Airline Competition and Network Structure*

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Abstract

This paper characterizes the equilibria in airline networks and their welfare implications in an unregulated environment. Competing airlines may adopt either fully-connected (FC) or hub-and-spoke (HS) network structures; and passengers exhibiting low brand loyalty to their preferred carrier choose an outside option to travel so that markets are partially served by airlines. In this context, carriers adopt *hubbing* strategies when costs are sufficiently low, and asymmetric equilibria where one carrier chooses a FC strategy and the other chooses a HS strategy may arise. Quite interestingly, flight frequency can become excessive under HS network configurations.

Keywords: fully-connected networks; hub-and-spoke networks; brand loyalty; fully-served markets; partially-served markets

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1 Introduction

Before the deregulation of the airline sector (that took place during the 1980s in the US and during the 1990s in Europe), carriers faced constraints in fares and route structures and competition was concentrated in service quality (flight frequency). The deregulation introduced a new source of competition focused on airfares. In this new competitive environment with fares determined by market forces, carriers also became free to make strategic network choices. The success of hub-and-spoke structures in the years following the deregulation led to a concentration of traffic on the spoke routes producing an increase of flight frequency, as documented in Morrison and Whinston (1995) and commented in Brueckner (2004).[1]

Brueckner and Flores-Fillol (2007) (hereafter BF) present a simple duopoly model of schedule competition in a single market, where airlines compete both in fares and scheduling decisions. This dual-competition pattern is studied in a setting that captures the most important elements characterizing the airline sector after its deregulation. Nevertheless, the analysis needs to be completed to allow for network choices in a multi-market framework. The wide-ranging network reorganization observed after the deregulation with the adoption of *hubbing* strategies, supports the idea of introducing this element into the analysis.

Thus, in an unregulated context where carriers may organize their networks either fully-connected (FC) or hub-and-spoke (HS),[2] this paper aims at applying the simple duopoly model of schedule competition introduced by BF to capture optimal network choices and analyze their welfare implications. The comparison between the two network categories is studied in Brueckner (2004) for the monopoly case and we extend this analysis to a duopoly setting.

In its modeling, this paper tries to capture the important elements characterizing the airline sector after its deregulation. Airlines compete in airfares and scheduling decisions, travelers exhibit *brand loyalty* (i.e., they have a utility gain from using a particular airline) and markets are partially served by airlines. In addition, cost per seat realistically falls with aircraft size, capturing the presence of *economies of traffic density* (i.e., economies from operating a larger aircraft) that are unequivocal in the airline industry.

In fact, brand loyalty is an important element of the airline industry, especially since the proliferation of frequent-flyer-programs and worldwide alliances (although brand loyalty may also reflect idiosyncratic consumer preferences for particular aspects of airline service that may differ across carriers). In this framework, the possibility of having partially-served markets by airlines is achieved by introducing in the analysis an outside option that can be interpreted as an alternative transport mode such as automobile, train or ship. In this way, passengers with low brand loyalty do not undertake air travel and make use of the outside option. Differently from BF, there is single group of passengers and the relevant margin of choice (either airline/airline; or airline/outside option) is determined endogenously depending on the cost of the outside option relative to the frequency-airfare pair offered by each carrier.[3]

Thus, the originality of the present paper lies in putting together the elements in BF and Brueckner (2004) that constitute the building blocks of the unregulated airline sector in a way that carriers are free to make strategic network choices in a competitive context where city-pair markets may be uncovered by airlines. Under this specification, the aim of the paper is twofold. On the one hand, it attempts to describe the possible equilibria in airline networks when carriers decide between FC and HS network strategies. In this vein, the paper links fare-and-frequency choices and uncovered markets with the network structures arising in equilibrium. On the other hand, the paper provides a welfare analysis so as to assess the results obtained in equilibrium under the different network specifications.

Our main findings can be summarized as follows. In a framework where air-transport costs are sufficiently low, carriers adopt hubbing strategies, as happened following the deregulation of the industry. As costs increase, economies of traffic density weaken and airlines' incentives to pool passengers from several markets into the same plane vanish. Consequently, FC structures occur in equilibrium when costs are sufficiently high. In addition, asymmetric configurations where one carrier chooses a FC strategy and the other chooses a HS strategy may arise without introducing any explicit asymmetry (neither in costs nor in demand parameters). This result captures the actual coexistence of alternative network strategies in the airline industry.

The analysis of the social optimum reveals that frequencies characterizing FC network structures are suboptimal, confirming the results in BF and Brueckner (2004). This finding seems to be accurate in a single-market setting but not in the current unregulated environment where most carriers organize their networks in a HS manner. Quite interestingly, flight frequency can become excessive under HS network configurations when markets are partially served. This outcome constitutes an explanation to the apparent overprovision of frequencies in the current airline unregulated environment, which is closely related to the adoption of hubbing strategies (that caused an increase in flight frequencies).

There are some previous contributions to the analysis of airline networks that mostly focus on the phenomenon of hubbing that became an issue in the airline sector after the deregulation when airlines started to pool passengers from several markets into the same plane. In this vein, Oum *et al.* (1995) find out that hubbing reduces costs and is typically a dominant strategy for carriers. From a more general approach, Hendricks *et al.* (1999) show that HS networks are likely to arise when carriers do not compete aggressively. Barla and Constantatos (2005), in a setting where each airline decides on its capacity under demand uncertainty, observe that HS networks help the firm to lower its cost of excess capacity in the case of low demand and to improve its capacity allocation in the case of high demand. Finally, with a numerical example, Alderighi *et al.* (2005) suggest the possibility of asymmetric equilibria when the size of the internal markets is large.

The plan of the paper is as follows. Section 2 presents the network and computes the equilibrium frequency and airfare in each of the three scenarios. Section 3 analyzes the equilibria in airline networks given the results in Section 2 (proceeding by backwards induction). Section 4 characterizes the social optimum, comparing optimal frequencies and traffic levels to those emerging in equilibrium. Finally, a brief concluding section closes the paper. All the proofs are provided in Appendix B.

2 The Model

We assume the simplest possible network with three cities (A, B and H), two airlines (1 and 2) and three city-pair markets (AH, BH and AB) as shown in Figure 1.[4] AH and BH are always served nonstop and AB can be served either directly or indirectly with a one-stop trip via hub H, depending on airlines' network choices. Travel in market AB can be also carried out by means of an outside option that can be interpreted as an alternative transport mode such as automobile, train or ship. Passenger population size in each of the city-pair markets is normalized to unity and it is assumed that all the passengers travel, but a proportion of them may not undertake air-travel in market AB whenever they prefer the outside option.[5] For instance, let us consider three Spanish cities like Barcelona, Alicante and Palma de Mallorca. Both Iberia and Spanair airlines serve the three city-pair markets (typically nonstop), but passengers willing to travel between Barcelona and Alicante have a significant outside option since they can also take a fast train (i.e., *Euromed* train) connecting both cities.

In the model, utility for a consumer traveling by air is given by $c + service\ quality + travel\ benefit$, where c is consumption expenditure and equals $Y - p_i$ for consumers using airline i with $i = 1, 2$. Y denotes income and is assumed to be uniform across consumers without loss of generality, and p_i is airline i's fare. *Service quality* measures flight flexibility and is determined by the frequency of flights offered by a particular airline that enhances passenger's utility.[6] Finally, as in BF, $travel\ benefit$ has two components: b, equal to the gain from travel, and a, the airline brand-loyalty variable.

Without brand loyalty, the airline with the most attractive frequency/fare combination would attract all the passengers in the market. However, in presence of brand loyalty, consumers are presumed to have a preference for a particular carrier, which means that an airline with an inferior frequency/airfare combination can still attract some passengers. Following Brueckner and Whalen (2000), this approach is formalized by specifying a utility gain from using airline 1 rather than airline 2, denoted a, and assuming that this gain is uniformly distributed over the range $[-\alpha/2, \alpha/2]$, so that half the consumers prefer airline 1 and half prefer airline 2. Therefore, a varies across consumers. Interestingly, α is a measure of (exogenous) product differentiation in the sense that a small α indicates similar products and thus small gain from using one airline or the other; whereas a big α allows for significant utility gains depending on passenger's preferred carrier. Redefining $Y + b \equiv y$, utility from air travel on carrier 1 is given by $y + f_1 - p_1 + a$ with $a > 0$ for passengers loyal to this carrier. Note that all consumers value flight frequency equally under the present approach and the utility coefficient for f is normalized to 1 (heterogeneity arises instead through brand loyalty).

The analysis that follows derives the demand function and introduces airline's cost structure. It is just presented for carrier 1 for simplicity reasons. The corresponding expressions for carrier 2 come up simply by interchanging 1 and 2 subscripts.

City-pair markets AH and BH (that are identical) are always fully served by airlines since there is no outside option. Thus, passengers will fly with carrier 1 when $y+f_1-p_1+a > y+f_2-p_2$, or when

$$a > p_1 - p_2 - f_1 + f_2. \tag{1}$$

Quite intuitively, for the consumer to choose airline 1, the minimum required brand-loyalty level increases with airline 1's airfare and decreases with its frequency, relative to the ones determined by carrier 2. Otherwise, the consumer will choose airline 2. Then, carrier 1's traffic is given by

$$q_1 = \int_{p_1-p_2-f_1+f_2}^{\alpha/2} \frac{1}{\alpha} da, \tag{2}$$

where $1/\alpha$ gives the density of a. Carrying out the integration, we obtain the following expression:

$$q_1 = \frac{1}{2} - \frac{1}{\alpha}(p_1 - p_2 - f_1 + f_2). \tag{3}$$

In market AB, a passenger making use of the outside option perceives a utility equal to $y-g$, where g stands for the (fixed) cost of the outside option. Nevertheless, when the outside option is very expensive, it becomes irrelevant since air travel is always chosen by all the passengers. In this case, the demand function in market AB has the same form as (3), i.e.,

$$Q_1 = \frac{1}{2} - \frac{1}{\alpha}(P_1 - P_2 - F_1 + F_2), \tag{4}$$

where capital letters denote airfares and frequencies in market AB.

On the other hand, when the outside option is sufficiently cheap, it attracts some passengers (i.e., markets become partially served by airlines) and carriers compete against the outside option. In this context, passengers opt for air-travel when $y+F_1-P_1+a > y-g$ or when

$$a > P_1 - F_1 - g. \tag{5}$$

Then, carrier 1's traffic is given by

$$Q_1 = \int_{P_1-F_1-g}^{\alpha/2} \frac{1}{\alpha} da = \frac{1}{2} - \frac{1}{\alpha}(P_1 - F_1 - g). \tag{6}$$

Since passenger population size in each of the city-pair markets equals the unity, the traffic that will make use of the outside option will be $1 - Q_1 - Q_2$.

To characterize the equilibrium in airfares and frequencies, we need to specify airline's cost structure. It is important to point out that costs borne by airlines are route-dependent (and not market-dependent),[7] so that they depend on the number of links operated by the airline.[8] A *flight's operating cost* is given by $\theta f_1^{xy} + \tau s_1^{xy}$ where f_1^{xy} and s_1^{xy} stand for carrier 1's plane frequency and aircraft size (i.e., the number of seats) along a certain route $xy = AH, BH, AB$. Parameters θ and τ are the marginal cost per departure (or aircraft-operation cost) and the marginal cost per seat, respectively.

Cost per departure (θf_1^{xy}) is increasing with frequency because airport slots are scarce and therefore an increase in congestion results in higher landing fees during peak hours as argued in Heimer and Shy (2006). This cost consists of fuel for the duration of the flight, airport maintenance, renting the gate to board and disembark the passengers, landing and air-traffic control fees.

As in BF and Brueckner (2004), it is assumed that all seats are filled, so that load factor equals 100% and therefore $s_1^{xy} = q_{1,tot}^{xy}/f_1^{xy}$, i.e., aircraft size can be determined residually dividing airline's total traffic on a route by the the number of planes. Note that cost per seat, that can be written $\theta q_{1,tot}^{xy}/(s_1^{xy})^2 + \tau$, visibly decreases with s_1^{xy} capturing the presence of economies of traffic density (i.e., economies from operating a larger aircraft) that are unequivocal in the airline industry.

Therefore, carrier 1's total cost from operating in route xy is $f_1^{xy}(\theta f_1^{xy} + \tau s_1^{xy})$ or equivalently

$$C_1^{xy}(q_{1,tot}^{xy}, f_1^{xy}) = \theta(f_1^{xy})^2 + \tau q_{1,tot}^{xy}. \tag{7}$$

Since routes AH and BH are identical, then $C_1^{AH}(\cdot) = C_1^{BH}(\cdot)$.

Airline 1's equilibrium airfares and frequencies depend on 1's network choice but also on the network configuration adopted by the other airline. The next step consists in computing this equilibrium using (3), (4), (6) and (7), and ascertaining the critical values of g making the outside option relevant in each network scenario: (FC,FC), (HS,HS) and (FC,HS). In order to compute the equilibrium fares and frequencies, we need to distinguish two different potential situations depending on the cost of the outside option (g) in market AB:

(*i*) *Market AB fully served by airlines (high g).* In this case, the outside option can be disregarded since it is never employed and then the relevant margin of choice for passengers is airline 1/airline 2.
(*ii*) *Market AB partially served by airlines (low g).* The outside option becomes attractive for low brand-loyalty travelers and the relevant margin of choice for passengers becomes preferred airline/outside option.

Hence, airlines only compete against each other when markets are fully-served. When this is not the case, airlines compete against the outside option. The network scenarios (FC,FC) and (HS,HS) yield a symmetric equilibrium and the results are just presented for carrier 1 for simplicity reasons.

Note that, after knowing the fare-and-frequency choice under each possible network scenario, we will explore in Section 3 the incentives for carriers to implement a certain network configuration.

2.1 The (FC,FC) Network Scenario

With this network configuration, all the three city-pair markets are served nonstop[9] since both airlines offer a direct flight between cities A and B and airline 1's profit function is computed by adding revenues and subtracting costs:

$$\pi_1 = 2 \underbrace{p_1 q_1}_{R_1^{AH}=R_1^{BH}} + \underbrace{P_1 Q_1}_{R_1^{AB}} - 2[\underbrace{\theta(f_1)^2 + \tau q_1}_{C_1^{AH}=C_1^{BH}}] - [\underbrace{\theta(F_1)^2 + \tau Q_1}_{C_1^{AB}}]. \tag{8}$$

(*i*) *Market AB fully served by airlines (high g).* The three markets are symmetric and thus, $p_1 = P_1$, $f_1 = F_1$, $q_1 = Q_1$ and $C_1^{AH} = C_1^{BH} = C_1^{AB} = \theta(f_1)^2 + \tau q_1$. Hence, (8) becomes $\pi_1 = 3[p_1 q_1 - \theta(f_1)^2 - \tau q_1]$ and using (3) we obtain

$$\pi_1 = 3[(p_1 - \tau)(\frac{1}{2} - \frac{1}{\alpha}(p_1 - p_2 - f_1 + f_2)) - \theta(f_1)^2]. \tag{9}$$

Airline 1 chooses p_1 and f_1 simultaneously[10] to maximize (9) yielding

$$f_1^* = F_1^* = \frac{1}{4\theta} \text{ and } p_1^* = P_1^* = \tau + \frac{\alpha}{2}, \tag{10}$$

where superscript $*$ denotes equilibrium values where all markets are fully served. Quite naturally, the equilibrium frequency is decreasing with the aircraft-operation cost (θ). On the other hand, the airfare equals the marginal cost of a seat (τ) plus a markup that depends on the degree of product differentiation ($\alpha/2$) and, as differentiation disappears, the fare converges to the marginal cost recovering the Bertrand-equilibrium outcome, as in BF. Finally, each airline carries half of the population (i.e., $q_1^* = Q_1^* = 1/2$) since all the passengers undertake air travel.

It is important to recall that the values in (10) are the ones obtained in equilibrium as long as g is high and thus $y + F_1 - P_1 + a > y - g$ and $y + F_2 - P_2 - a > y - g$ hold for any value of a (when the contrary occurs, i.e., $y + F_1 - P_1 + a < y - g$ is possible for low values of a, airlines compete against the outside option and low brand-loyalty passengers do not use airlines' service). In equilibrium, $y + F_1^* - P_1^* + a = y + F_2^* - P_2^* - a$ occurs when $a = 0$ and thus, fully-served markets require $g > g_{FC} \equiv P_1^* - F_1^*$ as shown in Figure 2. Replacing P_1^* and F_1^* with their equilibrium values this threshold value of g becomes

$$g_{FC} = \tau + \frac{\alpha}{2} - \frac{1}{4\theta}. \tag{11}$$

(ii) Market AB partially served by airlines (low g). In this case, we need to differentiate markets AH and BH where airlines compete against each other, from market AB where the relevant margin of choice is preferred airline/outside option. From plugging (3) and (6) into (8) and maximizing, we obtain

$$F_1^{**} = \frac{\alpha/2 + g - \tau}{4\alpha\theta - 1},\ P_1^{**} = \frac{2\alpha\theta(\alpha/2 + g + \tau) - \tau}{4\alpha\theta - 1}, \tag{12}$$

where superscript $**$ denotes equilibrium values where market AB is partially served and $f_1^* = f_1^{**}$ and $p_1^* = p_1^{**}$ (see (10)). Quite naturally, the proportion of passengers choosing air travel increases with the cost of the alternative mode of transport (g). Unsurprisingly, when g rises, airlines gain monopoly power and react by increasing fares (P_1). Markets AH and BH are fully served with $q_1^{**} = q_2^{**} = 1/2$ and market AB is partially served since some passengers choose the outside option to travel (i.e., $Q_1^{**} + Q_2^{**} < 1$). These are the values obtained in equilibrium as long as g is low, i.e., $y + F_1^{**} - P_1^{**} + a < y - g$ and $y + F_2^{**} - P_2^{**} - a < y - g$ hold for low brand-loyalty passengers. As before, in equilibrium $y + F_1^{**} - P_1^{**} + a = y + F_2^{**} - P_2^{**} - a$ occurs when $a = 0$, and airlines start competing against the outside option when $g < g_{FC}$ (i.e., when the outside option is sufficiently cheap). Note that g_{FC} turns out to be exactly the expression in (11), i.e., $g_{FC} = P_1^* - F_1^* = P_1^{**} - F_1^{**} = \tau + \alpha/2 - 1/4\theta$.

When market AB is partially served, passengers with low values of brand loyalty (a) are the first ones to choose the outside option. Hence, there is a brand-loyalty threshold denoted by a_{FC}, delimiting the passengers that fly with their preferred carrier (passengers with $a > a_{FC}$) from those that make use of the outside option (passengers with $a < a_{FC}$), as shown in Figure 3. Hence, the threshold delimiting the relevant margin of choice can be expressed both in terms of a and g. From $y + F_1^{**} - P_1^{**} + a = y - g$, the threshold a_{FC} can be easily derived:

$$a_{FC} = P_1^{**} - F_1^{**} - g. \tag{13}$$

Note that, *when market AB is fully served,* then $a_{FC} = 0$ and $g = P_1^{**} - F_1^{**}$. Then, it is easy to check that $F_1^{**} = f_1^{**} = f_1^* = \frac{1}{4\theta}$ and $P_1^{**} = p_1^{**} = p_1^* = \tau + \frac{\alpha}{2}$.[11] Equivalently, when $g = g_{FC}$ we also recover the results with fully-served markets.

2.2 The (HS,HS) Network Scenario

With this network configuration, route AB is abandoned by both carriers and city-pair market AB is served through a two-segment trip with stop at the hub city H.[12] Thus, airline 1's profit is now

$$\pi_1 = 2 \underbrace{p_1 q_1}_{R_1^{AH}=R_1^{BH}} + \underbrace{P_1 Q_1}_{R_1^{AB}} - 2[\underbrace{\theta(f_1)^2 + \tau(q_1 + Q_1)}_{C_1^{AH}=C_1^{BH}}], \tag{14}$$

because $C_1^{AH}(q_{1,tot}^{AH}, f_1^{AH})$ is link-dependent and incorporates all the traffic passing through route AH, i.e., $q_{1,tot}^{AH} = q_1 + Q_1$.

(i) Market AB fully served by airlines (high g). Plugging (3) and (4) into (14) and maximizing, we obtain

$$f_1^* = \frac{3}{8\theta},\ p_1^* = \tau + \frac{\alpha}{2} \text{ and } P_1^* = 2\tau + \frac{\alpha}{2}. \tag{15}$$

Comparing these values with the ones obtained under the (FC,FC) scenario (see (10)), it is easy to check that frequencies are now higher since there is more traffic in each of the two active routes AH and BH (as in Brueckner, 2004); airfares in AH and BH markets are the same; and AB trips are now more expensive because they make use of two routes.[13] Following the same reasoning as in scenario (FC,FC), fully-served markets require $g > g_{HS}$ where

$$g_{HS} \equiv P_1^* - f_1^* = 2\tau + \frac{\alpha}{2} - \frac{3}{8\theta}. \qquad (16)$$

Graphically, this situation looks like the one under (FC,FC) depicted in Figure 2.

(ii) Market AB partially served by airlines (low g). When exclusion from air-travel is an issue, plugging (3) and (6) into (14) and maximizing yields

$$f_1^{**} = \frac{5\alpha/2 + g - 2\tau}{8\alpha\theta - 1},\ p_1^{**} = \tau + \frac{\alpha}{2} \text{ and } P_1^{**} = \frac{4\alpha\theta(\alpha/2 + g + 2\tau) + \alpha - 2\tau}{8\alpha\theta - 1}. \qquad (17)$$

From $P_1^{**} - f_1^{**}$ we obtain expression (16), i.e., $g_{HS} = P_1^* - f_1^* = P_1^{**} - f_1^{**} = 2\tau + \alpha/2 - 3/8\theta$. Finally, the brand-loyalty threshold under which passengers prefer the outside option is

$$a_{HS} = P_1^{**} - f_1^{**} - g. \qquad (18)$$

Note that, *when market AB is fully served,* then $a_{HS} = 0$ and $g = P_1^{**} - f_1^{**}$. Then, it is easy to check that $f_1^{**} = f_1^* = \frac{3}{8\theta}$ and $P_1^{**} = P_1^* = 2\tau + \frac{\alpha}{2}$.[14] Equivalently, when $g = g_{HS}$ we also recover the results with fully-served markets.

As suggested before, carriers under HS networks offer higher frequency than under FC networks (*frequency effect*), but they charge higher airfares in market AB because of the use of two routes to serve the market (*airfare effect*), as it is spelt out in the following lemma (where carrier subscripts are dropped).

Lemma 1 *From comparing (FC,FC) and (HS,HS), we observe*
i) frequency effect: $f_{HS}^* > f_{FC}^* = F_{FC}^*$ *(fully-served markets); and* $f_{HS}^{**} > f_{FC}^{**}, F_{FC}^{**}$ *(partially-served markets); and*
ii) airfare effect: $P_{HS}^* > P_{FC}^*$ *(fully-served markets); and* $P_{HS}^{**} > P_{FC}^{**}$ *(partially-served markets).*

Air-transport cost reduces flight frequency and increases fares charged by airlines and this affects the threshold values of g delimiting the relevant margin of choice. The following lemma, that arises from comparing g_{FC} and g_{HS}, states that uncovered markets are more likely to be observed under a certain network structure depending on the cost of air transport.

Lemma 2 *Partially-served markets are more likely to be observed under:*
i) HS structures when $\theta\tau > \frac{1}{8}$. *In this case* $g_{FC} < g_{HS}$ *and therefore* $a_{FC} < a_{HS}$*; and*
ii) FC structures when $\theta\tau < \frac{1}{8}$. *In this case* $g_{FC} > g_{HS}$ *and therefore* $a_{FC} > a_{HS}$.

Interestingly, these two lemmas are closely linked to each other. When air-transport cost is high (as in part i of the previous lemma), the *airfare effect dominates the frequency effect* and competition is softer under HS structures (because fares are higher under HS networks, and this is not compensated by frequencies). On the other hand, when air-transport cost is low (as in part ii of the previous lemma), the *frequency effect dominates the airfare effect* and competition is softer under FC structures (because frequencies are lower under FC networks, and this is not compensated by fares).

Consequently, uncovered markets that are characterized by softer competition (because airlines compete against the outside option) are more likely to be observed under HS configurations when air-transport cost is high; and under FC configurations when air-transport cost is low.

For instance, consider a high air-transport cost environment where $g_{FC} < g_{HS}$ (as in part i of the previous lemma). For any value of g such that $g_{FC} < g < g_{HS}$, airlines compete against the outside option under HS structures; but they compete against each other under FC networks, as shown in Figure 4. The opposite behavior is observed in a low air-transport cost environment for any g such that $g_{HS} < g < g_{FC}$.

2.3 The Asymmetric (FC,HS) Network Scenario

Along this subsection, we need to distinguish between carriers 1 and 2 because they make different network choices. Without loss of generality, we assume that carrier 1 adopts a FC network, whereas carrier 2 serves AB city-pair market through two-segment trips that stop at the hub city H. Hence, airlines' profits are given by

$$\pi_1 = 2\underbrace{p_1 q_1}_{R_1^{AH}=R_1^{BH}} + \underbrace{P_1 Q_1}_{R_1^{AB}} - 2[\underbrace{\theta(f_1)^2 + \tau q_1}_{C_1^{AH}=C_1^{BH}}] - [\underbrace{\theta(F_1)^2 + \tau Q_1}_{C_1^{AB}}] \text{ and}$$

$$\pi_2 = 2 \underbrace{p_2 q_2}_{R_2^{AH}=R_2^{BH}} + \underbrace{P_2 Q_2}_{R_2^{AB}} - 2[\underbrace{\theta(f_2)^2 + \tau(q_2 + Q_2)}_{C_2^{AH}=C_2^{BH}}].$$

(i) Market AB fully served by airlines (high g). Substituting (3), (4) and the corresponding terms for carrier 2 (analogous expressions with the 1 and 2 subscripts interchanged) in the profit functions, we obtain the following equilibrium values for carrier 1

$$f_1^* = \tfrac{3+2\theta\tau+12\alpha\theta(3\alpha\theta-2)}{2\theta A},\ F_1^* = f_1^* + \tfrac{\tau}{(6\alpha\theta-1)},\ p_1^* = \tau + 2\alpha\theta f_1^*,\ P_1^* = \tau + 2\alpha\theta F_1^*, \tag{19}$$

and for carrier 2

$$f_2^* = \tfrac{3(3\alpha\theta-1)-2\theta\tau}{2\theta(12\alpha\theta-5)},\ p_2^* = 2\tfrac{9\alpha^2\theta(2\alpha\theta+4\theta\tau-1)-22\alpha\theta\tau+\alpha+5\tau/2}{A},\ P_2^* = 2\tfrac{3\alpha^2\theta(6\alpha\theta+20\theta\tau-3)-38\alpha\theta\tau+\alpha+5\tau}{A}, \tag{20}$$

where $A = (6\alpha\theta - 1)(12\alpha\theta - 5)$. The main difference with respect to the previous (symmetric) scenarios is that now $y + F_1^* - P_1^* + a = y + f_2^* - P_2^* - a$ occurs for a value of a different from 0 because $F_1^* - P_1^* \neq f_2^* - P_2^*$. Let us denote by $\widehat{a}$ this brand-loyalty level:

$$\widehat{a} = \alpha \frac{6\alpha\theta(1 - 8\theta\tau) + 16\theta\tau - 1}{A}. \tag{21}$$

Therefore $y + F_1^* - P_1^* + \widehat{a} = y + f_2^* - P_2^* - \widehat{a}$; and thus the relevant margin of choice is airline 1/airline 2 when this utility exceeds $y - g$. From this expression we can derive the level of g that draws up the boundaries for a relevant outside option:

$$\overline{g}_{FC,HS} = \frac{3\alpha^2\theta(24\alpha\theta - 29) + 12\alpha\theta\tau(18\alpha\theta - 11) + 61\alpha/2 + 18\tau - 3/\theta}{2A}. \tag{22}$$

Consequently, fully-served markets require $g > \overline{g}_{FC,HS}$. In this case, the brand-loyalty threshold determining the relevant margin of choice for carrier 1 ($a_{FC,HS}^1 \equiv P_1^* - F_1^* - g$) differs from the one of carrier 2 ($a_{FC,HS}^2 \equiv P_2^* - f_2^* - g$), as shown in Figure 5.

(ii) Market AB partially served by airlines (low g). Substituting (3), (6) and the corresponding terms for carrier 2 (analogous expressions with the 1 and 2 subscripts interchanged) in the profit functions, we obtain

$$f_1^{**} = \tfrac{6\alpha(2\alpha\theta-1)+2\tau-g}{B},\ F_1^{**} = \tfrac{\alpha/2+g-\tau}{4\alpha\theta-1},\ p_1^{**} = 2\alpha\theta f_1^{**},\ P_1^{**} = \tau + 2\alpha\theta F_1^{**} \tag{23}$$

for airline 1 and

$$f_2^{**} = \frac{\alpha\theta(15\alpha+6g)-\frac{9}{2}\alpha+2\tau-g}{B},\ p_2^{**} = \tau + 2\alpha\frac{\alpha\theta(12\alpha\theta-5)-\theta(2\tau-g)+1/2}{B},$$

$$P_2^{**} = \tau + \frac{2\alpha\theta[4g(3\alpha\theta-1)+6\alpha^2\theta+\alpha-1/\theta]-\tau(6\alpha\theta-1)}{B} \tag{24}$$

for airline 2 with $B = 1+2\alpha\theta(24\alpha\theta-11)$. Shifting attention to utility functions, $y+F_1^{**} - P_1^{**} + a = y + f_2^{**} - P_2^{**} - a$ occurs for $a = \widehat{\widehat{a}}$ and airlines start competing against the outside option when $y + F_1^{**} - P_1^{**} + \widehat{\widehat{a}} = y + f_2^{**} - P_2^{**} - \widehat{\widehat{a}} < y - g$ holds with

$$\widehat{\widehat{a}}=\frac{\alpha}{2}\frac{6\alpha\theta(1-8\theta\tau)+16\theta\tau-1}{5+2\alpha\theta(48\alpha\theta-25)}. \tag{25}$$

From this expression we can derive the threshold level for g ensuring a relevant outside option:

$$\underline{g}_{FC,HS} = \frac{2\alpha^2\theta(48\alpha\theta-55)+12\alpha\theta\tau(24\alpha\theta-13)+35\alpha+18\tau-3/\theta}{2(5+2\alpha\theta(48\alpha\theta-25))}.$$

Therefore, some low-brand loyalty travelers will not fly for $g < \underline{g}_{FC,HS}$. Graphically, this situation looks like the one depicted in Figure 5 but with different threshold values.

Consequently, we have shown in the above analysis that different critical values of g arise in the two considered frameworks (i.e., (i) *Market AB fully served by airlines;* and (ii) *Market AB partially served by airlines*).

Hence, we are left with four critical values of g (i.e., g_{FC}, g_{HS}, $\overline{g}_{FC,HS}$ and $\underline{g}_{FC,HS}$) that determine five regions, that are fully characterized in Lemma 3. This lemma asserts that, depending on the cost of air transport, uncovered markets are more likely to be observed under a certain network structure, completing the insight anticipated in Lemmas $1-2$.

Lemma 3 *Partially-served markets are more likely to be observed under:*
i) HS structures when $\theta\tau > \frac{1}{8}$. *In this case* $g_{FC} < \overline{g}_{FC,HS} < \underline{g}_{FC,HS} < g_{HS}$*; and*
ii) FC structures when $\theta\tau < \frac{1}{8}$. *In this case* $g_{FC} > \overline{g}_{FC,HS} > \underline{g}_{FC,HS} > g_{HS}$.

We can observe the regions resulting from plotting these threshold values of g in Figures $6-7$. Note that g determines the relevant margin of choice in the market since high values of g (i.e., expensive outside option) yield to a perfect duopoly where airlines compete against each other (see Region A in both figures); whereas low values of g (i.e., attractive outside option) yield to monopolistic situations where carriers compete against the outside option (see Region E in both figures). Region A portrays the simplest scenario because the outside option does not play any role and airlines exercise market power to affect the division of a fixed amount of traffic between them. In this case, airlines exert no monopoly power over any passenger, although they can still affect the division of the fixed traffic pool by their choices of fares and frequencies. On the other extreme, in Region E the outside option is always relevant. Under these circumstances, carriers enjoy monopolistic power and passengers with low brand-loyalty make use of the outside option.

For intermediate values of g, the order of the threshold values depends on the magnitude of air-transport costs. This result fleshes out and reinforces the intuition revealed by Lemmas $1-2$, confirming that softer competition under HS structures emerges with high air-transport costs (part i of the previous lemma); while softer competition under FC structures emerges with low air-transport costs (part ii of the previous lemma).

Hence, airlines decide whether to compete against each other in each of the regions (that are determined by the cost of the outside option g). When they compete against each other, the relevant margin of choice is airline 1/airline 2 and markets are fully served (in this case there is a high $F_i - P_i$ for $i = 1, 2$). On the other hand, when they do not compete against each other, the relevant margin is preferred airline/outside option and markets are partially served (in this case there is a low $F_i - P_i$ for $i = 1, 2$). The relevant margin of choice is thus determined endogenously depending on the cost of the outside option relative to the frequency-airfare-pair offered by each carrier in each scenario.[15] These decisions are Nash-proved and, when both airlines decide not to compete against each other, none of them is interested in deviating by lowering fares (or increasing frequency) to capture some travelers loyal to the rival carrier.

Finally, in Regions C (in Figure 6) and C' (in Figure 7), the relevant margin of choice under (FC,HS) is unclear.[16]

3 Equilibria in Airline Networks

In this section, taking into consideration the optimal fare-and-frequency choice under each possible network scenario, we take notice of the possible equilibria in networks (i.e., we proceed by backwards induction). Carriers decide simultaneously and independently between two strategies: either to adopt a FC or a HS network structure. Nevertheless, before drawing any conclusion, we need to assert the *relevant region* (R) in our analysis where fares and travel volumes are positive; and second-order and non-arbitrage conditions[17] are satisfied. In R, all the results are comparable.

Definition 1 *R is the region in the space $\{\alpha, \theta, \tau, g\}$ ensuring positive airfares and travel volumes; and compliance with second-order and non-arbitrage conditions in all the scenarios.*

In R, we require $\tau < \overline{\tau} \equiv (1 + 2\alpha\theta)/8\theta$ and $\alpha\theta > 7/12$ to hold. See Appendix A for the details. Given the structure of the suggested game, the following results are simply obtained by comparing $\pi(FC, FC)$ and $\pi(HS, FC)$ on the one hand; and $\pi(HS, HS)$ and $\pi(FC, HS)$ on the other hand, where the first element between parentheses gives the own-network strategy and the second element indicates the rival's strategy.

The equilibria in airline networks are presented for each possible case. Firstly, we make a distinction depending on the magnitude of air-transport costs, (i.e., either $\theta\tau > \frac{1}{8}$ or $\theta\tau < \frac{1}{8}$); and secondly, we study each possible region depending on the value of g as depicted in Figures 6 and 7.

Therefore, we have six possible cases: (i) Region A: *market AB fully served by airlines*; (ii) Region E: *market AB partially served by airlines*; (iii) region B: *market AB partially served under (HS,HS)*; (iv) Region D: *market AB partially served under (HS,HS) and (FC,HS)*; (v) Region B': *market AB partially served under (FC,FC)*; and finally (vi) Region D': *market AB partially served under (FC,FC) and (FC,HS).*

Remember that Regions A and E emerge for any value of $\theta\tau$ (see Figures $6-7$), whereas Regions B and D appear for $\theta\tau > \frac{1}{8}$ and Regions B' and D' come out for $\theta\tau < \frac{1}{8}$. The case $\theta\tau < \frac{1}{8}$ could be seen as more realistic in the current unregulated environment where market competition keeps a downward pressure on airline costs.

To understand how the different equilibria arise in the figures (see Figures $8-10$), it suffices to remember that it is enough to compare $\pi(FC, FC)$ and $\pi(HS, FC)$ on the one hand; and $\pi(HS, HS)$ and $\pi(FC, HS)$ on the other hand. When we observe $\pi(FC, FC) >$

$\pi(HS, FC)$, there is an equilibrium of the type (FC,FC); when $\pi(HS, HS) > \pi(FC, HS)$ happens, the configuration (HS,HS) arises as an equilibrium in networks; and finally when both $\pi(HS, HS) < \pi(FC, HS)$ and $\pi(FC, FC) < \pi(HS, FC)$ are observed, the equilibrium is asymmetric because carriers' network choices do not coincide.

These profit comparisons determine some critical values for the marginal cost per seat (τ^*) that depend on the other parameters of the model and delimit the different equilibrium areas. From the comparison between $\pi(FC, FC)$ and $\pi(HS, FC)$ we obtain τ_1^*; and from $\pi(HS, HS)$ and $\pi(FC, HS)$ we obtain τ_2^* and τ_3^* (but in some figures τ_3^* does not appear because $\tau_3^* > \overline{\tau}$).

3.1 High Air-Transport Cost ($\theta\tau > \frac{1}{8}$)

Figures $8-9$ portray the detailed equilibria in presence of high air-transport costs, where the following order of threshold values for τ is observed: $\tau_1^* < \tau_2^* < \tau_3^*$. Thus, the equilibrium in airline networks is: (HS,HS) for $\tau < \tau_1^*$; the multiple equilibria {(HS,HS),(FC,FC)} for $\tau \in (\tau_1^*, \tau_2^*)$; (FC,FC) for $\tau \in (\tau_2^*, \tau_3^*)$; and finally the structure {(HS,HS),(FC,FC)} again for $\tau > \tau_3^*$.

Once profits are compared, the constraint $\tau > \frac{1}{8\theta}$ has to be taken into account because it rules out some possible equilibrium areas. Only those areas compatible with the aforementioned constraint are relevant in the analysis.[18] The precise location of $\tau = \frac{1}{8\theta}$ in Figures $8-9$ depends on the value of the parameters. $\frac{1}{8\theta} \in (\tau_1^*, \min\{\tau_3^*, \overline{\tau}\})$ is always observed but both $\frac{1}{8\theta} < \tau_2^*$ and $\frac{1}{8\theta} > \tau_2^*$ are possible. Independently of this ambiguity, (FC,FC) is always a possible equilibrium for any value of τ. Therefore, our first result states that, in presence of high transport cost, FC structures prevail in equilibrium.

Proposition 1 *When $\theta\tau > \frac{1}{8}$, (FC,FC) always arises as an equilibrium in airline networks for any value of τ.*

This result can help to explain the high-cost environment existing before the deregulation where carriers used to operate FC. We know from Lemmas $1-3$ that competition is softer under HS configurations in presence of high air-transport costs. Since FC configurations prevail in equilibrium, we can ascertain a bias towards network structures implying higher levels of competition (i.e., networks where carriers compete against each other).

3.2 Low Air-Transport Cost ($\theta\tau < \frac{1}{8}$)

The possible equilibria in airline networks when air-transport costs are low are depicted in Figures 8 and 10. Once profits are compared and the constraint $\tau < \frac{1}{8\theta}$ has been taken into account, we appreciate that HS structures dominate in equilibrium and that asymmetric equilibria may occur for certain parameter values.

In fact, most of the areas where (FC,FC) appeared as an equilibrium in the previous case are now ruled out and (HS,HS) always emerges as a possible equilibrium (but not for any value of τ). The following result states that, in presence of low transport costs, HS structures prevail in equilibrium.

Proposition 2 *When $\theta\tau < \frac{1}{8}$, (HS,HS) always arises as an equilibrium in airline networks.*

When the cost of air transport is low, carriers operate HS. This is what we observed after the deregulation of the airline sector when carriers became free to make strategic network choices in a competitive framework that exerted a downward pressure over air-transport costs. As before, we detect a bias towards network structures implying higher levels of competition since competition is softer under FC configurations in presence of low air-transport costs (from Lemmas $1-3$) and HS configurations prevail in equilibrium.

The order of the critical values for the marginal cost per seat (τ^*) is the same as in the previous case; except in Region B' (see Figure 10) where $\tau_2^* < \tau_1^*$ occurs and an asymmetric equilibrium $\{$(FC,HS),(HS,FC)$\}$ comes up for $\tau \in (\tau_2^*, \tau_1^*)$.

Corollary 1 *Under $\theta\tau < \frac{1}{8}$, an asymmetric equilibrium where one carrier adopts a FC network and the other operates HS is possible for intermediate values of cost per seat and competition intensity.*

In this realistic framework in which markets may be partially served, asymmetric configurations where one carrier chooses a FC strategy and the other chooses a HS strategy may arise when cost per seat and competition intensity and are moderate. In particular, this behavior is detected on Region B' where market AB is partially served under (FC,FC) network configurations. Therefore asymmetric equilibria are possible without introducing any explicit asymmetry (neither in costs nor in demand parameters). This result is relevant to the extent that it captures the actual coexistence of alternative network strategies in the airline industry, as pointed out by Alderighi *et al.* (2005).

Summing up, in a framework characterized by low air-transport costs, carriers adopt hubbing strategies, as happened following the deregulation of the industry. As costs increase, economies of traffic density weaken and airlines' incentives to pool passengers from several markets into the same plane disappear. In this environment, carriers prefer to avoid multi-segment markets and they choose to serve city-pair markets directly to minimize the use of expensive routes. Consequently, FC structures occur in equilibrium when costs are sufficiently high. In addition, asymmetric network configurations may arise in equilibrium when air-transport costs are low.

4 The Social Optimum

With the equilibria in airline networks understood, attention now shifts to welfare analysis where a social planner decides flight frequency and traffic so as to maximize social surplus, that is computed as the sum of total utility and airline profit. Social surplus depends on airlines' network choices. Nevertheless, the social optimum is independent of the interaction between carriers, i.e., there is an optimum for carriers operating FC and another optimum for HS carriers. As in the equilibrium analysis, we need to distinguish between the situation with fully-served markets and the situation where market AB is partially served.

4.1 The (FC,FC) Scenario

Total utility for carrier 1 in market AH (or BH) can be written

$$U_1^{AH}(FC) = \int_0^{\alpha/2} (y + f_1 - p_1 + a)\frac{1}{\alpha}da = \frac{y + f_1 - p_1}{2} + \frac{\alpha}{8}. \tag{26}$$

Assuming that market AB is partially served by airlines, some passengers will not undertake air travel since they will choose the outside option to travel between cities A and B:

$$U_1^{AB}(FC) = \underbrace{\int_{a^*}^{\alpha/2} (y + F_1 - P_1 + a)\frac{1}{\alpha}da}_{Air\ traffic} + \underbrace{\int_0^{a^*} (y - g)\frac{1}{\alpha}da}_{Outside\ option\ traffic} =$$

$$= \frac{y}{2} + (F_1 - P_1)\left(\frac{1}{2} - \frac{a^*}{\alpha}\right) + \frac{\alpha}{8} + \frac{(a^*)^2}{2\alpha} - g\frac{a^*}{\alpha}, \tag{27}$$

where a^* denotes the air-travel/outside option loyalty margin, since airlines compete against the outside option in city-pair market AB. Note that, *when market AB is fully served by airlines,* then $a^* = 0$ and $U_1^{AB}(FC) = U_1^{AH}(FC)$.

Carrier 1's total profit equals

$$\pi_1(FC) = \underbrace{2(p_1 - \tau)\int_0^{\alpha/2} \frac{1}{\alpha} da - 2\theta(f_1)^2}_{market\ AH\ and\ market\ BH} + \underbrace{(P_1 - \tau)\int_{a^*}^{\alpha/2} \frac{1}{\alpha} da - \theta(F_1)^2}_{market\ AB} =$$

$$= (p_1 - \tau) + (P_1 - \tau)\left(\frac{1}{2} - \frac{a^*}{\alpha}\right) - 2\theta(f_1)^2 - \theta(F_1)^2. \tag{28}$$

Note that the 2 factor appears because markets AH and BH are identical; and that in market AB we only consider those passengers that undertake air travel (because airlines do not obtain any profit from passengers making use of the outside option).

Adding utilities and profits for both carriers we obtain

$$W(FC, FC) = 4U_1^{AH}(FC) + 2U_1^{AB}(FC) + 2\pi_1(FC) =$$

$$= 2\left\{ \underbrace{3y\int_0^{\alpha/2} \frac{1}{\alpha} da}_{Income} - \underbrace{g\int_0^{a^*} \frac{1}{\alpha} da}_{Outside\ option's\ cost} + \underbrace{2\int_0^{\alpha/2} \frac{a}{\alpha} da + \int_{a^*}^{\alpha/2} \frac{a}{\alpha} da}_{Average\ brand\text{-}loyalty\ benefits} + \right.$$

$$\left. \underbrace{+2f\int_0^{\alpha/2} \frac{1}{\alpha} da + F\int_{a^*}^{\alpha/2} \frac{1}{\alpha} da}_{Frequency\ benefits} - \underbrace{[\tau(2\int_0^{\alpha/2} \frac{1}{\alpha} + \int_{a^*}^{\alpha/2} \frac{1}{\alpha} da) + \theta(2f^2 + F^2)]}_{Costs} \right\},$$

where we can eliminate the subscripts since both carriers are identical. Notice that the two first elements give income from the three markets and the cost of the outside option for those passengers that do not make use of air transport. The last three terms are the average brand-loyalty benefits, the frequency benefits and the costs for those passengers undertaking air travel. The 2 factor is necessary because two airlines are present. Performing the integration, we obtain

$$= 3y - 2g\tfrac{a^*}{\alpha} + \tfrac{3\alpha}{4} - \tfrac{(a^*)^2}{\alpha} + 2f + 2F(\tfrac{1}{2} - \tfrac{a^*}{\alpha}) - 2\tau - 2\tau(\tfrac{1}{2} - \tfrac{a^*}{\alpha}) - 4\theta f^2 - 2\theta F^2, \quad (29)$$

The planner chooses a^* which determines the optimal air traffic, along with flight frequencies to maximize (29). Observe that airfares do not appear in the expression because they are a transfer between airlines and air travelers. The first-order condition for choice of a^* yields

$$a^* = \tau - F^{SO} - g \equiv a_{FC}^{SO}, \quad (30)$$

indicating that, for AB-market air travelers, the marginal cost is exactly balanced by the benefits from brand loyalty and frequency and the outside-option's cost. By comparing (13) with (30) it is easy to check that $a_{FC} > a_{FC}^{SO}$ since $P^{**} > \tau$ (as it can be seen by inspection). Therefore, too many passengers make use of the outside option to travel in equilibrium, as can be observed in Figure 11.

From (30) and the first-order condition for f and F, we obtain

$$f^{SO} = \frac{1}{4\theta},\ F^{SO} = \frac{\alpha/2 - \tau + g}{2\alpha\theta - 1}. \quad (31)$$

Using these results, the social optimum and equilibrium are easily compared from expressions (10), (12) and (31).

Proposition 3 *With market AB being partially served by airlines under (FC,FC), the equilibrium has efficient flight frequency in markets AH and BH (i.e.,* $f_{FC}^{**} = f_{FC}^{SO}$*), sub-optimal flight frequency in market AB (i.e.,* $F_{FC}^{**} < F_{FC}^{SO}$*) and too few air travelers (i.e.,* $a_{FC} > a_{FC}^{SO}$*).*

The proposition shows that the equilibrium is inefficient only in city-pair market AB where markets are partially served and airlines compete against the outside option. In this case, each airline has effective monopoly power over its passengers, whose next best choice is the outside option. In a familiar fashion, this monopoly power leads to a suboptimal level of traffic with underprovision of frequency. Consistently, there are too few air travelers in equilibrium, as in BF and Brueckner (2004).

By contrast, in markets AH and BH, the entire population undertakes air travel in equilibrium, so that there is no inefficient allocation of passengers. Equilibrium frequencies are efficient and airlines exert no monopoly power over any passenger. Nevertheless, the exercise of market power can still affect the distribution of a fixed amount of traffic between carriers through their relative choices of fares and frequencies. This result is also in line with the one in BF that considers only one FC market.[19]

Note that, *when market AB is fully served by airlines,* then $a^* = 0$ (or equivalently $U_1^{AH}(FC) = U_1^{AB}(FC)$ and $F = f$) and the welfare function becomes $W(FC, FC) = 3(y + \frac{\alpha}{4} + f - \tau - 2\theta f^2)$. Then it is easy to check that $f^{SO} = F^{SO} = \frac{1}{4\theta}$,[20] as portrayed in the following corollary.

Corollary 2 *With fully-served markets under (FC,FC), the equilibrium flight frequency is efficient (i.e.,* $f_{FC}^* = f_{FC}^{SO} = \frac{1}{4\theta}$*).*

Again, as in BF, frequency is socially optimal when the relevant margin of choice is airline 1/airline 2.

4.2 The (HS,HS) Scenario

Total utility for carrier 1 in market AH (or BH) is the same as $U_1^{AH}(FC)$ since this market is always served nonstop and there is no outside option. *Assuming that market AB is partially served by airlines,* airline 1's utility in this market is logically given by $U_1^{AB}(FC)$ but replacing F_1 by f_1 since now there is no direct connection between cities A and B. On the profits side,

$$\pi_1(HS) = 2(p_1 - \tau)\int_0^{\alpha/2} \frac{1}{\alpha} da + (P_1 - 2\tau)\int_{a^*}^{\alpha/2} \frac{1}{\alpha} da - 2\theta(f_1)^2, \qquad (32)$$

where the main differences with respect to $\pi_1(FC)$ are the element $(P_1 - 2\tau)$ that captures that the AB market needs to use two routes to be served $(AH + BH)$; and the absence of aircraft-operation cost in route AB since this route is not operated under a HS configuration. Welfare is therefore given by

$$W(HS,HS) = 4U_1^{AH}(HS) + 2U_1^{AB}(HS) + 2\pi_1(HS) =$$

$$= 2\left\{ \underbrace{3y\int_0^{\alpha/2}\frac{1}{\alpha}da}_{Income} - \underbrace{g\int_0^{a^*}\frac{1}{\alpha}da}_{Outside\ option's\ cost} + \underbrace{2\int_0^{\alpha/2}\frac{a}{\alpha}da + \int_{a^*}^{\alpha/2}\frac{a}{\alpha}da}_{Average\ brand\text{-}loyalty\ benefits} + \right.$$

$$\left. \underbrace{+2f\int_0^{\alpha/2}\frac{1}{\alpha}da + f\int_{a^*}^{\alpha/2}\frac{1}{\alpha}da}_{Frequency\ benefits} - \underbrace{[\tau(2\int_0^{\alpha/2}\frac{1}{\alpha} + 2\int_{a^*}^{\alpha/2}\frac{1}{\alpha}da) + 2\theta f^2]}_{Costs} \right\} =$$

$$= 3y - 2g\tfrac{a^*}{\alpha} + \tfrac{3\alpha}{4} - \tfrac{(a^*)^2}{\alpha} + 2f + 2f(\tfrac{1}{2} - \tfrac{a^*}{\alpha}) - 2\tau - 4\tau(\tfrac{1}{2} - \tfrac{a^*}{\alpha}) - 4\theta f^2. \qquad (33)$$

The interpretation of this expression is similar to the one for $W(FC,FC)$, except for the differences in U_1^{AB} and π_1 previously commented.

The first-order condition for choice of a^* yields now

$$a^* = 2\tau - f^{SO} - g \equiv a_{HS}^{SO}, \qquad (34)$$

where the sole difference with respect to the FC case is the 2 factor revealing the use of two routes to serve market AB. As in the FC scenario, $a_{HS} > a_{HS}^{SO}$ since $P^{**} > 2\tau$ and hence there are too few air travelers (same situation as the one depicted in Figure 11). From (34) and the first-order condition for f, we obtain

$$f^{SO} = \frac{3\alpha/2 - 2\tau + g}{4\alpha\theta - 1}. \qquad (35)$$

The following proposition compares the equilibrium frequency with the social optimum under the HS scenario (i.e., expressions (35) and (17)).

Proposition 4 *With market AB being partially served by airlines under (HS,HS), there is an underprovision of flight frequency when g is high; but both overprovision and underprovision of frequency can be observed when g is low.*

Before the deregulation, airlines faced constraints in fares and route structures and competition was concentrated in service quality (flight frequency). This competition focused on flight frequency was thought to generate excessive frequencies. After the deregulation, with airlines free to compete on airfares, softer competition on frequency was expected but exactly the opposite occurred in many routes. The explanation comes from considering another important consequence of the deregulation: the adoption of HS networks. In fact, HS networks and the concentration of traffic on the spoke routes generated an increase in flight frequency. The result in Proposition 4, compared to the one in Proposition 3, reflects the transition from FC to HS configurations occurred after the deregulation, since frequencies can become excessive in presence of partially-served markets (i.e., we need g is sufficiently low such that the outside option is effectively taken into account by passengers). Thus, uncovered markets seem to constitute an important element explaining the apparent overprovision of frequencies in the current airline unregulated environment.

Brueckner (2004) considers network choice in a monopoly setting; and BF brings up a competitive setting with partially-served markets in a single FC market. These two papers detect an underprovision of frequencies, a finding that seems to be precise in a single-market setting but not in the current unregulated environment where most carriers organize their networks in a HS manner. In a competitive context where carriers are free to make strategic network choices and city-pair markets may be uncovered by airlines, we can explain the extensive network reorganization observed after the deregulation with the adoption of hubbing strategies and the noticeable excess of flight frequency in many HS routes.

Note that, *when market AB is fully served by airlines,* then $a^* = 0$ and $W(HS, HS) = 3y + \frac{3\alpha}{4} + 3f - 4\tau - 4\theta f^2$. In this case, the maximization yields $f^{SO} = \frac{3}{8\theta}$ as shown in the following corollary.[21]

Corollary 3 *With fully-served markets under (HS,HS), the equilibrium flight frequency is efficient (i.e., $f^*_{HS} = f^{SO}_{HS} = \frac{3}{8\theta}$).*

Corollaries $2-3$ highlight the idea that uncovered markets are an important element to take into account when trying to analyze frequency efficiency in the airline industry. Without this element, frequencies are always optimal independently of airlines' network structure.[22]

The optimal values of flight frequency and traffic for the asymmetric scenario (FC,HS) are simply the ones obtained under (FC,FC) for the carrier operating FC and the ones obtained under (HS,HS) for the HS airline.[23]

5 Concluding Remarks

This paper has presented airline network choice in a competitive unregulated environment. Allowing airlines to make strategic network choices, along with the possibility of having uncovered markets such that low brand-loyalty passengers make use of an outside option, yields interesting results. HS structures prevail in presence of low air-transport costs and asymmetric equilibria arise without introducing any explicit asymmetry in the model. In addition, with HS configurations air travelers are too few and flight frequency can become excessive.

As it has been shown, the deregulation of the sector had important network implications for airlines. Following our results, hubbing strategies should prevail in the current competitive context characterized by low costs. Although the remarkable dynamism of the airline industry could result in a different context in the next future, the considered setup is sufficiently broad to incorporate new features into the analysis. For instance, if the integration trend observed in the last years keeps stepping forward, a new environment with few major players (around *SkyTeam, Star Alliance* and *oneworld*) seems to be possible. This would lead to a further rationalization of routes and the consequent cost saving that would exaggerate the phenomenon of hubbing, as predicted by our model.

A possible extension would be to introduce constraints to airport growth stemming from land availability or noise regulation in airports' surroundings.[24] This restrictions are already present and could become a major factor affecting airlines' network strategy and slowing down hubbing processes.

Another natural extension of the considered analysis involves introducing explicit asymmetries (in cost parameters) to differentiate among legacy carriers, regional operators (that offer higher service quality) and low-cost carriers. This analysis could probably provide some insights about the distinctive network choices characterizing each carrier type. Moreover, the presence of low-cost carriers could mitigate the excessive exclusion arising in the presented model.

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Notes

[1]Morrison and Whinston showed that a route-weighted measure of flight frequency rose by 9.2 percent between 1977 and 1983, generating passenger benefits in excess of \$10 billion per year.

[2]FC network structures provide direct connections in every city-pair market, whereas HS refer to networks organized around a main airport (hub).

[3]In BF there is an exogenous proportion of high and low-type passengers depending on their valuation of travel. High types choose between the two carriers and low types choose between their preferred airline and an outside option.

[4]The same network is considered in Brueckner (2004) and Oum *et al.* (1995) since it is the simplest possible structure allowing for comparisons between HS and FC configurations.

[5]Allowing for the use of an outside option in city-pair markets AH and BH, would imply that all the markets may be partially served. Although the framework would be more general, we restrict this possibility to market AB for simplicity since the results of the analysis do not change qualitatively.

[6]Introducing frequencies additively in the utility function simplifies the analysis with respect to the approach in BF, where higher frequencies reduce the cost of schedule delay. A similar formulation to ours is suggested in Heimer and Shy (2006).

[7]As suggested in Brueckner and Spiller (1991) under a very different specification.

[8]The trend consisting in restructuring airline networks around hubs that occured after the deregulation (hubbing), consisted precisely in trying to attain cost savings from eliminating secondary routes by pooling passengers from several markets into the same plane.

[9]Multi-segment trips can be ignored in this scenario since cheaper direct connections are available in all the markets.

[10]In BF, the simultaneous and the sequential choice of f and p are compared. In the sequential case, frequency is smaller than in the simultaneous-choice case; and fares involve a smaller markup over marginal seat cost.

[11]When $a_{FC} = 0$ then $g = P_1^{**} - F_1^{**}$. Substituting in P_1^{**} and F_1^{**}, we get $P_1^{**} = \frac{\alpha^2\theta + 2\alpha\theta F_1^{**} + \tau(2\alpha\theta - 1)}{2\alpha\theta - 1}$ and $F_1^{**} = \frac{\alpha/2 + P_1^{**} - \tau}{4\alpha\theta}$. From these expressions, we obtain directly the results with fully-served markets in (10).

[12]It could be argued that multi-segment trips are more costly for passengers that need to make stops and wait for connecting planes. Adding a disutility parameter into the travelers' utility when trips are HS is easy to implement (and could be seen as more realistic). However, it does not provide any additional insight (and introduces a new parameter) since the results are qualitatively equivalent with the exception that the equilibrium areas where carriers operate HS are smaller.

[13]Under (HS,HS), there is no F_1 since the route AB is ruled out.

[14]When $a_{HS} = 0$, then $g = P_1^{**} - f_1^{**}$ and, substituting in P_1^{**} and f_1^{**} we get $P_1^{**} = \frac{2\alpha^2\theta - 4\alpha\theta f_1^{**} + \alpha + 2\tau(4\alpha\theta - 1)}{4\alpha\theta - 1}$ and $f_1^{**} = \frac{5\alpha/2 + P_1^{**} - 2\tau}{8\alpha\theta}$. From these expressions, we obtain directly the results with fully-served markets in (15).

[15]This differs from BF, where there is an exogenous proportion of high and low-type passengers depending on their valuation of travel. In BF, high types choose between the two carriers whereas low types choose

between their preferred airline and an outside option.

[16]In Regions C and C', each particular value of g has a utility associated to it, i.e., $y + F_1^{**} - P_1^{**} + a$ for carrier 1; and $y + F_2^{**} - P_2^{**} - a$ for carrier 2.

In these regions, airlines face competition *both* from the rival airline *and* from the outside option. More precisely, when airlines compete more aggressively (i.e., carriers increase $F^{**} - P^{**}$), the relevant margin of choice is airline 1/airline 2. On the other hand, when airlines reduce $F^{**} - P^{**}$, they compete against the outside option and the relevant margin becomes preferred carrier/outside option.

[17]It is important to note that, while p_i and P_i are set independently, fares must satisfy a non-arbitrage condition. This condition says that an AB passenger must not be able to travel cheaper by purchasing two separate tickets (in routes AH and BH), and it is written $P_i < 2p_i$ for $i = 1, 2$.

[18]Therefore, the constraint $\tau > \frac{1}{8\theta}$ has to be taken into account for a double reason: firstly because it is fundamental in determining the existence of partially-served markets in each region (it differentiates between the cases in Figures 6 and 7); and secondly because it rules out some possible equilibrium regions arising from profit comparisons (as shown in Figures $8 - 9$).

[19]In BF the inefficiency only affects the low-type passengers (that choose between their preferred airline and the outside option).

[20]Applying $a^* = 0$ and using (30) and (31), we recover the social-optimum value with fully-served markets.

[21]Applying $a^* = 0$ and using (34) and (35), we recover the social-optimum value with fully-served markets.

[22]A different question would be to study network efficiency. Typically, when the outside option is sufficiently attractive, HS structures are more efficient that FC structures because they endow with higher frequency and more passengers undertake air travel.

[23]Assuming without loss of generality that carrier 1 adopts a FC network and that carrier 2 operates HS, welfare is given by $W(FC, HS) = 2U_1^{AH}(FC) + U_1^{AB}(FC) + \pi_1(FC) + 2U_2^{AH}(HS) + U_2^{AB}(HS) + \pi_2(HS) = \frac{1}{2}W(FC, FC) + \frac{1}{2}W(FC, FC) = 3y - g\frac{a_1^*}{\alpha} - g\frac{a_2^*}{\alpha} + \frac{3\alpha}{4} - \frac{(a_1^*)^2}{2\alpha} - \frac{(a_2^*)^2}{2\alpha} + f_1 + f_2 + F_1(\frac{1}{2} - \frac{a_1^*}{\alpha}) + f_2(\frac{1}{2} - \frac{a_2^*}{\alpha}) - 2\tau - \tau(\frac{1}{2} - \frac{a_1^*}{\alpha}) - 2\tau(\frac{1}{2} - \frac{a_2^*}{\alpha}) - 2\theta f_1^2 - \theta F_1^2 - 2\theta f_2^2$, and the social optimum for carrier 1 is the one given in the (FC,FC) scenario, whereas the one for airline 2 is the one provided in the (HS,HS) case.

[24]Brueckner and Girvin (2006) study the effect of airport noise regulation, focusing on flight frequency and aircraft "quietness". However, the implications of such regulation on airline network structure, remain to be studied.

Figures

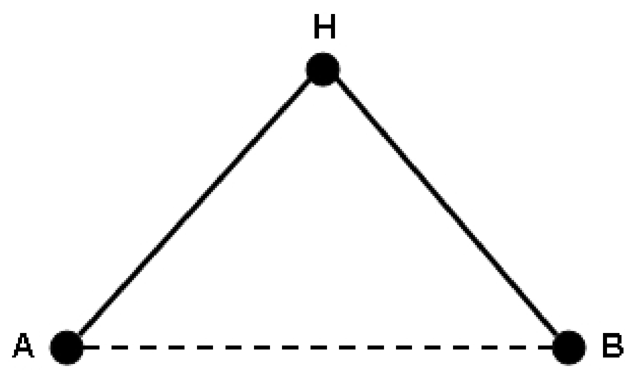

Figure 1: Network structure

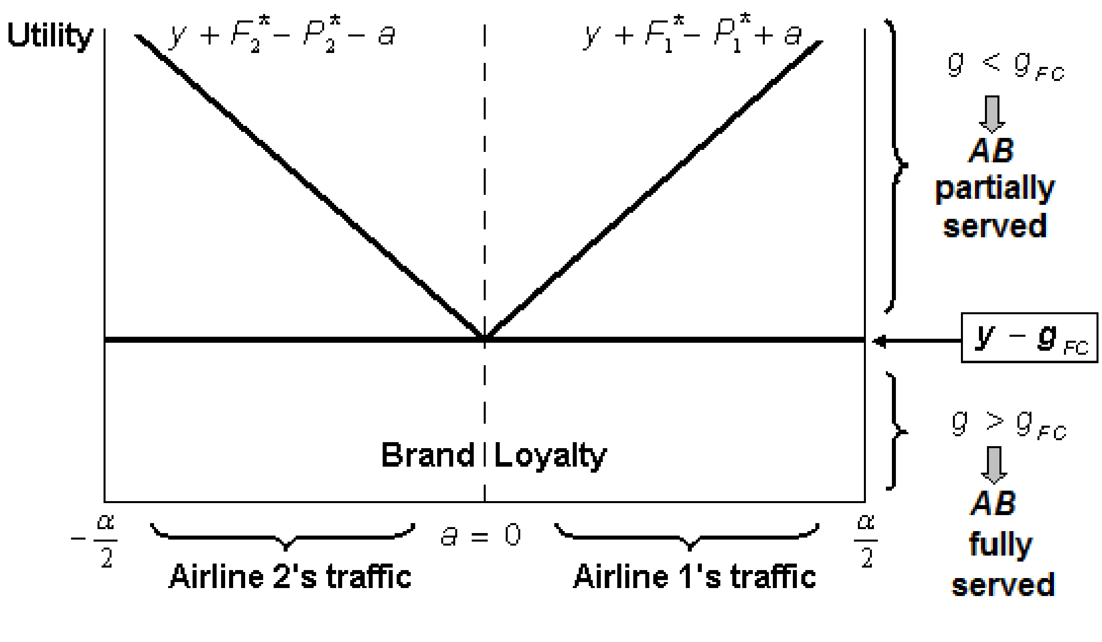

Figure 2: Utilities in scenario (FC,FC)

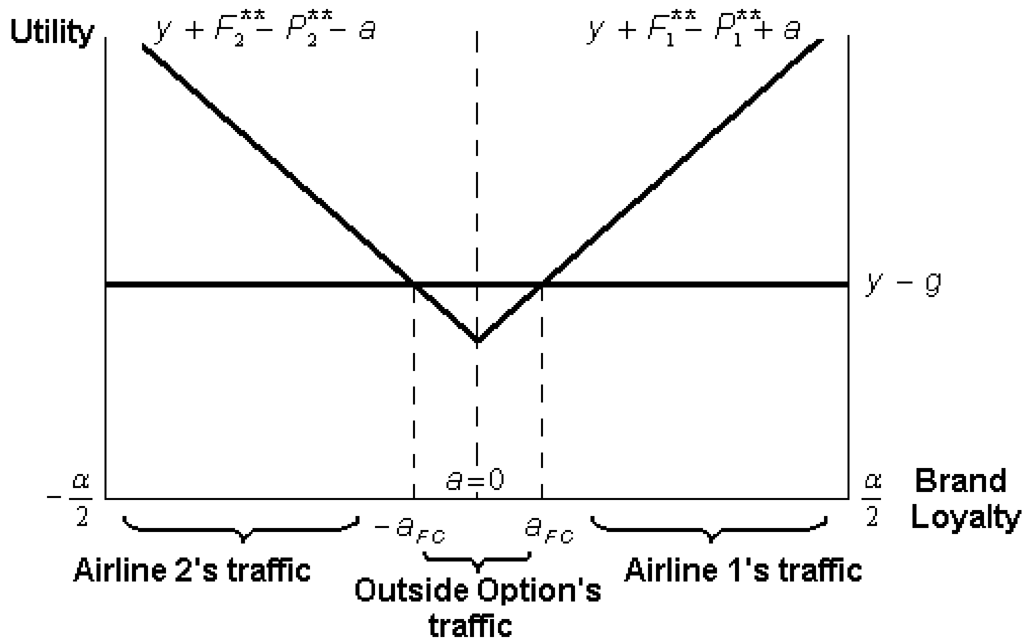

Figure 3: A relevant outside option under (FC,FC)

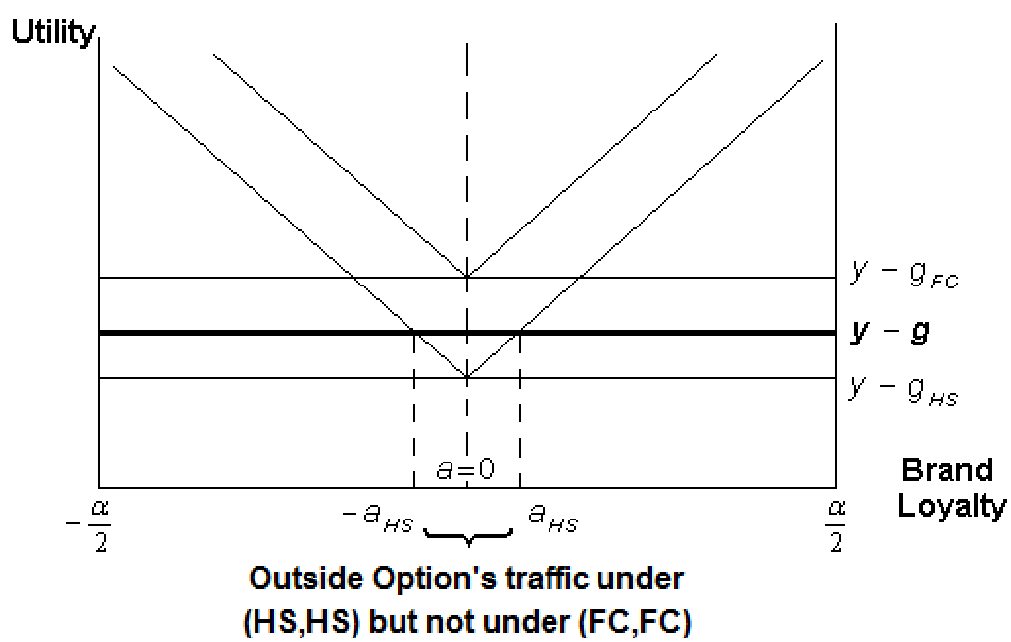

Figure 4: Case $g \in (g_{FC}, g_{HS})$ when $\theta\tau > \frac{1}{8}$

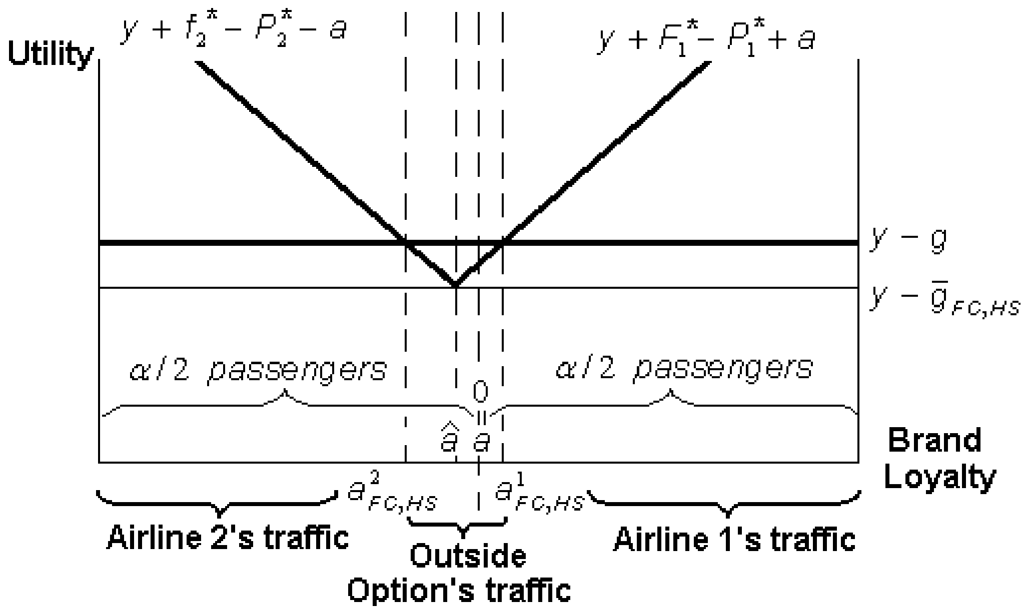

Figure 5: Scenario (FC,HS) with fully-served markets

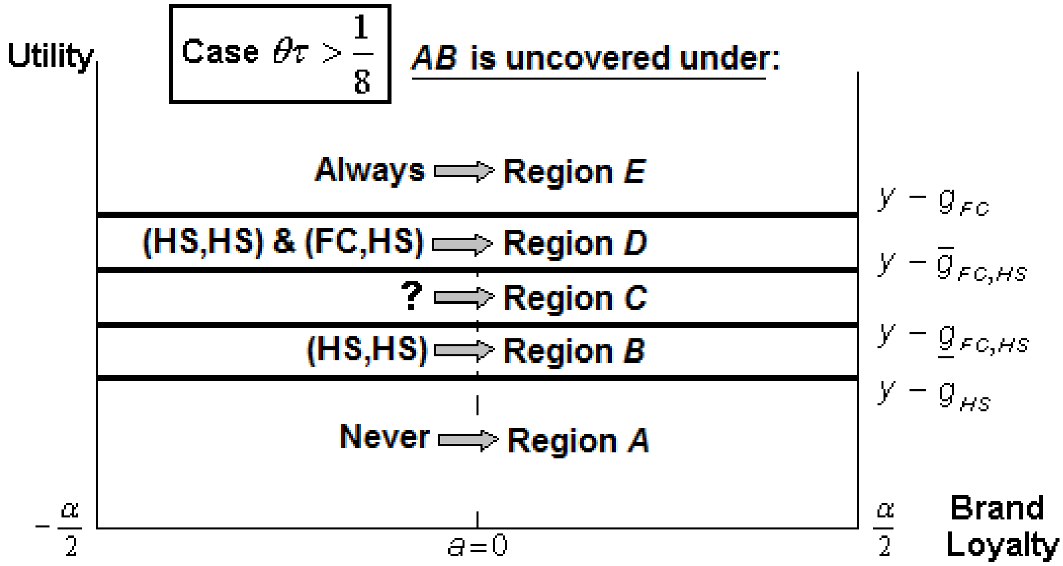

Figure 6: Regions with high air-transport costs

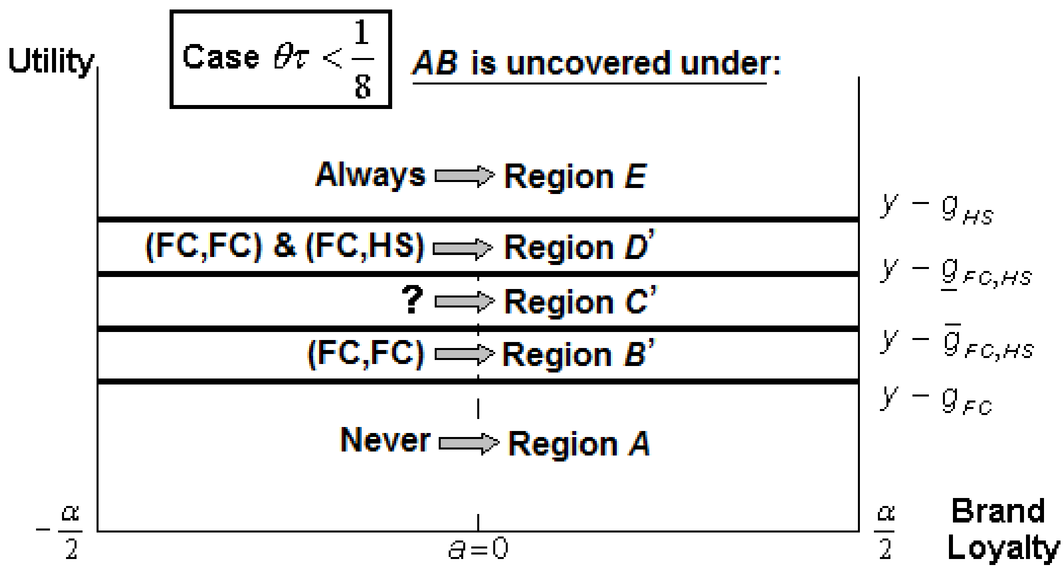

Figure 7: Regions with low air-trasport costs

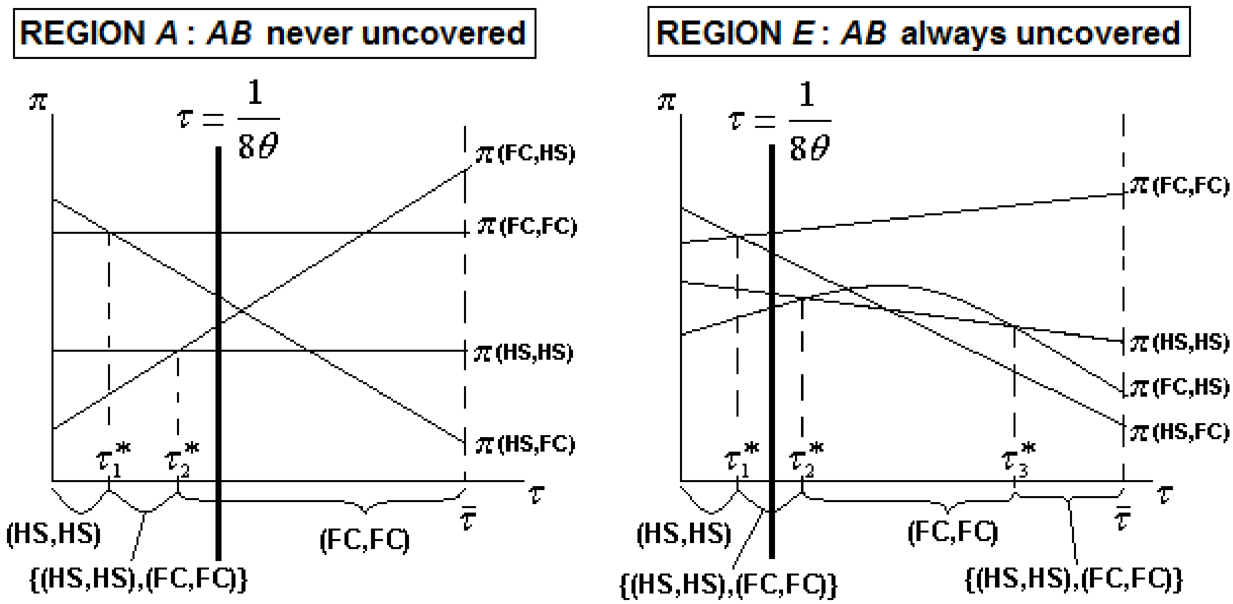

Figure 8: Equilibrium in Regions A and E (both for $\theta\tau > \frac{1}{8}$ and $\theta\tau < \frac{1}{8}$)

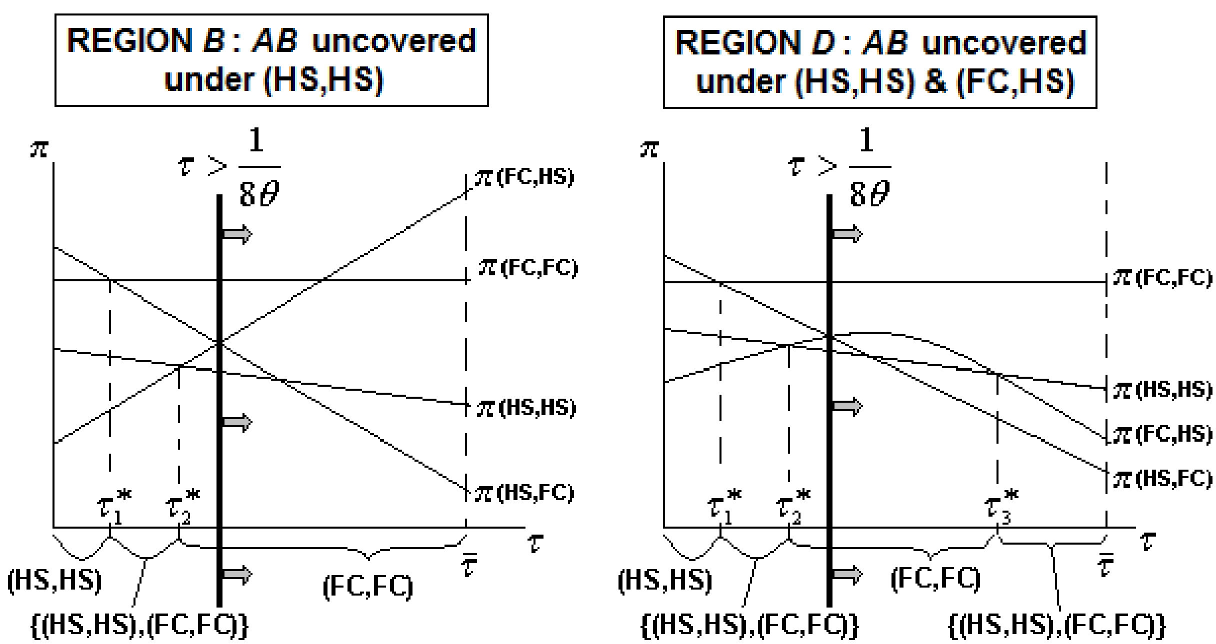

Figure 9: Equilibrium in Regions B and D (for $\theta\tau > \frac{1}{8}$)

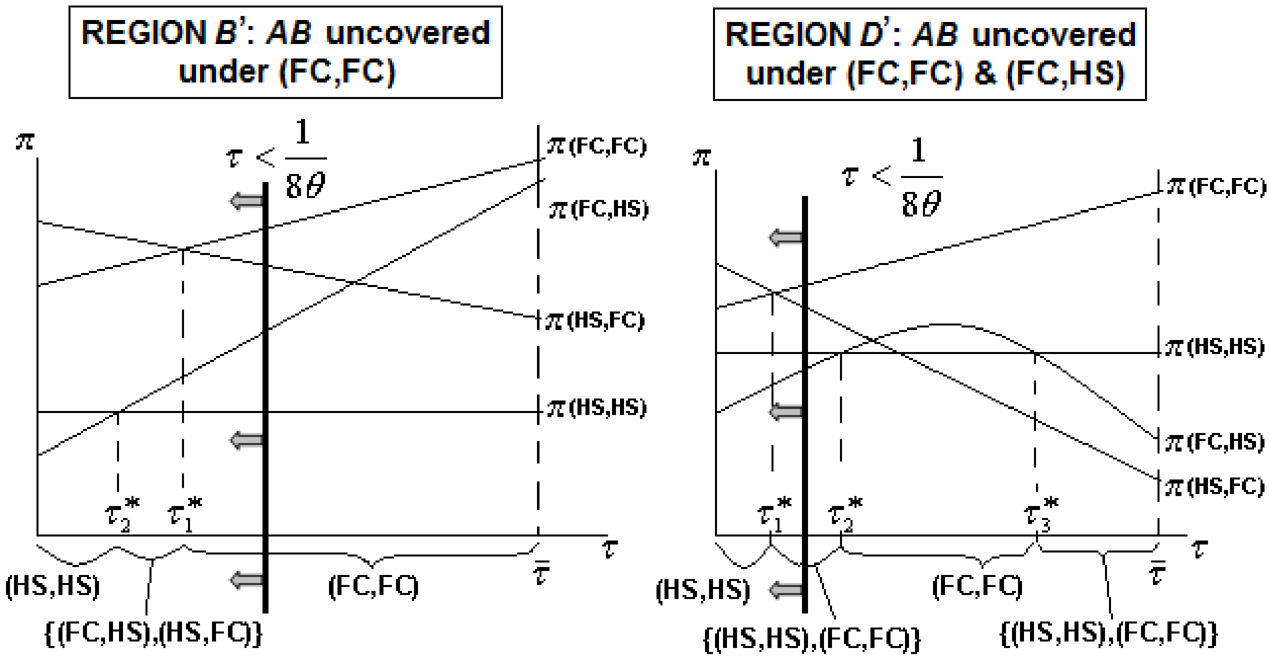

Figure 10: Equilibrium in Regions B' and D' (for $\theta\tau < \frac{1}{8}$)

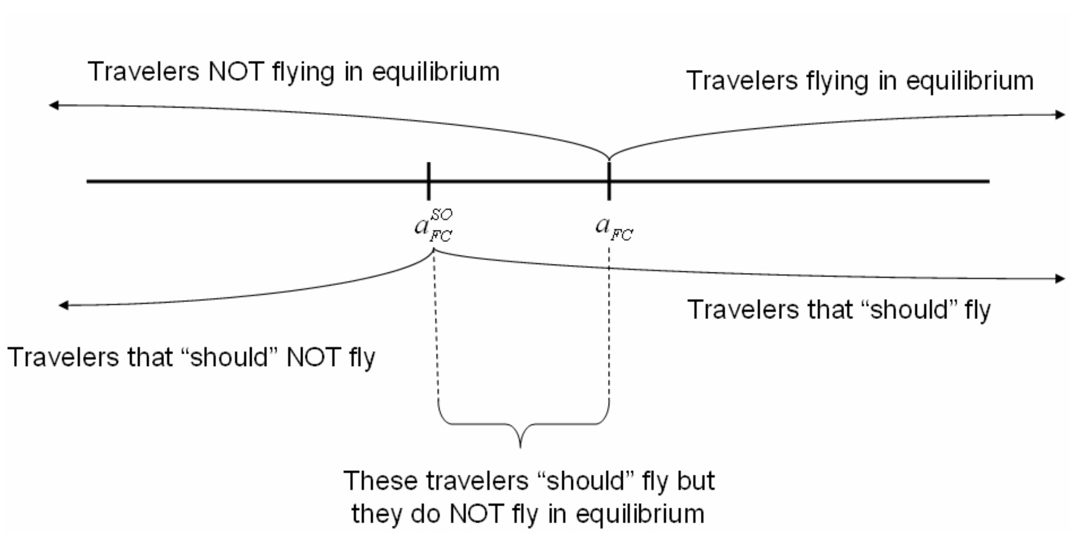

Figure 11: Too few air travelers in equilibrium

A Appendix: The Relevant Region (R)

Some restrictions on the parameters of the model need to be observed to ensure positive airfares, travel volumes and compliance with second-order and the non-arbitrage conditions in all the scenarios. The precise details of the computations are available from the author upon request.

(a) Second-order conditions (both for partially and fully served AB market). The Hessian matrix is definite negative for $\alpha\theta > 1/4$ in the (FC,FC) case; for $\alpha\theta > 3/8$ in the (HS,HS) scenario. Naturally, in the (FC,HS) case, the restrictions are $\alpha\theta > 1/4$ for carrier 1 and $\alpha\theta > 3/8$ for carrier 2.

(b) Positivity conditions (i.e., $p_i, P_i, q_i, Q_i, f_i, F_i > 0$ for $i = 1, 2$).
(i) *Market AB fully served by airlines (high g).* The (FC,FC) and the (HS,HS) scenarios do not impose any restriction. The asymmetric setting (FC,HS) requires $\alpha\theta > 1/2$ and $\tau < \tau_1 \equiv (6\alpha\theta - 1)/4\theta$ (from $Q_2 > 0$).
(ii) *Market AB partially served by airlines (low g).* In this case and given that there are four parameters, we provide sufficient conditions for $\alpha\theta$ (i.e., lower bounds) and τ (i.e., upper bounds) that hold for any value of g. The (FC,FC) scenario requires $\tau < \tau_2 \equiv \alpha/2$ (from F_1^{**} and $Q_1^{**} > 0$); the case (HS,HS) imposes $\tau < \tau_3 \equiv (1 + 2\alpha\theta)/8\theta$ (from $Q_1^{**} > 0$); and finally from the setting (FC,HS) we get $\tau < \tau_3$ as under (HS,HS) (now from $Q_2^{**} > 0$) and $\alpha\theta > 7/12$ (from p_1^{**} and $q_1^{**} > 0$) with $\alpha > g$ that seems reasonable since α needs to be sufficiently high.

(c) Non-arbitrage conditions (i.e., $P_i \leq 2p_i$ for $i = 1, 2$ for airlines operating HS networks, to ensure that no AB passenger is able to break down the trip into two parts).
(i) *Market AB fully served by airlines (high g).* There are no further restrictions in any scenario when $\alpha\theta > 1/2$.
(ii) *Market AB partially served by airlines (low g).* The asymmetric setting (FC,HS) does not impose any restriction; and under (HS,HS) networks, the non-arbitrage condition imposes $g < (3\alpha\theta + 4\theta\tau - 1)/2\theta$ which is always satisfied when $g < g_{HS}$.

The intersection of all the aforementioned constraints leaves us with $\alpha\theta > 7/12$ and $\tau < \tau_3 \equiv (1 + 2\alpha\theta)/8\theta$ since $\tau_3 < \tau_1, \tau_2$, and we redefine $\tau_3 \equiv \overline{\tau}$. ■

B Appendix: Proofs

Proof of Lemma 1.

Prior to any proof, it is necessary to specify that we restrict attention to the parameter region where the variables are positive and this requires $\alpha\theta > 7/12$ and $\tau < \overline{\tau} \equiv (1+2\alpha\theta)/8\theta$ (this is thoroughly explained in Appendix A).

$i)$ *Frequency effect.*

▶ $f^*_{HS} > f^*_{FC} = F^*_{FC}$ is straightforward (fully-served markets);

▶ $f^{**}_{HS} > f^{**}_{FC}$ holds for $g > 2\tau - \frac{\alpha}{2} - \frac{1}{4\theta}$ and this is always true because $g > 0$ and $2\tau - \frac{\alpha}{2} - \frac{1}{4\theta} < 0$ for $\tau < \overline{\tau}$.

▶ $f^{**}_{HS} > F^{**}_{FC}$ holds for $g < \widehat{g} = \frac{\tau}{4\alpha\theta} + \frac{3\alpha}{2} - \frac{1}{2\theta}$. When g is low, we know that $g < g_{FC} = \tau + \frac{\alpha}{2} - \frac{1}{4\theta}$. Therefore, it is sufficient to show $g_{FC} < \widehat{g}$. The latter inequality requires $\tau < \alpha$ which holds because $\tau < \overline{\tau} < \alpha$ for $\alpha\theta > 1/6$.

$ii)$ *Airfare effect.*

▶ $P^*_{HS} > P^*_{FC}$ is straightforward (fully-served markets);

▶ $P^{**}_{HS} > P^{**}_{FC}$ holds for $\tau > \widetilde{\tau} = \frac{\alpha+2\alpha\theta g-3\alpha^2\theta}{16\alpha^2\theta^2-6\alpha\theta+1}$. Therefore, it is sufficient to show that $\widetilde{\tau} < 0$ because $\tau > 0$. The denominator of $\widetilde{\tau}$ is obviously positive; and the numerator is negative for $g < \widetilde{g} = \frac{3\alpha}{2} - \frac{1}{2\theta}$. It is easy to check that $g_{FC} < \widetilde{g}$ requires $\tau < \frac{4\alpha\theta-1}{4\theta}$ and $\overline{\tau} < \frac{4\alpha\theta-1}{4\theta}$ for $\alpha\theta > 1/2$. Therefore, the numerator of $\widetilde{\tau}$ is negative and the proof is completed. ■

Proof of Lemma 2.

Straightforward. ■

Proof of Lemma 3.

$i)$ Case $\theta\tau > \frac{1}{8}$.

▶ $g_{FC} < \overline{g}_{FC,HS}$ holds for $\theta\tau > \frac{6\alpha\theta-1}{16(3\alpha\theta-1)}$ or equivalently for $6\alpha\theta(8\theta\tau-1) > 16\theta\tau - 1$ and this implies $\alpha\theta > \frac{16\theta\tau-1}{6(8\theta\tau-1)}$ since $\theta\tau > \frac{1}{8}$. It can be checked that, $\alpha\theta > \frac{16\theta\tau-1}{6(8\theta\tau-1)}$ is always observed in R.

▶ $\overline{g}_{FC,HS} < \underline{g}_{FC,HS}$ is true for $\theta\tau > \frac{6\alpha\theta-1}{16(3\alpha\theta-1)}$ and therefore it always hold (because $\alpha\theta > \frac{16\theta\tau-1}{6(8\theta\tau-1)}$ is always observed in R).

▶ $\underline{g}_{FC,HS} < g_{HS}$ requires $\theta\tau > \frac{3(1+16\alpha^2\theta^2-10\alpha\theta)}{8(1+48\alpha^2\theta^2-22\alpha\theta)}$. Since $\frac{1}{8} > \frac{3(1+16\alpha^2\theta^2-10\alpha\theta)}{8(1+48\alpha^2\theta^2-22\alpha\theta)}$ in R, the inequality is always respected.

$ii)$ Case $\theta\tau < \frac{1}{8}$.

▶ $g_{FC} > \overline{g}_{FC,HS}$ holds for $\theta\tau < \frac{6\alpha\theta-1}{16(3\alpha\theta-1)}$ or equivalently for $6\alpha\theta(8\theta\tau-1) < 16\theta\tau - 1$ and this implies $\alpha\theta > \frac{16\theta\tau-1}{6(8\theta\tau-1)}$ since $\theta\tau < \frac{1}{8}$. Thus, the initial inequality always holds (because $\alpha\theta > \frac{16\theta\tau-1}{6(8\theta\tau-1)}$ is always observed in R).

► $\overline{g}_{FC,HS} > \underline{g}_{FC,HS}$ is true for $\theta\tau < \frac{6\alpha\theta-1}{16(3\alpha\theta-1)}$ and therefore it always hold (just proved above).

► $\underline{g}_{FC,HS} > g_{HS}$ requires $\theta\tau < \frac{3(1+16\alpha^2\theta^2-10\alpha\theta)}{8(1+48\alpha^2\theta^2-22\alpha\theta)}$ or equivalently for $\alpha\theta(192\alpha\theta^2\tau - 88\theta\tau - 24\alpha\theta + 15) < 3 - 8$ The element between parentheses is negative for $\theta\tau < \frac{1}{8}$ in the relevant region R, so that the inequality is always respected. ■

Proof of Propositions 1 – 2 and Corollary 1.

Let us rename $\pi(FC, FC) \equiv \pi_A$, $\pi(HS, HS) \equiv \pi_B$, $\pi(FC, HS) \equiv \pi_C$ and $\pi(HS, FC) \equiv \pi_D$. To derive the equilibria in networks, we simply need to compute $\pi_B - \pi_C$ and $\pi_A - \pi_D$ in the relevant region R. Remember that τ_1^* is obtained from $\pi_A - \pi_D$; and that τ_2^* and τ_3^* are obtained from $\pi_B - \pi_C$. The precise details of the computations are available from the author upon request.

We need to compute the profits both for the cases of fully and partially-served markets.

(i) Market AB fully served by airlines (high g). With fully-served markets, profits are given by $\pi_A^F = \frac{3(4\alpha\theta-1)}{16\theta}$, $\pi_B^F = \frac{3(8\alpha\theta-3)}{32\theta}$, $\pi_C^F = \frac{3(4\alpha\theta-1)[9(2\alpha\theta-1)F+8F\tau+G\tau^2]}{4\theta(12\alpha\theta-5)^2(6\alpha\theta-1)^2}$, $\pi_D^F = \frac{3(3\alpha\theta-1)H-4\theta H\tau+I\tau^2}{4\theta(12\alpha\theta-5)^2(6\alpha\theta-1)^2}$ where F, G, H, $I > 0$ and superscript F stands for "fully-served markets".

$F = 3(6\alpha\theta-1)^2(2\alpha\theta-1)(4\alpha\theta-1)$, $G = 24\theta^2(4\alpha\theta-1)[8\alpha\theta(3\alpha\theta-2)+3]$,

$H = (6\alpha\theta-1)^2(3\alpha\theta-1)(8\alpha\theta-3)$, $I = 4\theta^2\{12\alpha\theta[\alpha\theta(24\alpha\theta-19)+4]-1\}$.

(ii) Market AB partially served by airlines (low g). With uncovered markets, profits depend on g and are given by $\pi_A^P = \frac{1+2\theta^2\left[9\alpha^2+4(g-\tau)(\alpha+g-\tau)-4\alpha/\theta\right]}{8\theta(4\alpha\theta-1)}$, $\pi_B^P = \theta\frac{9\alpha^2+4(g-2\tau)(\alpha+g-2\tau)-3\alpha/\theta}{2(8\alpha\theta-1)}$, $\pi_C^P = \frac{J+Kg+Lg^2-M\tau-N\tau^2-Rg\tau}{4\alpha(4\alpha\theta-1)[1+2\alpha\theta(24\alpha\theta-11)]^2}$, $\pi_D^P = \frac{S+Tg+Ug^2-V\tau+W\tau^2-Xg\tau}{2[1+2\alpha\theta(24\alpha\theta-11)]^2}$ where J, K, L, M, N, R, S, T, U, V, W, $X > 0$ and superscript P stands for "partially-served markets".

$J = \alpha^3\theta\{4\alpha\theta[\alpha\theta(48\alpha\theta(108\alpha\theta-155)+3889)-875]+289\}$,

$K = 4\alpha^2\theta\{4\alpha\theta[\alpha\theta(144\alpha\theta(4\alpha\theta-5)+337)-71]+25\}$,

$L = 12\alpha\theta\{4\alpha\theta[\alpha\theta(16\alpha\theta(12\alpha\theta-11)+51)-5]+1\}$,

$M = 4\alpha^2\theta\{4\alpha\theta[\alpha\theta(48\alpha\theta(12\alpha\theta-19)+529)-131]+49\}$,

$N = 4\alpha\theta\{4\alpha\theta[\alpha\theta(48\alpha\theta(84\alpha\theta-101)+2039)-351]+87\}-8$,

$R = 8\alpha\theta\{4\alpha\theta[\alpha\theta(48\alpha\theta(12\alpha\theta-11)+161)-19]+5\}$,

$S = 3\alpha^2\theta\{4\alpha\theta[\alpha\theta(216\alpha\theta-227)+87]-59\}+12\alpha$, $T = 4\alpha\theta\{4\alpha\theta[3\alpha\theta(24\alpha\theta-11)-5]+11\}$,

$U = 4\theta\{12\alpha\theta[\alpha\theta(24\alpha\theta-19)+4]-1\}$, $V = 8\alpha\theta\{6\alpha\theta[2\alpha\theta(72\alpha\theta-53)+21]-5\}+4$,

$W = 32\alpha\theta^2[18\alpha\theta(8\alpha\theta - 5) + 13]$, $X = 8\theta\{2\alpha\theta[12\alpha\theta(24\alpha\theta - 17) + 37] - 1\}$.

Now, we have all the needed information to compute the equilibrium in airline networks in each possible situation.

► $g > \max\{g_{FC}, g_{HS}\}$ (Region A: market AB never uncovered $\Rightarrow \pi_A^F, \pi_B^F, \pi_C^F, \pi_D^F$).
π_A^F and π_B^F are independent of τ whereas π_C is an increasing and convex function of τ and π_D is decreasing (for low values of τ) and concave function of τ. Hence π_A^F and π_D^F intersect for two values of τ, i.e., τ_1 and τ_2 with $\tau_1 < \tau_2$. It can be observed that $\tau_2 > \overline{\tau}$ and thus it is disregarded. On the other hand, π_A^F and π_D^F intersect for two values of τ, i.e., τ_3 and τ_4 with $\tau_3 < \tau_4$. Clearly $\tau_3 < 0$ and thus it is disregarded. Therefore, we are left with τ_1 and τ_4 and it can be checked that $0 < \tau_1 < \tau_4 < \overline{\tau}$. Let us redefine $\tau_1 \equiv \tau_1^*$ and $\tau_4 \equiv \tau_2^*$ to simplify notation. The precise expressions for τ_1^* and τ_2^* are available from the author upon request.
· When $\tau \in (0, \tau_1^*)$, $\pi_B^F > \pi_C^F$ and $\pi_A^F < \pi_D^F$ and thus the equilibrium is (HS,HS).
· When $\tau \in (\tau_1^*, \tau_2^*)$, $\pi_B^F > \pi_C^F$ and $\pi_A^F > \pi_D^F$ and thus the equilibrium is {(HS,HS),(FC,FC)}.
· When $\tau \in (\tau_2^*, \overline{\tau})$, $\pi_B^F < \pi_C^F$ and $\pi_A^F > \pi_D^F$ and thus the equilibrium is (FC,FC).
Finally, since typically $\tau_2^* < \frac{1}{8\theta} < \overline{\tau}$, there is a unique equilibrium (FC,FC) when $\theta\tau > \frac{1}{8}$, whereas the three aforementioned equilibria exist with $\theta\tau < \frac{1}{8}$ (as depicted in Figure 8).

► $g < \min\{g_{FC}, g_{HS}\}$ (Region E: market AB always uncovered $\Rightarrow \pi_A^P, \pi_B^P, \pi_C^P, \pi_D^P$).
Function π_C^P shifts downwards with respect to π_C^F (since $\partial\pi_C/\partial g > 0$) and becomes concave with respect to τ since $L < N$ and $g(\underset{+}{\tau})$, as shown in Figure 8, crossing π_B^P twice (i.e., τ_3 and τ_4 with $\tau_3 < \tau_4 < \overline{\tau}$). Function π_D^P shifts downwards (since $\partial\pi_D/\partial g > 0$) with respect to π_D^F and becomes convex since $U, W > 0$, crossing π_A^F twice (i.e., τ_1 and τ_2 with $\tau_1 < \tau_2$). As in Region A, $\tau_2 > \overline{\tau}$ and thus it is disregarded.
We are left with τ_1, τ_3 and τ_4 and it can be checked that $0 < \tau_1 < \tau_3 < \tau_4 < \overline{\tau}$. Again, we redefine $\tau_1 \equiv \tau_1^*$, $\tau_3 \equiv \tau_2^*$ and $\tau_4 \equiv \tau_3^*$ and the precise expressions for τ_1^*, τ_2^* and τ_3^* are available from the author upon request.
· When $\tau \in (0, \tau_1^*)$, $\pi_B^P > \pi_C^P$ and $\pi_A^P < \pi_D^P$ and thus the equilibrium is (HS,HS).
· When $\tau \in (\tau_1^*, \tau_2^*)$, $\pi_B^P > \pi_C^P$ and $\pi_A^P > \pi_D^P$ and thus the equilibrium is {(HS,HS),(FC,FC)}.
· When $\tau \in (\tau_2^*, \tau_3^*)$, $\pi_B^P < \pi_C^P$ and $\pi_A^P > \pi_D^P$ and thus the equilibrium is (FC,FC).
· When $\tau \in (\tau_3^*, \overline{\tau})$, $\pi_B^P > \pi_C^P$ and $\pi_A^P > \pi_D^P$ and thus the equilibrium is {(HS,HS),(FC,FC)}.
The precise location of $\tau = \frac{1}{8\theta}$ in Figure 8 depends on the value of the parameters and both $\frac{1}{8\theta} < \tau_2^*$ and $\frac{1}{8\theta} > \tau_2^*$ are possible but $\frac{1}{8\theta} > \tau_1^*$ is always observed. Figure 8 considers

the case $\tau_1^* < \frac{1}{8\theta} < \tau_2^*$.

► $g \in (\underline{g}_{FC,HS}, g_{HS})$ with $\theta\tau > \frac{1}{8}$ (Region B: market AB uncovered under (HS,HS) $\Rightarrow$ $\pi_A^F, \pi_B^P, \pi_C^F, \pi_D^F$).
τ_1^* is the same as in Region A. Function π_B^P, that is independent of τ for $g = g_{HS} = 2\tau + \frac{\alpha}{2} - \frac{3}{8\theta}$ (when $\pi_B^P = \pi_B^F$), shifts downwards (since $\partial\pi_B^P/\partial g > 0$) and typically becomes downward sloping for $g < g_{HS}$ and low τ, as shown in Figures 8−9. Then, τ_2^* is smaller than before (but always bigger than τ_1^*) and consequently the equilibrium {(HS,HS),(FC,FC)} becomes a bit smaller than in Region A.

► $g \in (\overline{g}_{FC,HS}, g_{FC})$ with $\theta\tau < \frac{1}{8}$ (Region B': market AB uncovered under (FC,FC) $\Rightarrow$ $\pi_A^P, \pi_B^F, \pi_C^F, \pi_D^F$).
τ_2^* is the same as in Region A. Function π_A^P, that is independent of τ for $g = g_{FC} = \tau + \frac{\alpha}{2} - \frac{1}{4\theta}$ (when $\pi_A^P = \pi_A^F$), shifts downwards (since $\partial\pi_A^P/\partial g > 0$) and typically becomes upward sloping for $g < g_{FC}$ and low τ, as shown in Figures 8 and 10. Since the intercept effect is stronger than the slope effect, τ_1^* is bigger than before and always surpasses τ_2^*, giving rise to the asymmetric equilibrium {(FC,HS),(HS,FC)}.

► $g \in (g_{FC}, \underline{g}_{FC,HS})$ with $\theta\tau > \frac{1}{8}$ (Region D: market AB uncovered under (HS,HS) and (FC,HS) $\Rightarrow \pi_A^F, \pi_B^P, \pi_C^P, \pi_D^P$).
π_C^P and π_D^P behave as commented in Region E. Although π_A^F is now different, it does not affect the equilibrium regions. Figure 9 considers the case $\tau_2^* < \frac{1}{8\theta} < \tau_3^*$ (but $\tau_1^* < \frac{1}{8\theta} < \tau_2^*$ is also possible).

► $g \in (g_{HS}, \overline{g}_{FC,HS})$ with $\theta\tau < \frac{1}{8}$ (Region D': market AB uncovered under (FC,FC) and (FC,HS) $\Rightarrow \pi_A^P, \pi_B^F, \pi_C^P, \pi_D^P$).
π_C^P and π_D^P behave as commented in Region E. Although π_B^F is now different, it does not affect the equilibrium regions, as shown in Figure 10. ■

Proof of Proposition 3.
From expressions (12) and (31), it can be checked that $f_{FC}^{**} = f_{FC}^{SO} = \frac{1}{4\theta}$ and that $F_{FC}^{**} = \frac{\alpha/2+g-\tau}{4\alpha\theta-1} < F_{FC}^{SO} = \frac{\alpha/2+g-\tau}{2\alpha\theta-1}$ is always true. ■

Proof of Corollary 2.
Straightforward. ■

Proof of Proposition 4.

When market AB is uncovered by airlines (i.e., $g < g_{HS} \equiv 2\tau + \frac{\alpha}{2} - \frac{3}{8\theta}$), it can be checked from expressions (17) and (35) that $f_{HS}^{**} = f_{HS}^{SO}$ occurs for $g = \frac{8\theta\tau - 1 - 2\alpha\theta}{4\theta}$ and $\frac{8\theta\tau - 1 - 2\alpha\theta}{4\theta} < g_{HS} \equiv 2\tau + \frac{\alpha}{2} - \frac{3}{8\theta}$. In addition, $\frac{8\theta\tau - 1 - 2\alpha\theta}{4\theta} > 0$ for $\tau > (2\alpha\theta - 1)/8\theta$ and this is possible because $(2\alpha\theta - 1)/8\theta < \overline{\tau}$ always holds. Therefore, when $g < \frac{8\theta\tau - 1 - 2\alpha\theta}{4\theta}$ then $f_{HS}^{**} > f_{HS}^{SO}$ (overprovision); and when $g \in \left(\frac{8\theta\tau - 1 - 2\alpha\theta}{4\theta}, g_{HS}\right)$ then $f_{HS}^{**} < f_{HS}^{SO}$ (underprovision). ■

Proof of Corollary 3.

Straightforward. ■