How tight should one’s hands be tied?
Fear of floating and credibility of exchange rate regimes

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Abstract
This paper analyzes the linkages between the credibility of a target zone regime, the volatility of the exchange rate, and the width of the band where the exchange rate is allowed to fluctuate. These three concepts should be related since the band width induces a trade-off between credibility and volatility. Narrower bands should give less scope for the exchange rate to fluctuate but may make agents perceive a larger probability of realignment which by itself should increase the volatility of the exchange rate. We build a model where this trade-off is made explicit. The model is used to understand the reduction in volatility experienced by most EMS countries after their target zones were widened on August 1993. As a natural extension, the model also rationalizes the existence of non-official, implicit target zones (or fear of floating), suggested by some authors.

Keywords: Fear of floating, target zones, exchange rate arrangements, rational expectations, credibility.

JEL classification: E52, E58, F31, F33.

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CÆSAR: Cowards die many times before their deaths;
The valiant never taste of death but once.
Of all the wonders that I yet have heard,
It seems to me most strange that men should fear,
Seeing that death, a necessary end,
Will come when it will come.

1 Introduction

In the mid nineties, Obstfeld and Rogoff [26] predicted a world of widely floating exchange rates, given the removal of controls to the international capital mobility. In contrast, recent analyses on exchange regimes have highlighted a common feature in exchange rate policies labeled as *fear of floating*: *de jure* free floaters strongly intervene to soften the fluctuations of the nominal exchange rate (see Calvo and Reinhart [9], Reinhart [27], Fischer [16] and Levy-Yeyati and Sturzenegger [24]). Intermediate regimes seem to be defining the current world so that completely fixed or fully flexible rates are rarely observed. The general aim of this paper is to provide an explanation as to why countries have found it optimal to choose this intermediate alternative.

For this purpose, we introduce a highly simplifying but useful analytical starting point: *any exchange regime is a particular case of a target zone*. On one extreme, fixed rates can be seen as target zones with band widths equal to zero. On the other extreme, pure floating rates are equivalent to fluctuation bands with a width tending to infinity. In between, dirty floats can be seen as implicit target zones with finite bands while target zone regimes impose those bands explicitly. Viewed in this manner, the choice of an exchange regime consists of deciding the width of a band of fluctuation.

In particular, this paper analyzes the linkages between the credibility of a regime, the volatility of the exchange rate and the width of the band where the exchange rate is allowed to fluctuate. It has been argued that different exchange regimes, that is, different band widths, trade off price stability with monetary independence. In this trade off, one of the most important ingredients are the beliefs that market participants have about the incentives of the central bank to defend the band, and these beliefs themselves affect the behavior of the exchange rate. More rigid systems, that is, narrower bands, impose ex-ante limits to the fluctuation of the exchange rate but also agents may perceive a higher risk of realignment in the future. These expectations should feed into the behavior of the exchange rate which itself determines the probability of a change in the system. Broader bands give more scope for the exchange rate to fluctuate but markets may consider the likelihood of a realignment low. In this paper, we analyze the incentives central banks have to renege from announced regimes and compute the endogenous relation between the width of the band, the volatility of the exchange rate and the probability of a change in the central parity of a target zone.
As an example, consider the experience with the European Monetary System (EMS). This system was designed to foster stability of exchange rates among European currencies. In the language of Giavazzi and Pagano [17], by tying the hands of central banks, countries participating in the ERM could reach a certain degree of credibility for their monetary policies due to the discipline effect. Thus when, on August 3rd 1993, the EMS widened the band of fluctuation for the exchange rates, from ±2.25% to ±15%, some analysts predicted the demise of the EMS, given that the wider band could motivate higher volatility of exchange rates.\(^1\) The wider band appeared as a contradiction at the very moment when the Treaty of Maastricht was imposing severe convergence criteria to EMU candidates. It seemed that the door was left open to access an environment with more scope for monetary discretion, exchange rates instability and larger inflation.

However, market participants and researchers have documented two related stylized facts after August 1993: a fall in exchange rate volatility (see Obstfeld [25]) and a substantial improvement in private beliefs about the long run sustainability of the EMS. Gómez-Puig and García-Montalvo [18] estimate an EMS credibility indicator, based on the Markov switching methodology. For the post 1993 period, they find that credibility increased for most of the currencies. Ledesma et alii [23], using a wide variety of credibility tests for the EMS, also conclude that credibility increased after the widening of the bands.\(^2\)

As surveyed in Stockman [30], proponents of narrow band target zones suggest them as substitutes of a monetary commitment. Giavazzi and Pagano [17] argue that exchange arrangements like the ERM of the EMS contributed to import credibility for domestic monetary policies, because differential inflation translates into real exchange rate appreciation, reducing the policy maker’s incentive to inflate. Examples of these narrow bands are abundant. A quick glance into the recent regime classification of Reinhart and Rogoff [28] reveals that most of the countries have at some point had an exchange arrangement involving a band (see their Appendix III). They sample market-determined exchange rates for 153 countries from 1946 through 2001. Among these, the most popular exchange rate regime has been the crawling peg or narrow crawling band, which account for over 26 percent of the observations. In the Western Hemisphere, explicit bands account for 42 percent of the observations.

On the other hand, proponents of flexible rates argue that the central bank may loose valuable short run monetary independence with a pegged exchange regime (see, among others, Svensson [35], Obstfeld and Rogoff [26] and Stockman [30]). A narrow target zone is simply one form of monetary policy rule and

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\(^1\)The Spanish Peseta and the Portuguese Escudo moved from a ±6% to a ±15% band.

\(^2\)This also seems to be the perception shared by the European Central Bank [15], as asserted by President Wim Duisenberg: "[...] The second way in which ERM II will contribute to a stable monetary environment within the EU is by limiting short-term exchange rate volatility. As I have already mentioned, the widening of the fluctuation bands has reduced the incentive for market participants to test the sustainability of the narrow margins. A two-way risk arises as exchange rates move away from central rates, thereby discouraging speculative attacks. The most recent experience is encouraging in this sense. In 1997, exchange rate volatility among ERM currencies has been the lowest since 1992."
some institutional reforms or other monetary policy rules could be used alternatively. If an anti-inflation policy is not very credible inside the country then, instead, why should market participants appreciate that the discipline imposed by a narrow exchange regime will become a credible device outside?

From a theoretical point of view, explicit modeling of band widening has received little attention. Bartolini and Prati [3] develop a model where the central bank targets the exchange rate within a wide band in the short run but attempts at keeping the exchange rate fluctuating in a narrow zone in the long run. More recently, Chamley [10] has proposed a strategic game with speculators and a central bank. Speculators learn about the possibility of a successful attack by looking at the position of the exchange rate within the band. The sustainability of a target zone regime increases by widening the bands of fluctuation. In both models, the idea of sustainability refers to the concept of survival time rather than credibility.

In the present paper, the concept of credibility is identified with the probability, perceived by the market, that the central bank will enforce defense mechanisms to keep the currency within the committed zone. Rather than trying to predict the timing of the collapse of a target zone, as usual in the literature on currency crises, we suggest a rational expectations equilibrium that allows us to identify the degree to which the central bank defends the currency at the margins of the zone. Our model follows the cost-benefit analysis proposed by Obstfeld [25], where the central bank calibrates the opportunity costs from defending relative altering the regime. Rational expectations allows us to check whether the monetary authority has ex post incentives to defend the currency at a given probability of defense. Thereby, credibility is endogenous in our model. Since, in the absence of capital controls, sooner or later narrow band regimes suffer currency crises, we will claim that the adoption of wide band target zones will permit a substantial gain in credibility, enabling an environment for financial and monetary stability.

Finally, the model is also used to further explore the idea of implicit bands within official bands as estimated by Labhard and Wyplosz [21]. These authors suggest that European central banks were able to optimally fine tune their desired degree of monetary independence after the August 3rd 1993 reform. Their estimations of the implicit bands targeted by the monetary authorities are reproduced in Table 1, in the column “Implicit bands”. Some countries kept on holding a tighter band (Netherlands and Belgium), compatible with the former ±2.25% system, but for most of the countries the narrow band system was no longer optimal. Hence, the ±15% regime served as a benchmark reference under which all EMS countries were supposed to adjust the ranges of fluctuations for their currencies (see Table 1). Apart from the August 1993 ERM case, Table 2 quotes some other examples of band widening from the mid nineties. Data have been borrowed from Reinhart and Rogoff’s [28] Appendix III. It seems as

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3 In the words of Calvo and Reinhart [9], we attempt to provide a rationale to the fear of floating.

4 Chung and Tauchen [12] and Chen and Giovannini [11] have also estimated implicit bands for the narrow band period.
though these countries were searching for an optimal width. Current exchange agreements then go from $±2.25$ for the Danish Krone (versus the Euro), up to $±39.2\%$ for the Israeli New Sheqel (versus a basket of currencies).

To capture this notion of implicit bands, we solve the model for a subgame perfect equilibrium where the central bank is given a menu of alternative band widths. Any of those bands produces an endogenous equilibrium credibility. Backward induction then reveals market traders the cheapest regime, which in turn signals the optimal band to be credibly committed by the central bank.

The next section sets up a target zone model and the definition of a rational expectation equilibrium. A computation exercise is carried out in the third section. The results show that there is a trade-off between credibility and volatility as the band width increases. The optimal band is also determined in this section. The last section summarizes the most relevant results.

2 The model

The model represents a highly stylized dynamic stochastic general equilibrium economy. It can be thought of as a reduced form version of more complicated fully optimizing models. Time is discrete. First, consider an equation specifying equilibrium in the money market:

$$m_t - p_t = \varphi y_t - \gamma i_t + \xi_t,$$

(1)

where $m_t$ is money supply, $p_t$ is the domestic price level of $y_t$, a tradable good, $i_t$ is the domestic interest rate of a one period of maturity bond, and $\xi_t$ is some shock to money demand. They are all expressed in logs, with the exception of the interest rate. Parameters $\varphi$ and $\gamma$ are both positive: money demand increases with output because of a transaction motive and there is an implicit liquidity preference behavior, meaning that money can be a substitute for a bond that returns a nominal interest $i_t$.

Let $x_t$ denote the log of the nominal exchange rate, expressing the price of one unit of foreign currency in terms of domestic currency. The (log) real exchange rate is given by

$$q_t = x_t - p_t + p_t^*,$$

(2)

where $p_t^*$ is the foreign price (variables with star will denote the foreign analogue).

Call $d_t \equiv i_t - i^*$, the interest rate differential, where $i^*$ is the foreign (constant) rate, and assume perfect capital mobility, risk aversion and the uncovered interest rate parity condition (UIP)

$$d_t = E_t \{x_{t+1} - x_t\} + r_t,$$

(3)
where $E_t$ is the expectation operator conditional on information available at time $t$. The exogenous state variable $r_t$ represents the foreign risk premium, governed by a first order Markov process

$$r_t = r_{t-1} + \varepsilon_t.$$  \hspace{1cm} (4)

The expected rate of depreciation must compensate for the interest rate differential plus the foreign premium. The white noise $\{\varepsilon_t\}$ is supposed to be Gaussian, $\varepsilon_t \sim N(0,\sigma^2)$, for convenience.\footnote{The foreign risk premium plays a key role in the current paper. In general, UIP does not hold (see Ayuso and Restoy [1]). Conventional target zone models consider that deviations from UIP are negligible in target zones (see Svensson [33]). A common practice in some credibility tests of target zones relies in this idea (e.g., the simplest test, see Svensson [31], and the drift adjustment method of Bertola and Svensson [8]). However, Bekaert and Gray [6] find that the risk premia in a target zone are sizable and should not be ignored. They argue that this might be the reason of why the credibility tests run on EMS at the beginning of the nineties failed in anticipating the 1992-93 turbulences.}

Using (2), (3) and (1) one obtains

$$x_t = f_t + \gamma E_t \{x_{t+1} - x_t\},$$  \hspace{1cm} (5)

$$f_t = m_t + v_t,$$

$$v_t = \theta_t + \gamma r_t,$$

$$\theta_t = q_t - p^*_t - \varphi y_t + \gamma i^* - \xi_t.$$

Total fundamentals $f_t$ amount to an endogenous process, $m_t$, plus an exogenous process, $v_t$. We will assume that both the central bank and traders can observe the realization of $\{v_t\}$.

The monetary authority must choose the path for money $\{m_t\}$ so as to maximize its preferences subject to the evolution of $\{v_t\}$. The objective is to set up these preferences to capture the degree of monetary independence that arises under alternative exchange rate regimes. As in Svensson [34], the concept of monetary independence is associated with the interest rate variability. At one extreme, and in the absence of realignments, a fixed rate eliminates monetary independence. At the other extreme, a managed float regime provides the highest degree of independence. In between, a target zone gives some scope to focus the monetary policy on domestic problems. To capture this idea, the central bank (henceforth, CB) preferences are modeled to evaluate the trade-off between interest rate variability versus exchange rate variability

$$J = \frac{1}{2} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (d_t - \rho_0)^2 + \lambda (x_t - c_0)^2 \right] \right\}. \hspace{1cm} (6)$$

The long run desirable target for the interest rate differential is $\rho_0$, that we will later assume as zero, for convenience. The target for the exchange rate is the central parity $c_0$ around which deviations are punished by the relative penalty $\lambda$. The factor $\beta \in (0,1)$ is a time discount rate.
We think of a very short maturity term for the bond in the UIP, say a few days, a week or a month at most. The idea is that the CB controls some monetary aggregate \( \{ m_t \} \) to target \( \{ d_t, x_t \} \). Output realizations and real fluctuations are observed with some delay, and not available by the time monetary policy is decided so, the only available information at any period is \( \{ \theta_t, r_t \} \). From (3) and (5) it is easy to show that \( m_t \) will respond one to one to the shock \( \theta_t \) and the problem reduces to deciding how to split the shock \( r_t \) between \( d_t \) and \( x_t \).

In what follows, the target zone regime will be characterized by the triple \( \{ \lambda \geq 0, w \geq 0, \alpha \in [0, 1] \} \). The first element is related to the preferences in (6). The number \( w \) is the band width. The last term \( \alpha \) is the probability that the CB defends the currency when it is outside the band and measures the credibility of the exchange regime. We suppose that, within the bands, the CB defends the currency with probability 1. As stated in the introduction, the purpose of this paper is to construct a model able to generate a consistent target zone solution, in the sense that the CB finds it optimal to defend the target zone at the margins by probability \( \alpha \). There are not further incentives to renounce from the target zone ex-post.

The exchange rate is always determined as a function of the shock \( r_t \). At any time \( t \), the timing of the game is as follows:

1. The shock \( r_t \) is realized.

2. If for that value of the shock the exchange rate should be outside a band \([c_t - w, c_t + w]\), for instance, say it is above the upper limit \( c_t + w \), the CB can do any of two actions:
   - It defends the currency with probability \( \alpha \). This means that the exchange rate is pegged at the edge of the band, \( x_t = c_t + w \), and the central parity is not altered, \( c_t = c_{t-1} \).
   - It realigns the currency with probability \((1 - \alpha)\). In this case, the central bank devalues the central parity by \( \mu \), i.e. \( c_t = c_{t-1} + \mu \), and situates the exchange rate at \( x_t = c_{t-1} + \max\{w, \mu\} \), that is, the rate jumps to the new central parity if the realignment rate is higher or equal than the width, \( \mu \geq w \). Otherwise, with overlapping zones, \( \mu < w \), the parity is altered by \( \mu \) but the exchange rate remains at \( c_{t-1} + w \). This condition is necessary in order to avoid jumps to the interior of the band, i.e. an appreciation under a devaluation.

3. If the realization of the shock makes the exchange rate be within the target zone, the CB solves a minimization problem. The process is as follows:
   - The central parity is left unaltered: \( c_t = c_{t-1} \).
   - Forward looking agents form expectations \( E_t x_{t+1} \).
   - For given \( \{ E_t x_{t+1}, r_t, \rho_t, c_t \} \), the CB chooses \( \{ x_t, d_t \} \) by minimizing the loss (6) subject to the arbitrage condition (3), the relation in (4) and the additional restriction \( x_t \in [c_t - w, c_t + w] \).
4. Finally, for given shock $\theta_t$ in the money demand equation (5), the CB supplies the optimal quantity of money $m_t$ supporting the pair $\{x_t, d_t\}$.

In the loss function (6), the variable $\rho_t$ has been defined as the long run desirable target for the interest rate differential. The timing that has just been described implies that the exchange rate is a function of the variable $r_t$, the foreign risk premium, that must be fluctuating within a zone before an intervention is called for. Since $r_t$ is a return on an asset, it takes the form of an interest rate. Therefore, the zone for this risk premium is an interval centered at $\rho_t$. In fact, the rate of depreciation of $x_t$ uniquely depends on the difference of $r_t$ from $\rho_t$. That is to say, when $r_t$ hits the upper limit of its zone, the exchange rate hits also its upper limit, $x_t = c_t + w$. Let $u(r_t - \rho_t)$ be the function linking the exchange rate to the fundamental process $r_t$ within the band. For the exchange rate to belong to $[c_t - w, c_t + w]$, the shock must be inside another band $[\rho_t - r, \rho_t + r]$. At $r_t = \rho_t \pm r$ two level conditions must be satisfied

$$u(-r) = -w,$$

and

$$u(r) = w.$$  

When a devaluation comes about (see step 2, bullet 2 in previous timing), the central parity is devalued by $\mu$, and the new band for the fundamental is updated (see appendix A.1 for an explanation of how the process $\{\rho_t\}_{t=0}^{\infty}$ is updated).

In what follows, consider that the starting central parity is $c_0 = 0$ and the starting center is $\rho_0 = 0$. The first order condition for values of $r_t \in [-\tau, \tau]$ is

$$(1 + \lambda) x(r_t) = r_t + E_t[x(r_{t+1})].$$

The function $x(r_{t+1})$ satisfies

$$x(r_{t+1}) = c_t \begin{cases} 
+\max\{w, \mu\} & \text{if } r_{t+1} > \tau \text{ with probability } 1 - \alpha \\
+w & \text{if } r_{t+1} > \tau \text{ with probability } \alpha \\
+u(r_{t+1}) & \text{if } r_{t+1} \in [-\tau, \tau] \\
-w & \text{if } r_{t+1} < -\tau \text{ with probability } \alpha \\
-\max\{w, \mu\} & \text{if } r_{t+1} < -\tau \text{ with probability } 1 - \alpha.
\end{cases}$$

The unknown continuous function $u(r_t)$ represents the CB’s best response when the risk premium lies within its band $[-\tau, \tau]$. Outside of it, condition (9) does no longer hold and the trade-off is not optimal.\(^6\)

\(^6\)The assumption that the exchange rate can only jump when a realignment occurs simplifies matters. Bekaert and Gray [6] construct an empirical model where jumps can be taken at any point within the zone (“several within-the-band jumps are larger than several realignment jumps”). There are other alternatives to model the realignment rate. For instance, the setting in Bertola and Caballero [7] is similar to ours, where agents perceive a constant size of the realignments, and the exchange rate takes a jump to the new central parity at realignments. Ball and Roma [2] and Svensson [33] assume that changes in the central parity follow a Poisson process.
In order to find out the particular form of the function $u(r_t)$, the band for the fundamental $[-\pi, \pi]$ must be computed, given the level conditions (7) and (8). This guarantees continuity of $u(r_t)$ from expression (10). Given that the first order conditions are only fulfilled in the interior of the bands, (9) can then be rewritten as

$$(1 + \lambda) u(r_t) = r_t + E_t[x(r_{t+1})].$$

(11)

Forward recursion is used to solve for $u(r_t)$ with the first order condition (11) subject to (10). Appendix A.2 provides all the details for computation of the solution.

From condition (11) and (3), it is straightforward to see that the model produces a positive relation between the exchange rate of depreciation within the band and the interest rate differential:

$$d_t = \lambda [u(r_t - \rho_t) - c_t] + \rho_t,$$

(12)

Outside the band, the trade-off is not optimal and expression (12) does not hold.

As an illustration, consider the following particular case depicted in figure 1. Further details of the properties of the function $u(r_t)$ can be found in Rodriguez and Rodríguez [29]. We consider a preference parameter $\lambda = 0.5$ and a standard deviation $\sigma = 0.002$. As in the EMS case, we select a width $w = \pm 2.25\%$, whose expected realignment rate is given by $\mu = \pm 4.5\%$ (these values are conveniently justified in the following section). The exchange rate function is calculated for five values of the probability of defense, $\alpha$: $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and $1$. These functions are positively sloped for $r_t$. The higher is $\alpha$, the flatter the curve, that is, the exchange rate response to $r_t$ is smoother, the bigger is credibility. Hence, a credible band helps stabilize the exchange rate within the zone. For values of $\alpha$ close to $0$, the slope of that function increases, implying that the exchange rate is more sensible to variations in the risk premium. Then, volatility will increase as long as $\alpha$ decreases.

Figure 1 here

2.1 The Concept of Equilibrium

The rational expectations equilibrium of this model amounts to finding a fixed point for the probability of defense, $\alpha$. The idea is as follows. For a given set of parameters ($\lambda$, $\sigma$, and $w$), and a prior about the probability $\alpha$, market participants may compute the \textit{ex-post} incentives the central bank has to renounce from the target zone regime. To estimate those incentives agents calculate the losses for the central bank derived from defending the zone as compared to its losses from realigning the band for any possible realization of the shock. Then, they compute the probability of the realizations for which losses from defending are smaller than those of realigning. The rational expectations equilibrium imposes that this probability should be equal to $\alpha$. If it is not equal, market participants deduce that they were not estimating consistently the probability of defending and update it consequently.
To understand how this equilibrium is computed consider the following. Imagine that at some point in time \( t = \tau \) the shock exceeds one of its boundaries, say the upper one, \( r_\tau = \tilde{r} > \rho_\tau + \tau \). The central bank has two options. If the currency is defended, neither the central parity \( c_\tau \) nor the center \( \rho_\tau \) are altered, so \( c_\tau = c_{\tau-1} \) and \( \rho_\tau = \rho_{\tau-1} \). Yet, for convenience, we assume that both \( c_\tau \) and \( \rho_\tau \) start at 0. Then, the current exchange rate is set at \( x_\tau = c_\tau + w \), and the interest rate differential is given by

\[
d(\tilde{r}) = E_\tau [x_{\tau+1} - x_\tau] + r_\tau = E_\tau (r_{\tau+1}) | r_\tau = \tilde{r} ] -(c_\tau + w) + \tilde{r}.
\]  
(13)

Hence, the CB pays a loss from then on of

\[
L^D_\tau (\tilde{r}) = \frac{1}{2} [d(\tilde{r})^2 + \lambda w^2] + \frac{1}{2} E^D_\tau \left[ \sum_{j=1}^{\infty} \beta^j \left( d^2_{\tau+j} + \lambda x^2_{\tau+j} \right) \right],
\]  
where \( E^D_\tau \) is the conditional expectation operator, given that the CB defends at time \( t = \tau \).

Instead, if the CB decides to devalue, the central parity and the exchange rate are moved to \( x_\tau = c_\tau = c_{\tau-1} + \mu \), and the new center for the risk premium is moved to \( \rho_\tau = \tilde{r} \). From (12), the interest rate differential will be given by

\[
d(\tilde{r}) = \tilde{r}.
\]  
(15)

Then, the CB receives a loss

\[
L^R_\tau (\tilde{r}) = \frac{1}{2} [\tilde{r}^2 + \lambda \mu^2] + \frac{1}{2} E^R_\tau \left[ \sum_{j=1}^{\infty} \beta^j \left( d^2_{\tau+j} + \lambda x^2_{\tau+j} \right) \right],
\]  
where \( E^R_\tau \) is the conditional expectation operator, given that the CB realigns at time \( t = \tau \).

These two values lead us to define, for any value \( \tilde{r} \), a cost function from defending as

\[
G (\tilde{r}) = L^D_\tau (\tilde{r}) - L^R_\tau (\tilde{r}).
\]  
(17)

If, for some \( r \notin [-\tau, \tau] \), \( G (r) \) is negative, the costs from realigning will overcome that from defending and the CB will keep the exchange rate at its primitive margins. Otherwise, when \( G (r) > 0 \), realigning will be preferred to defending for that particular value of the shock. Indifference stands for the case \( G = 0 \). Since the objective function is quadratic, the function \( G (r) \) is an increasing function of the shock \( r \), that is, the costs from defending relative to the costs from realigning increase as the value of the shock increases. Notice the relative costs of defending are only conditional on the value of the shock; they do not depend on previous costs. So, once the shock \( r \) has been reached, the function \( G (r) \) determines whether it is preferable to defend or to realign. That is why we do not write the subindex \( \tau \) anymore. Given the monotonicity of the function
there must be a value of the risk premium, call it \( r^* \), such that for all \( r_t > r^* \) it is preferable to realign, that is, \( G(r) > 0 \) for all \( r > r^* \). Then, market participants can determine the probability of reaching such a value which will be related to the probability of defending. With this information, they can evaluate whether the assumed \( \alpha \), the subjective probability of defense, was correct or not.

Furthermore, for given \( r_0 = \rho_0 = 0 \), denote by \( \zeta(r^*) \) the probability that a random walk stream, \( \{ r_t \} \), always wanders inside a moving corridor of width \( 2r^* \geq 0 \), centered at \( \{ \rho_t \} \), i.e.

\[
\zeta(r^*) = \Pr \left\{ \{ r_t - \rho_{t-1} \in [-r^*, r^*] \} \right\}_{t=1}^{\infty} | r_0 = 0 ,
\]

and analogously define \( \zeta(\pi) \) as

\[
\zeta(\pi) = \Pr \left\{ \{ r_t - \rho_{t-1} \in [-\pi, \pi] \} \right\}_{t=1}^{\infty} | r_0 = 0 ,
\]

such that \( r^* \geq \pi > 0 \), and where process \( \{ \rho_t \} \) is updated as follows

- \( \rho_t = \rho_{t-1} \), for \( -r^* < r_t - \rho_{t-1} < r^* \).
- \( \rho_t = r_t \), otherwise.\(^7\)

Notice that the function \( \zeta(r^*) \) is monotonically increasing. This implies that

\[
\lim_{r^* \to \infty} \zeta(r^*) = 1 , \quad \lim_{r^* \to 0} \zeta(r^*) = 0 .
\]

These concepts lead us to a rational expectation solution. Intuitively, agents observe that there exists a shock making the CB indifferent between defending and realigning. Equilibrium is found when the proportion of times that this shock is perceived to occur equals the proportion of times the currency is observed to be defended. A more formal equilibrium concept follows:

**Definition 1:** *(Rational expectations equilibrium)* A target zone system defined by the parameters \( (\sigma, \lambda, w, \alpha) \) is said to be a rational expectation equilibrium, if there exists a risk premium \( |r^*| \geq |\pi| \), such that

\[
G(r^*) = 0 ,
\]

and

\[
\frac{\zeta(r^*) - \zeta(\pi)}{1 - \zeta(\pi)} = \alpha .
\]

where \( \pi \) is the implied band width of the zone.

\(^7\)Appendix A.1 reports further details on how the center \( \rho_t \) is updated under a more general case when the realignment rate is not necessarily higher than the band width.
Condition (19) makes agents perceive that the CB is indifferent between realigning or defending. Condition (20) implies that market traders perceive the CB is defending with a probability $\alpha$ whenever a marginal intervention is called for. On the one hand, the central bank will not realign as long as the shock is not larger than $r^*$ in absolute value which happens with probability $\zeta(r^*)$. On the other hand, the market anticipates that realignment does not occur if the risk premium stays with its band, which happens with probability $\zeta(\tau)$, or if the shock leaves its band and the central bank defends the target zone which happens with probability $\alpha [1 - \zeta(\tau)]$. Obviously, in equilibrium these two probabilities of defending should be equal, that is,

$$\zeta(r^*) = \zeta(\tau) + \alpha [1 - \zeta(\tau)],$$

and solving for $\alpha$ gives condition (20).

Imagine that condition (20) is matched, but condition (19) is not. For instance, imagine there is a value for $r$, say $\tilde{r}$, such that $G(\tilde{r}) < 0$ and $[\zeta(\tilde{r}) - \zeta(\tau)]/[1 - \zeta(\tau)] = \alpha$. Market participants perceive that the central bank would be strictly better off from defending and letting the risk premium to deviate further from its center. Hence, there is still room for the risk premium to be higher, say $\tilde{r} + \Delta > \tilde{r}$, before the central bank generates a devaluation. Since the function $\zeta(\tilde{r})$ is monotonically increasing, this means that

$$\frac{\zeta(\tilde{r} + \Delta) - \zeta(\tau)}{1 - \zeta(\tau)} > \alpha,$$

that is, the perception by the market of the proportion of times the currency will be observed to be defended, out of the margins of the zone, should be higher than $\alpha$. Agents will revise their estimation of the probability of defense by increasing their subjective probability of defense, say to $\tilde{\alpha} > \alpha$. With a larger value of $\alpha$, the function $u(r_t - \rho)$ becomes flatter and the limit of the band for the risk premium, $\tau$, larger. Given that $\zeta(\tilde{r} + \Delta) < 1$, this movement of $\tau$ reduces the left-hand side of (21). Notice, the response of the market increasing $\alpha$ moves the economy towards the equilibrium by closing the gap in (21).

### 3 Results

Since the model does not permit a closed form solution, numerical alternatives are given as tentative answers. This requires to rely results on some prespecified parameters. This section is organized in four parts. The first justifies the parameters for the numerical computations. The second part use the previous model to endogenize the credibility of the target zone, as represented by the probability of defense $\alpha$. Estimations of conditional volatilities are given in the third subsection. The final subsection addresses to the question of determining the optimal band.
3.1 Parameters

The following values for the parameters are used in the numerical exercises of this section. We think of a month as the time frequency. As in Svensson [34], the time discount factor is set to $\beta = 0.90^{12}$. We use this value to ease the calculations but the main results of the paper do not hinge on it.

Three bands are chosen, $w = \pm 2.25\%$, $\pm 6.00\%$ and $\pm 15\%$, the official widths experienced in the ERM. For the last case, it is well known that EMS monetary authorities targeted implicit widths much narrower than the $\pm 15\%$ officially announced, as estimated by Labhard and Wyplosz [21] (see also Bartolini and Prati [3]). In fact, when the Spanish Peseta and the Portuguese Escudo were realigned on March 6th 1995, the edges of the zone had not been reached. In both cases, the realignment rate ($\mu = 7.5\%$ and $3.5\%$, respectively) were smaller than the official widths.

Appendix B suggests some estimates for $\sigma$ and $\mu$. A VAR estimation lead to conclude that $\sigma = 0.01$ can be a reasonable value for this variance. This value parallels some other estimations for the risk premium standard deviation. The value of the constant realignment rate $\mu$ is estimated depending on the band width, $w$. Thus, we conclude that $\mu = \pm 4.5\%$ and $\mu = \pm 6.3\%$, for widths $w = \pm 2.25$ and $w = \pm 6\%$, respectively, are consistent with the EMS history.

For the wide band $\pm 15\%$ we use $\mu = \pm 7.5\%$, as the Spanish Peseta was so devalued in March 6th 1995.

Finally, regarding the preference parameter $\lambda$, we have used a grid ranging from $10^{-8}$ to 1.

3.2 Credibility

The implementation of the rational expectation equilibrium goes as follows. For particular values for $\sigma$, $\lambda$ and $w$, we guess the equilibrium probability of defense $\alpha$, and solve the model, that is, we find the function $u$, and the bounds for the risk premium band, $r$. Second, we compute the value of $\tilde{r}$ from expression (20). Third, we simulate $N = 1000$ runs of $T = 500$ observations for the shock starting at the value $\tilde{r}$. Fourth, the relative costs of defending versus realigning are computed for each run in the simulation so the cost function $G(\tilde{r})$ is approximated as the arithmetic mean over the $N = 1000$ simulated runs. If the estimated $G(\tilde{r})$ is positive (negative), we scale down (up) the initial value of $\alpha$ and repeat the process until convergence is reached (see appendix C for the details of this iterative process).

Table 3 collects the equilibrium values of $\alpha$ for different combinations of the preference parameter $\lambda$, and the three widths considered. In our search of values, we find that $\lambda$ should be smaller or equal than 0.1 in order to produce

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8 Expected realignment rates are usually estimated through some measure of accumulated competitiveness losses, e.g. the growth rates of the CPI, the Industrial Price Index or the Unit Labor Cost Index. In general, devaluations are not sufficient to dampen for the real appreciation (Giavazzi and Pagano [17]). By estimating different realignment rates for the different band widths, we simply want to extract some figures that reasonably describe the ERM history.
equilibrium values of $\alpha$ strictly in the $[0, 1]$ interval. In complementary exercises (not reported here), when $\sigma$ has been increased, this range for $\lambda$ has increased: when the risk premium suffers a bigger volatility, a central bank needs a higher $\lambda$ in order to preserve a certain degree of credibility.

The main result to be highlighted from Table 3 is that the equilibrium probability of defense $\alpha$ increases as the band widens. The market perceives that this situation might discourage the monetary authority to reneg from the target zone arrangement. Thus, credibility increases with the band width. Another interesting result is that the system is more credible, the larger the preference weight attached to the variability of the exchange rate, $\lambda$. If the central bank exhibits in its preferences a strong targeting of the exchange rate around the central parity, the exchange rate variability will be lower. The preference parameter $\lambda$ punishes any deviation from the central parity. Then, the central bank finds it consistent to commit the target zone arrangement with a higher probability.

Although the quantitative outcomes of this exercise are highly sensible to the realignment rate $\mu$, the qualitative results should remain the same independently of the particular value of $\mu$ or any other parameter. Here, we are assuming that as the band gets wider, the realignment rate increases too which increases the cost of changing the central parity and makes the system more credible. The choice of making $\mu$ dependent of the width of the band has been made to match what we observe in the data. However, if $\mu$ were constant the same effects would appear. This is due to the fact that as the band widens, the probability of incurring in the fixed costs of realigning decrease.

Table 3 here

### 3.3 Volatility

Is the model consistent with the experience of the EMS? As pointed out in the introduction, after the widening of the bands in 1993, researchers found that volatility of the exchange rate had decreased (see Gómez-Puig and García-Montalvo [18], Ledesma el alii [23] and Ayuso el alii [1]). To see this, we compute the standard deviation of the exchange rate, for $t = 1, 2...36$ periods ahead, conditional on information available at time 0

$$\sigma_{x,t} = \sqrt{E_0 \left\{ [x_t - E_0(x_t)]^2 \right\}}.$$  \hfill (22)

If we consider that the starting conditional value for the risk premium is $r_0 = 0$, it is easy to show that the conditional expectation is $E_0(x_t) = c_0 = 0$. The standard deviation for the interest rate differential has a similar form:

$$\sigma_{d,t} = \sqrt{E_0 \left\{ [d_t - E_0(d_t)]^2 \right\}}.$$  \hfill (23)

Again, for $r_0 = 0$, it is easy to show that the conditional expectation is $E_0(d_t) = \rho_0 = 0$. 

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Figure 2 shows the conditional standard deviation in (22) for the different time horizons and the three band widths used in the EMS. The value used for $\lambda$ is 0.0001. We observe that the volatility of the exchange rate 19 periods ahead in the system with the width of ±15% is smaller than the one with the zone of ±2.25%. The reason for this result is in Table 3 where we find that the probability of a realignment in the first system is 85 percent while with the narrow band is only 23 percent. We also observe that these reductions in volatility from moving to the wide band increase as the time horizon is larger. With a larger probability of realignment future discrete jumps in the exchange rate are more likely.

Figure 3 presents the same computations with $\lambda = 0.1$. With this value for the central bank preferences wider bands present higher volatility. In this case, it is in the interest of the central bank to foster credibility and market participants know that the monetary authority will be likely to defend the band often. In fact, Table 3 shows that all bands are equally credible. The only difference stands for the different reaction of the exchange rate to movements in the shock. This response is given by the slope of the function $u$ which is smaller for narrower bands.

Finally, Figure 4 presents the volatility of the interest rate differential for $\lambda = 0.0001$. We observe this volatility decreases with the widening of the band. A broader band allows for more monetary independence, that is, the central bank enjoys more degrees of freedom to adjust the nominal interest rate to domestic conditions. With a wider zone the exchange rate may now absorb more variability from the risk premium shock. At the same time, a wider band is also more credible, this helps stabilizing forward looking expectations and, through the UIP, the interest rate differential.

Figures 2-4 here

3.4 The optimal band width

This subsection explores the idea of Labhard and Wyplosz [21] about non-official bands inside official bands. Their estimates suggest that, while the widths of the target zones were widened on August 1993, most of EMS countries found it optimal to target a tighter band than ±15% (see Table 1): The Netherlands kept on pursuing a close targeting to the DM, while Belgium enforced a band similar to that of the previous period. The rest of countries preferred to gain more degrees of freedom for their monetary policies, probably due to the need of meeting the convergence criteria imposed by the Maastricht Treaty.

The exercise of this subsection rationalizes the choice of this nonofficial bands by solving the model for a subgame perfect equilibrium. Given a value for the parameters $\lambda$, $\beta$, and $\sigma$, an equilibrium probability $\alpha$ can be found for any band width $w \geq 0$. This defines a continuum of equilibria within which one can compute the ex-ante costs from such strategy as measured by (6). Any of those widths conforms the strategy set of the CB. A credible Nash equilibrium
is determined by choosing the strategy that returns the minimum ex-ante cost. This is the optimal band width.

In order to perform this exercise, a word has to be said about the rate of realignment. At the beginning of this section, we suggested a positive correlation between the band width and the realignment rate, on the basis of EMS history. We mapped the value of $\mu = 4.5\%$ to $w = \pm 2.25\%$, and $\mu = 6.3\%$ to $w = \pm 6\%$. In order to keep the same schedule in the parameters, we make the realignment rate vary linearly with the band matching these previous points. This takes us to the function

$$\mu = 0.0342 + 0.48w.$$  

Denote $J(w)$ the costs derived from the rational expectation equilibrium associated with the width $w$, given the rest of parameters. The optimal band width is found as

$$w^* \equiv \arg\min_w J(w).$$

Let $\alpha^*$ denote the equilibrium probability of defense associated with the optimal band $w^*$. These steps have been repeated for thirteen different values of $\lambda$ for a grid ranging from 0.0008 to 0.01.

Table 4 includes the equilibrium values for the optimal band and the associated probabilities of defense $\alpha^*$ for each of the values of $\lambda$. As the central bank dislikes more the volatility of the exchange rate (for large values of $\lambda$), the optimal band becomes smaller at the same time that more credible. In this sense, the August 3rd 1993 reform can be viewed as a benchmark regime within which all the EMS countries could optimally adjust their targeted zones where the exchange rates were allowed to fluctuate. An important corollary of this result is that, while this optimal fine-tuning of bands can be achieved when the official band is $\pm 15\%$, the official narrow width period of $\pm 2.25\%$ could only make feel uncomfortable those EMS currencies who wished to use a bigger optimal degree of monetary independence. The column of Table 1 labelled “Maastricht” collects the average number of criteria not satisfied by the countries in the sample of Labhard and Wyplosz [21] from 1993 until 1996. With the exception of Ireland we see that countries that decided to let the exchange rate to fluctuate in wider bands were the ones that did not satisfy a larger number of criteria and therefore needed to make more adjustments in their domestic economies.\footnote{Ireland had held a long tradition (from 1826) of anchoring the Irish Pound versus the currency of the U.K., its main trading partner. As the U.K. left the EMS discipline on September 1992, the economic gains from participating in the ERM could only come through a credible enough arrangement. The broad implicit band of Table 1 allowed for accommodation of the strong depreciation of the British Pound (BP) versus the Deutshe Mark by the end of 1994 and early in 1995 (see Ledesma et alii [22]). In contrast, the sharp depreciation of the BP on September 1992, in combination with the tight $\pm 2.25\%$ official band, led the Irish authorities to strongly realign on 1st February 1993 (+11%).}

Table 4 here
4 Conclusions

The choice of an exchange regime can be viewed as the choice of a band width of a target zone. This apparently strong assertion has been the guiding line of the current paper. In fact, we observe that target zones are the most common exchange agreements in recent history. The most important currencies in the world are traded under regimes with bands of fluctuation, officially or unofficially. The importance of the band width is twofold: first, it determines the degree of monetary independence, that is, the degree of flexibility the central bank has in order to adapt monetary policy to home conditions and second, it determines the credibility of the exchange regime. Both items are heavily related.

In this paper, it has been suggested a possible explanation of why wide bands could be more credible and less volatile than narrow bands, as reported by the empirical literature on the EMS. There are two opposite forces when moving to a wider band. First, we get closer to the free float which increases volatility but, second, the credibility of the band may increase and this, in turn, reduces volatility. At the same time, a wider band allows more room for the central bank to use the interest rate. By endogenizing the probability of defense we have showed that a target zone can gain in credibility by widening the band width. Thus, our results support the arguments of the proponents of flexible exchange rates: the central bank may not only reap the benefits from monetary independence but also may take advantage in credibility by widening the band width.

However, this widening needs not lead to one of the extreme regimes, that is, the free floating. Our model has also a normative implication that serves as a valid rationale for the estimates of implicit bands, found by Labhard and Wyplosz [21], or for the fear of floating, as dubbed by Calvo and Reinhart [9]. Officially, countries can declare themselves as floaters within a broad band, but market participants may appreciate a fear of floating that leads the central bank to target an unofficial implicit band. Hence, that fear is rational.

A Solution of the Target Zone

In this appendix we show how to make the computations for the solution of the target zone regime. Forward recursion in the first order condition (9) is used to solve for $u(r_t)$ subject to (10). Let the expected rate of depreciation out of the band be given by

$$\delta_{\tau,\tau-1} = [\alpha w + (1 - \alpha) \max \{ w, \mu \}] \times [\Pr (r_{t+\tau} > \tau |F_{\tau}) - \Pr (r_{t+\tau} < -\tau |F_{\tau-1})]$$

Forward iteration of (9) leads to the following general solution

$$u(r_t) = \frac{r_t}{1 + \lambda} + \frac{1}{1 + \lambda} \sum_{\tau=1}^{\infty} \frac{F [r_{t+\tau} |F_{\tau}]}{(1 + \lambda)^{-\tau}} + \sum_{\tau=1}^{\infty} \frac{F [\delta_{\tau,\tau-1} |F_{\tau-1}]}{(1 + \lambda)^{-\tau}}.$$
where the sequence $F_{\tau \geq 0}$ represents filtered information sets of the next form:

$$F_0 = \{ r_t \},$$  \hspace{1cm} (26)  

$$F_{\tau \geq 1} = \{ \{ r_{t+n} \in [-\tau, \tau] \} \}_{n=1}^\tau, r_t \}.  \hspace{1cm} (27)$$

On the other hand, the components in (25) are given by:

$$F_{\tau} \{ \delta_{1,0} | F_0 \} = [\alpha w + (1 - \alpha) \max \{ w, \mu \} ] \times [\Pr (r_{t+1} > r_t) - \Pr (r_{t+1} < -r_t) ]$$  \hspace{1cm} (28)

$$F_{\tau} \{ r_{t+1} | F_1 \} = \int_{-\tau}^\tau r_{t+1} \phi (r_{t+1} | r_t) dr_{t+1},$$  \hspace{1cm} (29)

for $\tau = 1$, and

$$F_{\tau} \{ \delta_{\tau-1} | F_{\tau-1} \} = \int_{-\tau}^\tau \int_{-\tau}^\tau \delta_{r,\tau-1} \phi (r_{t+\tau-1}, ..., r_{t+1} | r_t) dr_{t+\tau-1} ... dr_{t+1},$$  \hspace{1cm} (30)

$$F_{\tau} \{ r_{t+\tau} | F_{\tau} \} = \int_{-\tau}^\tau \int_{-\tau}^\tau r_{t+\tau} \phi (r_{t+\tau}, ..., r_{t+1} | r_t) dr_{t+\tau} ... dr_{t+1},$$  \hspace{1cm} (31)

for $\tau = 2, 3, \ldots$, where

$$\phi (r_{t+\tau}, ..., r_{t+1} | r_t) = \left( 2\pi \sigma^2 \right)^{-\tau/2} \exp \left[ \frac{-1}{2\sigma^2} \sum_{n=1}^{\tau} (r_{t+n} - r_{t+n-1})^2 \right].$$  \hspace{1cm} (32)

This general solution is consistent with (9) and (10). In order to determine a particular solution, it is necessary to identify the value of $\tau$ for which $u(\tau) = w$.

### A.1 Updating of the center $\rho_t$

The center $\rho_t$ signals the value for which the fundamental $r_t$ makes the exchange rate equal the central parity. This center $\rho_t$ is updated as follows. For values of $r_t$ belonging to its band, the regime is not altered. Thus, nor $c_t$ nor $\rho_t$ are moved. Now consider a shock that exceeds its upper band, $r_t > \tau$, and the CB decides to devalue. Then, we know that the central parity jumps at $c_t = c_{t-1} + \mu$, and the exchange rate is pegged at $x_t = c_t = \max \{ w, \mu \}$. At this particular moment, the depreciation rate within the new band is $x_t - c_t = \max \{ w, \mu \} - \mu$. The new center $\rho_t$ compatible with this situation is to set

$$\rho_t = r_t - \bar{\delta},$$  \hspace{1cm} (33)

where $\bar{\delta}$ denotes the value that meets the following condition:

$$u (\bar{\delta}) = \max \{ w, \mu \} - \mu.$$  \hspace{1cm} (34)
• If \( w \leq \mu \), the exchange rate jumps and equals the new central parity, and the depreciation within band is 0. Then, \( \delta = 0 \) and the new center is \( \rho_t = r_t \).

• If \( w > \mu \), the exchange rate equals its own previous upper limit, and the depreciation is \( w - \mu > 0 \). The new center is \( \rho_t = r_t - \delta \).

When the shock exceeds the lower bound, \( r_t < -\tau \), the new center is pegged at
\[
\rho_t = r_t - \delta.
\] (35)

A similar condition arises to determine \( \delta \):
\[
u(\delta) = -\max\{w, \mu\} + \mu.
\] (36)

Notice that \( u(\delta) = -u(\delta) \), then \( \delta = -\delta \). Again, when \(-\mu < -w\), we have that \( \delta = -\delta = 0 \).

### A.2 A numerical approximation

The general solution (25) involves a collection of integrals where only (28) and (29) enjoy an explicit form. Numerical solutions are requested for the remaining ones. Here, we propose a method that discretizes variable \( r_t \) on \( K \) values (\( K \geq 3 \) odd) within the interval \([-\tau, \tau] \) as
\[
\begin{align*}
\tau &= r_1, r_2, \ldots, r_K, \\
r_1 &= -\tau, \\
r_k &= r_{k-1} + h, \\
h &= 2\tau / (K - 1) > 0, \\
r_K &= \tau, \\
r_{K+1} &= 0.
\end{align*}
\] (37) (38) (39)

In practice, we have used \( K = 101 \).

Let \( P, Q, R, S \) and \( T \) be row vectors (\( 1 \times K \)), adopting the following form
\[
\begin{align*}
P_1 &= \Phi\left(\frac{\tau + h/2 - r_t}{\sigma}\right) - \Phi\left(\frac{\tau - r_t}{\sigma}\right), \\
P_k &= \Phi\left(\frac{r_k + h/2 - r_t}{\sigma}\right) - \Phi\left(\frac{r_k - h/2 - r_t}{\sigma}\right), \\
P_K &= \Phi\left(\frac{\tau - r_t}{\sigma}\right) - \Phi\left(\frac{\tau - h/2 - r_t}{\sigma}\right)
\end{align*}
\] (40)

with \( r_t \) given and
\[ Q_k = 1 - \Phi \left( \frac{\tau - r_k}{\sigma} \right) - \Phi \left( \frac{-\tau - r_k}{\sigma} \right), \]  
\[ R_k = \left[ \Phi \left( \frac{\tau - r_k}{\sigma} \right) - \Phi \left( \frac{-\tau - r_k}{\sigma} \right) \right] r_k + \sigma \left[ \phi \left( \frac{-\tau - r_k}{\sigma} \right) - \phi \left( \frac{\tau - r_k}{\sigma} \right) \right], \]  
for \( k = 1, 2, \ldots, K \), and

\[ T = [\alpha w + (1 - \alpha) \max \{w, \mu\}] Q \]  

where \( \phi \) and \( \Phi \) represent, respectively, the Gaussian pdf and cdf.

For the first period ahead, and only for this period, integrals (28) and (29) have explicit form:

\[ E[\delta_{1,0} | F_0] = [\alpha w + (1 - \alpha) \max \{w, \mu\}] \left[ 1 - \Phi \left( \frac{\tau - r_1}{\sigma} \right) - \Phi \left( \frac{-\tau - r_1}{\sigma} \right) \right] \]

\[ E[r_{t+1} | F_1] = \left[ \Phi \left( \frac{\tau - r_t}{\sigma} \right) - \Phi \left( \frac{-\tau - r_t}{\sigma} \right) \right] r_t + \sigma \left[ \phi \left( \frac{-\tau - r_t}{\sigma} \right) - \phi \left( \frac{\tau - r_t}{\sigma} \right) \right]. \]

For the second period ahead, a numerical approximation is given by

\[ E[\delta_{2,1} | F_1] = TP', \]
\[ E[r_{t+2} | F_2] = RP'. \]

This implies that the value of the integrals at \( t+2 \) is determined for any possible mean at \( t+1 \), \( r_{t+1} \in [-\tau, \tau] \), and any of these means is weighted by a probability \( P \), given a starting value \( r_t \) at \( t \).

The loop becomes harder as the period ahead increases over three. In order to solve this problem, we develop a backward recursion algorithm, for which the last period integral is firstly solved and then proceed backward up to the first one. Thereby, consider the following non negative matrix \( M \in \mathbb{R}^{K \times K} \)

\[ M_{1,l} = \Phi \left( \frac{-\tau + h/2 - r_l}{\sigma} \right) - \Phi \left( \frac{-\tau - r_l}{\sigma} \right), \]
\[ M_{k,l} = \Phi \left( \frac{r_k + h/2 - r_l}{\sigma} \right) - \Phi \left( \frac{r_k - h/2 - r_l}{\sigma} \right), \]
\[ M_{K,l} = \Phi \left( \frac{-\tau - r_l}{\sigma} \right) - \Phi \left( \frac{\tau - h/2 - r_l}{\sigma} \right), \]
for \( k, l = 1, 2, \ldots K \), with the following properties: the sum over each column gives a row vector \( m_c \in \mathbb{R}^{1 \times K} \), with all its components lying within the \((0, 1)\) interval

\[
m_c(l) = \sum_{k=1}^{K} M_{kl}, \quad \int_{-\tau}^{\tau} \phi(s|r_l) \, ds \in (0, 1) \quad (44)
\]

for \( l = 1, 2, \ldots, K \). The proof follows trivially. This property is sufficient to verify the Hawkins-Simon condition (Brauer-Solow Theorem).

Matrix \( M \) contains the transition probabilities in the intermediate periods from \( \tau \) up to \( \tau + 1 \), within the interval \([-\tau, \tau]\) and for any conditional mean belonging to \([-\tau, \tau]\). Thus, the solution for the third period is given by

\[
E[\delta_{3,2} | \mathcal{F}_2] = TM P',
\]

\[
E[r_{t+3} | \mathcal{F}_3] = RM P'.
\]

Again, the value of the integrals are first determined at \( t + 3 \) for any possible mean at \( t + 2 \), \( r_{t+2} \in [-\tau, \tau] \), given by the columns of \( M \). The vector \( P \) closes the calculation for any possible mean at \( t + 1 \), \( r_{t+1} \in [-\tau, \tau] \), for given a starting value \( r_t \).

For \( \tau = 2, 3, \ldots \), further generalization gives a sequence:

\[
E[\delta_{\tau,\tau-1} | \mathcal{F}_{\tau-1}] = TM^{\tau-2} P',
\]

\[
E[r_{t+\tau} | \mathcal{F}_\tau] = RM^{\tau-2} P'.
\]

Plugging these values into (25), one obtains

\[
u(r_t) = \frac{r_t}{1 + \lambda} + \frac{E[r_{t+1} | \mathcal{F}_t]}{(1 + \lambda)^2} + \frac{E[\delta_{1,0} | \mathcal{F}_0]}{1 + \lambda} \sum_{\tau=2}^{\infty} \frac{RM^{\tau-2} P'}{(1 + \lambda)^\tau}.
\]

Let the vector of eigenvalues be given by \( \eta(M) \), and call \( \eta^*(M) \) the Frobenius root, \textit{i.e.} the maximum eigenvalue. From (44) we know that \( m_c(l) < 1 \), this is a sufficient condition to verify the Hawkins-Simon condition (see Brauer-Solow Theorem). In turn, verification of the Hawkins-Simon condition implies that \( \eta^*(M) < 1 \), (see Hawkins and Simon [19] and [20]). Then, matrix \((I - M)^{-1}\) exists, it is non negative and can be written as

\[
(I - M)^{-1} = \sum_{j=0}^{\infty} M_j
\]

This gives rise to a convenient simplification

\[
\mathcal{M} = \sum_{\tau=2}^{\infty} (1 + \lambda)^{-\tau} M^{\tau-2} = \frac{1}{1 + \lambda} [(1 + \lambda) I - M]^{-1}.
\]
The general solution becomes

\[ u(r_t) = \frac{\Omega (r_t)}{1 + \lambda} + \Delta (r_t), \tag{46} \]

with

\[ \Omega (r_t) \equiv r_t + \frac{E [r_{t+1} | F_t]}{1 + \lambda} + R M P', \]

\[ \Delta (r_t) \equiv \frac{E [\delta_{t,0} | F_{t-1}]}{1 + \lambda} + T M P'. \]

Application of level conditions (7) and (8) gives the particular solution

\[ \frac{\Omega (r)}{1 + \lambda} + \Delta (r) = w. \]

Finally, the exchange rate expectation is

\[ E_t x_{t+1} = \sum_{\tau=1}^{\infty} \frac{F [r_{t+\tau} | F_t]}{(1 + \lambda)^\tau} + (1 + \lambda) \sum_{\tau=1}^{\infty} \frac{F [\delta_{t,\tau-1} | F_{t-1}]}{(1 + \lambda)^\tau}, \]

or using the previous approximation

\[ E_t x_{t+1} = \Omega (r_t) + (1 + \lambda) \Delta (r_t) - x_t. \tag{47} \]

Once we know the expression for the exchange rate and the expectation, the interest rates differential is obtained from the uncovered interest parity condition as

\[ d_t = \Omega (r_t) + (1 + \lambda) \Delta (r_t) - x_t. \tag{48} \]

## B Estimation of \( \sigma \) and \( \mu \)

Bekaert [4] reports an unconditional variance for a time invariant risk premium of 10.6222, for the Dollar/Yen rate. This is equivalent to a monthly standard deviation of \( \sigma = 0.008 \) basis points per month. Bekaert and Gray [6] estimate a mean for the foreign risk premium of 3.02% and standard deviation of 3.28%, in yearly terms, for the FF/DM exchange rate, with fluctuations between the \([-3.30\%, +31.45\%]\) interval. For flexible rates, Bekaert [5] concludes that this standard deviation is about 10%.

As a first approximation, we have used the ARCH-in-mean estimation by Domowitz and Hakkio [14], for monthly observations 1973-6-1982-8, of Stirling Pound, French Franc, German Marc, Japanese Yen and Swiss Franc, all against the USA Dollar. Table B.1 reports the results from a Montecarlo simulation with calculations of the standard deviations for a first difference of \( r_t \). Values goes from a minimum of 0% for the Swiss Franc (i.e. no variability in its risk premium), to a maximum of 1% per month for the DM versus the dollar.
As a second approximation, we estimate a \( n \)-variate VAR

\[
z_t = a + \sum_{i=1}^{k} A_i z_{t-i} + \eta_t
\]

with \( \eta_t \sim N(0, \Sigma) \) for every \( t \), serially uncorrelated, and \( \Sigma \) not diagonal, where \( z_t \) is a \((n \times 1)\) vector whose first element is given by \( \Delta x_t \equiv x_t - x_{t-1} \). Three different specifications are given:

- **Model 1**: \( z_t = (\Delta x_t, \Delta i_t, \Delta i^*_t)' \),
- **Model 2**: \( z_t = (\Delta x_t, i_t - i^*_t)' \),
- **Model 3**: \( z_t = (\Delta x_t, i_t - i^*_t, \pi_t - \pi^*_t)' \),

with \( i_t \) and \( i^*_t \) are the domestic and the German interest rates, respectively, measured by 1-month interbank rates. Notice that \( \Delta x_t \) collects within band jumps and realignments. In model 1, variables are first differenced to preserve stationarity. In models 2 and 3, stationarity is also reached through the interest rate differential. On the other hand, \( \pi_t \) and \( \pi^*_t \) stand by domestic and German inflation rates, as measured by deseasonalized CPI's. Both interest rates and inflation rates are natural candidates to forecast the exchange rate. Since no restrictions are imposed on the coefficients of matrices \( A_i \)'s, OLS can be applied to estimate (49). We use monthly observations of these variables for Belgium, Denmark, France, Ireland, Italy, The Netherlands, Spain, Portugal and United Kingdom. All exchange rates are expressed in terms of one German Mark. Observations cover the final period of the EMS, from July 1989 to December 1998 (114 observations).

Thus, the one period ahead expectation is given by

\[
\tilde{E}_t (z_{t+1}) = \tilde{a} + \sum_{i=1}^{k} \tilde{A}_i z_{t+1-i}.
\]

for which the first row of (50) yields an estimate for the expectation

\[
\tilde{E}_t (\Delta x_{t+1}) = \tilde{E}_t (x_{t+1} - x_t).
\]

This estimate is used to calculate deviations from UIP as

\[
\tilde{r}_t = i_t - i^*_t - \tilde{E}_t (\Delta x_{t+1}).
\]

Finally, we compute the standard deviation \( \tilde{\sigma} \) as

\[
\tilde{\sigma} = std (\Delta \tilde{r}_t).
\]

Results are collected in table B.2. A LR-test has been used to identify the \( VAR(k) \) order. Countries are organized according to its \( \sigma \)-estimate in model
1, but this ordinality (and cardinality as well) is more or less preserved under models 2 and 3. At the first sight, countries who suffered the biggest speculative attacks in September 1992 happen to have also the largest $\sigma$, regardless the preference $\lambda$ revealed by their respective central banks. Portugal is a borderline case, and perhaps should be a priori associated to the lowest half of the table. The mean value of these last four rows suggests a $\sigma = 1\%$ as representative of those currencies that experienced serious credibility losses by September 1992. This estimate coincides with the maximum given in the article of Domowitz and Hakkio [14].

Table B.2 here

Table B.3 collects data of realignment rates for the different currencies participating in the ERM of the EMS, except the Dutch guilder. Columns represent the percentage of realignment. Pooling these data for each band, it seems that $\mu = \pm 4.5\%$ and $\mu = \pm 6.3\%$, for widths $w = \pm 2.25$ and $w = \pm 6\%$, respectively, are consistent with the EMS history.

Table B.3 here

C Numerical search of equilibrium

Expressions (19) and (20) define a system of two equations on $(\alpha, \bar{r})$, that is, a function $f (\alpha, \bar{r}) : [0, 1] \times \mathbb{R} \to \mathbb{R}^2$, with

$$f (\alpha, \bar{r}) = \begin{pmatrix} \frac{\zeta (\bar{r}) - \zeta (r)}{\zeta (\bar{r}) - \zeta (r)} - \alpha \\ G (\bar{r}, \alpha; \{\lambda, \sigma, \mu\}) \end{pmatrix},$$

now the function $G (\cdot)$ is expressed to mean that it is depending on $(\alpha, \bar{r})$, for given $\{\lambda, \sigma, \mu\}$. Therefore, the problem to solve is to find a fixed point $(\alpha, \bar{r})$, such that $f (\alpha, \bar{r}) = 0$.

The following procedure is a four step iterative numerical algorithm to solve this fixed point problem.

- **1st step.** This step generates two values, indexed with $n = 1, 2$.

  Guess two starting values for $\alpha$, say $\alpha_{(1)}$ and $\alpha_{(2)}$, such that
  
  $$0 \leq \alpha_{(1)} < \alpha_{(2)} < 1,$$

  define the profiles $P_{(1)} = \{\lambda, w, \alpha_{(1)}\}$ and $P_{(2)} = \{\lambda, w, \alpha_{(2)}\}$, and calculate the ranges $[-\tau_{(1)}, \tau_{(1)}]$ and $[-\tau_{(2)}, \tau_{(2)}]$, for which

  $$\bar{x} = u (\tau_{(1)} | P_{(1)}) = w,$$

  $$\bar{x} = u (\tau_{(2)} | P_{(2)}) = w.$$
Calculate the values $\tilde{r}(1)$ and $\tilde{r}(2)$ for which

\[ \alpha(1) = \frac{\zeta(\tilde{r}(1)) - \zeta(\bar{r}(1))}{1 - \zeta(\bar{r}(1))}, \]

\[ \alpha(2) = \frac{\zeta(\tilde{r}(2)) - \zeta(\bar{r}(2))}{1 - \zeta(\bar{r}(2))}. \]

Compute the gain functions in both cases conditioned on their respective values, $\tilde{r}(1)$ and $\tilde{r}(2)$. If $G(1) = G(\tilde{r}(1); \mathcal{P}(1))$ is negative and $G(1) = G(\tilde{r}(2); \mathcal{P}(2))$ is positive, then proceed through step number 2. Otherwise, find a duet of starting values, $\{0 \leq \alpha(1) < \alpha(2) < 1\}$ yielding $\{G(1) < 0, G(2) > 0\}$.

- **2nd step.** This step generates values indexed with $n = 3, 4, ...$

Update $\alpha$ with a linear projection as

\[ \alpha(n) = \alpha_{\inf}(n) - \frac{\alpha_{\sup}(n) - \alpha_{\inf}(n)}{G_{\sup}(n) - G_{\inf}(n)} \]

where

\[ G_{\inf}(n) = \max \{G^-\}, \]

\[ G_{\sup}(n) = \min \{G^+\}, \]

with $G^-$ and $G^+$ being the sets that collect the sequence of negative and positive gains, respectively:

\[ G^- = \left\{ G(\tilde{r}(i); \mathcal{P}(i))_{i=1,...,n-1} / G(\tilde{r}(i); \mathcal{P}(i)) \leq 0 \right\}, \]

\[ G^+ = \left\{ G(\tilde{r}(i); \mathcal{P}(i))_{i=1,...,n-1} / G(\tilde{r}(i); \mathcal{P}(i)) > 0 \right\}, \]

and $\{\alpha_{\inf}(n), \alpha_{\sup}(n)\}$ is the pair of probabilities associated to $G_{\sup}(n)$ and $G_{\inf}(n)$, respectively. Calculate the range $[-\bar{r}(n), \bar{r}(n)]$, for which

\[ u(\bar{r}(n)|\mathcal{P}(n)) = w. \]

Calculate the value $\tilde{r}(n)$ that solves

\[ \alpha(n) = \frac{\zeta(\tilde{r}(n)) - \zeta(\bar{r}(n))}{1 - \zeta(\bar{r}(n))}. \]

Calculate $G(n) = G(\tilde{r}(n); \mathcal{P}(n))$.

- **3rd step.** If $\alpha(n+1) \neq \alpha(n)$ go again to 2nd step. For practical purposes, we give a numerical tolerance of $10^{-5}$ for the difference $|\alpha(n+1) - \alpha(n)|$. Otherwise, if such a tolerance level has been reached, the process is finished.
## D Tables

### Table 1

<table>
<thead>
<tr>
<th>Country</th>
<th>Implicit band</th>
<th>Official band</th>
<th>Maastricht</th>
</tr>
</thead>
<tbody>
<tr>
<td>Netherlands</td>
<td>±0.85%</td>
<td>±15%</td>
<td>1.75</td>
</tr>
<tr>
<td>Belgium</td>
<td>±2.06%</td>
<td>±15%</td>
<td>2.00</td>
</tr>
<tr>
<td>Denmark</td>
<td>±6.65%</td>
<td>±15%</td>
<td>1.50</td>
</tr>
<tr>
<td>Portugal</td>
<td>±7.89%</td>
<td>±15%</td>
<td>3.75</td>
</tr>
<tr>
<td>Spain</td>
<td>±10.08%</td>
<td>±15%</td>
<td>3.25</td>
</tr>
<tr>
<td>Ireland</td>
<td>±12.95%</td>
<td>±15%</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: Labhard and Wyplosz [21] and Deustche Bundesbank [13]

### Table 2

<table>
<thead>
<tr>
<th>Country</th>
<th>Date</th>
<th>Width change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>January 20, 1997</td>
<td>±12.5% → ±5.00%</td>
</tr>
<tr>
<td></td>
<td>June 25, 1998</td>
<td>±5.00% → ±2.75%</td>
</tr>
<tr>
<td></td>
<td>September 16, 1998</td>
<td>±2.75% → ±3.50%</td>
</tr>
<tr>
<td></td>
<td>December 22, 1998</td>
<td>±3.50% → ±8.00%</td>
</tr>
<tr>
<td></td>
<td>September 2, 1999</td>
<td>±8.00% → Float</td>
</tr>
<tr>
<td>Cyprus</td>
<td>August 1, 1999</td>
<td>±2.25% → ±15.0%</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>February 28, 1996</td>
<td>±2.00% → ±7.50%</td>
</tr>
<tr>
<td></td>
<td>May 27, 1997</td>
<td>±7.50% → Float</td>
</tr>
<tr>
<td>Ecuador</td>
<td>March 25, 1998</td>
<td>±5.00% → ±10.0%</td>
</tr>
<tr>
<td></td>
<td>September 14, 1998</td>
<td>±10.0% → ±15.0%</td>
</tr>
<tr>
<td></td>
<td>February 12, 1999</td>
<td>±15.0% → Float</td>
</tr>
<tr>
<td>Hungary</td>
<td>May 2001</td>
<td>±2.25% → ±15.0%</td>
</tr>
<tr>
<td>Israel</td>
<td>March 1, 1990</td>
<td>±3.00% → ±5.00%</td>
</tr>
<tr>
<td></td>
<td>July 26, 1993</td>
<td>±5.00% → ±6.00%</td>
</tr>
<tr>
<td></td>
<td>December 30, 2000</td>
<td>±6.00% → ±39.2%</td>
</tr>
<tr>
<td>Poland</td>
<td>October 29, 1998</td>
<td>±10.0% → ±12.5%</td>
</tr>
<tr>
<td></td>
<td>March 24, 1999</td>
<td>±12.5% → ±15.0%</td>
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<tr>
<td></td>
<td>April 12, 2000</td>
<td>±15.0% → Float</td>
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<tr>
<td>Sri Lanka</td>
<td>November 3, 2000</td>
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</tr>
<tr>
<td></td>
<td>December 11, 2000</td>
<td>±8.00% → ±10.0%</td>
</tr>
<tr>
<td></td>
<td>January 22, 2001</td>
<td>±10.0% → Float</td>
</tr>
<tr>
<td>Uruguay</td>
<td>January 4, 2002</td>
<td>±3.00% → ±6.00%</td>
</tr>
</tbody>
</table>

Source: Reinhart and Rogoff [28]
### Table 3
Equilibrium results for $\alpha$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$w = \pm 2.25%$</th>
<th>$w = \pm 6.0%$</th>
<th>$w = \pm 15%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-1}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$1 \times 10^{-2}$</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>$1 \times 10^{-3}$</td>
<td>0.51</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>$5 \times 10^{-4}$</td>
<td>0.39</td>
<td>0.76</td>
<td>0.85</td>
</tr>
<tr>
<td>$1 \times 10^{-4}$</td>
<td>0.23</td>
<td>0.71</td>
<td>0.85</td>
</tr>
</tbody>
</table>

### Table 4
Optimal band widths

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$w(\lambda)$</th>
<th>$\alpha(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008</td>
<td>15%</td>
<td>0.8223</td>
</tr>
<tr>
<td>0.0009</td>
<td>14%</td>
<td>0.8189</td>
</tr>
<tr>
<td>0.0010</td>
<td>11%</td>
<td>0.8694</td>
</tr>
<tr>
<td>0.0020</td>
<td>11%</td>
<td>0.9945</td>
</tr>
<tr>
<td>0.0030</td>
<td>8%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0040</td>
<td>6%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0050</td>
<td>5%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0060</td>
<td>4%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0070</td>
<td>3%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0080</td>
<td>3%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0090</td>
<td>2%</td>
<td>1.0000</td>
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<tr>
<td>0.0100</td>
<td>2%</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0500</td>
<td>1%</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Table B.1: Montecarlo simulation of $\sigma$

<table>
<thead>
<tr>
<th>Country</th>
<th>$\text{std} \left( \Delta r_t \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom</td>
<td>0.0022</td>
</tr>
<tr>
<td>France</td>
<td>0.0044</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0101</td>
</tr>
<tr>
<td>Japan</td>
<td>0.0046</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0</td>
</tr>
</tbody>
</table>

Table B.2: VAR estimation of $\sigma$

<table>
<thead>
<tr>
<th>Country</th>
<th>Model 1 $k$</th>
<th>$\bar{\sigma}$</th>
<th>Model 2 $k$</th>
<th>$\bar{\sigma}$</th>
<th>Model 3 $k$</th>
<th>$\bar{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Netherlands</td>
<td>4</td>
<td>0.00055</td>
<td>2</td>
<td>0.00040</td>
<td>1</td>
<td>0.00026</td>
</tr>
<tr>
<td>Belgium</td>
<td>4</td>
<td>0.00321</td>
<td>4</td>
<td>0.00323</td>
<td>4</td>
<td>0.00325</td>
</tr>
<tr>
<td>France</td>
<td>4</td>
<td>0.00330</td>
<td>5</td>
<td>0.00305</td>
<td>5</td>
<td>0.00312</td>
</tr>
<tr>
<td>Portugal</td>
<td>12</td>
<td>0.00560</td>
<td>9</td>
<td>0.00499</td>
<td>10</td>
<td>0.00556</td>
</tr>
<tr>
<td>Denmark</td>
<td>6</td>
<td>0.00575</td>
<td>7</td>
<td>0.00566</td>
<td>7</td>
<td>0.00592</td>
</tr>
<tr>
<td>Spain</td>
<td>5</td>
<td>0.00628</td>
<td>7</td>
<td>0.00763</td>
<td>5</td>
<td>0.00674</td>
</tr>
<tr>
<td>Ireland</td>
<td>4</td>
<td>0.00898</td>
<td>2</td>
<td>0.00474</td>
<td>3</td>
<td>0.00746</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>3</td>
<td>0.01301</td>
<td>5</td>
<td>0.01216</td>
<td>3</td>
<td>0.01337</td>
</tr>
<tr>
<td>Italy</td>
<td>3</td>
<td>0.01680</td>
<td>4</td>
<td>0.01704</td>
<td>4</td>
<td>0.01729</td>
</tr>
</tbody>
</table>

Table B.3: Realignment rates in the ERM\(^\text{10}\)

<table>
<thead>
<tr>
<th>Date</th>
<th>FF</th>
<th>IRP</th>
<th>BF</th>
<th>DK</th>
<th>ITL</th>
<th>SP</th>
<th>ESC</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-Sep-1979</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>5.00</td>
<td>2.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-Nov-1979</td>
<td></td>
<td>0.14</td>
<td></td>
<td>5.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23-Mar-1981</td>
<td></td>
<td>-0.14</td>
<td></td>
<td></td>
<td>6.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-Oct-1981</td>
<td>8.76</td>
<td>5.50</td>
<td>5.50</td>
<td>5.50</td>
<td>8.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22-Feb-1982</td>
<td></td>
<td>9.29</td>
<td></td>
<td>3.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21-Mar-1983</td>
<td>8.20</td>
<td>9.33</td>
<td>3.94</td>
<td>2.93</td>
<td>8.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-Jul-1985</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-Apr-1986</td>
<td>6.19</td>
<td>3.00</td>
<td>1.98</td>
<td>1.98</td>
<td>3.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-Aug-1986</td>
<td></td>
<td>8.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12-Jan-1987</td>
<td>3.00</td>
<td>3.00</td>
<td>0.98</td>
<td>3.00</td>
<td>3.00</td>
<td>8.22</td>
<td></td>
</tr>
<tr>
<td>8-Jan-1990</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.70</td>
<td>6.95</td>
<td></td>
</tr>
<tr>
<td>14-Sep-1992</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.25</td>
</tr>
<tr>
<td>17-Sep-1992</td>
<td></td>
<td>5.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14-May-1993</td>
<td></td>
<td>8.70</td>
<td></td>
<td></td>
<td></td>
<td>6.95</td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>6.46</td>
<td>5.86</td>
<td>3.10</td>
<td>3.84</td>
<td>5.54</td>
<td>6.78</td>
<td>6.67</td>
</tr>
<tr>
<td><strong>std</strong></td>
<td>3.39</td>
<td>3.41</td>
<td>3.01</td>
<td>1.26</td>
<td>2.42</td>
<td>1.75</td>
<td>0.40</td>
</tr>
</tbody>
</table>

\(^{10}\)FF stands for the French franc, IRP the Irish punt, BF the Belgium franc, DK the Danish
References


Krona, ITL the Italian lira, SP the Spanish peseta and PESC the Portuguese escudo. The Italian lira ITL participated in the narrow ±2.25% band from 12th-January-1987 until 14th-September-1992, when it left the ERM.


Figure 1:
Function $u(r)$ ($\lambda = 0.5$)
Figure 4: $\lambda = 10^{-4}$
Exchange rate standard deviation

Time horizon

- $w = \pm 2.25\%$
- $w = \pm 6\%$
- $w = \pm 15\%$