the proof of existence and uniqueness for the n-player contest game are more involved because not all contestants will always exert a strictly positive effort level in equilibrium.[19]

[17]Similar results are known, for example, from the literature on optimal auction design: A revenue maximizing auction implies also the favoring of weak bidders (comp. McAfee and McMillan 1989).

[18]Nti (2004) introduces a 2-player contest game with different valuations and a CSF of the form $p_i(e) = \frac{\alpha_i e_i + \gamma_i}{\sum_{i=1,2} \alpha_i e_i + \gamma_i}$. In this set-up, total equilibrium effort is maximized if $\gamma_1 = \gamma_2 = 0$ and the multiplicative parameters (α_1, α_2) balance the heterogeneity of the valuations.

[19]This implies that first-order conditions that were used in the two-player contest to characterize the equilibrium are not feasible here because the equilibrium might be non-interior. The approach

Additionally, an assumption is needed in the n-player contest because for a non-linear CSF with parameter $r < 1$ it is not possible to derive closed form solutions.[20] As the existence of closed form solutions is crucial for the comparative analysis of the policy alternatives, it is assumed from now on that the CSF is linear with $r = 1$.

The expected utility of the risk-neutral contestant i in the n-player contest can then be expressed as:

$$u_i(e_i, e_{-i}) = \frac{\alpha_i^P e_i}{\sum_{j \in N} \alpha_j^P e_j} V - \beta_i e_i \text{ for all } i \in N \text{ and for } P \in \{ET, AA\}. \quad (9)$$

It is also assumed that the contestants are ordered with respect to their marginal cost parameter: $\beta_1 \leq \beta_2 \leq \ldots \leq \beta_n$ with the normalization $\beta_1 = 1$.

The equilibrium of this contest game will be derived in appendix A.1, based on the observation that the contest game can be interpreted as an aggregative game with its convenient properties. The following equation provides an expression of the equilibrium effort for those m contestants of the set $M \subseteq N$ that are active, i.e. that exert a positive equilibrium effort:

$$e_i^*(P) = \frac{1}{\alpha_i^P} \left(1 - \frac{\beta_i}{\alpha_i^P} \frac{(m-1)}{\sum_{j \in M} \frac{\beta_j}{\alpha_j^P}} \right) \frac{(m-1)V}{\sum_{j \in M} \frac{\beta_j}{\alpha_j^P}} \text{ for all } i \in M \text{ and } P \in \{ET, AA\}. \quad (10)$$

Set M is indirectly defined by the following inequality:

$$(m-1)\frac{\beta_i}{\alpha_i^P} < \sum_{j \in M} \frac{\beta_j}{\alpha_j^P} \text{ for all } i \in M \text{ and } P \in \{ET, AA\}. \quad (11)$$

Using the specification of the weights for the AA and ET policy and the characterization of the active set, the following Lemma describes the set of participating contestants for each policy option.

Lemma 1 *Under the policy ET the active set $M \subseteq N$ of contestants is implicitly defined by the following inequality:*

$$(m-1)\beta_i < \sum_{j \in M} \beta_j \text{ for all } i \in M. \quad (12)$$

Under policy AA all contestants will be active.

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that is instead applied is based on the notion of 'share functions' as defined in Cornes and Hartley (2005) which has the advantage that the existence proof of equilibrium is reduced to a simple fixed point argument in $\Re^2$.

[20] Cornes and Hartley (2005) give existence results for this class of games by analyzing the properties of implicit equilibrium conditions. They also show that total equilibrium effort is increasing in r.

As equilibrium effort is given by eq. (10), the two policies can now be compared with respect to the aggregated equilibrium effort $E_P^* = \sum_{i \in M} e_i^*(P)$ that they induce. However, Lemma 1 already reveals that the comparison between the two policy options will not be as straight forward as in the two-player contest game because the total equilibrium effort depends on the distribution of the cost parameter that determines the active set.

The following notation will simplify the characterization of the relevant distribution for a subset $J \subseteq N$ of contestants: the arithmetic mean of the cost parameters of agents of set J will be denoted as $\bar{\beta}_J = \frac{1}{j}\sum_{i \in J} \beta_i$ (where $\bar{\beta} = \bar{\beta}_N$ to facilitate notation), and the harmonic mean respectively as: $\beta_J^H = \left[\frac{1}{j}\sum_{i \in J} \frac{1}{\beta_i}\right]^{-1}$.

The subsequent proposition states the condition under which policy AA generates higher aggregated effort.

Proposition 3 *In the n-player contest game the sum of equilibrium effort levels is higher under policy AA than under policy ET if:*

$$\frac{\bar{\beta}_M}{\beta_N^H} > \frac{\frac{m-1}{m}}{\frac{n-1}{n}}. \tag{13}$$

Proof: Calculation of the sum of equilibrium effort for each policy under consideration of lemma 1 yields $E_{AA}^* = \frac{n-1}{n^2} V \sum_{i \in N} \frac{1}{\beta_i}$ and $E_{ET}^* = \frac{m-1}{\sum_{i \in M} \beta_i} V$. Reformulating the inequality $E_{AA}^* > E_{ET}^*$ leads to condition (13).□

The following intuitive explanation is provided for the condition in Proposition 3 which is afterwards clarified by a numerical example. As already observed in the two-player contest game, AA in general induces higher competitive pressure because contestants are more similar than under ET. Increasing the number of active contestants therefore yields higher total effort for both policies because this implies more intense competition. However, inducing heavily discriminated contestants to participate comes at a non-negligible cost, especially for the AA policy because by lemma 1 all participants will be active under AA. This effect is less profound for ET because highly discriminated contestant will not participate under ET.

Numerical Example: Consider the following contest game with three contestants that have marginal costs of $(\beta_1, \beta_2, \beta_3) = (1, 2, 2)$. The underlying dispersion is measured by the coefficient of variation (defined as $CV = \sigma(\beta)/\bar{\beta}$) which is in this case $CV \approx 0.2828$. For these parameters AA will generate $E_{AA}^* \approx 0.4444$ that is higher than the aggregated effort under ET which is $E_{ET}^* = 0.4$. If a fourth contestant with $\beta_4 = 2.43$

(which yields nearly the same level of dispersion $CV \approx 0.2828$) is added the difference between AA and ET becomes even more profound: $E^*_{AA} \approx 0.4522$ versus $E^*_{ET} \approx 0.4038$ and $E^*_{AA} - E^*_{ET} \approx 0.0483$. Note also that both policies induce higher total effort in comparison with the three player example. However, if the fourth contestant is highly discriminated ($\beta_4 = 10$) this would imply a decline of total effort in the case of AA: $E^*_{AA} \approx 0.3938$. This decline is less intense in case of ET because here the fourth player will not participate. As only the first three contestants will be active under ET, the result is identical to the three player contest game considered before, i.e. $E^*_{ET} = 0.4$. Comparing both values shows that for this four player constellation the result of the policy analysis has been reversed because now $E^*_{AA} < E^*_{ET}$.

This example demonstrates that the key factor for the outcome of the policy comparison is the distribution of the discrimination parameter in combination with the number of contestants. In general it can be stated that either a low number of contestants or a sufficient low dispersion makes it more probable that AA will induce more total effort than ET because then the set of active contestants tends to be similar for both policies.[21] The exact relation between the distribution of discrimination parameters and the number of players is described by the inequality[22] in Proposition 3 in combination with the characterization of the active set in Eq. (11).

An additional remark with respect to the relation between Proposition 1 and 3 should be in order. Applying Proposition 3 to a two-player contest game would yield the same result as Proposition 1 because condition (13) holds irrespective of the distribution of cost parameters in the two-player case: For the optimally designed vector of policy parameters both contestants will exert positive equilibrium effort, i.e. set M and N coincide. Therefore, Proposition (3) is satisfied without further restriction because condition (13) can be reduced to $\bar{\beta} > \beta^H_N$ which is always true (comp. the proof for Proposition 4).

For the two-player contest game Proposition 1 also contained a statement that com-

[21]The observation that affirmative action might imply a distortion of the participation decision of individuals (which could finally dominate the effect of increased competitive pressure) has also empirical relevance: In an econometric analysis of bid preferences in highway procurement auctions (Marion 2007), it is shown that preferential treatment implies a decline in competitive pressure because non-preferred bidders switched to procurement auctions without bid preference program.

[22]Note, that the left hand side of condition (13) is lower than one for m small and larger than one for m large (where m is determined according to condition (11)). Inspection of the right hand side reveals that it is always lower or equal to one. This confirms the qualitative statement that condition (13) is likely to hold if the number of contestants is relatively small or the distribution is not too dispersed.

pares individual equilibrium effort under each policy. However, a comparable result for the n-player contest is not possible because the set of active agents depends on the underlying distribution of the marginal cost parameter. Hence, the additional assumption of full participation by all contestants under both policies shall be considered to get some further insights into the individual equilibrium behavior. This assumption would imply that the dispersion of cost parameters is sufficiently low such that also under policy ET all contestant would be active.

The following Proposition mirrors Proposition 1 for this class of restricted distributions of the marginal cost parameter. Although the sum of equilibrium effort in this special case is higher under the optimal AA policy versus the optimal ET policy (without any further conditions) as in the two-player case, the result with respect to individual equilibrium effort is different: In the n-player contest game the set of contestants that individually exert higher equilibrium effort under policy AA than under ET is restricted to contestants with either very low marginal cost or higher than average marginal cost.

Proposition 4 *If all contestants in the n-person contest game are active under policy ET, then (i) the sum of equilibrium effort levels is higher under policy AA, and (ii) the individual equilibrium effort of all contestants with marginal cost parameter* $\beta \in \left[1, \frac{1}{n-1}\bar{\beta}\right) \cup \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$ *is higher under policy AA, while it is lower for contestants with* $\beta \in \left(\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right)$*. For contestants with* $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$ *the individual equilibrium effort is the same under both policies.*

Proof: If all contestants are active set M and N coincide, and condition (13) simplifies to $\bar{\beta} > \beta_N^H$. This inequality is always satisfied which proves the first part of the proposition.
For the second part the following inequality has to be analyzed: $e_i^*(AA) > e_i^*(ET)$. Simplifying this expression yields after some algebra the following inequality:

$$\left(\sum_{j \in N} \beta_j\right)^2 - n^2 \beta_i \left(\sum_{j \in N} \beta_j - (n-1)\beta_i\right) > 0. \tag{14}$$

This inequality is satisfied if $\beta_i \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$, where the lower bound stems from the assumption that $\beta_i \geq 1$ for all $i \in N$, or if $\beta_i \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$, where the upper bound comes from the assumption of full participation under the optimal ET policy. The left hand side of Eq. (14) is equal to zero for $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$. Continuity of the left hand side of Eq. (14) in β_i implies the condition for the reversed inequality. Note also that the first interval could be empty if $n-1 > \bar{\beta}$ which depends on the relevant underlying distribution. $\square$

The set of contestants that individually exert more effort under the AA policy is not connected. The following argumentation provides an intuition for this result: Consider first a (potentially hypothetical) contestant k with a marginal cost that is identical to the mean of the total distribution: $\beta_k = \bar{\beta}$. Under policy AA this would imply that contestant k is favored by $\alpha_k^{AA} = \bar{\beta}$. Normalizing the vector $(\alpha_1^{AA}, \ldots, \alpha_n^{AA})$ yields the equivalent vector $\alpha' = (\alpha_1^{AA}/\bar{\beta}, \ldots, \alpha_n^{AA}/\bar{\beta})$. For contestant k this would imply no distortion under AA because $\alpha'_k = \alpha_k^{AA}/\bar{\beta} = 1$. Additionally, he knows that under AA contestants with higher marginal costs than him are favored (in average) to the same amount as contestants with lower marginal costs are handicapped. Therefore, his equilibrium effort level is not altered. Contestants with higher marginal costs than contestant k are favored under α', i.e. their efficiency of effort in the CSF is increased ($\alpha'_i > 1$ for $i > k$) which implies that they exert higher effort level. The contrary is true for contestants with less marginal cost than contestant k: they are handicapped ($\alpha'_i < 1$ for $i < k$) which reduces their efficiency of effort and therefore also their equilibrium effort. However, there exists a counter effect for contestants with very low marginal cost which becomes dominant for some cut-off value. This counter effect is due to increasing competitive pressure for those highly effective contestants because they are more handicapped under AA than their competitors. The cut-off marginal cost value is exactly at $\beta_c \equiv \bar{\beta}/(n-1)$. Contestants that have a lower cost parameter than contestant c will therefore exert higher equilibrium effort under AA than under ET.

5 An Extension: Group Contests

In the last section the implementation of the AA policy was based on a bias of the CSF that was individually specified for each contestant. However, the implementation of affirmative action policies is usually not based on individual characteristics, but on group membership, e.g. minority, sex, race etc. Reasons for this phenomenon could be incomplete information with respect to individual discrimination, or simply the fact that group members are sufficiently homogeneous to treat them identical. In the following section the latter aspect is analyzed while in the next section the informational requirements of the contest designer are relaxed.

The following model is a simplified version of the n-player contest game with the additional assumption that the n contestants belong either to group A or B that each consists of $n_A \geq 2$ and $n_B \geq 2$ members. The members of each group are assumed to be identical, i.e. face the same marginal cost parameter which is normalized for the non-discriminated group A such that $\beta_i = \beta_A = 1$ for all $i \in A$ and $\beta_i = \beta_B$ for all $i \in B$ where $\beta_B > 1$. It should be emphasized that this specification is already covered by the model of section 4 which implies that $\alpha_i^{ET} = \alpha^{ET}$ and $\alpha_i^{AA} = \beta_i$ for all $i \in N$. The main objective is therefore another clarification of Proposition 3 and the interplay between total effort and the active set of contestants. Additionally, the simplified model presented here can be considered as the starting point of the generalized model in the next section.

At first, the active set under the optimally designed vector parameters for the ET policy has to be determined (for AA all contestants will always be active). Denote the number of active contestants of A by m_A, and m_B for group B. Starting with the less discriminated group A, it is obvious that all members of A are active because condition (11) reduces to $1 < \frac{m_A}{m_A - 1}$ which is trivially satisfied for all $m_A \leq n_A$. Hence, all members of group A will be active under ET.

Considering the members of group B, condition 11 becomes $\beta_B < \frac{n_A + m_B \beta_B}{n_A + m_B - 1}$ which can be simplified to:

$$\beta_B < \frac{n_A}{n_A - 1}. \tag{15}$$

Notice that the last condition does not depend on m_B anymore which implies that this condition either holds for all or for none of the members of B. Based on the number of group A-members and the marginal cost parameter of group B the following two cases are possible:

1. If condition (15) is satisfied both groups are active under ET.
2. Otherwise, only members of group A are active under ET.

Based on these two cases the aggregated equilibrium effort level under the optimal designed vector of policy parameters under policy AA and ET can now be compared. In case 1 all contestants are active such that Proposition 4 can be used directly to conclude that AA induces higher aggregated effort than ET. The same proposition gives conditions for each discrimination level under which AA induces more individual equilibrium effort than ET. As $\beta_A = 1$ and $\beta_B > \bar{\beta}$ this implies that $\beta_A \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$, $\beta_B \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$, and that there exists no contestant i such that $\beta_i \in \left[\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right]$. However, it remains to be checked whether the first interval is non-empty, i.e. if

$1 < \frac{1}{n-1}\bar{\beta}$ or not. This inequality is satisfied if $\beta_B > ((n_A + n_B)^2 - 2n_A)/n_B - 1$. If this is the case, all individuals will individually exert higher equilibrium effort under AA. Otherwise, only group B members will increase their individual effort.

For the second case Proposition 3 is applicable, which provides condition (13) to compare the aggregated equilibrium effort[23] under the optimal specified policy parameters for AA and ET. This condition simplifies for the contest game considered here to the following expression:

$$\beta_B < \frac{n_A(n_A + n_B - 1)}{n_A(n_A + n_B - 2) - n_B} \equiv \beta^*. \tag{16}$$

The intuitive explanation that was given in the last section is that condition 13 is likely to hold if either the level of dispersion is sufficiently low or the number of contestants is relatively small. For the case considered here this can be verified explicitly for the simplified condition given in Eq. (16). In fact, it is satisfied if either β_B is low in comparison to β^* (which coincides with low dispersion), or if n_A and n_B are sufficiently low (it can be checked that β^* is decreasing in n_A and n_B).

It should also be noticed that condition (16) is not trivial in the sense that, for instance, satisfying condition (15) automatically implies condition (16) because it can be shown that $\beta^* > \frac{n_A}{n_A - 1}$. Hence, there are cases in which it is possible that, although not all contestants are active, the sum of equilibrium effort is higher under AA than under ET.

5.1 A Partially Informed Contest Organizer

In this section the previous contest game with groups is generalized by relaxing the assumption on homogeneity within groups and on complete information of the contest organizer with respect to individual characteristics of the contestants. From now on the contestants again face different individual marginal costs that are common knowledge for the contestants. However, the contest designer is only partially informed about the heterogeneity of the contestants because, by assumption, she can only observe the group membership of each contestant and is supposed to know an aggregated measure of heterogeneity given by $\bar{\beta}_A = \frac{1}{n_A}\sum_{j\in A}\beta_j$ for all contestants in group A and $\bar{\beta}_B = \frac{1}{n_B}\sum_{j\in B}\beta_j$ for all contestants in group B, respectively. Group B is assumed to

[23] Proposition 3 does not mention individual equilibrium effort. For the simple contest game analyzed here, the analytical solutions for individual equilibrium effort can be compared easily to show that members of group A exert individually less effort under AA than under ET while members of group B trivially exert more (because they are not active under ET).

be weaker in aggregated terms: $\bar{\beta}_B > \bar{\beta}_A$.

The specification of the equal treatment policy (Definition 1) for this framework remains as before ($\alpha_i^{ET} = \alpha^{ET}$ for all $i \in N$) because it is defined for all contestants identically (and therefore also irrespective of group membership). However, the definition of affirmative action has to be adapted to the limited informational knowledge of the contest organizer because Definition 2 is based on complete information. As the contest organizer can only observe group membership, she is restricted to compensate only for the aggregated (group-specific) level of discrimination. Definition 2 has to be revised respectively where the normative justification remains as in Section 2.

Definition 3 *A policy is called **affirmative action (AA')** in a contest game with a partially informed contest designer if:*

$$\bar{\beta}_A e_i = \bar{\beta}_B e_j \Rightarrow p_i(e) = p_j(e) \text{ for } i \in A, j \in B. \tag{17}$$

The following transformation of variables which respects now the limited information of the contest organizer is useful to proceed in the same line as in the discussion of Definition 2: $z_i = \bar{\beta}_i e_i$ where $\bar{\beta}_i$ can only take two values: $\bar{\beta}_i = \bar{\beta}_A$ for $i \in A$ and $\bar{\beta}_i = \bar{\beta}_B$ for $i \in B$. The requirement formalized in Eq. (17) then implies that for all $z_i = z_j$ it must be true that $p_i(e) = p_j(e)$ for $i \in A$ and $j \in B$. Using the linear CSF as in Eq. 2 with $r = 1$ yields then the following specification of weights $(\alpha_1^{AA'}, \ldots, \alpha_n^{AA'})$:

$$\alpha_i^{AA'} = \bar{\beta}_i \text{ for all } i \in \{A, B\}. \tag{18}$$

An alternative interpretation of this limited information case would be to assume two sources for the heterogeneity of the contestants: one, for which the contestants are not held responsible (i.e. the discrimination of group B as a whole with $\beta_B > \beta_A$), and a second individual one for which the contestants are held ethically responsible. An example would be the following cost function: $c_i(e_i) = (\beta_A + \gamma_i)e_i$ if $i \in A$ (analogously for $i \in B$) where the idiosyncratic parameter γ_i could be positive or negative.[24] The objective of affirmative action is then limited to balance solely the difference between β_A and β_B and not the differences between all the individual parameters γ_i for all $i \in N$.[25]

The comparison between policy ET and AA' is complex for this kind of set-up because not all contestants will always be active under AA' (Lemma 1 does not hold anymore

[24]With this kind of cost function, where $\beta_i = \bar{\beta}_A + \gamma_i$, it is generally not the case that $\bar{\beta}_A = \sum_{i \in A} \beta_i$. However, the important point is that$\bar{\beta}_A$ is known by the contest organizer.

[25]I thank Caterina Calsamiglia for suggesting this interpretation.

in this framework), i.e. there might be distributions where for each policy option different sets of contestants are active. A condition that guarantees that the sum of equilibrium effort is higher under AA' than under ET (parallel to Proposition 3) would depend on the number of active contestants under each policy options and on the underlying distribution of marginal cost parameters in both groups.[26]

To reduce the complexity of the policy comparison, the same special case as in the last section shall be considered, i.e. it is assumed that all contestants are active under both policy options. This implies the analysis is restricted to distributions with a sufficiently low degree of dispersion such that condition (11) is satisfied for all contestants under ET and AA'. As each contestant takes the effort level of the competitors as given, the aggregated equilibrium effort can be calculated as usually, i.e. $E_P^* = \sum_{i \in N} e_i(P)$ for $P = \{ET, AA'\}$. The following result about the consequences of optimal affirmative action AA' is possible:

Proposition 5 *If all contestants are active under policy ET and AA' in a contest game with a partially informed contest organizer, then (i) the sum of equilibrium effort levels is higher under AA' than under ET , and (ii) the individual equilibrium effort is higher under AA' than under ET for all contestants $i \in A$ with discrimination level*

$$\beta_{i \in A} < \frac{n}{(n-1)} \frac{\bar{\beta}_A \bar{\beta}}{(\bar{\beta} + \bar{\beta}_A)}$$

and for all contestants $i \in B$ with discrimination level

$$\beta_{i \in B} < \frac{n}{(n-1)} \frac{\bar{\beta}_B \bar{\beta}}{(\bar{\beta} + \bar{\beta}_B)}$$

Proof: For the first part the following inequality has to be analyzed: $E_{AA'}^* > E_{ET}^*$. If all contestants are active, this inequality is reduced to: $n_A/\bar{\beta}_A + n_B/\bar{\beta}_B > n^2/(\sum_{i \in N} \beta_i)$. This inequality can be further reduced to $n_A n_B (\bar{\beta}_A - \bar{\beta}_B)^2 > 0$ which is always satisfied by assumption.

For the second part the individual equilibrium effort has to be compared. Starting with a member of group A, the inequality $e_{i \in A}^*(AA') > e_{i \in A}^*(ET)$ can be reduced to $\beta_{i \in A} < \frac{(n_A + n_B)\bar{\beta}_A \bar{\beta}}{(n_A + n_B - 1)(\bar{\beta} + \bar{\beta}_A)}$ with the analogous derivation for members of group B.$\square$

[26] The condition for $E_{AA'}^* > E_{ET}^*$ is in fact:

$$\frac{\bar{\beta}_{M_{AA'}}}{\beta_{M_{ET}}^H} > \frac{m_{ET} - 1}{m_{ET}} \frac{m_{AA'}}{m_{AA'} - 1}$$

where M_P denotes the active set under policy $P \in \{ET, AA'\}$. As the characterization of the active set will now depend on the distribution of each group and also its interrelation, an intuitive interpretation of this condition seems to be overly complex and is therefore omitted.

Proposition 5 is intuitive because policy AA' also levels the playing field in an aggregative sense: although contestants might not win the contest game if they exert identical effort under AA', the heterogeneity between the groups is lower under AA' than under ET because the discriminated group is favored in the average. This ameliorates the disincentive effects due to the differences in cost functions for the two groups. The assumption on full participation implies then increased competitive pressure between the two groups which is translated to higher aggregated equilibrium effort.

However, contrary to the full information case individual equilibrium effort increases only for those contestants whose marginal costs are below a specific cut-off parameter. The reason is that under policy AA' higher discrimination does not imply a higher bias in favor of affected contestants proportional to their level of discrimination (as it is the case under policy AA with a fully informed contest organizer). Therefore, policy AA' remains relative ineffective for those contestants with high level of discrimination. However, under AA' higher competitive pressure in the aggregate also has incentive augmenting effects for contestants with relatively low marginal costs that will increase equilibrium effort under AA'. The exact threshold level for those group of contestants is given by the two inequalities in Proposition 5.

6 An Alternative Definition of Affirmative Action

The two policy option specified in Definition 1 and 2 are formulated with respect to the CSF in the sense that a specific constellation of effort and marginal cost parameter for two contestants should yield a similar probability of winning the contest game. As the outcome of the contest game is the relevant equalisandum, those definitions can be described as procedural notions of fairness. However, an alternative end-state notion of fairness would be focused instead on equality of expected equilibrium utility. In the case of affirmative action policy it could be alternatively argued that the outcome of the contest game for each individual should reflect the ethical perception of the heterogeneity in the following sense: if the contestants are perceived to be different because they are discriminated (for which they cannot be held ethically responsible) then the contest outcome should be as if they would be de facto homogenous. Hence, the expected utility in equilibrium should be identical for all contestants. This gives rise to the following alternative 'end-state notion' of affirmative action:

Definition 4 *A policy is called* ***affirmative action with respect to expected equilibrium utilities (AU)*** *if the expected utility for each contestant in the contest* *game is identical in equilibrium:*

$$u_i(e_i^*(AU), e_{-i}^*(AU)) = u_j(e_j^*(AU), e_{-j}^*(AU)) \text{ for all } i \neq j. \tag{19}$$

The equilibrium utility for the two-player contest is derived in Eq. (7). For the n-player contest with linear CSF it is

- for all active contestants $i \in M$: $u_i^*(e_i^*(P), e_{-i}^*(P)) = \left(1 - \frac{\beta_i}{\alpha_i^P} \frac{(m-1)}{\sum_{j \in M} \frac{\beta_i}{\alpha_i^P}}\right)^2$,
- for all non-active contestants $i \notin M$: $u_i^*(e_i^*(P), e_{-i}^*(P)) = 0$.

Condition (19) immediately implies that the set of non-active contestants must be empty, i.e. all contestants will be active under AU. Closer inspection of the expression for equilibrium utility for the two- and n-player contest also reveals that condition (19) is satisfied if $\alpha_i^{AU} = \beta_i^r$ for all $i \in N$ which coincides directly with policy AA. Hence, the different interpretations of the normative objective of affirmative action do not yield different policies for the class of contest games considered here.

7 Concluding Remarks

The implementation of affirmative action policies is a highly controversial topic in public policy discussion. One of the frequent critical remarks is focused on the potential disincentives for effort contribution that could be generated by affirmative action policies. It is argued that there might exist discouraging effects on targeted and non-targeted groups that could finally imply a reduction of effort levels.

This claim is analyzed for a stylized contest game where contestants could be heterogeneous because of past discrimination. If this is the case then, from a normative perspective, the contest rule should be biased in favor of discriminated contestants to induce a level playing field. This affirmative action bias is implemented through the specification of different individual effort weights that are tailored to the individual discrimination parameter of each contestant in such a way that the normative objective is satisfied. At the same time the biased contest rule leads to a change in the incentive structure of the game that affects the optimal effort choice by each contestant. Hence, the consequences of the implementation of affirmative action can be analyzed with respect to the equilibrium effort that this policy induces. Using this approach it can be shown that for the two-player contest game a trade-off between affirmative action and aggregated effort does not only not exist but that both objectives are also closely related. The result for the n-players case and the case with a partially informed contest designer is not as straight-forward: a trade-off is unlikely to exist if the participation decision of the contestants is not altered substantively through the implementation of the affirmative action policy. The results for the individual comparison of equilibrium effort are highly dependent on the distribution of the discrimination parameters.

However, the general idea of how the implementation of affirmative action affects the incentives with respect to effort contribution can be summarized in the following way:[27] Discrimination is a source of heterogeneity between individuals in competitive situations. The implementation of appropriate affirmative action ameliorates (at least in the aggregate) this heterogeneity and makes individuals more similar. This increases competitive pressure and therefore induces higher effort by all participants. However, this argumentation only works if discriminated individuals are in fact the weak ones,[28] if they are identifiable, and if participation effects are not too important. If these requirements are satisfied the critique that affirmative action instruments have disincentivating effects on contestants and therefore adverse consequences for total effort seems to be unjustified.

A Appendix

A.1 Equilibrium in the n-Player Contest Game

To construct the share function of contestant i, his expected utility function has to be transformed in such a way that the contest can be interpreted as an aggregative game in which the utility function of contestant i can be expressed as $\pi_i(z_i, Z)$, where $Z = \sum_{i \in N} z_i$. Consider the following transformation that yields a transformed utility function that is strategically equivalent to Eq. (9): denote $z_i = \alpha_i^P e_i$ which can be

[27] This argumentation must not be restricted to the specific model of contest games considered here. In fact, in Che (2000) it is shown that the effort reducing effect of asymmetries, the so called 'preemption' effect, also exists for difference-form contests that include all-pay auctions as a special case.

[28] In a previous version of this paper the contestants were also heterogeneous with respect to valuation for which they were held ethically responsible. In this case the result in Proposition 2 only holds if discriminated players are sufficiently weak because otherwise the preferential treatment of discriminated players with high valuation would increase the de-facto heterogeneity (a phenomenon which is also coined 'reverse discrimination').

inverted to $e_i = z_i/\alpha_i^P$ for all $i \in N$. The resulting transformed expected utility function for contestant i, which has the aggregative game property, has then the following form:

$$\pi_i(z_i, Z) = \frac{z_i}{Z} - \delta_i^P z_i \text{ for all } i \in N \text{ and for } P \in \{ET, AA\} \tag{20}$$

where $\delta_i^P = \frac{\beta_i}{\alpha_i^P V}$ and Z defined as above. This transformed contest game is now covered by the model in Cornes and Hartley (2005). The share function can therefore be constructed in an analogous way by deriving the first order condition:

$$z_i \left(\frac{Z - z_i}{Z^2} - \delta_i^P \right) = 0 \text{ for } z_i \geq 0. \tag{21}$$

The best response z_i^* of player i can be expressed in terms of the aggregated equilibrium effort:[29] $z_i^*(Z) = \max\{Z - \delta_i^P Z^2, 0\}$. Finally, define player i's share function as her relative contribution

$$s_i(Z) = \frac{z_i^*(Z)}{Z} = \max\{1 - \delta_i^P Z, 0\}. \tag{22}$$

In equilibrium the aggregated effort Z^* is implicitly defined by the condition that the individual share functions sum up to one:

$$S(Z^*) = \sum_{i \in N} s_i(Z^*) = 1 \tag{23}$$

Theorem 1 in Cornes and Hartley (2005) states that a solution to this equation exists and is unique by observing that the aggregated share function $S(Z)$ is continuous and strictly decreasing for positive Z, and that it has a value higher than one for Z sufficiently small and equal to zero for Z sufficiently large.

Equation (22) already indicates that contestants with a high level of δ might have an equilibrium share of zero, i.e. they might prefer to stay non-active. Note that due to the definitions of AA and ET the order of the contestants according to δ_i^P coincides[30] for both policies with the one based on marginal costs because $\delta_1^P \leq \delta_2^P \leq \ldots \leq \delta_n^P$.

Now the the set of active contestants $M \subseteq N$ can be characterized, i.e. the m players with strict positive share in equilibrium. From Eq. (22) it is obvious that in equlibrium $Z^* < 1/\delta_i^P$ for all $i \in M$. Combining Eq. (22) and (23) yields $Z^* = \frac{m-1}{\sum_{j \in M} \delta_j^P}$. The last two expressions yield the condition that indirectly defines the set $M \subseteq N$ of active

[29] It should be obvious that the best response and also the share functions depends on the policy parameter P. But as the finally implemented policy does not affect the proof of equilibrium existence and uniqueness, it is suppressed in this section for notational convenience.

[30] in a weak sense for the AA weights because $\delta_i^{AA} = \delta_j^{AA}$ for $i \neq j$.

contestants that consists out of those m contestants with the lowest δ values that satisfy the following inequality:

$$(m-1)\delta_i^P < \sum_{j \in M} \delta_j^P \text{ for all } i \in M \text{ and for } P \in \{ET, AA\}\,. \tag{24}$$

From the definition of the share function in Eq. (22) the equilibrium effort level of contestant i can be calculated as $e_i^*(P) = z_i^*/\alpha_i^P = s_i(Z^*)Z^*/\alpha_i^P$. Inserting the expression for Z^* leads to Eq. (10).[31]

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[31] Stein (2002) derives a similar expression in a rent-seeking framework where the contestants are heterogeneous with respect to valuations instead of marginal costs.

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