Corrigendum: Stable Matchings and Preferences of Couples*

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Abstract: We correct an omission in the definition of our domain of weakly responsive preferences introduced in Klaus and Klijn (2005) or KK05 for short. The proof of the existence of stable matchings (KK05, Theorem 3.3) and a maximal domain result (KK05, Theorem 3.5) are adjusted accordingly.
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1 Introduction

In a recent paper Nakamura (2005, Example 1) shows that the weak responsiveness condition of KK05 is not sufficient to ensure stability. Two results in KK05 are affected by the omission Nakamura detected in our definition of weak responsiveness, namely Theorems 3.3 and 3.5. In this corrigendum we add a small extra condition to our definition of weak responsiveness that is in line with our intuitive motivation and description of weak responsiveness as presented in KK05. We also provide short adjustments of the proofs of KK05's Theorems 3.3 and 3.5.

Nakamura (2005) introduces a logically equivalent adjustment of weak responsiveness (called reasonable responsiveness) and reformulates KK05's Theorems 3.3 and 3.5 as new results (Nakamura, 2005, Theorems 1 and 2). Nakamura's adjusted proof of KK05's Theorem 3.5 is different from the proof we present here.

2 Corrections: Weak Responsiveness and Domain Maximality

In order to exclude that for a couple two undesirable positions are combined to a desirable allotment, we add condition (iii) to our KK05 definition of weak responsiveness.

Weakly responsive preferences: Couple $c = (s_k, s_l)$ has *weakly responsive* preferences if there exist preferences $\succeq_{s_k}$ and $\succeq_{s_l}$ such that

(i) for all $h \in H$,

$(u, h) \succ_c (u, u)$ if and only if $h \succ_{s_l} u$,

$(h, u) \succ_c (u, u)$ if and only if $h \succ_{s_k} u$,

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(ii) for all $h_p, h_q, h_r \in H \cup \{u\}$,

$[h_p \succeq_{s_k} u,\ h_q \succeq_{s_l} u$, and $h_p \succ_{s_k} h_r$ imply $(h_p, h_q) \succ_c (h_r, h_q)]$ and

$[h_p \succeq_{s_l} u,\ h_q \succeq_{s_k} u$, and $h_p \succ_{s_l} h_r$ imply $(h_q, h_p) \succ_c (h_q, h_r)]$, and

(iii) for all $h', h'' \in H$, $h' \neq h''$, $u \succeq_{s_k} h'$ and $u \succeq_{s_l} h''$ imply $(u, u) \succ_c (h', h'')$.

Nakamura (2005) showed that KK05's Theorem 3.3 is not true without the omitted condition (iii). When introducing weak responsiveness in KK05 we explained (see KK05, page 82, lines 9-12) that "The idea of this extension [*i.e.*, from responsiveness to weak responsiveness] is that the exact associated preferences that deal with the comparison of unacceptable positions are irrelevant with respect to stability since an agent can always replace any unacceptable position with unemployment." Without condition (iii), the only case where the agents in a couple may not want to replace their unacceptable positions by unemployment occurs when the combination of them is acceptable – a case that is now excluded by (iii). We would like to emphasize that since condition (iii) is still in line with our intuition that motivated weak responsiveness, we do not find it necessary to change our nomenclature. Our corrected weak responsiveness is logically equivalent to Nakamura's (2005) "reasonable responsiveness" condition.[1]

With the corrected notion of weak responsiveness KK05's Theorem 3.3 is correct and the adjustment of the proof minimal.

KK05's Theorem 3.3 (Stability for weakly responsive preferences)
Let (P^H, P^C) be a couples market where couples have weakly responsive preferences. Then, any matching that is stable for an associated singles market $(P^H, P^S(P^C))$ is also stable for (P^H, P^C). In particular, there exists a stable matching for (P^H, P^C).

Proof. The first 13 lines of the proof are identical to the proof of Theorem 3.3 in KK05. Then the next two lines should be adjusted as follows (the change is marked in bold face).

" Assume $h_p \prec_{s_k} u$ and $h_q \prec_{s_l} u$. Then by weak responsiveness **(iii)**, $(\mathbf{u}, \mathbf{u}) \succ_{\mathbf{c}} (\mathbf{h_p}, \mathbf{h_q})$. Using ... "

The remaining part of the proof is identical to the proof of Theorem 3.3 in KK05. □

Also KK05's Theorem 3.5 is correct with the corrected notion of weak responsiveness. The adjustment of the proof is minimal by adding an extra case that deals separately with a violation of weak responsiveness condition (iii). We also add a case to the proof that we forgot to deal with when condition (ii) is violated. Changes in the proof are again indicated in bold face.

KK05's Theorem 3.5 (Maximal Domain I)
For couples markets with restricted strictly unemployment averse couples, the domain of weakly responsive preferences is a maximal domain for the existence of stable matchings.

Proof. We prove the theorem by constructing a counter example for each possible violation of weak responsiveness. Assume that couple $c_1 = (s_1, s_2)$'s preferences are restricted strictly unemployment averse, but not weakly responsive. Consider $\succeq_{s_1}$ and $\succeq_{s_2}$ satisfying weak responsiveness condition (i) **(note that such preferences always exist)**. Since couple c_1's preferences are not weakly responsive, $\succeq_{s_1}$ and $\succeq_{s_2}$ **do not** satisfy weak responsiveness conditions [**(ii) and (iii)**], ***i.e.*, [not (ii)] or [not (iii)]**.

[1] Our correction of the weak responsiveness condition allows for a short and straightforward adjustment of the proofs, in particular the proof of KK05's Theorem 3.5.

Case (a): Suppose condition (ii) is violated.

First, assume that the violation of (ii) is such that there exist $h_p, h_q, h_r \in H \cup \{u\}$ **such that (1)** $[h_p \succeq_{s_1} u$**,** $h_q \succeq_{s_2} u$**, and** $h_p \succeq_{s_1} u \succ_{s_1} h_r$ **imply** $(h_r, h_q) \succ_{c_1} (h_p, h_q)]$ **or (2)** $[h_p \succeq_{s_2} u$**,** $h_q \succeq_{s_1} u$**, and** $h_p \succeq_{s_2} u \succ_{s_2} h_r$ **imply** $(h_q, h_r) \succ_{c_1} (h_q, h_p)]$**.**

Without loss of generality assume (1). Then, by restricted strict unemployment aversion, $(h_p, h_q) \succeq_c (u, h_q) \succeq_c (u, u)$**. Furthermore, since** $u \succ_{s_1} h_r$**,** $(u, u) \succ_{c_1} (h_r, u)$**. Hence,** $(h_r, h_q) \succ_{c_1} (h_r, u)$ **and** $h_r, h_q \in H$**.**

Now, for h_r, h_q **and** $h \in H \backslash \{h_r, h_q\}$ **we specify** $P(h_r) = s_1, s_3, \emptyset, \ldots$**,** $P(h_q) = s_3, s_2, \emptyset, \ldots$**, and** $P(h) = \emptyset, \ldots$**. Couple** $c_2 = (s_3, s_4)$ **has restricted strictly unemployment averse responsive preferences based on** $P(s_3) = h_r, h_q, u, \ldots$ **and** $P(s_4) = u, \ldots$**. Note that for any individually rational matching** μ**,** $\mu(c_2) \in \{(h_r, u), (h_q, u), (u, u)\}$**. Assume that** μ **is stable. If** $\mu(c_2) = (u, u)$**, then** μ **is blocked by** $(c_2, (h_q, u))$**. If** $\mu(c_2) = (h_q, u)$**, then** $\mu(c_1) = (u, u)$**. Hence,** μ **is blocked by** $(c_2, (h_r, u))$**. If** $\mu(c_2) = (h_r, u)$**, then** $\mu(c_1) = (u, h_q)$ **(by restricted strict unemployment aversion,** $(u, h_q) \succeq_{c_1} (u, u)$**). Hence,** μ **is blocked by** $(c_1, (h_r, h_q))$**. Thus all candidates for a stable matching are blocked.**

Second, assume that the violation of (ii) involves neither $u \succ_{s_1} h_r$ **nor** $u \succ_{s_2} h_r$**. Then,** it follows that ... The remainder of this part of the proof is as in KK05, p. 85, line 23 – p. 86, line 28.

Case (b): Suppose condition (iii) is violated.

Then, there exist $h', h'' \in H$**,** $h' \neq h''$**, such that** $u \succeq_{s_1} h'$**,** $u \succeq_{s_2} h''$**, and** $(h', h'') \succ_{c_1} (u, u)$**.**

Now, for h', h'' **and** $h \in H \backslash \{h', h''\}$ **we specify** $P(h') = s_1, s_3, \emptyset, \ldots$**,** $P(h'') = s_3, s_2, \emptyset, \ldots$**, and** $P(h) = \emptyset, \ldots$**. Couple** $c_2 = (s_3, s_4)$ **has restricted strictly unemployment averse responsive preferences based on** $P(s_3) = h', h'', u, \ldots$ **and** $P(s_4) = u, \ldots$**. Note that for any individually rational matching** μ**,** $\mu(c_2) \in \{(h', u), (h'', u), (u, u)\}$**. Assume that** μ **is stable. Then,** $\mu(c_1) \in \{(h', h''), (u, u)\}$**. If** $\mu(c_2) = (u, u)$**, then** μ **is be blocked by** $(c_2, (h'', u))$**. If** $\mu(c_2) = (h'', u)$**, then** $\mu(c_1) = (u, u)$**. Hence,** μ **is blocked by** $(c_2, (h', u))$**. If** $\mu(c_2) = (h', u)$**, then** μ **is blocked by** $(c_1, (h', h''))$**. Thus all candidates for a stable matching are blocked.** □

The added Case (b) in the proof of Theorem 3.5 also provides a simple example that weak responsiveness condition (iii) is a necessary condition for stability. In contrast to Nakamura's (2005) Example 1, we only utilize two hospitals and only one violation of condition (iii) [in Nakamura's Example 1 four hospitals are needed and two couples preferences are in violation of weak responsiveness condition (iii)].[2]

References

[1] B. Klaus and F. Klijn, Stable matchings and preferences of couples, *Journal of Economic Theory* **121** (2005), 75-106.

[2] T. Nakamura, A simple example of instability and a new condition for stability in matching with couples, Working paper (2005).

[2]We could also easily adjust the example to three agents, namely a couple $c_1 = (s_1, s_2)$ and a single agent s_3.