On the Identification of Monetary (and Other) Shocks*

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Abstract

The purpose of this paper is twofold. First, we construct a DSGE model which spells out explicitly the instrumentation of monetary policy. The interest rate is determined every period depending on the supply and demand for reserves which in turn are affected by fundamental shocks: unforeseeable changes in cash withdrawal, autonomous factors, technology and government spending. Unexpected changes in the monetary conditions of the economy are interpreted as monetary shocks. We show that these monetary shocks have the usual effects on economic activity without the need of imposing additional frictions as limited participation in asset markets or sticky prices.

Second, we show that this view of monetary policy may have important consequences for empirical research. In the model, the contemporaneous correlations between interest rates, prices and output are due to the simultaneous effect of all fundamental shocks. We provide an example where these contemporaneous correlations may be misinterpreted as a Taylor rule. In addition, we use the sign of the impact responses of all shocks on output, prices and interest rates derived from the model to identify the sources of shocks in the data.

1 Introduction

The monetary transmission mechanism has been a predominant topic in modern Macroeconomics. In practice, this research agenda has proceeded as follows.

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The main aim of the theoretical literature has been to construct models that reproduce, at least qualitatively, the way the economy is believed to respond to monetary shocks. Most of these models share the same design. For money to have real effects on economic activity some sort of rigidity is assumed, either in the agents' ability to adjust their portfolio of assets or in the determination of some nominal price. In these models, monetary policy is described as a lump-sum transfer of money or as variations in Taylor rules,[1] i.e. a policy rule that specifies the interest rate as a function of the state of the economy, typically described by inflation and an output gap measure. Monetary policy shocks are then unexpected transfers of money or random terms in the Taylor rule, respectively. Because these models do not make any distinction between different monetary aggregates or between different asset maturities, they presume that the central bank has perfect control either of a broad measure of money or of a long-term interest rate.

On the other hand, a recurrent theme in the empirical literature has been the identification of monetary policy shocks through VAR techniques. The main strategy in many of these papers has been the decomposition of the variance matrix of disturbances estimated from a VAR on output, prices, a short term interest rate and other policy variables like the level of reserves. A common practice is to use a lower triangular decomposition with the short term rate ordered after output and prices. This way, the interest rate equation has the form of a Taylor rule and its disturbance is identified as the monetary policy shock. Another route has been taken by proponents of factor analysis. The purpose of this scheme is to describe the covariance structure among variables in terms of a few unobservable common factors.

We believe the connections between the theoretical strand of this literature and its empirical counterpart are very loose. This is for several reasons:

1. As noted above, theoretical models work under the assumption of central banks directly controlling a long term interest rate or a broadly defined monetary aggregate. However, this is far from true in reality. Monetary policy is conducted using instruments designed to affect 'operating targets', namely, short-term interest rates and narrowly defined money. In particular, nowadays most central banks usually lend out liquidity via open market operations and define their operating target in terms of the overnight interbank interest rate.

2. Although most of the theoretical work concentrates on the responses of output and inflation on changes in long-term rates and broad definitions of money, the empirical papers look at those responses stemming from unexpected changes in short term rates and reserves. Still, no attempt has been done at the theoretical level to seriously analyze the connection between the two sets of variables and their role in the propagation mechanism.

3. Taylor rules still are the central construct in both empirical and theoretical work. In this kind of setup, this literature has not yet provided a sensi-

[1] See Taylor [7].

ble explanation about what monetary policy shocks mean. The typical explanations run from changes in preferences of central bankers to measurement errors in the data collected by the monetary authority. In any case, the literature has not shown how such explanations can generate the types of shocks we measure, at least theoretically. Besides, it is not clear why central banks keep being a source of uncertainty in our economies, a source of uncertainty regarded as important by many economists.

4. From a theoretical perspective, the exclusion restrictions imposed by empirical researchers are far from being justified. Instead, theoretical models usually predict all fundamental shocks affecting all endogenous variables contemporaneously. This critique, already made by Canova and Pina [2], is important in the sense that the Cholesky decomposition is just one out of infinitely many possible decompositions of the VAR disturbances. Furthermore, once the monetary shocks are computed with triangular schemes, the other "fundamental shocks" that these strategies deliver have no clear interpretation as the shocks that we usually deal with in our theoretical models, namely, technology, fiscal, preferences, and the like.

5. Finally, the main drawback of factor analysis is that the factors obtained with this scheme usually have no economic content.

The aim of this paper is to fill some of these gaps. For that, we first construct a DSGE model which spells out explicitly the procedure by which monetary policy is done in actual economies. To get a feeling about the issues involved consider the way monetary policy is conducted in the model. At the beginning of each period the central bank sets the level of reserves via open market operations. The interest rate is determined later in the period depending on the supply and demand for reserves. The supply of reserves may be affected randomly by changes in cash withdrawals by the public and/or transfers of reserves by the Treasury not compensated by the central bank. The demand for reserves may be shifted by variations in the demand for deposits by the clients of the bank due to technology and demand shocks affecting economic activity. Hence, the model specifies how the interest rate deviates from its expected value. We show how these deviations are a function of all fundamental shocks in the economy.

This model has several advantages over other macro models. First, this economy presents a monetary non-neutrality without the need of imposing further frictions than the ones required to have money valued in general equilibrium. Thus, we do not rely on sticky prices or a limited participation of agents in asset markets.[2] The demand for reserves due to reserve requirements and settlement accounts for payment systems combined with the need of firms to finance their factor remunerations before they get the receipts from selling their product is

[2] Coleman, Gilles and Labadie [5] build a similar model of the reserve market. However, they impose a limited participacion structure and specify monetary policy as a conventional feedback rule for the interest rate.

sufficient to let monetary policy affect economic activity.[3]

Second, the model includes several fundamental shocks. We use the responses of our variables of interest to these shocks in the identification strategy at the end of the paper. The aim of this process is to construct series for shocks interpretable from a theoretical perspective. Furthermore, we show how several shocks can be interpreted as "monetary policy shocks" in the sense of generating the responses usually associated with these type of disturbances. An econometrician using data generated by the model would not be able to distinguish between shocks originated at the central bank when supplying reserves through open market operations from other fundamental shocks, such as changes in the cash to deposit ratio, not compensated by the central bank.

All our fundamental shocks affect all endogenous variables contemporaneously. So, we follow the sign restriction approach developed by Canova and de Nicoló [1] in the identification of shocks. For us, the contemporaneous nature of the relation between endogenous variables is of crucial importance for the way we interpret and identify monetary shocks. We provide an example to illustrate this point. In particular, we show that correlations between the interest rate, output and prices can arise such that they might be misinterpreted as a Taylor rule with a random term which is usually labelled a "monetary policy shock".

The paper is organized as follows. In section 2 we present the model. In section 3 we illustrate how the model works and solve for the policy functions. In section 4 we present a simple example with the implications of the model for the identification of shocks. In section 5 we apply the identification scheme derived from the model to the data. A discussion in chapter 6 concludes by highlighting the potential of the model as a laboratory for future research.

2 The Model

2.1 The Setup

In this model economy there are four different types of agents: a continuum of households in the unit interval, competitive firms, competitive banks, and a government. All **households** are identical and composed of a consumer and a worker. Each household ranks stochastic streams of consumption (c_t) and labor supply (n_t) according to the utility function

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(c_t, n_t\right) \right] \tag{1}$$

with

$$U\left(c_t, n_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma} - \Psi \frac{n_t^{1+\psi}}{1+\psi}. \tag{2}$$

[3] The same is true in the flexible price version of the model of Heer and Schabert [6] who built another general equilibrium model with open-market operations.

Households begin each period with financial wealth A_t^h. The timing of events is as follows. Shocks are realized at the beginning of the period. After shocks are known, an asset market opens where the consumer divides the household's wealth A_t^h depositing an amount S_t to an illiquid savings account and leaving the remaining as liquid assets, Z_t,

$$A_t^h \geq S_t + Z_t. \tag{3}$$

Then a goods market opens where the consumer decides how much to consume c_t. The nominal price of consumption goods is P_t. At the same time, a labor market opens and the worker decides how many units of labor to supply n_t. The consumer's purchases of goods are subject to the finance constraint

$$P_t c_t \leq Z_t + W_t n_t - P_t \tau_t \equiv M_t, \tag{4}$$

where $W_t n_t$ is his wage income, which the firm where the worker works transfers at the beginning of the period, and $P_t \tau_t$ is a lump-sum tax deducted by the government. In the goods market, liquid assets M_t are split between cash, X_t, and checkable deposits, D_t. To simplify matters, we assume that the currency-to-deposit ratio is determined exogenously by the random variable η_t, that is,

$$M_t = D_t + X_t, \tag{5}$$

with

$$X_t = \eta_t D_t. \tag{6}$$

Using (4) and (5), (6) yields

$$D_t = \frac{Z_t + W_t n_t - P_t \tau_t}{1 + \eta_t},\ X_t = \eta_t \frac{Z_t + W_t n_t - P_t \tau_t}{1 + \eta_t}. \tag{7}$$

Thus, the household pays the fraction $1/(1+\eta_t)$ of total consumption spending $P_t c_t$ with its deposits and the fraction $\eta_t/(1+\eta_t)$ with cash.[4]

The household receives all its non-labor income at the end of the period. It earns interest at rate i_t^s on the amount S_t in its savings account. The funds in the deposit account earn no interest. Finally, as the owner of one of the representative banks and firms it collects the profits $\Pi_t^F + \Pi_t^B$. Hence, the financial wealth of the household evolves as:

$$A_{t+1}^h = D_t + X_t - P_t c_t + (1 + i_t^s) S_t + \Pi_t^F + \Pi_t^B. \tag{8}$$

The problem of the household is to choose sequences for consumption, c_t, labor supply, n_t, liquid assets, Z_t, and savings, S_t, to maximize its utility (1) given prices and subject to constraints (3), (4), (7) and (8).

Firms produce using labor (n_t^d) through the production function

$$y_t = f(q_t, g_t, n_t^d) = q_t g_t^{\alpha_g} (n_t^d)^{1-\alpha}, \tag{9}$$

[4] Of course, given the rate of return dominance of other assets, liquid assets are only held for transaction purposes and depleted totally within the period.

where q_t is a technology shock and g_t is productive government expenditures which firms take as given.[5] Producers start each period with no financial funds. Since they have to pay their wage bill in advance, producers ask for a loan, L_t, from a bank at the interest rate i_t^l. Loans are assumed to be default free. These wages are transferred to the checking account of workers at the beginning of the period. The problem of the firm is then to choose labor, n_t^d, to maximize profits

$$\Pi_t^F = P_t f(q_t, g_t, n_t^d) - (1 + i_t^l) W_t n_t^d$$

given prices.

Banks are in the business of providing deposits to households and intermediating liquid assets between consumers and producers. They also hold the financial assets of the economy from one period to the other. The liability side of their balance sheets is comprised of savings deposits, S_t, and checkable deposits of households, D_t. Banks hold assets in the form of reserves, R_t, government bonds, B_t, and loans to firms, L_t. Thus,

$$B_t + L_t + R_t = S_t + D_t. \tag{10}$$

The return on government bonds is i_t^b. Reserves earn no interest. Banks demand them because they are subject to a reserve requirement by which they have to hold a fraction ρ of checking deposits as reserves,

$$R_t \geq \rho \times D_t. \tag{11}$$

Profits are given by:

$$\Pi_t^B = i_t^b B_t + i_t^l L_t - i_t^s S_t. \tag{12}$$

The problem of the bank is to determine the composition of assets and liabilities to maximize profits given prices.

The **Government** is divided into two departments: the fiscal authority manages government consumption and public debt while the monetary authority decides about the split of public debt into liquid and illiquid liabilities. Government consumption g_t is assumed to be exogenous and stochastic. Public expenditure is composed of government consumption plus interest payments on government bonds ($i_{t-1}^b B_{t-1}$) and it is financed by means of the lump-sum tax τ_t on households so

$$P_t g_t + i_{t-1}^b B_{t-1} = P_t \tau_t.$$

The monetary authority is able to affect the interest rate of the economy by deciding on the composition of government liabilities, A_t, into illiquid bonds, B_t, and liquid high-powered money, i.e. the monetary base, H_t. This monetary base is used either as reserves (including vault cash), R_t, as cash in the hands of

[5] Here, we assume capital to be fixed at 1. Profits of firms, which represent capital's remuneration, are transferred to households at the end of the period.

the public, X_t, or changed by autonomous factors (float or Treasury deposits), V_t. Remember that $X_t = \eta_t(Z_t + W_t n_t - P_t \tau_t)/(1+\eta_t)$. Hence, we have

$$R_t + \eta_t \left(\frac{Z_t + W_t n_t - P_t \tau_t}{1+\eta_t} \right) + V_t = H_t, \tag{13}$$

or

$$R_t = H_t - V_t - \eta_t \left(\frac{Z_t + W_t n_t - P_t \tau_t}{1+\eta_t} \right). \tag{14}$$

This is one of the central equations of the model. It says that the amount of reserves in the hands of the banks to satisfy reserve requirements, R_t, can change for four distinct reasons. First, the central bank can affect it through open market operations that change the initial monetary base, H_t, in order to induce changes in the interest rate. Second, it is affected by shocks to reserves which may come from the Treasury and float, V_t. Third, a higher demand for cash due to a change in the cash/deposit ratio η_t also induces movements in the total amount of reserves. Finally, all shocks affecting Z_t, W_t, P_t and n_t modify the amount of cash that is withdrawn and hence the level of reserves.

Expression (14) is important for several reasons. First, it summarizes the problem of the central bank. Assume the central bank has a desired level for the interest rate. The actual level of the interest rate determined in the market on period t is going to be related to R_t. Thus, the problem of the central bank is to determine the size of the open market operation, i.e. how much to change H_t, to compensate for the rest of shocks in the economy providing enough monetary base to cover demand for reserves and cash at the desired interest rate. Second, this expression is also important because it summarizes all sources of monetary shocks in the model which affect the interest rate. In this sense, in a world where the central bank wants the target rate to be fixed at a particular value, but has imperfect knowledge about the different shocks hitting the economy or does not fully compensate the shocks it has information about, we will observe the interest rate to be moving in response to shocks not compensated by the central bank. Below we will make explicit the information set of the central bank and the rule it follows in terms of the instruments under its control. Then we construct a monetary shock accordingly.

Finally, assume total government liabilities, $B_t + H_t$, are constant so that the role of monetary policy is to change its composition between liquid and illiquid liabilities. Then, the evolution of total government liabilities in private hands, A_t, is described by:

$$A_{t+1} = A_t - (V_{t+1} - V_t). \tag{15}$$

Note that in equilibrium the financial wealth A_t^h of the households corresponds to these liabilities of the government which are the only financial assets that survive from one period to the next. Hence, the growth rate of government liabilities used in period t by the system is

$$\mu_{t+1} = \frac{A_{t+1}}{A_t} = \frac{1+v_t}{1+v_{t+1}} \tag{16}$$

where lower case letters refer to nominal variables normalized by A_t.

2.2 Derivation of Equilibrium Conditions

Let $\xi = (q, g, \eta, v)$ be the vector of shocks. Dividing by A_t, the stationary problem of the household/firm may be rewritten as

$$J(a, \xi, h) = \max_{c,n,z,s,n^d} \left\{ U(c, n) + \beta E \left[J(a', \xi', h') \right] \right\}, \tag{17}$$

subject to

$$pc \leq z + wn - p\tau, \tag{18}$$

$$z + s \leq a, \tag{19}$$

$$\mu' a' = z + wn - p\tau - pc + (1 + i^s)\, s + pf(q, g, n^d) - (1 + i^l) wn^d + \pi^b, \tag{20}$$

with

$$\mu' = \frac{1 + v}{1 + v'}, \tag{21}$$

and lower case letters refer to nominal variables normalized by A with

$$a = \frac{A^h}{A}. \tag{22}$$

Also, banks maximize their profits

$$\max\ \pi_t^b = i_t^b b_t + i_t^l l_t - i_t^s s_t, \tag{23}$$

subject to

$$b_t + l_t + r_t = d_t + s_t, \tag{24}$$

and

$$r_t = \rho \times d_t, \tag{25}$$

which is equivalent to

$$b_t + l_t - s_t = (1 - \rho)\, d_t. \tag{26}$$

Perfect competition in the banking industry implies that interest rates equalize:

$$i_t^b = i_t^l = i_t^s = i_t. \tag{27}$$

Using the functional forms in (2) and (9), and after imposing market clearing in the labor market, the **equations determining equilibrium** are the optimum decisions by households and firms,

$$\frac{1}{p_t c_t^\gamma} = \beta(1 + i_t) E_t \left[\left(\frac{1 + v_{t+1}}{1 + v_t} \right) \frac{1}{p_{t+1} c_{t+1}^\gamma} \right], \tag{28}$$

$$(1 - \alpha) q_t g_t^{\alpha_g} = (1 + i_t)\, \Psi n_t^{\alpha + \psi} c_t^\gamma, \tag{29}$$

$$y_t = q_t g_t^{\alpha_g} n_t^{1-\alpha}, \tag{30}$$

market clearing in the goods market,

$$c_t + g_t = y_t, \tag{31}$$

and market clearing in the reserve market,

$$h_t = v_t + \left(\frac{\rho + \eta_t}{1 + \eta_t}\right) p_t c_t. \tag{32}$$

These are five equations to determine $\{y_t,\ c_t,\ n_t,\ p_t,\ i_t\}$ as functions of the policy instrument, h_t, and the fundamental shocks q_t, g_t, η_t, and v_t.

In what follows we will assume the shocks to be i.i.d. and specified as:

1. technology,
$$q_t = q_{ss} \exp(\widehat{q}_t);\ \widehat{q}_{t+1} = \varepsilon^q_{t+1}, \tag{33}$$
2. government expenditures
$$g_t = g_{ss} \exp(\widehat{g}_t);\ \widehat{g}_{t+1} = \varepsilon^g_{t+1}, \tag{34}$$
3. cash demand or cash to deposit ratio,
$$\eta_t = \eta_{ss} \exp(\widehat{\eta}_t);\ \widehat{\eta}_{t+1} = \varepsilon^\eta_{t+1}, \tag{35}$$
4. autonomous factors
$$v_t = v_{ss} \exp(\widehat{v}_t);\ \widehat{v}_{t+1} = \varepsilon^v_{t+1}, \tag{36}$$

with $\varepsilon^k_t \sim N(0, \sigma^2_k)$, $k = q,\ g,\ \eta,\ v$. Variables with the subindex ss denote steady state values. Notice hatted variables have the interpretation of percentage deviations with respect to steady states.

2.3 Solution of the Model and Calibration

Log-linearizing equations (28) to (32) around the steady state and substituting for n_t and c_t, leads to a system of equations in our variables of interest, namely output, prices and the interest rate. The solution of this system expressed as deviations with respect to the steady state can be written in the following form:[6]

$$\widehat{y}_t = \theta_{yq}\widehat{q}_t + \theta_{yg}\widehat{g}_t + \theta_{y\eta}\widehat{\eta}_t + \theta_{yv}\widehat{v}_t + \theta_{yh}\widehat{h}_t, \tag{37}$$

$$\widehat{p}_t = \theta_{pq}\widehat{q}_t + \theta_{pg}\widehat{g}_t + \theta_{p\eta}\widehat{\eta}_t + \theta_{pv}\widehat{v}_t + \theta_{ph}\widehat{h}_t, \tag{38}$$

$$\widehat{i}_t = \theta_{iq}\widehat{q}_t + \theta_{ig}\widehat{g}_t + \theta_{i\eta}\widehat{\eta}_t + \theta_{iv}\widehat{v}_t + \theta_{ih}\widehat{h}_t. \tag{39}$$

There are 18 parameters in the model. These are the parameters for preferences (β, γ, Ψ, ψ), technology (α, α_g), reserve requirements (ρ), shocks (q_{ss}, g_{ss}, η_{ss}, v_{ss}, σ_q, σ_g, σ_η, σ_v) and the ones associated with the rule determining the monetary policy instrument (h_t) to be specified below. For the calibration described in Appendix B, the coefficients of the system (37) to (39) are:

[6] See the Appendix for the details of the log-linearization and for the definition of the θ coefficients.

Table 1
Impact coefficients (model)

	Shocks				
	$\widehat{q}_t$	$\widehat{g}_t$	$\widehat{\eta}_t$	$\widehat{v}_t$	$\widehat{h}_t$
$\widehat{y}_t$	0.9359	0.2044	-0.1388	-0.0016	0.2175
$\widehat{p}_t$	-1.2132	0.0312	-0.4620	-0.0040	0.7241
$\widehat{i}_t$	-2.9531	0.0761	1.0892	0.0131	-1.7069

So, an increase in the technology shock (q) raises output at the same time that it reduces prices and the interest rate. Furthermore, a fiscal shock (g) rises output, prices and the interest rate. The cash demand shock (η) decreases prices and output and increases the interest rate as the autonomous factor shock (v) does while the official reserves (h) behave in the opposite direction. A decrease in η is expansionary because it increases the liquid liabilities of commercial banks in relation to their illiquid ones and, therefore, it has the same qualitative effects of a decrease in v or an increase in h. Unlike h and v which change the total liquid liabilities of the system, movements in η only change the composition of the monetary base between reserves and cash. In any case, these three shocks affect the composition of government liabilities which is what determines the interest rate.

Several points are in order here. First, changes in the monetary conditions of the economy (as measured by η, v or h) have real effects without imposing additional frictions like limited participation in the asset market or sticky prices. This result is due to the combination of the CIA constraint together with the fact that the monetary authority controls the composition of the government liabilities which affect the nominal interest rate through the reserve requirement. Second, shocks η, v together with changes in the monetary policy instrument h imply the same effects we usually attribute to monetary shocks. An econometrician, with data on the endogenous variables y, p, and i will not be able to identify these three sources of innovations.

In the next section we will make explicit the rule the central bank uses to determine the instrument h_t as well as the information the central bank may have when making this policy decision. Then, we will apply different identification schemes to see which identification method is able to recover the fundamental shocks of the economy.

2.4 Specification of monetary policy

Assume the central bank determines $\widehat{h}_t$ at the beginning of the period using a feedback rule to set a target for the interest rate equal to

$$i_t^{TR} = i_{ss} + \delta_y E_t^{CB}(\widehat{y}_t) + \delta_p E_t^{CB}(\widehat{p}_t) - \varepsilon_t^{TR}, \tag{40}$$

where δ_y, $\delta_p \geq 0$. In this expression E_t^{CB} represents the expectation operator with the information set of the central bank at the moment of the policy decision on $\widehat{h}_t^{CB}$ and ε_t^{TR} is a monetary policy shock to the target rate. Thus, we could write the expected deviation evaluated at the beginning of the period of the interest rate from its steady state value as

$$E_t^{CB}(\widehat{i}_t) = \widehat{i}_t^{TR} = \begin{pmatrix} \delta_y & \delta_p & -1 \end{pmatrix} \begin{pmatrix} E_t^{CB}(\widehat{y}_t) \\ E_t^{CB}(\widehat{p}_t) \\ \varepsilon_t^{TR} \end{pmatrix}. \tag{41}$$

Using (37) to (39) we can write

$$\begin{aligned} E_t^{CB}(\widehat{i}_t) &= \begin{pmatrix} \theta_{iq} & \theta_{ig} & \theta_{i\eta} & \theta_{iv} & \theta_{ih} \end{pmatrix} \begin{pmatrix} E_t^{CB}(\widehat{\xi}_t) \\ \widehat{h}_t \end{pmatrix} = \widehat{i}_t^{TR} = \\ &= \begin{pmatrix} \delta_y & \delta_p \end{pmatrix} \begin{pmatrix} \theta_{yq} & \theta_{yg} & \theta_{y\eta} & \theta_{yv} & \theta_{yh} \\ \theta_{pq} & \theta_{pg} & \theta_{p\eta} & \theta_{pv} & \theta_{ph} \end{pmatrix} \begin{pmatrix} E_t^{CB}(\widehat{\xi}_t) \\ \widehat{h}_t \end{pmatrix} \\ &\quad -\varepsilon_t^{TR}. \end{aligned}$$

So, the value of $\widehat{h}_t$ to achieve the desired target rate has to be

$$\begin{aligned} \widehat{h}_t &= \frac{\delta_y\theta_{yq} + \delta_p\theta_{pq} - \theta_{iq}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} E_t^{CB}(\widehat{q}_t) + \frac{\delta_y\theta_{yg} + \delta_p\theta_{pg} - \theta_{ig}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} E_t^{CB}(\widehat{g}_t) \\ &\quad + \frac{\delta_y\theta_{y\eta} + \delta_p\theta_{p\eta} - \theta_{i\eta}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} E_t^{CB}(\widehat{\eta}_t) + \frac{\delta_y\theta_{yv} + \delta_p\theta_{pv} - \theta_{iv}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} E_t^{CB}(\widehat{v}_t) \\ &\quad - \frac{1}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} \varepsilon_t^{TR}. \end{aligned} \tag{42}$$

Substituting $\widehat{h}_t$ back into (37) to (39) we obtain the system as a function of the fundamental shocks of the economy ($\widehat{q}_t$, $\widehat{g}_t$, $\widehat{\eta}_t$, $\widehat{v}_t$, and ε_t^{TR}) after including the feedback rule.

To make rule (42) operational we have to specify the information set of the central bank at the time of the open market operation. A natural assumption is that the central bank has perfect knowledge about the monetary conditions of the economy but has poor information about the current state of real variables. Thus, we assume that only $\widehat{\eta}_t$ and $\widehat{v}_t$ are known by the central bank at the time of the policy decision so that $E_t^{CB}(\widehat{q}_t) = E_t^{CB}(\widehat{g}_t) = 0$ but $E_t^{CB}(\widehat{\eta}_t) = \widehat{\eta}_t$ and $E_t^{CB}(\widehat{v}_t) = \widehat{v}_t$. Then (42) becomes

$$\begin{aligned} \widehat{h}_t &= \frac{\delta_y\theta_{y\eta} + \delta_p\theta_{p\eta} - \theta_{i\eta}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} \widehat{\eta}_t + \frac{\delta_y\theta_{yv} + \delta_p\theta_{pv} - \theta_{iv}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} \widehat{v}_t \\ &\quad - \frac{1}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}} \varepsilon_t^{TR}. \end{aligned} \tag{43}$$

Substituting in (37) to (39) yields the system

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix} = \begin{pmatrix} \theta_{yq} & \theta_{yg} & \lambda_{y\eta} & \lambda_{yv} & \theta_{yTR} \\ \theta_{pq} & \theta_{pg} & \lambda_{p\eta} & \lambda_{pv} & \theta_{pTR} \\ \theta_{iq} & \theta_{ig} & \xi_{i\eta} & \xi_{iv} & \theta_{iTR} \end{pmatrix} \begin{pmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \varepsilon_t^{TR} \end{pmatrix} = \Theta \begin{pmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \varepsilon_t^{TR} \end{pmatrix},$$

where

$$\lambda_{kj} = \frac{\theta_{kj}\theta_{ih} - \theta_{kh}\theta_{i\lambda}}{\theta_{ih} - \delta_y\theta_{yh} - \delta_p\theta_{ph}}$$

and

$$\xi_{ij} = \delta_y\lambda_{yj} + \delta_p\lambda_{pj}$$

for $k = y$, p, i and $j = q$, g, η, v.

Let Γ be a diagonal matrix with the variances of the shocks

$$E\left[\begin{pmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \varepsilon_t^{TR} \end{pmatrix}\begin{pmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \varepsilon_t^{TR} \end{pmatrix}'\right] = \Gamma.$$

Then, we can compute the variance-covariance matrix of the endogenous variables implied by the model.

$$\Sigma = E\left[\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix}\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix}'\right] = \Theta\Gamma\Theta' = \Phi\Phi',$$

and write the system in terms of unit variance shocks (e_t^q, e_t^g, e_t^{TR}) as

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix} = \Phi \begin{pmatrix} \widehat{e}_t^q \\ \widehat{e}_t^g \\ \widehat{e}_t^\eta \\ \widehat{e}_t^v \\ \widehat{e}_t^{TR} \end{pmatrix}.$$

For the simulations below, we will make the coefficients of the Taylor rule equal to

$$\delta_y = \delta_p = 0,$$

so there is no feedback from the state of the economy. In this case, the coefficients of the matrix Φ are:

Table 2
Impact coefficients in the model ($\times 100$)

	Shocks				
Variable	$\widehat{e}_t^q$	$\widehat{e}_t^g$	$\widehat{e}_t^\eta$	$\widehat{e}_t^v$	$\widehat{e}_t^{TR}$
$\widehat{y}_t$	0.1403	0.7156	0.0000	0.0000	0.0509
$\widehat{p}_t$	-0.1819	0.1095	0.0000	0.0020	0.1696
$\widehat{i}_t$	-0.4429	0.2666	0.0000	0.0000	-0.4000

This table shows the impact coefficients of unit variance shocks. The central bank is assumed to know η and v at the time of the policy decision and the variance decomposition is:

Table 3
Variance decomposition in the model

	Shocks				
Variable	$\widehat{e}_t^q$	$\widehat{e}_t^g$	$\widehat{e}_t^\eta$	$\widehat{e}_t^v$	$\widehat{e}_t^{TR}$
$\widehat{y}_t$	0.0368	0.9582	0.0000	0.0000	0.0048
$\widehat{p}_t$	0.4480	0.1623	0.0000	0.0000	0.3895
$\widehat{i}_t$	0.4592	0.1663	0.0000	0.0000	0.3744

This table shows the variance decomposition of unit variance shocks. The central bank is assumed to know η and v at the time of the policy decision

From this table we see the main shock affecting output in the model is the government expenditure shock. The monetary shock is important in determining the variability of interest rates and prices while the technology shock also affects prices and the interest rate. Furthermore, because η and v are included in the information set of the central bank at the time of policy decisions, these shocks do not affect endogenous variables anymore and the model behaves as if only three shocks drive total volatility. Thus, in the identification schemes below we will be looking for a supply shock ($\widehat{e}_t^q$), a real demand shock ($\widehat{e}_t^g$), and a nominal demand shock ($\widehat{e}_t^{TR}$). Also notice that long run restrictions are of no use for identification purposes here.

3 Identification of shocks in the model

The coefficients in the matrix Φ indicate the percentage deviation of y, p and i induced by a change of one standard deviation in q, g, and e^{TR}. An empirical researcher would construct Σ from data on output, prices and interest rates. If we assume that the model is the data generating process his estimated variance covariance matrix of the variables of interest would converge asymptotically to the true Σ. As we are only interested in a theoretical exercise let us just assume that the econometrician can use the true Σ to recover the structural shocks. Identification of structural shocks means using the estimated Σ to find a particular Φ, such that $\Phi\Phi' = \Sigma$ by imposing enough restrictions on Φ to make it unique.[7] The exercise to be carried out here is to review several identification schemes to see how close they are in identifying the true responses to shocks.

3.1 Zero-Restrictions

The Cholesky decomposition of Σ delivers a unique lower triangular matrix Φ_{CHOL}

$$\Phi_{CHOL} = \begin{bmatrix} \phi_{p1} & 0 & 0 \\ \phi_{y1} & \phi_{y2} & 0 \\ \phi_{i1} & \phi_{i2} & \phi_{i3} \end{bmatrix}, \tag{44}$$

[7] See Christiano, Eichenbaum and Evans [3] as a general survey about identification schemes applied in the literature.

so,

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix} = \Phi_{CHOL} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{mt} \end{pmatrix}. \tag{45}$$

In this expression u_{it}, $i = 1, 2, m$ represent the three structural shocks estimated through this identification scheme. This procedure can be used if one accepts a recursive structure meaning that only shock u_1 affects contemporaneously the first variable, shocks u_1 and u_2 affect the second variable, and so on. It is usually assumed that shocks u_1 and u_2 are in the information set of the central bank who can react contemporaneously to them in setting the interest rate. In this sense, the shock u_m is interpreted as a monetary policy shock which does not affect prices and output within the same period.[8] From the last row of this system a feedback rule of the central bank relating $\widehat{i}$, $\widehat{y}$, and $\widehat{p}$ can be derived as

$$\widehat{i}_t = \vartheta_y \widehat{y}_t + \vartheta_p \widehat{p}_t + \phi_{im} u_{mt}. \tag{46}$$

To obtain this policy rule, use the first row of (45) to substitute for u_{1t}

$$u_{1t} = \frac{\widehat{y}_t}{\phi_{y1}}, \tag{47}$$

in the second row to get the value of u_{2t}

$$u_{2t} = \frac{1}{\phi_{p2}} \widehat{p}_t - \frac{\phi_{p1}}{\phi_{p2}\phi_{y1}} \widehat{y}_t. \tag{48}$$

Then substitute both u_{1t} and u_{2t} in the third row

$$\widehat{i}_t = \frac{\phi_{i1}\phi_{p2} - \phi_{i2}\phi_{p1}}{\phi_{y1}\phi_{p2}} \widehat{y}_t + \frac{\phi_{i2}}{\phi_{p2}} \widehat{p}_t + \phi_{im} u_m. \tag{49}$$

Hence

$$\vartheta_y = \frac{\phi_{i1}\phi_{p2} - \phi_{i2}\phi_{p1}}{\phi_{y1}\phi_{p2}}; \ \vartheta_p = \frac{\phi_{i2}}{\phi_{p2}}. \tag{50}$$

Table 4 shows the estimated impact coefficients that results from applying the Cholesky decomposition.

Table 4
Estimated impact coefficients ($\times 100$): Cholesky decomposition

	Shocks		
Variable	u_{1t}	u_{2t}	u_{mt}
$\widehat{y}_t$	0.7310	0.0000	0.0000
$\widehat{p}_t$	0.0841	0.2585	0.0000
$\widehat{i}_t$	0.1480	0.1140	0.6264

This table shows the impact coefficients of unit variance shocks computed from a Cholesky decomposition of the variance covariance matrix

[8] See Christiano, Eichenbaum and Evans [4] as a prominent example of that approach.

This decomposition generates the estimated coefficients for the rule:

$$\vartheta_y = 0.15 \text{ and } \vartheta_p = 0.44.$$

Although the central bank in this economy does not follow a contemporaneous feedback rule (i.e. the true coefficients of the rule are $\delta_y = \delta_p = 0$), the imposition of the triangular structure in the decomposition identifies such a rule from the contemporaneous endogenous correlation between output, prices and the interest rate. Notice the estimated rule responds more aggressively to inflation than to output as is usually demanded to provide stability in monetary models with Taylor rules. Also notice the estimated impact coefficients of the monetary shock differ from the true ones.

In fact, the misspecification of the Taylor rule is a general phenomenon. Table 5 shows the pairs of estimated coefficients (ϑ_y;ϑ_p) for different values of the feedback parameters δ_y and δ_p.

Table 5
Estimated coefficients in Taylor rule

	δ_p			
δ_y	0.0	0.5	1.0	1.5
0.00	(0.15;0.44)	(0.12;0.87)	(0.09;1.20)	(0.08;1.43)
0.25	(0.14;0.51)	(0.11;0.93)	(0.09;1.24)	(0.07;1.46)
0.50	(0.14;0.58)	(0.11;0.98)	(0.08;1.27)	(0.07;1.49)
0.75	(0.13;0.65)	(0.10;1.03)	(0.08;1.31)	(0.07;1.52)
1.00	(0.13;0.72)	(0.10;1.08)	(0.08;1.35)	(0.06;1.55)

This table shows the estimated coefficients (ϑ_y;ϑ_p) of the Taylor rule for different values of the feedback parameters δ_y and δ_p.

We observe this scheme underestimates the coefficient on output and overestimates the coefficient on inflation. Also, notice the estimated rule assumes the central bank responds contemporaneously to output and prices while in the model it only responds to some fundamental shocks affecting these variables.

Finally, we can compute the variance decomposition associated with this identification scheme:

Table 6
Estimated variance decomposition: Cholesky decomposition

	Shocks		
Variable	u_{1t}	u_{2t}	u_{mt}
$\widehat{y}_t$	1.0000	0.0000	0.0000
$\widehat{p}_t$	0.0957	0.9042	0.0000
$\widehat{i}_t$	0.0512	0.0304	0.9182

This table shows estimated variance decomposition of unit variance shocks computed from a Cholesky decomposition of the variance covariance matrix.

Because shocks u_1 and u_2 cannot be associated with fundamental disturbances we cannot comment on them. However, we observe how this identification scheme completely misses the importance of the monetary shock, the only one the scheme is designed to identify, in generating volatility. By construction it underestimates its contribution for output and prices. It also overestimates its contribution to interest rate variability.

3.2 Sign Restrictions

Another possibility is to estimate the matrix Φ directly using sign restrictions implied by theoretical arguments. Instead of imposing zero-restrictions on Φ, one can use the following approach proposed by Canova and de Nicoló [1]. In this approach orthogonalization and identification are separated from each other. First, an eigenvector decomposition orthogonalizes the shocks. Then the matrix of eigenvectors V multiplied by the diagonal matrix of square roots of the eigenvalues $D^{1/2}$ is taken as a candidate Φ. Finally, $VD^{1/2}$ is multiplied by different rotation matrices

$$Q_{m,n}(\omega) = \begin{bmatrix} 1 & 0 & \cdots & 0 & & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cos(\omega) & \cdots & -\sin(\omega) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sin(\omega) & \cdots & \cos(\omega) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{51}$$

where the subscript (m, n) indicates that only row m and column n are rotated by the angle $\omega \in [0, \pi]$. Each resulting $\Phi_{m,n}(\omega) = VD^{1/2}Q_{m,n}(\omega)$ with economically sensible impact coefficients is selected as candidate identification scheme.

It turns out the model proposed here delivers such signs for the effect on the variables of interest of an array of shocks. In particular, we could write the system as

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \end{pmatrix} = \begin{bmatrix} \phi_{ys} & \phi_{yd} & \phi_{ym} \\ \phi_{ps} & \phi_{pd} & \phi_{pm} \\ \phi_{is} & \phi_{id} & \phi_{im} \end{bmatrix} \begin{pmatrix} u_{st} \\ u_{dt} \\ u_{mt} \end{pmatrix},$$

In this identification, u_s is interpreted as a supply shock. This means it should affect positively output, and negatively prices and interest rates (as e_t^q did in the model). Second, u_d can be interpreted as a demand shock so that it affects positively output, prices and interest rates (as e_t^g in the model). Finally, u_m should be a monetary shock impacting output and prices positively while interest rates negatively (as e_t^{TR} did). These theoretically implied signs will be used below to construct series for orthogonalized shocks with an economic content. When we use this method, we divide the interval [0, π] in 500 points. Since there are 3 possible rotations in the 3×3 matrix Σ, this means 1500 possible combinations.

Of all possible rotations, 460 gave the right signs. Thus, Table 7 presents the average of them while Table 8 shows the standard deviation of the estimations as a measure of how reliable this method is in capturing the right coefficients.

Table 7
Estimated impact coefficients ($\times 100$): Sign restrictions

	Shocks		
Variable	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.2203	0.6576	0.1920
$\widehat{p}_t$	-0.1686	0.1071	0.1544
$\widehat{i}_t$	-0.3095	0.4098	-0.3484

This table shows the impact coefficients of unit variance shocks computed from the variance covariance matrix using a sign restriction scheme.

Table 8
Std. deviation of estimated impact coefficients: Sign restrictions

	Shocks		
Variable	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.0850	0.0000	0.0967
$\widehat{p}_t$	0.0683	0.0000	0.0741
$\widehat{i}_t$	0.1530	0.0000	0.1370

This table shows the standard deviation of the impact coefficients of unit variance shocks computed from the variance covariance matrix using a sign restriction scheme.

Table 9 shows the average variance decomposition across the identifications that satisfy the sign restrictions. Table 10 summarizes the variability in these identifications

Table 9
Estimated variance decomposition: Sign restrictions

	Shocks		
Variable	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.1043	0.8091	0.0864
$\widehat{p}_t$	0.4477	0.1552	0.3970
$\widehat{i}_t$	0.2788	0.3930	0.3280

This table shows the estimated variance decomposition of unit variance shocks computed from the variance covariance matrix using a sign restriction scheme.

Table 10
Std. deviation of estimated variance decomposition: Sign restrictions

	Shocks		
Variable	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.0640	0.0000	0.0640
$\widehat{p}_t$	0.2849	0.0000	0.2849
$\widehat{i}_t$	0.2041	0.0000	0.2041

This table shows the standard deviation of the estimated variance decomposition of unit variance shocks computed from the variance covariance matrix using a sign restriction scheme.

If we compare the numbers in Tables 7 and 9 with those in Tables 2 and 3 we see this identification method is able to approximate both the impact coefficients and the variance decomposition very well on average. However, Tables 8 and 10 warn that may be instances of large deviations with respect to the true coefficients.

4 Identification of Shocks in the Data

In this section we apply the identification schemes outlined above to the data. For that, we estimate a VAR(4) on the log of real GDP (y_t), the log of the consumer price index CPI (p_t), and the 3-month Treasury Bill rate (i_t). The details about these series can be found in Appendix A. We use the 3-month T-Bill rate instead of the usual Federal funds rate for several reasons. First, we want to use an interest rate that closely affects the economic activity, which is the implied assumption in our model. For that, a three month rate is a better choice than a daily rate. In this sense, we are also including the transmission of shocks through the yield curve usually not considered in the empirical literature. Second, the contemporaneous correlations that we want to analyze in this paper should be stronger in a longer maturity rate not so closely controlled by the central bank.

After estimating the VAR, the strategy is to decompose the variance covariance matrix of disturbances. The next sections apply the identification schemes reviewed in the previous section.

4.1 Cholesky Decomposition

First, we use a Cholesky decomposition. The resulting matrix of coefficients and the variance decomposition are included in the next two tables.

Table 11
Estimated coefficients (Data)
Cholesky identification scheme

	Shocks		
	u_{1t}	u_{2t}	u_{mt}
$\widehat{y}_t$	0.7195	0.0000	0.0000
$\widehat{p}_t$	0.0241	0.3226	0.0000
$\widehat{i}_t$	0.1612	0.1605	0.6086

Table 12
Variance decomposition (Data)
Choleski identification scheme

	Shocks		
Variance of	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	1.0000	0.0000	0.0000
$\widehat{p}_t$	0.0055	0.9945	0.0000
$\widehat{i}_t$	0.0616	0.0610	0.8774

Using (50) we find the coefficients of the feedback rule to be $\vartheta_y = 0.21$ and $\vartheta_p = 0.50$. We see that both coefficients are positive so that the central bank reacts to increases in output or inflation above trend by raising the interest rate. We can work out the conditions on the matrix Σ which ensure this result. From

$$\Sigma \equiv \begin{bmatrix} \sigma_{yy} & \sigma_{yp} & \sigma_{yr} \\ \sigma_{py} & \sigma_{pp} & \sigma_{pr} \\ \sigma_{ry} & \sigma_{rp} & \sigma_{rr} \end{bmatrix} = \begin{bmatrix} \phi_{p1} & 0 & 0 \\ \phi_{y1} & \phi_{y2} & 0 \\ \phi_{r1} & \phi_{r2} & \phi_{r3} \end{bmatrix} \begin{bmatrix} \phi_{p1} & \phi_{y1} & \phi_{r1} \\ 0 & \phi_{y2} & \phi_{r2} \\ 0 & 0 & \phi_{r3} \end{bmatrix}$$

and after some tedious algebra it is easy to see that the coefficients in expression (50) implies

$$sign\left(\vartheta_y\right) = sign\left[\frac{\sigma_{yr}\sigma_{pp} - \sigma_{yp}\sigma_{pr}}{\sigma_{pp} - \left(\sigma_{yp}\right)^2 / \sigma_{yy}}\right], \tag{52}$$

while

$$sign\left(\vartheta_p\right) = sign\left[\sigma_{pr}\sigma_{yy} - \sigma_{yp}\sigma_{yr}\right]. \tag{53}$$

Given that σ_{pp} and σ_{yy} are both positive, a sufficient condition for both ϑ_y and ϑ_p to be positive is to have the covariance between output and prices (σ_{yp}) small together with the covariance between prices and the interest rate (σ_{pr}) and output and the interest rate (σ_{yr}) both positive. Notice this result is completely independent on the existence of a Taylor rule. As long as the data shows these covariances, which is the case here, a triangular scheme will always estimate an interest rate equation with the form of such a rule. Also notice that, as the theoretical exercise has shown in the previous section, this estimation may misinterpret the comovement between output, prices and the interest rate as evidence for the existence of a monetary policy rule while this comovement may be driven by the joint responses of these variables to all fundamental shocks.

4.2 Sign-Shape Restrictions

We use the sign restrictions approach to identify structural shocks from the data. Again, we use 500 different angles for each of the 3 rotation matrices. As in the theoretical part, many of the rotations lead to valid impact coefficients. We find 872 rotations that produce the correct signs of the impact responses. In order to narrow down the number of identified shocks we can use restrictions on the shape of the implied impulse responses. We use the condition that reasonable impulse responses should not change sign in the first periods. Applying this condition we find 20 rotations consistent with the criterion of no sign-switch implying mean responses and standard deviations as shown in Tables 13 and 14.

Table 13
Average estimated coefficients (Data)
Sign and shape restrictions identification scheme

	Shocks		
	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.1539	0.6348	0.3016
$\widehat{p}_t$	-0.3088	0.0714	0.0646
$\widehat{i}_t$	-0.1728	0.4375	-0.4481

Table 14
Standard deviations of estimated coefficients (Data)
Sign and shape restrictions identification scheme

	Shocks		
	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.0000	0.0000	0.0000
$\widehat{p}_t$	0.0000	0.0000	0.0000
$\widehat{i}_t$	0.0000	0.0000	0.0000

The sign and shape restrictions approach is able to estimate very precisely the impact coefficients from the data. This high precision is also reflected in the low standard deviations of the variance decomposition:

Table 15
Average variance decomposition (Data)
Sign and shape restrictions scheme

	Shocks		
Variance of	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.0458	0.7785	0.1757
$\widehat{p}_t$	0.9111	0.0487	0.0402
$\widehat{i}_t$	0.0709	0.4534	0.4757

Table 16
Standard deviation of variance decomposition (Data)
Sign and shape restrictions scheme

	Shocks		
Variance of	u_{st}	u_{dt}	u_{mt}
$\widehat{y}_t$	0.0287	0.0000	0.0287
$\widehat{p}_t$	0.0598	0.0000	0.0598
$\widehat{i}_t$	0.0584	0.0000	0.0584

4.3 Estimated Impulse Response Functions and Monetary Shocks

More informative about the comparison between the two estimations is to look at the impulse response functions as well as the series for the monetary shocks that they imply. Figure 1 presents the responses of output, prices and the interest rate derived from a one standard deviation change in the monetary shock. For comparison with previous studies we also include the responses computed as in Christiano, Eichenbaum and Evans [4], denoted as CEE in the figure. They used a Cholesky decomposition of a 6 variable VAR(4) ordered as real GDP, price deflator, commodity prices, Federal funds rate, nonborrowed reserves and total reserves. From the graph we observe that the sign restriction estimation provides reasonable magnitudes as compared with the two triangular schemes. However, it does not present the price puzzle behavior typical of Cholesky decompositions. Notice the sign restriction estimation predicts a sizable contemporaneous response of output to a monetary shock at the same time that prices respond sluggishly.

We can compare the series for shocks estimated through the three methods. These series are reported in Figure 2. The correlation between the monetary shock estimated through the Cholesky decomposition of the 3-variable VAR and the one estimated using sign restrictions is 0.84. The correlation of the shocks estimated with the two triangular schemes is 0.71 while the correlation between the CEE shocks and the ones computed with the sign restriction is 0.65.

Next, we provide evidence that links the monetary policy shock to one of its possible sources, the central bank. For that, we regress the Federal Reserve holdings of government assets (FEDSEC) as well as nonborrowed reserves (NBR) on the shocks just estimated together with four lags of the dependent variable. Table 17 shows the results of the OLS regressions in the columns labelled "Sign restriction".

Table 17
Regression of policy variables on shocks

	Dependent variable, $k(t)$					
	Sign restriction		Cholesky		CEE	
	FEDSEC	NBR	FEDSEC	NBR	FEDSEC	NBR
constant	0.1167	-0.1289	0.0899	-0.1493	0.0601	-0.2515
	(0.846)	(-0.477)	(0.663)	(-0.547)	(0.439)	(-0.974)
$k(t-1)$	0.8083	1.2745	0.8321	1.2808	0.8281	1.2939
	(8.212)	(6.352)	(8.665)	(6.367)	(8.674)	(6.209)
$k(t-2)$	0.2151	-0.5096	0.1909	-0.5189	0.1953	-0.5391
	(1.777)	(-1.981)	(1.624)	(-1.987)	(1.702)	(-1.992)
$k(t-3)$	-0.1051	0.3408	-0.1009	0.3531	-0.0906	0.3577
	(-1.007)	(2.186)	(-0.966)	(2.220)	(-0.873)	(2.137)
$k(t-4)$	-0.0300	-0.1220	-0.0220	-0.1325	-0.0423	-0.1300
	(-0.432)	(-1.540)	(-0.322)	(-1.677)	(-0.604)	(-1.570)
Demand	-0.0247	-0.6243				
	(-0.127)	(-2.357)				
Supply	0.1705	0.2384				
	(1.161)	(0.877)				
Monetary	0.5571	0.8985	0.4220	1.4181	0.2338	1.2799
	(2.525)	(2.498)	(1.595)	(3.057)	(1.101)	(3.726)

Results from OLS regression of variable $k(t)$ on four lags and the fundamental shocks. T-statistics based on White's heteroskedasticity consistent variance matrix in parenthesis.

A monetary shock that lowers interest rates generates a statistically significant increase in both, the Federal Reserve holdings of government securities as well as nonborrowed reserves (both coefficients with p-values of 0.01). This finding supports the idea of the Fed reducing interest rates by buying securities in the open market. Furthermore, the monetary shock is the only one influencing the government assets in the hands of the Fed. On the other hand, nonborrowed reserves are also affected by the demand shock. A demand shock that raises output, interest rates and prices, also reduces nonborrowed reserves. From equation (14), we see that an increase in expenditures has a negative impact on reserves through the withdrawing of cash. Supply shocks do not affect either of the policy variables.

The table also shows the estimations using the monetary shocks identified by the Cholesky decomposition and the CEE scheme. In these cases, the monetary shocks do not significantly affect the Federal Reserve holdings of government securities and only have an impact on nonborrowed reserves. Thus, it is hard to justify these shocks measure innovations originating from the central bank.

One advantage of the sign restriction approach is that it allows us to estimate the response of the variables of interest to other shocks. Figure 3 shows the responses of output, prices and interest rates to the three shocks considered: a supply shock, a demand shock, and the monetary shock. A positive supply shock persistently raises output while reducing prices and the interest rate. A positive demand shock increases output, prices and the interest rate. A positive monetary shock that raises the interest rate, lowers output and prices.

Finally, we can plot the estimated series for the three orthogonal shocks estimated through the sign restrictions imposed by the model: the supply shock, the demand shock, and the monetary shock. This is done in Figure 4. Notice the supply shock is relatively small as compared to the demand and monetary shocks.

5 Discussion

The motivation of this paper has been the apparent separation between theoretical and applied work in the analysis of the monetary transmission mechanism. This separation is manifested most clearly along four dimensions: (i) the different set of variables assumed to be controlled by central banks in theoretical models and the ones they can directly influence in reality, (ii) the different set of variables used to analyze the monetary transmission mechanism in theoretical models as compared to the set used in empirical work, (iii) the lack of a sensible explanation as of what monetary policy shocks mean, and (iv) the need of a set of identifying restrictions fully generated by theory.

We believe the dynamic stochastic general equilibrium model constructed in this paper may be useful in responding to these challenges. The strategy has been to make a step forward towards incorporating the way monetary policy is implemented in an otherwise conventional model. The consequences of this step are staggering. On the one hand, the model helps us along the four dimensions mentioned in the previous paragraph. In the model, the central bank sets targets on a market interest rate and exerts its control by managing the supply of reserves in the economy. Thus, the model makes the connection between narrow definitions of money and broad ones, a distinction often emphasized in the empirical literature. Another distinction this literature stresses is between short term rates, close to central bank control but far from affecting economic activity directly, and long term rates farther from control by the monetary authority but directly influencing economic activity. Although the model does not have a term structure we believe it can be helpful in this dimension because it emphasizes the distinction between operating targets and market-determined rates and provides explanations as of why they may not be equal.

In fact, the difference between the value for the interest rate the central bank should have targeted and the value determined by the market is the crux of the matter for this paper because it is this difference what is typically used to measure monetary policy shocks. We argue that this difference is a consequence of the contemporaneous concurrence of a variety of shocks. Furthermore, we use the model to identify which part is attributable to changes in the monetary environment of the economy and, in this way, provide a rationale as of why these shocks occur and what they mean.

One consequence of the contemporaneous correlation between endogenous variables is that it may lead to a misidentification of the fundamental shocks hitting the economy. In particular, we provide an example where the endogenous reaction of output, prices and interest rates to fundamental shocks can induce cross-correlations which might be erroneously misinterpreted as a Taylor rule. This erroneous interpretation of the cross-correlations would treat the error term of the supposed Taylor rule as a monetary 'policy' shock while in the example it is just a combination of fundamental shocks. Furthermore, we show that stochastic deviations from a Taylor rule by the central bank may not be the only source of monetary shocks. In fact, we demonstrate how a variety of variables produce the same type of responses we associate with monetary disturbances so that an econometrician provided with data generated from the model would not be able to distinguish between them. These other variables are variations in autonomous factors, or random changes in the cash-to-deposit ratio. One implication of this finding is that instead of estimating Taylor rules to identify monetary shocks, one alternative strategy could be concentrating on the qualitative responses of endogenous variables to these types of shocks. Of course, in order to implement this strategy one needs to compare the responses of the economy to monetary shocks with the responses to other fundamental shocks, a point commonly forgotten in the theoretical literature. This is why we have included shocks stemming from the demand and supply of goods.

Finally, the incorporation of the implementation of monetary policy makes money nonneutral without the need to impose additional frictions as limited participation in the asset market or sticky prices. The only features needed is a reason to hold money in general equilibrium (here provided by the cash-in-advance constraint in consumption goods purchases), the need of firms to finance their wage bill before getting sales receipts, and a reserve requirement.

Further research can be built on the model presented here. One simplifying assumptions that need to be changed is the absence of a state variable such as capital. This is important because in an economy with capital as a state variable the interest rate target will be varying over time. However, we believe the same type of results will still hold. In any case, optimal policy issues could be readdressed with a more general model. Is there an optimal way to set the target? Does this policy mimic a Taylor rule for the target? The rich structure of the model also allows to address questions like: Given that the operational targets are volatile at the time of policy decisions, does this fact have any implications about the design of a rule in order to achieve an optimal policy? Do institutional aspects like the level of reserve requirements, the volatility of float and treasury deposits, the customs of using cash versus checks, etc. matter for how well the central bank can target the interest rate and therefore for the conduct of monetary policy? In summary, we think that the model presented here opens up a path to readdress many questions in monetary policy analysis and to ask many new ones.

References

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A Data

The data for the calibration is taken from the FRED database of the Federal Reserve Bank of St. Louis. We use the following series (acronyms in parenthesis): Output (y_t) is the "Real Gross Domestic Product" (GDPC1), the price level (P_t) is the "CPI price index" (CPIAUCSL), the interest rate (i_t) is the "3-Month Treasury Bill: Secondary Market Rate" (TB3MS) and government expenditures (g_t) are "Real Government Consumption Expenditures and Gross Investment" (GCEC1). Output and government expenditures are divided by "Civilian Noninstitutional Population" (CNP16OV) to make variables in per capita terms and logged. Employment (n_t) is computed as "Civilian Employment: Sixteen Years and Over" (CE16OV) divided by population. We estimate a VAR(4) of y_t, P_t, and i_t in levels and store the variance-covariance matrix of the residuals.

We also use the Flow of Funds data of the Board of Governors of the Federal Reserve System to compute series for the financial variables. Cash (X_t) is measured as "Currency Outside Banks" (FL713125005.Q). Deposits (D_t) is the sum of "Checkable Deposits in Commercial Banks" (FL763129205.Q) plus "Small Time and Savings Deposits in Commercial Banks" (FL763131005.Q) plus "Large Time deposits in Commercial Banks" (FL763135005.Q) plus "Checkable Deposits in Savings Banks" (FL443127005.Q) plus "Small Time and Savings Deposits in Savings Banks" (FL443131005.Q) plus "Large Time deposits in Savings Banks" (FL443135005.Q). Reserves (R_t) is computed as "Depository Institutions Reserves" (FL713113000.Q) plus "Vault Cash of Commercial Banks" (FL723025000.Q). Autonomous factors (V_t) are equal to "Federal Government deposits at the Monetary Authority" (FL713123105.Q) minus "Federal Reserve Float" (FL713022003.Q). Government bonds (B_t) is equal to "Total Treasury Securities" (FL893061505.Q) minus "Treasury Securities in State and Local Governments" (FL213061105.Q) "Treasury Securities in Foreign Hands" (FL263061105.Q) minus "Treasury Securities in Monetary Authority" (FL713061100.Q). To construct the series we start with the initial value of the stocks (table *ltab*) and add the seasonally adjusted increments (table *atab*).

For the identification of monetary policy shock with the CEE scheme, we use data on the "Effective Federal Funds Rate" (FEDFUNDS), "Nonborrowed reserves" (BOGNONBR), "Total Reserves" (TRARR), and a commodity price index.

The sample covers the period from the first quarter of 1960 until the first quarter of 2003. This is the period with observations for all variables. When only monthly data is available, the numbers are averaged to obtain quarterly series. For all empirical applications, the data was detrended using a linear trend.

B Calibration

Some parameters are taken directly from the data. This is the case of the reserve requirement (ρ) computed as the average reserves over deposits. Also, we use the sample averages of the government expenditure over output (g_t/y_t), the cash-to-deposit ratio (X_t/D_t), the fraction of the monetary base over total government liabilities (H_t/A_t), and the fraction of autonomous factors over total government liabilities (V_t/A_t) to approximate the equivalent ratios at the steady state, g_{ss}/y_{ss}, η_{ss}, h_{ss}, and v_{ss}, respectively.

The average technology shock (q_{ss}) is arbitrarily fixed at 1. The time discount (β) is set so that the steady state value of the interest rate matches the average interest rate (i_t) over the sample. The disutility of labor (Ψ) is computed so that labor at the steady state matches the average of employment (n_t) and the capital elasticity of output (α) is determined as the fraction of income remunerating labor equal to 0.33. Finally, the rest of parameters (γ, ψ, α_g, σ_q, σ_g, σ_η, σ_v, and σ_{TR}) are set to approximate the variance-covariance matrix of the model to that of the residuals found in the data. This matrix is

$$\Sigma = \begin{bmatrix} 0.5177 & 0.0173 & 0.1160 \\ 0.0173 & 0.1047 & 0.0557 \\ 0.1160 & 0.0557 & 0.4222 \end{bmatrix}.$$

The next two tables show the values assigned to the parameters of the model.

Table A.1
Values for parameters in preferences, technology and reserve requirement

Symbol	Value	Meaning
β	0.99	Discount factor
γ	3.40	Inverse of the intertemporal elasticity of consumption
ψ	1.90	Inverse of the wage elasticity of labor supply
Ψ	13.3	Disutility of labor
α	0.33	Capital elasticity of output
α_g	0.15	Government expenditure elasticity of output
ρ	0.03	Reserve requirement

Table A.2
Values for distribution of shocks and monetary policy

Symbol	Value	Meaning
q_{ss}	1.0000	Average technology shock
σ_q	0.0015	Std. deviation of technology shock
g_{ss}	0.1171	Average government shock
σ_g	0.0350	Std. deviation of government shock
η_{ss}	0.0855	Average cash demand shock
σ_η	0.0001	Std. deviation of cash demand shock
v_{ss}	0.0015	Average autonomous factor shock
σ_v	0.0130	Std. deviation of autonomous factor shock
h_{ss}	0.2541	Average official reserves
δ_y	0.0000	Output coefficient in Taylor rule
δ_p	0.0000	Inflation coefficient in Taylor rule
σ_{TR}	0.0040	Std. deviation of monetary policy shock

C Log-linearization

The first order conditions of the system are

$$\frac{1}{p_t c_t^\gamma} = \beta(1+i_t)E_t\left[\left(\frac{1+v_{t+1}}{1+v_t}\right)\frac{1}{p_{t+1}c_{t+1}^\gamma}\right], \tag{54}$$

$$(1-\alpha)q_t g_t^{\alpha_g} = (1+i_t)\,\Psi n_t^{\alpha+\psi} c_t^\gamma, \tag{55}$$

$$y_t = q_t g_t^{\alpha_g} n_t^{1-\alpha}, \tag{56}$$

market clearing in the goods market,

$$c_t + g_t = y_t, \tag{57}$$

and market clearing in the reserve market,

$$h_t = v_t + \left(\frac{\rho+\eta_t}{1+\eta_t}\right) p_t c_t. \tag{58}$$

C.1 Steady state

At the steady state, all real and normalized nominal variables are constant. Denote such levels by the subindex ss. The system (54) to (58) becomes

$$i_{ss} = \frac{1}{\beta} - 1,$$

$$(1+i_{ss})\Psi c_{ss}^{\gamma} n_{ss}^{\alpha+\psi} = (1-\alpha) q_{ss} g_{ss}^{\alpha_g},$$
$$y_{ss} = q_{ss} g_{ss}^{\alpha_g} n_{ss}^{1-\alpha},$$
$$c_{ss} = y_{ss} - g_{ss},$$

and

$$h_{ss} = v_{ss} + \left(\frac{\rho+\eta_{ss}}{1+\eta_{ss}}\right) p_{ss} c_{ss}.$$

C.2 Linearization

In the linearization, hat variables correspond to percentage deviations with respect to the steady state. The exception is the interest rate which is measured as the difference with respect to the value at the steady state. The linearized system of equations is as follows:

$$\widehat{p}_t + \frac{1}{1+i_{ss}}\widehat{i}_t + \gamma\widehat{c}_t = \frac{v_{ss}}{1+v_{ss}}\widehat{v}_t + E_t\left[\widehat{p}_{t+1} + \gamma\widehat{c}_{t+1} - \frac{v_{ss}}{1+v_{ss}}\widehat{v}_{t+1}\right],$$

$$\frac{1}{1+i_{ss}}\widehat{i}_t + (\psi+\alpha)\widehat{n}_t + \gamma\widehat{c}_t = \widehat{q}_t + \alpha_g\widehat{g}_t,$$
$$\widehat{y}_t - (1-\alpha)\widehat{n}_t = \widehat{q}_t + \alpha_g\widehat{g}_t,$$
$$y_{ss}\widehat{y}_t - c_{ss}\widehat{c}_t = g_{ss}\widehat{g}_t,$$

and

$$\widehat{p}_t + \widehat{c}_t = \frac{(\rho-1)\eta_{ss}}{(\rho+\eta_{ss})(1+\eta_{ss})}\widehat{\eta}_t - \frac{v_{ss}}{(h_{ss}-v_{ss})}\widehat{v}_t + \frac{h_{ss}}{(h_{ss}-v_{ss})}\widehat{h}_t.$$

This provides a system of five unknown linear functions ($\widehat{y}_t$, $\widehat{p}_t$, $\widehat{i}_t$, $\widehat{n}_t$, $\widehat{c}_t$) of the shocks ($\widehat{q}_t$, $\widehat{g}_t$, $\widehat{\eta}_t$, $\widehat{v}_t$) and the policy variable ($\widehat{h}_t$).

For the case of *i.i.d.* shocks the system becomes

$$\widehat{p}_t + \frac{1}{1+i_{ss}}\widehat{i}_t + \gamma\widehat{c}_t = \frac{v_{ss}}{1+v_{ss}}\widehat{v}_t \tag{59}$$

$$\frac{1}{1+i_{ss}}\widehat{i}_t + (\psi+\alpha)\widehat{n}_t + \gamma\widehat{c}_t = \widehat{q}_t + \alpha_g\widehat{g}_t, \tag{60}$$

$$\widehat{y}_t - (1-\alpha)\widehat{n}_t = \widehat{q}_t + \alpha_g\widehat{g}_t, \tag{61}$$

$$y_{ss}\widehat{y}_t - c_{ss}\widehat{c}_t = g_{ss}\widehat{g}_t, \tag{62}$$

and

$$\widehat{p}_t + \widehat{c}_t = \frac{(\rho-1)\eta_{ss}}{(\rho+\eta_{ss})(1+\eta_{ss})}\widehat{\eta}_t - \frac{v_{ss}}{(h_{ss}-v_{ss})}\widehat{v}_t + \frac{h_{ss}}{(h_{ss}-v_{ss})}\widehat{h}_t. \tag{63}$$

or

$$\Phi \begin{bmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \\ \widehat{n}_t \\ \widehat{c}_t \end{bmatrix} = \Omega \begin{bmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\Psi}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \widehat{h}_t \end{bmatrix}$$

with

$$\Phi = \begin{bmatrix} 0 & 0 & \frac{1}{1+i_{ss}} & \psi+\alpha & \gamma \\ y_{ss} & 0 & 0 & 0 & -c_{ss} \\ 1 & 0 & 0 & -(1-\alpha) & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{1+i_{ss}} & 0 & \gamma \end{bmatrix}$$

and

$$\Omega = \begin{bmatrix} 1 & \alpha_g & -1 & 0 & 0 & 0 \\ 0 & g_{ss} & 0 & 0 & 0 & 0 \\ 1 & \alpha_g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(\rho-1)\eta_{ss}}{(\rho+\eta_{ss})(1+\eta_{ss})} & -\frac{v_{ss}}{(h_{ss}-v_{ss})} & \frac{h_{ss}}{(h_{ss}-v_{ss})} \\ 0 & 0 & 0 & 0 & \frac{v_{ss}}{(h_{ss}-v_{ss})} & 0 \end{bmatrix}.$$

Then, the solution is

$$\begin{bmatrix} \widehat{y}_t \\ \widehat{p}_t \\ \widehat{i}_t \\ \widehat{n}_t \\ \widehat{c}_t \end{bmatrix} = \Phi^{-1}\Omega \begin{bmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \widehat{h}_t \end{bmatrix} = \Theta \begin{bmatrix} \widehat{q}_t \\ \widehat{g}_t \\ \widehat{\eta}_t \\ \widehat{v}_t \\ \widehat{h}_t \end{bmatrix}$$

or

$$\widehat{y}_t = \theta_{yq}\widehat{q}_t + \theta_{yg}\widehat{g}_t + \theta_{y\eta}\widehat{\eta}_t + \theta_{yv}\widehat{v}_t + \theta_{yh}\widehat{h}_t,$$

$$\widehat{p}_t = \theta_{pq}\widehat{q}_t + \theta_{pg}\widehat{g}_t + \theta_{p\eta}\widehat{\eta}_t + \theta_{pv}\widehat{v}_t + \theta_{ph}\widehat{h}_t,$$

$$\widehat{i}_t = \theta_{iq}\widehat{q}_t + \theta_{ig}\widehat{g}_t + \theta_{i\eta}\widehat{\eta}_t + \theta_{iv}\widehat{v}_t + \theta_{ih}\widehat{h}_t.$$

$$\widehat{n}_t = \theta_{nq}\widehat{q}_t + \theta_{ng}\widehat{g}_t + \theta_{n\eta}\widehat{\eta}_t + \theta_{nv}\widehat{v}_t + \theta_{nh}\widehat{h}_t,$$

$$\widehat{c}_t = \theta_{cq}\widehat{q}_t + \theta_{cg}\widehat{g}_t + \theta_{c\eta}\widehat{\eta}_t + \theta_{cv}\widehat{v}_t + \theta_{ch}\widehat{h}_t.$$

The first three rows form the system (37) to (39).

FIGURE 1: Impulse responses to a monetary policy shock

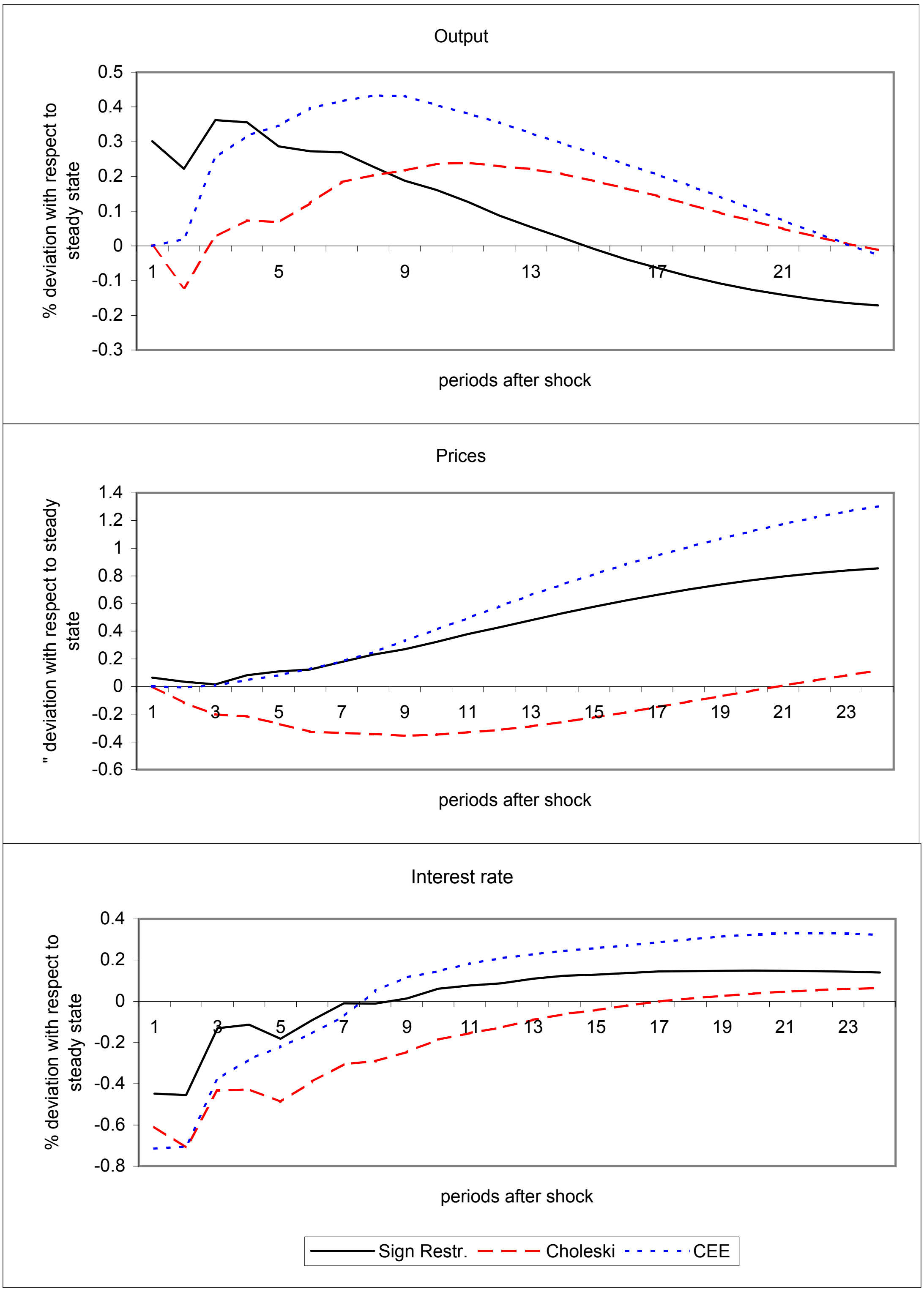

FIGURE 2: Monetary policy shocks identified with differnet schemes

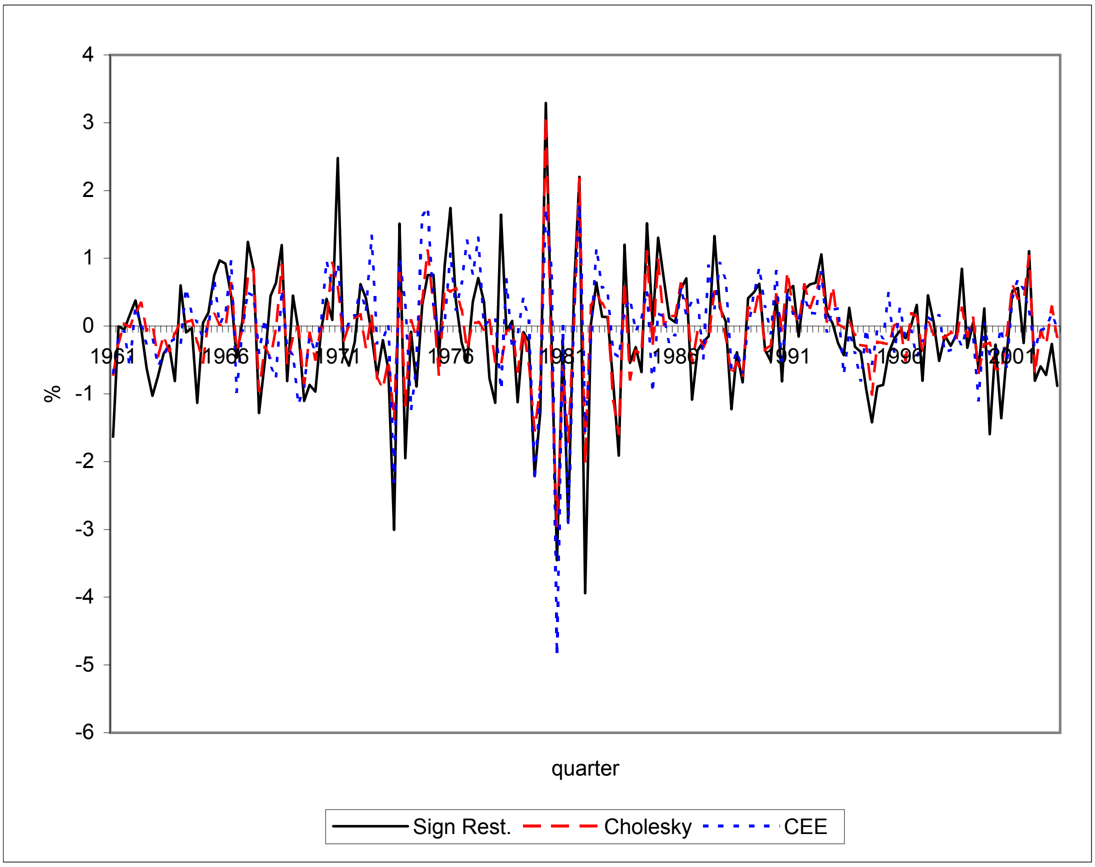

FIGURE 3: Impulse responses to three shocks estimated with sign-shape identification scheme

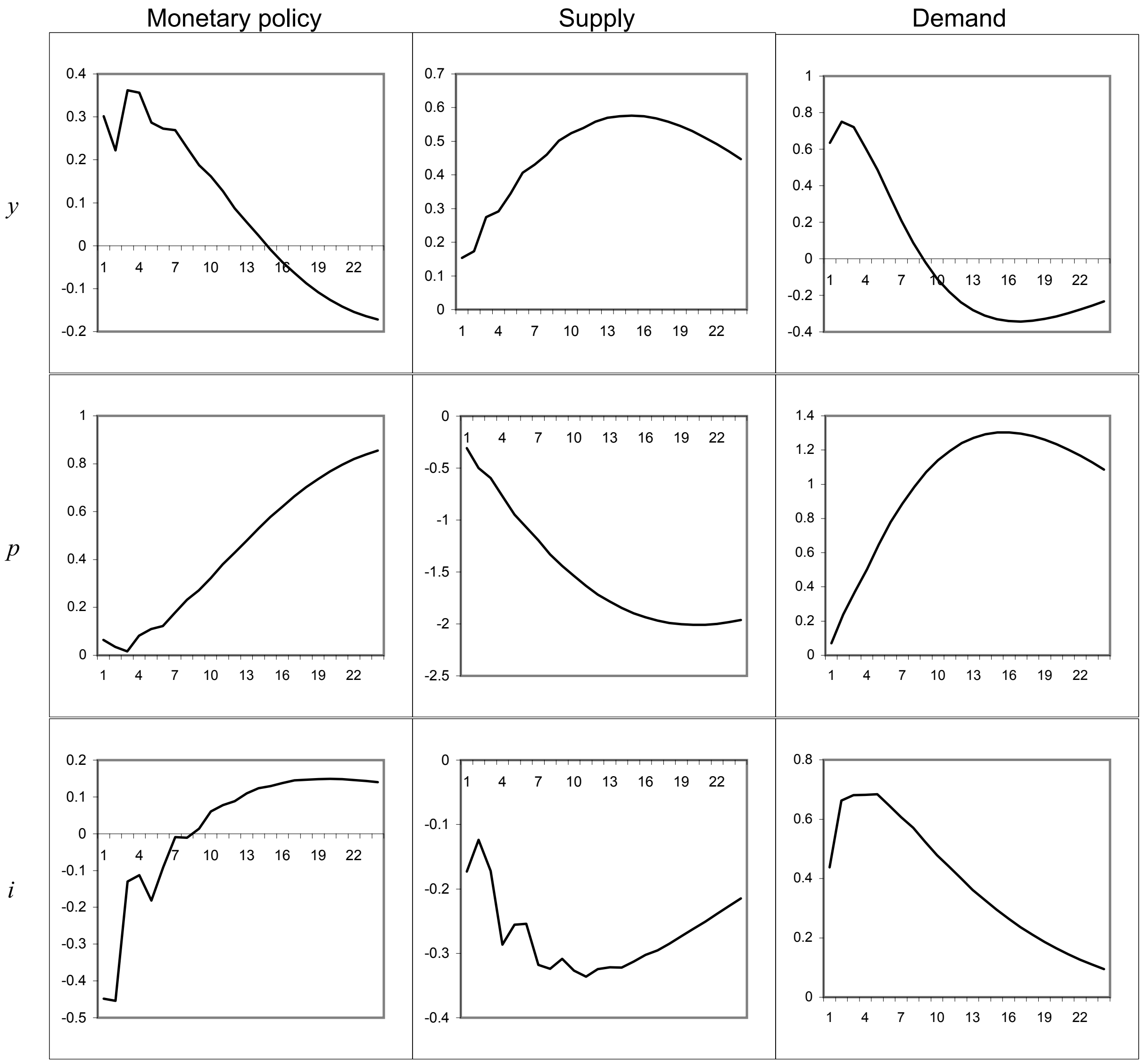

FIGURE 4: Fundamental shocks identified with the sign restriction scheme