Sincere Voting with Cardinal Preferences: Approval Voting*

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Abstract

We discuss sincere voting when voters have cardinal preferences over alternatives. We interpret sincerity as opposed to strategic voting, and thus define sincerity as the optimal behaviour when conditions to vote strategically vanish. When voting mechanisms allow for only one *message type* we show that this optimal behavior coincides with an intuitive and common definition of sincerity. For voting mechanisms allowing for multiple *message types*, such as approval voting (AV), there exists no conclusive definition of sincerity in the literature. We show that for AV, voters' optimal strategy tends to one of the existent definitions of sincerity, consisting in voting for those alternatives that yield more than the average of cardinal utilities.

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1 Introduction

In this paper we discuss what sincere voting means under different voting mechanisms when voters have cardinal preferences over alternatives. A definition of sincerity is important since it allows to compare the properties of different voting rules with respect to voters' strategic behavior. Under different voting mechanisms, and given voters' preferences over alternatives, voters may be better able to favour the election of preferred outcomes by behaving strategically instead of sincerely and thus, manipulate the voting mechanism.[1] In order to provide a general definition of sincerity, our approach is to consider this strategic component of voting and eliminate it.

There exists ample literature on the definition of sincerity for different voting mechanisms and on which voting rules may achieve it.[2] Brams and Fishburn (1978) define sincere voting as non-strategic behavior in which individuals vote "directly in accordance with their preferences". The problem arises because translating preferences over alternatives to sincere votes may not be direct under some voting rules, since they may demand to structure votes in a different format than preferences may be specified.

Since the majority of the voting literature limits the analysis to ordinal preferences over alternatives, votes are structured in the same format as preferences and thus, this problem has not been highlighted.[3] However, it seems plausible to assume that voters may be able to quantify differences between alternatives and thus, they may have cardinal preferences over them. Under cardinal preferences, if a voting mechanism exactly required all cardinal information, the definition of "sincere voting" would be straightforward. A sincere "vote" would just be the declaration of the cardinal utility that each alternative gives to a voter.

Consider the following example. There are three alternatives x, y and z that yield the following utilities to a voter: $U(x) = 0.8$, $U(y) = 0.5$ and $U(z) = 0.1$. A voting rule that required all cardinal information would have associated as "sincere voting" the revelation of utilities $0.8, 0.5$ and 0.1 respectively.

However, the majority of voting mechanisms only require (partial) ordinal information from voters and thus a definition of sincerity is more complex. Votes may be understood as messages since they transmit information on the desirability of the alternatives for the voters. The translation of cardinal utilities to non-cardinal votes may then depend on the number (and type) of messages each voting mechanism allows.

If the voting mechanism only allows for one possible message type then identifying sincere behavior is not so problematic. A sincere vote would be the one that intuitively "best represents" the order of the cardinal preferences, given the restrictions of the voting mechanism. For example, the plurality rule is a clear case of a voting rule that allows for only one message type, since voters can only choose between singletons (with the meaning of a

[1] Any voting rule is subject to strategic voting behaviour when its range has at least three alternatives and there are no dictators (Gibbard (1973), Satterwhaite (1975).

[2] Starting with Farquharson (1969).

[3] See, for example, Arrow (1951), Fishburn (1973) and Nurmi (1987).

superior alternative, since the aggregation process will consider positively such singletons). Thus sincere voting under Plurality Rule (PR) would intuitively fit with voting (in the top set) for the alternative that yields highest utility to the voter. In our example, a sincere voter under PR would then declare her real preferences by voting for alternative $\{x\}$. Intuitively, any other possible message, for example, $\{z\}$ would be a worse representation of the voter's real cardinal preferences and thus, would not be sincere.

There are however several voting rules that allow for more than one message type. In such cases, there is no clear intuition of what the best representation of cardinal preferences would be. A paradigmatic example of such rules is Approval Voting (AV), in which the decision of whether to include an alternative among the "approved" ones or not may naturally depend on the difference in cardinal utility between alternatives. In our example, if the voter was only allowed to approve her "best alternative" (to choose from the set of singletons of $2^{\{x,y,z\}}$) then voting $\{x\}$ would intuitively be sincere as previously mentioned. On the other hand, if the voter was only allowed to vote for pairs of alternatives (which is what Negative Voting would do), then voting $\{x, y\}$ would be sincere as it best fits with her cardinal preferences given the restrictions. However, as AV allows voters to specify any subset of alternatives as the set of approved options, it may not be clear whether voting $\{x\}$ or $\{x, y\}$ is the sincere message.

Previous literature has discussed at least two definitions of sincerity for AV. The first one specifies that if one alternative is voted in the top set (approved), all alternatives that yield higher cardinal utility to the individual should also be included in the top set to be considered as sincere.[4] Notice that this definition is somewhat weak as several messages would then be considered sincere. In our example, $\{x, y, z\}$ (meaning "all alternatives are approved"), ϕ (meaning "all alternatives are disapproved"), $\{x\}$ and $\{x, y\}$ would all be considered sincere under this weak definition. A second and more restrictive definition defines sincerity as voting for those alternatives that yield the individual more cardinal utility than the average of all alternatives.[5] In our numerical example, the only sincere voting representation would then be to vote for both x and y (i.e. $\{x, y\}$) since both provide more cardinal utility than the average ($0.8 > 0.47$ and $0.5 > 0.47$).

As we have seen, complexity in the translation of preferences over alternatives into votes creates ambiguity in defining sincere voting. We abstract from such ambiguity by considering a new approach. We consider a voter under a hypothetical situation in which conditions to behave strategically are diminished and define sincerity as her optimal voting strategy under such conditions.

Strategic voting implies balancing the relative preference for the different alternatives against the relative likelihood of influencing the outcome of the election.[6] Notice that whether a voter assesses that her vote may affect the outcome depends on how she thinks other voters will vote. Strategic behavior may thus be enhanced the more information voters have on the

[4] See Brams and Fishburn (1981) and Niemi (1984) .

[5] See, for instance, Merrill (1983), Merrill and Nagel (1987) and Hoffman (1982) who present some results characterizing this behavior under a very restrictive set of assumptions.

[6] See Fisher and Myatt (2002).

strategies of other voters. Weber (1978) goes as far as claiming that in settings where voters have little access to information concerning either the preferences of other voters or their intended behavior, voters can be presumed to vote sincerely, since the lack of information means there is no basis for voting "in some clever strategic way". Our first result shows that Weber's (1978) intuition is correct for any voting rule allowing for only one message type and three alternatives. We show that in this setting, the best strategy of a voter with no information on other voters' strategies is unique and independent of the size of the electorate. We thus define sincere voting behaviour as this optimal strategy for voting rules that allow for only one message type.

However, when voting rules allow for more than one message type things become more complex. In settings with three alternatives and taking AV as our benchmark example of such rules, our second result shows that the optimal strategy for any voter under AV when information on others' strategies is eliminated depends on the size of the electorate. Thus, it cannot be the case that we consider this behavior as a precise definition of sincerity, since how sincere a vote is should not vary with the number of voters. We show, however, that the optimal strategy when there is no information on others' strategies always satisfies (for any size of the electorate) the weak definition of sincerity in AV previously discussed.

We next introduce new conditions that may diminish strategic behaviour. A natural intuition is that the larger the electorate the lower the manipulative effect of a strategic vote on the outcome of the election may be. Therefore, following our approach, we check what the optimal strategy in AV is when the size of the electorate tends to infinity (and there is no information on others' strategies). We define sincere voting as the optimal strategy under such conditions. Our third result shows that the optimal strategy under such conditions coincides with the second and stronger definition of sincerity previously discussed in the literature. Our paper thus provides new support for the idea that sincerity in AV implies adding to the top set all alternatives that yield (cardinal) utility above the average of the utilities generated by all alternatives. Although this definition is not new, ours is not an intuitive definition but the result of a new methodology consisting in eliminating those features of the problem that generate strategic behaviour.

We have focused on the case of three alternatives x, y and z. Although this case is of course special, it is the simplest case that allows to differentiate between different voting rules while maintaining conditions for strategic voting to appear.[7]

The rest of the paper is organized as follows. Section 2 shows the notation and the basic assumptions made. Section 3 discusses sincerity in simple voting mechanisms (Result 1), while Section 4 discusses sincerity in complex voting mechanisms, in particular in approval voting (Results 2 and 3). Section 5 concludes.

[7] See Myerson and Weber (1993), Myerson (2002) and Dhillon and Lockwood (2004).

2 Notation and Definitions

Consider a set of n agents $\{1, 2, ..., n\}$ and a set of three alternatives $X = \{x, y, z\}$. Individuals are endowed with cardinal utilities over alternatives $U = (U_j(k))$ with $j \in \{1, 2, ..., n\}$, $k \in X$ and $U_j(k) \in [0, 1]$. For the elegance of the exposition, assume that there are not two alternatives providing the same utility to each agent.[8]

Assume there exists a set of messages M from which each agent has to choose one. Such message is the agent's vote and transmits information on her preferences. In this paper we consider sets of messages M containing either linear orders over X or subsets of X.[9]

Consider any bijective mapping $\sigma : X \to X$. Given a message $m \in M$, with m being a linear order, then $\sigma(m)$ is a linear order such that: $x\ \sigma(m) y \Leftrightarrow \sigma(x)\ m\ \sigma(y)$. Given a message $m \in M$, with m being a subset of alternatives $m = \{x_1, ..., x_t\}$, then $\sigma(m)$ denotes a subset of alternatives such that: $\sigma(m) = \{\sigma(x_1), ..., \sigma(x_t)\}$. We say that two messages m and m' belong to the same *message type* if there exists a bijective mapping $\sigma : X \to X$ such that $m' = \sigma(m)$. We now impose an additional condition on the valid sets of messages in order to avoid voting mechanisms to be biased towards alternatives: if the set of possible messages M contains a message m then it also contains any other message of the form $\sigma(m)$, i.e., $m \in M, \sigma : X \to X$ a bijective mapping $\Longrightarrow \sigma(m) \in M$. The class of messages which belong to the same message type as m is denoted by $[m]$.

A voting mechanism $V : M^n \to 2^{\{x,y,z\}}$ can be defined as the composition of a set of messages (among which the voters can choose one) and an aggregation process of the collected messages such that some alternatives are chosen. We refer to elements of M^n as $\mathbf{m} = (m_1, ..., m_n)$ with $m_j \in M$ for $j = 1, ..., n$. We naturally denote $\sigma(\mathbf{m}) = (\sigma(m_1), ..., \sigma(m_n))$. Finally, we denote, as usual, $\mathbf{m}_{-j} = (m_1, ..., m_{j-1}, m_{j+1}, ..., m_n)$.

A voting mechanism may allow a set of possible messages with several message types. We first classify voting mechanisms according to the number of message types associated to them. The crucial property to study sincerity will be whether voting mechanisms have a single or several message types associated to them.

Definition 1 *A voting mechanism is said to be simple if it only allows for one message type. Otherwise, it is said to be complex.*

Two examples of *simple* voting mechanisms are the Borda Rule and the Plurality Rule. In the former, the set of possible messages contains all linear orders over alternatives while in the latter, the set of possible messages contains all singletons, i.e., $M = \{\{x\}, \{y\}, \{z\}\}$.

A prime example of a *complex* voting mechanism is Approval Voting. We will formally define it below as we will discuss it thoroughly in the following.

Once we have discussed messages, we now briefly refer to the aggregation process. In particular, we now define some properties on how voting mechanisms may aggregate messages to select alternatives.

[8] Parallel results are obtained without such assumption, although proofs become tedious without adding further insights.

[9] Messages on linear orders or subsets of alternatives are the most common approach to voting.

Definition 2 *A voting mechanism V is Neutral in alternatives if for any permutation σ of the set of alternatives and any $\mathbf{m}$ in M^n, then $V(\sigma(\mathbf{m})) = \sigma(V(\mathbf{m}))$.*

Neutrality in alternatives implies that the names of the alternatives do not affect the selection of alternatives.

Our second definition refers to the monotonicity of the aggregation process. We distinguish between voting mechanisms composed by linear orders or subsets as messages.

Definition 3 *A voting mechanism V with M containing linear orders (respectively subsets of alternatives) is weakly monotonic if for any alternative x, for all $y, z \in X \backslash \{x\}$, for all $j \in \{1, ..., n\}$ and for any pair of messages' collections $\mathbf{m}$ and m' with $y\ m_j\ z \Longleftrightarrow y\ m'_j\ z$ and $x\ m_j\ y \Longrightarrow x\ m'_j\ y$, (respectively $y \in m \Longleftrightarrow y \in m'$ and $x \in m_j \Longrightarrow x \in m'_j$) , then:*

$x \in V(\mathbf{m}) \Longrightarrow x \in V(\mathbf{m}')$,

$\{x\} = V(\mathbf{m}) \Longrightarrow \{x\} = V(\mathbf{m}')$.

Our monotonicity condition is mild. It just implies that if an agent's message is modified such that it *favours* an alternative x, the voting mechanism responds accordingly. Thus, if x was in the elected set before modifying agent's message in a particular way, then it is also elected under the new message. Similarly, if x is the only elected alternative then it must also be the only elected alternative under the new message.

Finally, we define Approval Voting since we will use it throughly. Notice that it satisfies Neutrality in alternatives and Monotonicity.

Definition 4 *A voting mechanism V is Approval Voting if $M = 2^{\{x,y,z\}}$ and the selected alternatives are those that maximize the number of messages in which they appear.*[10]

Using the above definitions, our goal is to define sincere voting behaviour for voting mechanisms. We understand sincere voting behaviour as opposed to strategic behaviour. The latter comprises the possibility of favouring the election of preferred outcomes by misrepresenting *sincere* messages. There exist some conditions that facilitate the appearance of strategic behaviour. For instance, the influence of an individual agent's message on the outcome of the election or the amount of information agents have on others' preferences over alternatives. Our approach is to define sincere voting as the optimal voting strategy when the conditions that ease strategic behaviour are diminished. Since such approach requires to study how agents react to uncertainty, we impose the following two assumptions.

Assumption 1 *In the absence of information on other agents' preferences over alternatives, agents believe that any possible combination of others' messages is equally probable.*

[10] Merril and Nagel (1987) differentiate between balloting methods and the decision rules that produce an outcome. In that spirit, they would claim that AV is our balloting method, while, given our definition, the outcome of the election is decided under Plurality Rule. Our definitions consider both characteristics of voting rules, i.e., the set of available messages and the way to aggregate them.

Formally, for all j and for all $m_{-j} \in M^{n-1}$, $p_j(m_{-j}) = (\frac{1}{\sharp M})^{n-1}$ where $p_j(m_{-j})$ is the probability with which agent j beliefs other agents will transmit messages m_{-j}.

Notice that the probability each agent assigns to any combination of messages by other agents clearly depends on the cardinality of the set of messages. In particular, for the case of AV, $\forall j$, and for all $\mathbf{m}_{-j} \in M^{n-1}$, $p_j(\mathbf{m}_{-j}) = \left(\frac{1}{2^{\sharp X}}\right)^{n-1}$.

Assumption 2 *Given agents' beliefs, they maximize their expected utility over alternatives.*

Assumptions 1 and 2 are a simple way for voters to resolve the uncertainty about others' preferences. Notice that we aim to strengthen conditions that eliminate strategic voting and thus, our assumptions refer to cases in which agents can not form clear expectations about how others will vote. Moreover, these assumptions may have a behavioural support. Both assumptions are also the common starting point to define *k-levels of rationality* in the literature on degrees of cognitive complexity which has found certain experimental validity.[11]

3 Sincerity in simple voting mechanisms

We aim to define sincerity for simple voting mechanisms as the best response strategy when the possibility of strategic behaviour is diminished. Theorem 1 confirms the intuition that under such mechanisms sincerity implies transmitting pieces of ordinal information contained in agents' cardinal preferences over alternatives.

Theorem 1: *Let V* be a *simple voting mechanism satisfying Neutrality in alternatives and Weak Monotonicity. Assume, there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions* 1 *and* 2 hold*. Then, for any number of agents n, agent i's best response (sincere behaviour) is:*

- *For $M = [\overline{m}]$ with $\overline{m}$ being a linear order, the linear order such that $x \ m \ y \ m \ z \Leftrightarrow U_i(x) > U_i(y) > U_i(z)$.*

- *For $M = [\overline{m}]$ with $\overline{m}$ being a subset of alternatives, the subset of the $\sharp m$ alternatives which provide highest utility to agent i.*

Proof: We proceed to prove separately the cases in which the set of messages is the set of linear orders and the cases in which the set of messages is a collection of subsets of X.

- We first consider the case in which $M = \{\text{linear orders over } X\}$. Consider wlog. $U_i(x) > U_i(y) > U_i(z)$. Consider the linear order $x \ m \ y \ m \ z$ by m. We have to prove that m is agent i's best response independently of the number of agents in society.

[11] See Stahl (1993), Stahl and Wilson (1994, 1995), McKelvey and Palfrey (1995), Broseta, Costa-Gomes and Crawford (2001) and Goeree and Holt (2004).

We show that m is a better response than m', where $y\ m'\ x\ m'\ z$. To see this, let us analyze all the possible situations in which transmitting m' could be beneficial for agent i. Consider any combination of messages by the other agents in society, $\mathbf{m}_{-i}$. Then, given that the voting mechanism is Weakly Monotonic, we know that $x \in V(\mathbf{m}_{-i}, m') \Rightarrow x \in V(\mathbf{m}_{-i}, m)$ and $y \in V(\mathbf{m}_{-i}, m) \Rightarrow y \in V(\mathbf{m}_{-i}, m')$. We also know that $\{x\} = V(\mathbf{m}_{-i}, m') \Rightarrow \{x\} = V(\mathbf{m}_{-i}, m)$ and $\{y\} = V(\mathbf{m}_{-i}, m) \Rightarrow \{y\} = V(\mathbf{m}_{-i}, m')$. The following table specifies all possible outcomes of the election in which declaring m' instead of m may be beneficial for agent i. Any other combination of others' messages always yields a worse outcome when declaring m'. For instance, outcome $\{x, y\}$ whenever i states m yields lower utility than outcome $\{x, z\}$ whenever i states m', since $\frac{U_i(z)+U_i(x)}{2} < \frac{U_i(y)+U_i(x)}{2}$ and thus declaring m' would not be beneficial. Notice also that not every pair of outcomes can be associated to messages m and m'. For example, the outcome $\{y, z\}$ whenever i states m and outcome $\{x, y\}$ whenever i states m' is not possible since $x \in V(\mathbf{m}_{-i}, m')$ but $x \notin V(\mathbf{m}_{-i}, m)$.

Messages	Outcome							
m	$\{x,z\}$	$\{x,y,z\}$	$\{x,z\}$	$\{x,z\}$	$\{x,y,z\}$	$\{y,z\}$	$\{z\}$	$\{z\}$
m'	$\{x,y\}$	$\{x,y\}$	$\{x,y,z\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y,z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

Notice that under cases $3), 4)$ and $5)$, m' yields higher expected utility than m only when $U_i(y) > \frac{U_i(x)+U_i(z)}{2}$.

In order to prove that message m is a better response than m', we show that, for any of the previous cases (associated to a combination of messages by the others), there exists another combination of messages by the others such that:

1. Its probability of occurrence is larger.
2. The benefit from transmitting m instead of m' is larger than the benefit from transmitting m' instead of m in the initial case.

Consider the bijection $\sigma : X \Rightarrow X$, where $\sigma(x) = y, \sigma(y) = x$ and $\sigma(z) = z$. For k, $k \in \{1, ..., 8\}$, consider the combination of others' messages $\mathbf{m}_{-i}^k$ which makes transmitting m' beneficial with respect to m. Consider also the combination of others' messages $\sigma(\mathbf{m}_{-i}^k)$. By Assumption 1, individual i assigns the same probability to messages $\sigma(\mathbf{m}_{-i}^k)$ and m_{-i}^k. Since $\sigma(m) = m'$ and $\sigma(m') = m$, by Neutrality in alternatives, it must be that $V\left(m, \sigma(\mathbf{m}_{-i}^k)\right) = \sigma\left(V\left(m', \mathbf{m}_{-i}^k\right)\right)$ and $V\left(m', \sigma(\mathbf{m}_{-i}^k)\right) = \sigma\left(V\left(m, \mathbf{m}_{-i}^k\right)\right)$. Thus, we can compute parallel cases (with equal probability) to those of the previous table. The outcomes of the voting mechanism now are:

$Messages$	Outcome							
m	$\{x, y\}$	$\{x, y\}$	$\{x, y, z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x, z\}$
m'	$\{y, z\}$	$\{x, y, z\}$	$\{y, z\}$	$\{y, z\}$	$\{x, y, z\}$	$\{x, z\}$	$\{z\}$	$\{z\}$
Cases	1')	2')	3')	4')	5')	6')	7')	8')

For cases k', $k \in \{1, ..., 8\}$, the benefit obtained from declaring m instead of m' is, in all the cases, at least as large as the loss for the corresponding case k. Given that any of these cases has the same probability as its counterpart, m guarantees a expected utility at least as large as m'.

Showing that any other message m'' yields lower expected utility than m follows exactly the same reasoning.[12] Thus, m is agent i's best response.

- We now consider situations in which M is a family of subsets of X. In order to have a simple voting mechanism, there only exist four possibilities:

$$M_1 = \{\phi\}, M_2 = \{X\}, M_3 = \{\{x\}, \{y\}, \{z\}\} \text{ and } M_4 = \{\{x, y\}, \{x, z\}, \{y, z\}\}.$$

M_1 and M_2 are trivial cases given that agents can not decide which message to transmit. Plurality Rule is a prime example of a voting mechanism using M_3. Negative Voting (or Antiplurality) is an example of a voting mechanism using M_4.[13] We here prove the result for M_3 and leave the analogous proof for M_4 for the reader.[14]

Consider $M_3 = \{\{x\}, \{y\}, \{z\}\}$ and *wlog.* $U_i(x) > U_i(y) > U_i(z)$. We first show that transmitting $\{x\}$ is better than transmitting $\{y\}$. Consider any combination of messages in society, $\mathbf{m}_{-i}$. Then, given that the voting mechanism is Weakly Monotonic, we now that $x \in V(\mathbf{m}_{-i}, \{y\}) \Rightarrow x \in V(\mathbf{m}_{-i}, \{x\})$ and $y \in V(\mathbf{m}_{-i}, \{x\}) \Rightarrow y \in V(\mathbf{m}_{-i}, \{y\})$. Additionally, $\{x\} = V(\mathbf{m}_{-i}, \{y\}) \Rightarrow \{x\} = V(\mathbf{m}_{-i}, \{x\})$ and $\{y\} = V(\mathbf{m}_{-i}, \{x\}) \Rightarrow \{y\} = V(\mathbf{m}_{-i}, \{y\})$. The following table, which is in fact equivalent to the case of linear orders, specifies all possible outcomes in which transmitting $\{y\}$ may be beneficial for agent i.

$Messages$	Outcome							
$\{x\}$	$\{x, y\}$	$\{x, y\}$	$\{x, y, z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x, z\}$
$\{y\}$	$\{y, z\}$	$\{x, y, z\}$	$\{y, z\}$	$\{y, z\}$	$\{x, y, z\}$	$\{x, z\}$	$\{z\}$	$\{z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

[12] Since all the proofs rely in the same construction, for simplicity we explicitly exclude them. They are, however, available upon request.

[13] One is tempted to think that Negative Voting also uses M_3, given that agents transmit their least preferred alternative. However, for Negative Voting to satisfy weak monotonicity, its messages must be interpreted as transmitting all the alternatives but the least preferred one.

[14] We consider this theorem as a baseline for the results of the following section and thus, we explicitly avoid this part of the proof which can be easily derived from the one presented.

The analysis is parallel to the case of linear orders, but proving that $\{x\}$ strictly yields a larger expected utility than $\{y\}$. Reproducing the analysis with strategies $\{y\}$ and $\{z\}$ it can be shown that $\{y\}$ strictly yields a larger expected payoff than $\{z\}$.Thus, transmitting $\{x\}$ strictly yields a larger expected utility than $\{y\}$ and $\{z\}$, concluding the proof for M_3. □

We have therefore defined sincere voting behaviour under simple voting mechanisms. Voters' optimal strategy under no information conditions is to assign votes in a manner that maintains some ordinal information of their true preferences. Notice that in simple mechanisms this behaviour does not depend on the weight of an individual agent's vote on the outcome of the election. However, in the next section we show that the absence of information is not enough to guarantee a precise definition of sincerity for complex voting mechanisms. The reason is that best responses will depend, for instance, on the number of agents participating in the election.

4 Complex Voting Mechanisms: Approval Voting

We consider in this section voting mechanisms which allow for several message types. Approval Voting is a prototypical case of complex voting mechanisms. Under AV agents can transmit a large variety of messages. For example, in the case of three alternatives, AV allows for the set of messages $M = 2^{\{x,y,z\}}$. This set is composed by the following four different message types $M_1 = \{\phi\}$, $M_2 = \{X\}$, $M_3 = \{\{x\}, \{y\}, \{z\}\}$ and $M_4 = \{\{x, y\}, \{x, z\}, \{y, z\}\}$.

The message an agent chooses, and thus the message type used, naturally depends on the cardinal utility alternatives yield. A definition of sincere voting behaviour is thus more complicated. Brams and Fishburn (1981) and Niemi (1984) have previously defined sincerity in AV as given that an agent supports an alternative, she must also support all alternatives that are preferred to that one. Translating this argument to cardinal utilities and using our notation, we establish a definition of *weak sincerity*:

Definition 5 *Agent i's message m is Weak Sincere under AV if for all x, y such that $U_i(x) > U_i(y)$, $y \in m$ implies $x \in m$.*

We refer to such definition as *weak* because it does not determine a unique message as sincere, which may be an appealing property.

Weber (1978), Merrill (1983), Merrill and Nagel (1987) and Hoffman (1982) provide a stronger definition of sincerity in AV that uses cardinal utilities and uniquely determines one message as sincere. Using our notation, such definition can be expressed as follows:

Definition 6 *Agent i's message m is Strong Sincere under AV if for all $x \in X$:*

$$x \in m \Leftrightarrow U_i(x) \geq \frac{1}{3} \sum_{y \in X} U_i(y).$$

The strong definition of sincere voting under AV implies voting for those alternatives that yield more utility than the average of utilities. This definition, although intuitively appealing, has not been given a complete formal justification. In particular, it has been defined under a restrictive set of assumptions, such as imposing specific probabilities on the number of votes each alternative receives. As in the previous subsections, we obtain our results by precisely calculating these probabilities using a cognitive process based only on initial beliefs over individual votes. In the remainder of the paper, we show that the best response of an agent under conditions that diminish the possibility of behaving strategically is precisely voting for those alternatives that yield more than the average of utilities. Therefore, we provide stronger support for this second definition of sincerity, which uniquely determines which message is sincere in AV.

4.1 Dependence on the Size of the Electorate

We proceed here to obtain optimal voter behavior in the absence of information in order to achieve a proper definition of sincerity in AV. Theorem 2 shows, however, that optimal behavior is dependent on the number of individuals in a society, thus making impossible to achieve a non-contingent definition of sincerity. However, it also shows that any behavior (for any size of the electorate) satisfies the weak definition of sincerity in AV.

Theorem 2: *Let V be Approval Voting. Assume $U_i(x) > U_i(y) > U_i(z)$. Assume, there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions* 1 *and* 2 hold. *Then agent i transmits message $\{x, y\}$ if and only if $U_i(y) \geq \lambda(n)U_i(x) + (1 - \lambda(n))U_i(z)$ with $\lambda(n) \in (0, 1)$. Otherwise, agent i transmits message $\{x\}$.*

Proof: *Let $U_i(x) > U_i(y) > U_i(z)$.* In Theorem 1, we have proved that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\}, \{y\}, \{z\}\}$ whenever the domain of the voting rule V is M_3. Notice that using the same procedure as the proof of Theorem 1, we can indeed show that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\}, \{y\}, \{z\}\}$ whenever the domain of the voting rule V is $2^{\{x,y,z\}}$. Given that AV satisfies *Neutrality* in alternatives and *Weak Monotonicity*, we can ensure that $\{x\}$ is a best response among strategies in $M_3 = \{\{x\}, \{y\}, \{z\}\}$. A similar argument applies for M_4.

Therefore, the only messages worth considering are $\{\phi,\ X, \{x\}, \{x, y\}\}$. We first show that voting $\{x\}$ is always better than voting X (respectively ϕ). Suppose that voting X (respectively ϕ) leads to have $S \neq \{x\}$ as the set of elected alternatives.[15] If $x \in S \neq \{x\}$, then transmitting $\{x\}$ leads to have x as the unique elected outcome, which obviously dominates S for agent i. If $x \notin S$, transmitting $\{x\}$ leads either to have S as the set of elected outcomes or to have $S \cup \{x\}$ as the set of elected outcomes. This is clearly preferable to the outcome obtained when transmitting X (respectively ϕ). Thus we focus on $\{x, y\}$ and $\{x\}$. We present here the situations in which $\{x, y\}$ and $\{x\}$ could yield different outcomes:

[15] Obviously if $S = \{x\}$ then transmitting $\{x\}$ has the same effect on the election of outcomes.

$Messages$	Outcome					
$\{x\}$	$\{x\}$	$\{z\}$	$\{x,y\}$	$\{x,z\}$	$\{y,z\}$	$\{x,y,z\}$
$\{x,y\}$	$\{x,y\}$	$\{y,z\}$	$\{y\}$	$\{x,y,z\}$	$\{y\}$	$\{y\}$
Cases	1)	2)	3)	4)	5)	6)

The previous table shows all possible combinations of others agents' messages in which messages $\{x,y\}$ and $\{x\}$ yield different outcomes. In order for these situations to occur, the distribution of other agents' messages must satisfy the following conditions:

$$\begin{array}{ll} 1)\ a_x = a_y > a_z - 1 & 4)\ a_x + 1 = a_y + 1 = a_z \\ 2)\ a_x + 1 < a_y + 1 = a_z & 5)\ a_x + 1 < a_y = a_z \\ 3)\ a_x = a_y - 1 > a_z - 1 & 6)\ a_x + 1 = a_y = a_z, \end{array}$$

where a_k represents the number of times alternative k appears in other agents' messages, excluding agent i.

- Under Assumption 1, the probabilities P_q of each of these six conditions are:

$$P_1 = \frac{\sum_{t=0}^{n-1}\sum_{s=0}^{t}\binom{n-1}{t}\binom{n-1}{t}\binom{n-1}{s}}{(2^3)^{n-1}} \qquad P_2 = \frac{\sum_{t=2}^{n-1}\sum_{s=0}^{t-2}\binom{n-1}{s}\binom{n-1}{t-1}\binom{n-1}{t}}{(2^3)^{n-1}}$$

$$P_3 = \frac{\sum_{t=1}^{n-1}\sum_{s=0}^{t-1}\binom{n-1}{t-1}\binom{n-1}{t}\binom{n-1}{s}}{(2^3)^{n-1}} \qquad P_4 = \frac{\sum_{t=1}^{n-1}\binom{n-1}{t-1}\binom{n-1}{t-1}\binom{n-1}{t}}{(2^3)^{n-1}}$$

$$P_5 = \frac{\sum_{t=2}^{n-1}\sum_{s=0}^{t-2}\binom{n-1}{s}\binom{n-1}{t}\binom{n-1}{t}}{(2^3)^{n-1}} \qquad P_6 = \frac{\sum_{t=1}^{n-1}\binom{n-1}{t-1}\binom{n-1}{t}\binom{n-1}{t}}{(2^3)^{n-1}}$$

Hence, the expected utility of messages $\{x\}$ and $\{x,y\}$ under assumption 1 can be expressed in terms of these probabilities. For message $\{x\}$, the expected utility equals:

$$P_1 U_i(x) + P_2 U_i(z) + P_3\left(\frac{U_i(x)+U_i(y)}{2}\right) + P_4\left(\frac{U_i(x)+U_i(z)}{2}\right)$$
$$+P_5\left(\frac{U_i(y)+U_i(z)}{2}\right) + P_6\left(\frac{U_i(x)+U_i(y)+U_i(z)}{3}\right)$$

whereas for message $\{x,y\}$ the expected utility equals:

$$P_1\left(\frac{U_i(x)+U_i(y)}{2}\right)+P_2\left(\frac{U_i(y)+U_i(z)}{2}\right)+P_3U_i(y)+$$
$$P_4\left(\frac{U_i(x)+U_i(y)+U_i(z)}{3}\right)+P_5U_i(y)+P_6U_i(y)$$

Therefore, using Assumption 2, the condition for preferring to transmit $\{x, y\}$ has to be that it yields a higher expected value than transmitting $\{x\}$, i.e.,

$$\left(\tfrac{P_1}{2}+\tfrac{P_3}{2}+\tfrac{P_4}{6}+\tfrac{P_6}{3}\right)U_i(x)+\left(\tfrac{P_2}{2}+\tfrac{P_4}{6}+\tfrac{P_5}{2}+\tfrac{P_6}{3}\right)U_i(z)\leq\left(\tfrac{P_1}{2}+\tfrac{P_2}{2}+\tfrac{P_3}{2}+\tfrac{P_4}{3}+\tfrac{P_5}{2}+\tfrac{2P_6}{3}\right)U_i(y)$$

Denoting:

$$\begin{aligned}f(n)&=\left(\tfrac{P_1}{2}+\tfrac{P_3}{2}+\tfrac{P_4}{6}+\tfrac{P_6}{3}\right)\\ g(n)&=\left(\tfrac{P_2}{2}+\tfrac{P_4}{6}+\tfrac{P_5}{2}+\tfrac{P_6}{3}\right)\\ h(n)&=\left(\tfrac{P_1}{2}+\tfrac{P_2}{2}+\tfrac{P_3}{2}+\tfrac{P_4}{3}+\tfrac{P_5}{2}+\tfrac{2P_6}{3}\right),\end{aligned}$$

we can express the previous inequality as $\frac{f(n)}{h(n)}U_i(x)+\frac{g(n)}{h(n)}U_i(z)\leq U_i(y)$. Since $f(n)+g(n)=h(n)$, we only need to consider the function $\lambda(n)=\frac{f(n)}{h(n)}$ to conclude the proof. Since any $P_q : t=1,...,6$ is different from zero, it follows that $\lambda(n)\neq 1$ and $\lambda(n)\neq 0$. □

Corollary 7 *Let V be Approval Voting. Assume, there is no information on agents' preferences over alternatives $X=\{x, y, z\}$ and assumptions* 1 *and* 2 *hold. Then agent i's optimal behaviour satisfies the weak definition of sincerity.*

Proof. Notice that following Theorem 2, the only possible optimal messages are $\{x\}$ and $\{x, y\}$, and thus, it easily follows that the weak definition of sincerity always holds. ■

Whether the alternative yielding second highest utility to an agent is included in her transmitted message depends on its *relative* cardinal utility with respect to the utilities yielded by the most and least preferred alternatives. Such dependence rests on the weights measured by the function $\lambda(n)$, which varies with the size of the electorate n. For example, if $U(x)=0.9, U(y)=0.7$ and $U(z)=0.1$, basic calculus shows that an agent optimally transmits message $\{x\}$ when the size of the electorate is 2. On the other hand, the same agent transmits message $\{x, y\}$ when the size of the electorate is 3. It seems unreasonable that how sincere a voting strategy is depends on the size of the electorate. Thus, in the following subsection we further impose conditions to diminish the possibility of strategic voting in order to obtain sincere behaviour.

4.2 Sincerity in Approval Voting

The influence of an individual agent's vote on the outcome of an election diminishes the bigger the size of an electorate. Our last result shows that when the number of agents tends to infinity the previously defined strong definition of sincere approval voting arises as the best response of any agent.

In Theorem 2, we identified agents' best response in the absence of information. Under such conditions, to include in the transmitted message the alternative yielding the second highest utility partially depends on the size of the electorate through the weighting function $\lambda(n)$. Theorem 3 determines the limit of $\lambda(n)$ when n goes to infinity.

Theorem 3: *Let V be Approval Voting. Assume there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions* 1 *and* 2 hold, *then:*

$$\lambda(n) \underset{n\longrightarrow\infty}{\rightarrow} \frac{1}{2}.$$

Proof: Following notation introduced in the proof of Theorem 2, we want to prove that $\lambda(n) = \frac{f(n)}{h(n)} \underset{n\longrightarrow\infty}{\rightarrow} \frac{1}{2}$. Given that

$$f(n) + g(n) = h(n),$$

this is equivalent to proving,

$$\frac{f(n) - g(n)}{h(n)} \underset{n\longrightarrow\infty}{\rightarrow} 0.$$

Substituting values and after basic calculus,

$$\frac{f(n) - g(n)}{h(n)} = \frac{\frac{1}{2}(P_1 - P_5)}{h(n)} + \frac{\frac{1}{2}(P_3 - P_2)}{h(n)} \leq \frac{\frac{1}{2}(P_1 - P_5)}{\frac{1}{2}P_5} + \frac{\frac{1}{2}(P_3 - P_2)}{\frac{1}{2}P_2} = \frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2}.$$

Hence, proving

$$\frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2} \underset{n\longrightarrow\infty}{\rightarrow} 0,$$

implies $\frac{f(n)-g(n)}{h(n)} \underset{n\longrightarrow\infty}{\rightarrow} 0$.

Actually, we here prove that $\frac{P_1-P_5}{P_5} \underset{n\longrightarrow\infty}{\rightarrow} 0$ and $\frac{P_3-P_2}{P_2} \underset{n\longrightarrow\infty}{\rightarrow} 0$, which is stronger than what is needed. We start by proving that $\frac{P_1-P_5}{P_5} \underset{n\longrightarrow\infty}{\rightarrow} 0$.

Consider the following two standard properties of combinatorial numbers which apply to any non-negative integers k, i for $k \geq i$:

Property 1: $\begin{pmatrix} k \\ i \end{pmatrix} + \begin{pmatrix} k \\ i-1 \end{pmatrix} = \begin{pmatrix} k+1 \\ i \end{pmatrix}$.

Property 2 (symmetry): $\begin{pmatrix} k \\ i \end{pmatrix} = \begin{pmatrix} k \\ k-i \end{pmatrix}$.

By Property 1,

$$\lim_{n\longrightarrow\infty}\frac{P_1-P_5}{P_5}=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n-1}{t}+\binom{n-1}{t-1}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}=$$

$$=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n}{t}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}.$$

We only consider the cases in which n is even (a similar reasoning would follow for the case in which n is odd). From the last expression and using Property 2 we can derive,

$$\lim_{n\longrightarrow\infty}\frac{P_1-P_5}{P_5}=\lim_{n\longrightarrow\infty}\frac{\sum_{t=2}^{n-1}\left[\binom{n-1}{t}^2\left[\binom{n}{t}\right]\right]}{\sum_{t=2}^{n-1}\binom{n-1}{t}^2\sum_{s=0}^{t-2}\binom{n-1}{s}}=\lim_{n\longrightarrow\infty}\frac{A_1+A_2+A_3}{B_1+B_2+B_3}$$

where,

$$A_1=\sum_{t=2}^{\frac{n-2}{2}}\binom{n-1}{t}^2\left[\binom{n}{t}+\binom{n}{n-t-1}\right],$$

$$A_2=\binom{n-1}{n-2}^2\binom{n}{n-2},$$

$$A_3=\binom{n-1}{n-1}^2\binom{n}{n-1}.$$

$$B_1=\sum_{t=2}^{\frac{n-2}{2}}\binom{n-1}{t}^2\left[\sum_{s=0}^{t-2}\binom{n-1}{s}+\sum_{s=0}^{n-t-3}\binom{n-1}{s}\right],$$

$$B_2=\binom{n-1}{n-2}^2\sum_{s=0}^{n-4}\binom{n-1}{s},$$

$$B_3=\binom{n-1}{n-2}^2\sum_{s=0}^{n-3}\binom{n-1}{s}.$$

Notice that by Property 2, $\begin{pmatrix} n \\ n-t-1 \end{pmatrix} = \begin{pmatrix} n \\ t+1 \end{pmatrix}$. Applying Properties 1 and 2 to A_1, we obtain:

$$A_1 = \sum_{t=2}^{\frac{n-2}{2}} \begin{pmatrix} n-1 \\ t \end{pmatrix}^2 \begin{pmatrix} n+1 \\ t+1 \end{pmatrix}.$$

Notice that $\lim\limits_{n\longrightarrow\infty} \frac{A_1+A_2+A_3}{B_1+B_2+B_3}$ can be expressed as $\lim\limits_{n\to\infty} \frac{\sum_t a_t(n)}{\sum_t b_t(n)}$. By taking into account that $\lim\limits_{n\to\infty} \frac{\sum_t a_t(n)}{\sum_t b_t(n)} \leq \lim\limits_{n\to\infty} \frac{a_{k_n}}{b_{k_n}}$ where k_n is the value that maximizes $\frac{a_t(n)}{b_t(n)}$ for dimension n, it is sufficient to prove that $\lim\limits_{n\to\infty} \frac{a_{k_n}}{b_{k_n}} = 0$.

Notice that,

$$\frac{a_t}{b_t} = \frac{\begin{pmatrix} n-1 \\ t \end{pmatrix}^2 \begin{pmatrix} n+1 \\ t+1 \end{pmatrix}}{\begin{pmatrix} n-1 \\ t \end{pmatrix}^2 \left[\sum_{s=0}^{t-2} \begin{pmatrix} n-1 \\ s \end{pmatrix} + \sum_{s=0}^{n-t-3} \begin{pmatrix} n-1 \\ s \end{pmatrix}\right]}$$

and therefore,

$$\frac{a_{t+1}}{b_{t+1}} = \frac{\begin{pmatrix} n-1 \\ t+1 \end{pmatrix}^2 \begin{pmatrix} n+1 \\ t+2 \end{pmatrix}}{\begin{pmatrix} n-1 \\ t+1 \end{pmatrix}^2 \left[\sum_{s=0}^{t-1} \begin{pmatrix} n-1 \\ s \end{pmatrix} + \sum_{s=0}^{n-t-4} \begin{pmatrix} n-1 \\ s \end{pmatrix}\right]}.$$

Notice that $t+1 \leq \frac{n-2}{2}$ implies $t+2 \leq \frac{n}{2} \leq \frac{n+1}{2} \Longrightarrow \begin{pmatrix} n+1 \\ t+2 \end{pmatrix} \geq \begin{pmatrix} n+1 \\ t+1 \end{pmatrix}$ and $b_t - b_{t-1} = \begin{pmatrix} n-1 \\ t+2 \end{pmatrix} - \begin{pmatrix} n-1 \\ t-1 \end{pmatrix} > 0$, and therefore $\frac{a_t}{b_t} \leq \frac{a_{t+1}}{b_{t+1}}$ if and only if,

$$\frac{\begin{pmatrix} n+1 \\ t+1 \end{pmatrix}}{\begin{pmatrix} n-1 \\ t \end{pmatrix}^2 \left[\sum_{s=0}^{t-2} \begin{pmatrix} n-1 \\ s \end{pmatrix} + \sum_{s=0}^{n-t-3} \begin{pmatrix} n-1 \\ s \end{pmatrix}\right]} \leq$$

$$\frac{\begin{pmatrix} n+1 \\ t+2 \end{pmatrix}}{\begin{pmatrix} n-1 \\ t+1 \end{pmatrix}^2 \left[\sum_{s=0}^{t-1} \begin{pmatrix} n-1 \\ s \end{pmatrix} + \sum_{s=0}^{n-t-4} \begin{pmatrix} n-1 \\ s \end{pmatrix}\right]}.$$

Therefore,

$$\lim_{n\to\infty}\frac{a_t}{b_t} \leq \lim_{n\to\infty}\frac{\binom{n-1}{\frac{n-2}{2}}^2\binom{n+1}{\frac{n}{2}}}{\binom{n-1}{\frac{n-2}{2}}^2\left[\sum_{s=0}^{\frac{n-6}{2}}\binom{n-1}{s}+\sum_{s=0}^{\frac{n-4}{2}}\binom{n-1}{s}\right]} =$$

$$\underset{n\to\infty}{Lim}\frac{\binom{n-1}{\frac{n}{2}}}{\sum_{s=0}^{n-1}\binom{n-1}{s}-\binom{n-1}{\frac{n-2}{2}}-\binom{n-1}{\frac{n}{2}}-\binom{n-1}{\frac{n+2}{2}}} =$$

$$\lim_{n\to\infty}\frac{\binom{n+1}{\frac{n}{2}}}{2^{n-1}-\binom{n-1}{\frac{n-2}{2}}-\binom{n-1}{\frac{n}{2}}-\binom{n-1}{\frac{n+2}{2}}} = 0.$$

The proof for $\frac{P_3-P_2}{P_2} \underset{n\longrightarrow\infty}{\rightarrow} 0$ is similar, and thus we omit it. This concludes the proof. □

Theorem 3 says that as the size of the electorate increases, agents' best response consists in voting for those alternatives that yield more than the average of utilities. Given that we have eliminated the most important components of strategic behavior, namely information on others' preferences and the weight of an individual vote in determining the outcome, we interpret such best response as sincere voting behaviour under approval voting.

Notice that previous attempts to define sincere behaviour under approval voting did not differentiate between the implications of Theorems 2 and 3.[16] The reason is that they assumed that the probability of a tie between the number of votes that two alternatives received was equal to the probability of one of the alternatives surpassing the other by just one vote. As a by-product of our Theorem 3, we have shown that such assumption only holds true in the limit.

5 Discussion

Identifying sincere voting behaviour under a variety of voting rules is an important starting point in the discussion of adopting new voting mechanisms. A definition of sincerity is almost straightforward when simple voting mechanisms are considered. However, we have seen that under a complex voting mechanism such as Approval voting defining sincerity is cumbersome.

Approval Voting is a paradigmatic voting mechanism to study how cardinal utilities over alternatives affect sincere behaviour. We conjecture that the difficulty in defining sincerity arises as a consequence of the presence of several message types in complex mechanisms. Our intuition should be confirmed by studying other complex voting mechanisms.

[16] See, for instance, Hoffman (1982).

Our approach to define sincere voting behaviour consists in opposing sincere behavior to strategic behavior. We methodologically contribute to obtain a formal definition of sincerity, by omitting the elements that facilitate strategic behavior; namely, by increasing the size of the electorate and by eliminating information on other agents' preferences. The optimal behavior obtained under such conditions is thus what we define as sincere voting behaviour. We have shown that under Approval Voting sincere agents vote for those alternatives that yield more than the average of the utilities.

Notice that following our approach, the definition of sincerity coincides with the previously provided strong definition of sincerity. Our technical contribution consists in calculating the optimal voting behaviour by assessing explicitly the probability of each of the possible races between alternatives that can occur instead of assuming they all have the same probability. Therefore, we have provided stronger support to an intuitive definition of sincerity when agents have cardinal utilities over three alternatives.

Our aim in this paper has been to provide a definition of sincere voting behaviour. Nevertheless, this is not equivalent to identifying sincere voting from the results of an election. Knowledge on the cardinal value that the alternatives yield to the voters is required in empirical tests of our results. An experiment controlling for such utility values may be a worthwhile avenue to explore how individuals vote when informational conditions or on the influence of their votes are changed.

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