The Structural Approach of a Natrex Model on Equilibrium Exchange Rates

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Abstract

Following a general macroeconomic approach, this paper sets a closed micro-founded structural model to determine the long run real exchange rate of a developed economy. In particular, the analysis follows the structure of a Natrex model. The main contribution of this research paper is the development of a solid theoretical framework that analyse in depth the basis of the real exchange rate and the details of the equilibrium dynamics after any shock influencing the steady state. In our case, the intertemporal factors derived from the stock-flow relationship will be particularly determinant. The main results of the paper can be summarised as follows. In first place, a complete well-integrated structural model for long-run real exchange rate determination is developed from first principles. Moreover, within the concrete dynamics of the model, it is found that some convergence restrictions will be necessary. On one hand, for the medium run convergence the sensitivity of the trade balance to changes in real exchange rate should be higher that the correspondent one to the investment decisions. On the other hand, and regarding long-run convergence, it is also necessary both that there exists a negative relationship between investment and capital stock accumulation and that the global saving of the economy depends positively on net foreign debt accumulation. In addition, there are also interesting conclusions about the effects that certain shocks over the exogenous variables of the model have on real exchange rates.

Keywords: *macroeconomic approach, long run real exchange rate, Natrex models, equilibrium dynamics, convergence restrictions, productivity, foreign debt accumulation.*

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1. Introduction

Certainly, economists have made significant efforts trying to report convincing explanations about the exchange rate behaviour. The results, however, have not been up to the dimension of the work and, as Meese and Rogoff (1983) underline, too frequently they have turned out a little bit disappointing. The new research on the topic has echoed this fact revealing the necessity of alternative results. This is the case of the quite recent contributions to the structural approach, which have clearly shown up their preference for the long-run analysis and for stronger theoretical structures. The interest of these new contributions is in reality two-fold. On one hand, they avoid the cyclical and speculative short run behaviour, proved to be transitory and rather unpredictable and, on the other hand, they deliberately oversight the simplifying assumption of highly integrated markets. Instead, the approach deals with the characteristics of the internal and external economic equilibrium, as well as with the stability requirements of the system.

To a large extent, the most popular approaches to exchange rate determination have ended up, in one way or another, bringing up single-equation systems where the nominal or real exchange rate has been simply related to a set of possible exogenous explanatory variables. From an economic point of view, this sort of proceeding can be justified by the useful, but important, simplifications that a reduced version imposes on the theoretical complexity of a model and on the intuitive interpretation of it. Probably, the best known examples of this nature are the PPP hypothesis, along with its subsequent theoretical developments[1], and those that, taking a step forward, have included some macroeconomic fundamental determinants. This is for instance the case of the vast research based on the Balassa-Samuelson supply hypothesis –originally developed from separated contributions of Balassa (1964) and Samuelson (1964)– or the demand-side fundamental approach developed in De Gregorio, Giovannini and Wolf (1994). Recent contributions on the PPP approach are those of Engel and Rogers (2001) and Rey and Varachaud (2002) for the case of the European economies; Strong and Sharma (2002) and Taylor (2002) for a group of industrialised countries and Anoruo, Braha and Ahmad (2002) for the case of developing economies. There are also recent notable contributions on the Balassa-Samuelson hypothesis like MacDonald and Ricci (2001), DeLoach (2001) or Canzoneri et al (2002).

Regarding exchange rate determination, it cannot be either ignored the extensive research done from a merely econometric point of view. This sort of literature has evolved from simple univariate time-series techniques, and its subsequent multivariate extensions, to the new multivariate techniques for non-stationary variables. These ones, on their own, conform a complete battery of work that goes from simple uniequational exchange rate cointegrating relationships to the more complicated SVAR (structural vector autoregressive) approach. Recent outstanding contributions using the cointegration technique are Karfakis and Phipps (1999) for the case of the American dollar against the Australian dollar and Detken et al (2002a, 2002b) for a composite

[1] That is, the asset monetary approach to exchange rate determination.

index of the euro against its major trading partners. A good reference of the SVAR literature is the seminal work of Clarida and Gali (1994)[2] as well as that of MacDonald and Swagel (1998).

However, and although it would not be fair to refer to these theories as simply rough attempts to better understand the exchange rate behaviour, it is obvious to us that the great majority of them can be merely assumed as partial views of a more general global market equilibrium. The alternative can be found in the so-called structural approach, which follows in fact two main tracks, the partial equilibrium specification and the general equilibrium one.

The partial equilibrium approach, or macroeconomic balance approach, has focused exclusively on the trade balance component of the current account, so that it has completely avoided the existent feedback between the capital account and the real exchange rate determination. This feedback, clearly induced by the payments for serving the accumulation of net foreign debt, generates a persistence effect that cannot be ignored. On the contrary, the general equilibrium approach is based on complete macroeconomic models that take care, in one way or another, of the problem of long-run sustainability, being consequently conscious of the hysteresis phenomenon. The most known applications of this general equilibrium framework are the Williamson's fundamental equilibrium real exchange rate (FEER), the IMF's desired equilibrium exchange rate (DEER) and the natural equilibrium real exchange rate (NATREX) of Stein. Representative works of these approaches are Williamson (1994) for the FEER approach, Bayoumi et al (1994) for the case of the DEER and Stein and associates (1995) for the NATREX approach.

This paper aims to contribute to the theoretical literature on the structural macroeconomic approach analysing the real exchange rate from a dynamic general equilibrium perspective. In the paper, we set a closed micro-founded structural model to determine the long run real exchange rate of a developed economy. In particular, the analysis will follow the structure of a Natrex model[3], whose main peculiarities within the structural approach are twofold. On one hand, the own structure of the model presents a singular stock-flow interaction not known up to the moment. On the other hand, it introduces an exclusive distinction between the medium and the long-run equilibrium that consists on the following: while in the medium-run only the external and internal equilibrium conditions will be required, in the long-run it will be also necessary that the net foreign debt plus the capital stock reach their steady state levels.

It is interesting how many of the self-named structural models of exchange rate determination, included the different versions of the Natrex approach, have ended up, in one way or another, only partially deriving the equations that constitute the theoretical models, so the unfortunate result is that the approaches lack of well-integrated structures with relative frequency. Regarding the

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[2] Authors of the so-called CG model.

[3] Fist developed by Stein (1990, 1994).

implementation of equilibriums, an additional objection is also the systematic omission of some important theoretical restrictions that are in fact determinant for the structural characterisation of the approach. Finally, and not less important, there is the fact of the continuous absence of a formal discussion about the convergence and stability of the systems under analysis. In that sense, the main contribution of this paper is the development of a solid theoretical framework that analyses in depth the basis of the real exchange rate, as well as the details of the equilibrium dynamics after any shock influencing the steady state positions. In our case, the intertemporal factors derived from the stock-flow relationship will be particularly determinant.

The main results of the paper are related next. In first place, a complete well-integrated structural model for long-run real exchange rate determination is developed from first principles. Moreover, we find that for convergence reasons, there are some restrictions that the model will necessarily need to satisfy. For the medium run convergence, the trade balance sensitivity to changes in the real exchange rate should be higher, in absolute values, than the correspondent one for investment decisions[4]. Regarding the long-run, it is also necessary both, that there exists a negative relationship between investment and capital stock accumulation and that the global saving of the economy (integrating public and private sectors) depends positively on the net foreign debt accumulation. In addition, there are also interesting conclusions about the results that certain shocks over the exogenous variables of the model have on real exchange rates.

The structure of the paper is as follows. To start with, section two introduces the concept of equilibrium as well as the notion of stability used along the paper, which is set up according to the Natrex approach. Next, in order to find well-defined investment, consumption and trade balance equations that describe the behaviour of our fully rational economic agents, in section three it is developed the correspondent micro-founded optimising programs. To continue, section four introduces the definitive characterisation of the equilibrium model both in a medium and long-run horizon, as well as it is introduced a detailed analysis of the medium and long run convergence conditions of the system. Moreover, section four presents in addition interesting conclusions about the effect of some particular shocks over the fundamentals of equilibrium. Finally, section five concludes the paper.

2. Equilibrium, Sustainability and the Natrex Approach

There is a wide consensus regarding what can be understood by long-run equilibrium of the real exchange rate. In this respect, Nurkse (1945) had already laid the foundations of the concept considering that it is the real exchange rate that is consistent with the dual objectives of internal and external balance of an economy, given the values of other variables that could influence the established equilibrium. However, although the definition is conceptually clear, in practical terms it

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[4] Following Stein (1999), in this approach investment depends negatively on real exchange rates.

is not so easy to find an agreement about how to deal with the implementation of these theoretical equilibriums.

In the particular case of Natrex, the approach introduces a distinction between the medium and long-run equilibrium depending on if the considered stock variables of the model, that is, the capital stock and the accumulated stock of foreign assets, have the time to converge. The Natrex, defined as a medium run equilibrium, is understood as the real exchange rate that prevails once speculative and cyclical factors are absent and the unemployment is at its natural rate. Therefore, in this medium term approach the internal equilibrium will be characterised by a non-inflationary employment rate, while for the external equilibrium it is simply required a balance between the international flow of capitals. This medium run analysis does not enter however into considerations about long-run sustainability conditions.

Alternatively, in the long-run the Natrex approach introduces the additional requirement that the capital stock and the stock of foreign debt should reach their steady state levels. For the external balance condition it implies not only that the current account will need to be financed by foreign capitals, but that the capital inflow should be sustainable over time.

Regarding the exogenous factors driving the equilibrium, the most important ones within the Natrex approach are a thrift parameter measuring the preference for consumption and a measure of productivity. Such as Stein suggested, in the context of the Natrex models the standard responses of exchange rates to exogenous shocks are the following: a negative shock to the consumption preferences will clearly depreciate the currency in the medium-run, although it will lead to a real appreciation in the long-run. However, a negative productivity shock will depreciate the equilibrium exchange rate in the medium-run, whereas the long-run effect can be ambiguous and will depend on the speed of decumulation of net foreign debt.

3. Behavioural Equations of the Model

In this model there are decisions on consumption, production, savings and investment, which will determine the equilibrium of the economy. To start with, the main assumptions of the model are carefully detailed next.

3.1 Assumptions

a) Consumption and investment decisions are decentralised. That is, the case of a representative agent who decides both variables simultaneously is specifically omitted. In this approach families will choose consumption and savings, while firms decide production and investment.

b) Savings are canalised to the national or international investment requirements through the financial system. That is, the national and foreign families can make savings profitable buying national or foreign bonds. Moreover, it is introduced here the assumption of perfect international financial capital mobility, what implies a domestic rate of return tied to the international one.

c) In this model the labour supply is assumed offered inelastically inside the country, although it is free to migrate between sectors. Regarding the capital stock, it will grow depending on investment decisions.

d) The role of the relative price between national and foreign goods is introduced through the differentiation between goods of tradable and non-tradable nature. Non-tradable goods will be produced and consumed inside the country, while tradables are freely dealt in international homogeneous markets. They will be therefore indistinctly produced and consumed both inside and outside the country, being their price exogenously determined in international markets[5].

e) Prices are considered in relative terms. In particular, the price of each good is defined in terms of a numerary that, in this case, is given by a good of tradable nature. The model is therefore characterised by the absence of a role for money, which is completely neutral in the analysis.

f) Finally, it is assumed that it does not exist any kind of international transactional costs or distorting taxes.

3.2 Investment Decisions

Let us assume common Cobb-Douglas production functions for the national production of tradable (T) and non-tradable (N) goods respectively,

$$Q_T = f(K_T, L_T) = A^{1-\alpha} K_T^{\alpha} L_T^{1-\alpha} \tag{1}$$

$$Q_N = f(K_N, L_N) = A^{1-\beta} K_N^{\beta} L_N^{1-\beta} \tag{2}$$

Being Q the real GDP, K the national capital stock -assumed exclusively composed of tradable goods-, L the total labour force and parameter A a labour augmenting productivity factor equal for the two sectors of the economy.

To determine investment we will start off with the definition in real terms of the sectorial profit

[5] Note that in this particular case the terms of trade are assumed to be equal to one, so the real exchange rate can be approximated by the relative price of tradable to non-tradable goods. For an exhaustive analysis of this point see appendix 1.

equations, being all variables specified in terms of the numerary. These are expressions (3) and (4) respectively,

$$Pr_T = Q(K_T, L_T) - r\,K_T - \omega L_T \tag{3}$$

$$Pr_N = R\,Q(K_N, L_N) - r\,K_N - \omega L_N \tag{4}$$

where R is the real exchange rate approached by the price index ratio of non-tradable to tradable goods, and r and ω are the real interest rate and real wage respectively.

Moreover, the following existent relationships between the sectorial and global variables should be also taken into account,

$$K = K_T + K_N \tag{5}$$

$$L = L_T + L_N \tag{6}$$

In order to find the optimal levels of the labour force and capital stock, we should optimise equations (3) and (4) with respect to the two sectorial production factors. Conditions from (7) to (10) give us successively the obtained results,

$$\frac{\partial Pr_T}{\partial K_T} = \frac{\partial Q_T}{\partial K_T} - r = 0 \tag{7}$$

$$\frac{\partial Pr_N}{\partial K_N} = R\frac{\partial Q_N}{\partial K_N} - r = 0 \tag{8}$$

$$\frac{\partial Pr_T}{\partial L_T} = \frac{\partial Q_T}{\partial L_T} - \omega = 0 \tag{9}$$

$$\frac{\partial Pr_N}{\partial L_N} = R\frac{\partial Q_N}{\partial L_N} - \omega = 0 \tag{10}$$

Determining next the required derivatives from the correspondent production functions, the previous optimality conditions can give us final expressions for the labour and capital stock equilibrium levels,

$$K_T = A\,L_T\left(\frac{\alpha}{r}\right)^{1/1-\alpha} \tag{11}$$

$$K_N = A\,L_N\left(\frac{\beta R}{r}\right)^{1/1-\beta} \tag{12}$$

$$L_T = K_T\, A^{1-\alpha/\alpha} \left(\frac{1-\alpha}{\omega} \right)^{1/\alpha} \tag{13}$$

$$L_N = K_N\, A^{1-\beta/\beta} \left(\frac{1-\beta}{\omega/R} \right)^{1/\beta} \tag{14}$$

The optimal sectorial distribution can be also determined taking into account that the capital and labour remuneration, r and ω respectively, should equalise across sectors. For the capital stock, (11) and (12) will be the relevant expressions to work with, while (13) and (14) will be the ones for the case of the labour market. These subsequent equilibrium conditions are (15) and (16) respectively,

$$\frac{K_N^{\beta}}{L_N^{\beta}} = \frac{1}{A^{\alpha\text{-}\beta}} \frac{1}{R} \frac{\alpha}{\beta} \frac{L_T}{L_N} \frac{K_N}{K_T} \frac{K_T^{\alpha}}{L_T^{\alpha}} \tag{15}$$

$$\frac{K_T^{\alpha}}{L_T^{\alpha}} = A^{\alpha-\beta} R \frac{1-\beta}{1-\alpha} \frac{K_N^{\beta}}{L_N^{\beta}} \tag{16}$$

Moreover, when the capital stock has the time to redistribute between sectors, (15) and (16) can give us a unique equilibrium condition, shown next in (17),

$$\frac{K_T}{L_T} = \frac{\alpha}{\beta} \frac{1-\beta}{1-\alpha} \frac{K_N}{L_N} \tag{17}$$

At this point we can take advantage of equation (17) to determine expressions for the sectorial capital stocks as functions of the global one. They are easily obtained by the simple combination of the previous (17) equation with the (5) identity,

$$K_T = \frac{1}{1 + \frac{\beta}{\alpha} \frac{1-\alpha}{1-\beta} \frac{L_N}{L_T}} K \tag{18}$$

$$K_N = \frac{1}{1 + \frac{\alpha}{\beta} \frac{1-\beta}{1-\alpha} \frac{L_T}{L_N}} K \tag{19}$$

Let us assume next that the engine moving the capital stock is the labour augmenting productivity growth rate. In that case, the first differences of identity (5) with respect to A will allow us to specify the first approach of the required investment function,

$$I = A \left(\frac{dK_T}{dA} + \frac{dK_N}{dA} \right) \hat{A} \tag{20}$$

Taking into account expressions (11) and (12) for K_T and K_N respectively, and given L as a fixed value, the correspondent derivatives necessary to solve equation (20) are then easily obtained,

$$\frac{dK_T}{dA}=\frac{K_T}{A}+\frac{K_T}{L_T}\frac{dL_T}{dA} \tag{21}$$

$$\frac{dK_N}{dA}=\frac{K_N}{A}+\frac{K_N}{L_N}\frac{dL_N}{dA} \tag{22}$$

Substituting next (21) and (22) into the correspondent (20) equation, it is finally obtained a concrete expression representing the required investment function,

$$I=A\left[\frac{K_T}{A}+\frac{K_N}{A}+\left(\frac{K_T}{L_T}-\frac{K_N}{L_N}\right)\frac{dL_T}{dA}\right]\hat{A} \tag{23}$$

Or alternatively,

$$I=A\,L_T\frac{K_T}{A\,L_T}\left[1+\frac{K_N}{K_T}+\left(1-\frac{K_N}{K_T}\frac{L_T}{L_N}\right)\frac{A}{L_T}\frac{dL_T}{dA}\right]\hat{A} \tag{24}$$

Moreover, making use again of the (11) and (17) expressions, the previous (24) equation can still simplify to the next one,

$$I=A\,L_T\left(\frac{\alpha}{r}\right)^{1/1-\alpha}\left[1+\frac{K_N}{K_T}+\frac{\alpha-\beta}{\alpha(1-\beta)}\frac{A}{L_T}\frac{dL_T}{dA}\right]\hat{A} \tag{25}$$

In order to determine dL_T/dA we will start off with the derivative of equation (17) with respect to the A parameter,

$$\frac{dL_T}{dA}=\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{K_T}{K_N}L_N\left[\frac{1}{K_T}\left(\frac{dK_T}{dK}-\frac{K_T}{K_N}\frac{dK_N}{dK}\right)\frac{dK}{dA}+\frac{1}{L_N}\frac{dL_N}{dL_T}\frac{dL_T}{dA}\right] \tag{26}$$

Coming back to the use of the own (17) equation plus the (5) and (6) identities, (26) will subsequently simplify to equation (27),

$$\frac{dL_T}{dA}=\frac{\dfrac{L_T}{K_T}\left[\left(1+\dfrac{K_T}{K_N}\right)\dfrac{dK_T}{dK}-\dfrac{K_T}{K_N}\right]\dfrac{dK}{dA}}{1+\dfrac{L_T}{L_N}} \tag{27}$$

Finally, from the following (28) and (29) expressions, derived respectively from equations (17) and (18),

$$\frac{K_T}{K_N} = \frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N} \tag{28}$$

$$\frac{dK_T}{dK} = \frac{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}K \tag{29}$$

we can easily conclude the following,

$$\frac{dL_T}{dA} = 0 \tag{30}$$

And, consequently, the final expression for the investment function is given by equation (31),

$$I = A L_T \left(\frac{\alpha}{r}\right)^{1/1-\alpha}\left(1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}\right)\hat{A} \tag{31}$$

For simplicity reasons, the model will be presented with equations as ratios to GDP. Therefore, the next step should be to determine the explicative variables of production.

To start with, let us define the total real production Q (defined in terms of tradable good) as a function of the sectorial ones,

$$Q = Q_T + R Q_N \tag{32}$$

Given equation (15), which shows the optimal distribution of the capital stock between sectors, plus the (1) and (2) definitions of Q_T and Q_N respectively, the following relationship is easily obtained,

$$R Q_N = \frac{\alpha}{\beta}\frac{K_N}{K_T}Q_T \tag{33}$$

Now, the previous production function (32) can be simplified to the next (34) one,

$$Q = \left(K + \frac{\alpha-\beta}{\beta}K_N\right)\frac{Q_T}{K_T} \tag{34}$$

Moreover, taking into account the definition of K_N as it appears in (19), equation (34) can simplify again to the following,

$$Q = K\,\frac{\alpha}{\beta}\left(\frac{1+\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}\right)\frac{Q_T}{K_T} \tag{35}$$

In the same way, the ratio of Q_T to K_T can be determined as a function of the global capital stock taking into account the production function (1) and expression (18) that connects K_T with the global capital stock,

$$\frac{Q_T}{K_T} = \left(\frac{A\,L_T}{K}\right)^{1-\alpha}\left(\frac{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}\right)^{1-\alpha} \tag{36}$$

Substituting then (36) into (35), this one easily collapses to the new production function (37),

$$Q = A^{1-\alpha}L_T^{1-\alpha}K^{\alpha}\,\frac{1+\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}}{\left(1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}\right)^{\alpha}} \tag{37}$$

From (37), rearranging and grouping conveniently, it is determined a final expression for production where the capital stock appears as a ratio to GDP,

$$Q = A\,L_T\,(K/Q)^{\alpha/1-\alpha}\left[\frac{1+\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}}{\left(1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}\right)^{\alpha}}\right]^{1/1-\alpha} \tag{38}$$

So we can now specify the investment function as a ratio to GDP as it appears in (39),

$$\frac{I}{Q} = \frac{\hat{A}}{(K/Q)^{\alpha/1-\alpha}}\left(\frac{\alpha}{r}\right)^{1/1-\alpha}\left(\frac{1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}}{1+\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}}\right)^{1/1-\alpha} \tag{39}$$

Finally, we should look for the explanatory variables of the ratio of L_N over L_T. Making use of definitions (13) and (14) for L_T and L_N respectively, the following relationship is easily obtained,

$$\frac{L_N}{L_T} = R^{1/\beta} \ \frac{(1-\beta)^{1/\beta}}{(1-\alpha)^{1/\alpha}} \left(\frac{A}{\omega}\right)^{\alpha-\beta/\alpha\beta} \frac{K_N}{K_T} \tag{40}$$

So, substituting this one into (39), equation (40) can be redefined as follows,

$$\frac{I}{Q} = \frac{\hat{A}}{(K/Q)^{\frac{\alpha}{1-\alpha}}} \frac{\beta}{\alpha}\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \left(\frac{\frac{\alpha}{\beta} + R^{1/\beta} \ \frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}} \left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}} \frac{K_N}{K_T}}{1 + R^{1/\beta} \ \frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}} \left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}} \frac{K_N}{K_T}} \right)^{\frac{1}{1-\alpha}} \tag{41}$$

Given that the index A and real wages will grow at the same rate, and that after increments in the capital stock (every thing equal) the ratio K_N/K_T would keep constant, from equation (41) the main variables explaining the investment ratio are deduced. Summarising the most important factors we find that the ratio of the capital stock, the real interest rate and the real exchange rate are all variables influencing negatively the investment ratio, while improvements of the productivity index ($\hat{A}$) are alternatively boosting it. These results are summarised as follows,

$$\frac{I}{Q} = F_I \left(\hat{A}, \frac{K}{Q}, r, R \right) \tag{42}$$

Being $(F_I)'_{\hat{A}} > 0$, $(F_I)'_{K/Q} < 0$, $(F_I)'_r < 0$ and $(F_I)'_R < 0$. Appendix 2 proofs in detail the signs of the derivatives.

3.3 Consumption and Savings Decisions

Let us consider a representative family who decides between consumption and savings. Given that in this section we are interested in the national global levels of the two variables, in order to simplify we decide to undertake this particular problem working –at least temporarily– not in terms of tradable and non-tradable goods, but in aggregated terms.

In this case, it is assumed that the representative family choosing between consumption and savings is solving this dilemma by means of an intertemporal utility maximisation problem. The optimisation problem, without loss of generality, is simply specified over the horizon of two single periods. The periods, named present and future period, will be represented by the sub-indexes 1 and 2 respectively.

The families in this approach will not need to restrict their consumption to their current period incomes. That is to say, whenever the agents are able to predict future incomes and there are not liquidity constraints, they can change present for future consumption and vice versa. Moreover, the family can conclude its live with a predetermined quantity of assets inherited by the next generation. The only restriction imposed to this assumption is that the final quantity of accumulated assets should not evolve explosively over time.

Therefore, in order to induce convergence it is assumed that the decision to accumulate assets in the second period is going to be discouraged by the possession of some of them inherited from a previous generation. Moreover, we also find reasonable that this negative relationship is qualified by the families' preferences for consumption. In this respect, we are going to consider that a higher preference for consumption should cause a lower desire to accumulate assets at the end of the life, facilitating then an increment in the volume of consumption. From these assumptions the following relationship should occur,

$$\frac{a_2}{q_2} = f\left[\frac{a_0}{q_0}, \varphi\right] \quad f_1' < 0 \quad f_2' < 0 \tag{43}$$

The consideration that for the consumer the most important thing is not the possession of assets in absolute level but their proportion out of his total income is what motivates that a_0 an a_2 enter in (43) as ratios of the families' incomes. The parameter φ measures the preference for consumption.

To simplify the subsequent analysis, we will concretise the relationship (43) to a particular functional form. Being the linear specification the more comfortable one, (43) is finally defined as in (44),

$$\frac{a_2}{q_2} = -\alpha\left(\frac{a_0}{q_0}\right) - \varphi_1 \quad 0 < \alpha < 1 \tag{44}$$

To determine the intertemporal budget constraint we can start with the definition of savings taking into account that this variable has a dual perspective; from a financial point of view savings can be considered as the flow of accumulated assets, while from the point of view of consumption it can be defined as the differential between disposable income and the own quantity of consumption. These two aspects of savings have been reflected in expressions (45) and (46) respectively,

$$s_t = a_t - a_{t-1} \tag{45}$$

$$s_t = q_t + r_{t-1}\, a_{t-1} - c_t \tag{46}$$

Being s savings, a the total stock of accumulated assets, q the share of national production received by the family, r the real interest rate and c the level of consumption. Lower case means (with the exception of the real interest rate) that variables are defined for the sphere of activity of the family.

The total stock of assets a represents the family portfolio composed of both national bonds, named b, and foreign assets, named f or d alternatively. In particular, the name f is associated to the case when the family possesses rights over the foreign sector, while the name d is associated to the possession of foreign debt. The identity can be therefore indistinctly specified as in (47) or (48),

$$a_t = b_t + f_t \tag{47}$$

$$a_t = b_t - d_t \tag{48}$$

Working with expressions (45) and (46), the dynamic equation for the stock of total assets a is easily obtained and it remains as follows,

$$a_t = q_t - c_t + (1+r_{t-1})\, a_{t-1} \tag{49}$$

Particularising (49) for the periods one and two, the following relationships are consequently obtained,

$$a_1 = q_1 - c_1 + (1+r_0)\, a_0 \tag{50}$$

$$a_2 = q_2 - c_2 + (1+r_1)\, a_1 \tag{51}$$

These (50) and (51) equations are the base to determine the family's intertemporal budget constraint that appears in (52),

$$c_1 + \frac{c_2}{1+r_1} = q_1 + \frac{q_2}{1+r_1} + (1+r_0)\, a_0 - \frac{a_2}{1+r_1} \tag{52}$$

Regarding preferences, it is assumed a standard CES[6] utility function that depends positively on both the present and the future consumption as expression (53) shows,

$$U(c_1, c_2) = \left(c_1^{\gamma} + c_2^{\gamma}\right)^{1/\gamma} \tag{53}$$

The parameter γ is directly related to the intertemporal elasticity of substitution. In particular, appendix 3 displays the concrete relationship, shown in (54), connecting γ and the elasticity of

[6] CES is the acronym of constant elasticity of substitution.

substitution between the future and present consumption, named ε_{21}.

$$\varepsilon_{21} = \frac{1}{1-\gamma} \quad \gamma < 1 \quad \varepsilon'_{\gamma} > 0 \tag{54}$$

As (54) suggests, the parameter γ and ε_{21} move in the same direction. The intuition behind this elasticity of substitution is that in the case of a higher ε_{21} the consumer would remain indifferent renouncing to important quantities of present consumption in exchange of an additional unit of future consumption, and vice versa in the case of a lower ε_{21}.

We are now equipped with the appropriate elements to solve the consumer optimisation problem. The next step is therefore the resolution of the Lagrangian function given in equation (55),

$$\ell(c_1, c_2) = c_1^{\gamma} + c_2^{\gamma} - \lambda\left[c_1 + \frac{c_2}{1+r_1} - q_1 - \frac{q_2}{1+r_1} - (1+r_0)\,a_0 + \frac{a_2}{1+r_1}\right] \tag{55}$$

Equations (56), (57) and (58) are therefore the first order conditions of the problem,

$$\frac{\partial \ell}{\partial c_1} = c_1^{\gamma-1} - \lambda = 0 \tag{56}$$

$$\frac{\partial \ell}{\partial c_2} = c_2^{\gamma-1} - \frac{\lambda}{1+r_1} = 0 \tag{57}$$

$$\frac{\partial \ell}{\partial \lambda} = c_1 + \frac{c_2}{1+r_1} - q_1 - \frac{q_2}{1+r_1} - (1+r_0)\,a_0 + \frac{a_2}{1+r_1} = 0 \tag{58}$$

From (56) and (57) it is obtained the equilibrium condition (59) that relates the present and future consumption,

$$\frac{c_2}{c_1} = (1+r_1)^{1/1-\gamma} \tag{59}$$

Equation (58) simply provides the intertemporal budget constraint previously detailed in (52).

Eventually, with the determination of (59) we are provided with the necessary equations to settle down the optimal consumption and savings decisions. The equations to consider in this problem are therefore (46), (59) and the budget constraint (52).

In particular, fom (59) and (52), the expression (60) defining the present consumption is easily obtained. In the case of savings, it is (46) and the own (60) the ones used instead.

$$c_1 = \frac{(1+r_1)\left[\,q_1 + (1+r_0)\,a_0\,\right] + q_2 - a_2}{1 + r_1 + (1+r_1)^{1/1-\gamma}} \tag{60}$$

$$s_1 = q_1 + r_0\,a_0 - \frac{(1+r_1)\left[\,q_1 + (1+r_0)\,a_0\,\right] + q_2 - a_2}{1 + r_1 + (1+r_1)^{1/1-\gamma}} \tag{61}$$

However, given that we are interested in the national aggregated consumption and savings decisions, the obtained (60) and (61) individual equations will have to be extended to functions that represent the whole population. Expressions (62) and (63) show this aggregated versions, where we have also assumed equal interest rates between periods,

$$C_1 = \frac{(1+r)\left[\,Q_1 + (1+r)\,A_0\,\right] + Q_2 - A_2}{1 + r + (1+r)^{1/1-\gamma}} \tag{62}$$

$$S_1 = Q_1 + r\,A_0 - \frac{(1+r)\left[\,Q_1 + (1+r)\,A_0\,\right] + Q_2 - A_2}{1 + r + (1+r)^{1/1-\gamma}} \tag{63}$$

The capital letters means national levels and the total stock of assets A_2, from equations (44) and (48), is determined as follows,

$$A_2 = -Q_2\left[\alpha\left(\frac{A_0}{Q_0}\right) + \varphi_1\right] \tag{64}$$

$$A_0 = B_0 - D_0 \tag{65}$$

Given that in aggregated terms the variable B collapses to zero, (65) can be then reduced to the next simplified expression,

$$A_0 = -D_0 \tag{66}$$

Moreover, assuming that in the production function the capital stock enters with a lag, a production function for the level of Q_2 can be specified as follows,

$$Q_2 = F_Q(K_1) \tag{67}$$

Or, assuming constant returns to scale, it can be also settled as expression (68), where Q_2 is given as a ratio of the current level of production,

$$\frac{Q_2}{Q_1} = f_q\left(\frac{K_1}{Q_1}\right) \tag{68}$$

Both derivatives, $(F_Q)'_K$ and $(f_q)'_{K/Q}$, are considered greater than zero.

Now, making use of (64), (66) and (68), the consumption and saving equations (62) and (63) can be respectively reformulated as follows,

$$C_1 = \frac{Q_1 - Q_0\,(1+r)\dfrac{D_0}{Q_0} + \dfrac{Q_1}{1+r} f_q \left(1+\varphi_1 - \alpha \dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{69}$$

$$S_1 = Q_1 - Q_0\, r \frac{D_0}{Q_0} - \frac{Q_1 - Q_0\,(1+r)\dfrac{D_0}{Q_0} + \dfrac{Q_1}{1+r} f_q \left(1+\varphi_1 - \alpha \dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{70}$$

Or, alternatively, as in (71) and (72), if they are expressed as ratios to GDP,

$$\frac{C_1}{Q_1} = \frac{1-(Q_0/Q_1)\,(1+r)\dfrac{D_0}{Q_0} + \dfrac{f_q}{1+r} \left(1+\varphi_1 - \alpha \dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{71}$$

$$\frac{S_1}{Q_1} = 1-(Q_0/Q_1)\, r \frac{D_0}{Q_0} - \frac{1-(Q_0/Q_1)\,(1+r)\dfrac{D_0}{Q_0} + \dfrac{f_q}{1+r} \left(1+\varphi_1 - \alpha \dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{72}$$

From equations (71) and (72) we finally deduce the main variables explaining the saving and consumption ratios.

With regard to the consumption ratio, appendix 4 demonstrates that there exist unambiguous effects on behalf of all the main explanatory variables of consumption. Both, a higher capital stock as ratio to GDP and a higher preference for consumption will undoubtedly encourage the desire to consume while on the contrary, a higher interest rate or an increment in the amount of accumulated foreign debt are sure to provoke a continence of it.

Just the opposite reasoning can be deduced for the case of savings with the exception of both the real interest rate and the foreign debt variable. In this case, a change in the foreign debt ratio will affect in opposite directions the savings behaviour. The reason of it is that while higher foreign debt ratios will disincentive consumption, at the same time, the interest payment owed to the external sector will also decrease the level of disposable income. From a purely theoretical point of view our model cannot help us to forecast the sign relating the two variables. However, the empirical evidence seems to predict a positive relationship between foreign debt and saving, just the opposite to the case of consumption.

Moreover, appendix 5 shows how changes in the real interest rate are also ambiguous over the savings decisions. As it is standard in the literature, our approach to savings also predicts that a positive relationship between the real interest rates and savings is more probable in the case of the country being a net creditor. The probability decreases on the contrary when the country changes from being a net creditor to be a net debtor. Nevertheless, following the predictions of the empirical evidence the most probable result will be a positive relationship between the two variables.

All these results have been summarised in (73) and (74) respectively,

$$\frac{C}{Q} = F_C\left(\frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{73}$$

$$\frac{S}{Q} = F_S\left(\frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{74}$$

With $(F_C)'_{K/Q} > 0,\ (F_C)'_{D/Q} < 0,\ (F_C)'_r < 0,\ (F_C)'_\varphi > 0$ regarding the consumption ratio and $(F_S)'_{K/Q} < 0,\ (F_S)'_{D/Q} >^? 0,\ (F_S)'_r >^? 0,\ (F_S)'_\varphi < 0$ in the case of savings[7]. Appendix 4 and 5 proof in detail the signs of the derivatives.

3.4 Trade Balance and the Current Account

The trade balance is defined as the net income flow coming from the goods sold to the foreign sector (exports, X) once discounted the goods that the nationals acquire to the rest of the world (imports, M). Regarding the trade balance, the only relevant goods to be taken into account will be those that can be exchanged internationally. This is the reason why the trade balance is considered in analytical terms as the difference, shown in (75), between the national and foreign tradable goods exchanged with the rest of the world. In this section, it is again important the distinction between tradables and non-tradables left out in the analysis of consumption.

$$TB = X - M \tag{75}$$

In order to settle the trade balance determinants, we propose next the second step of an optimisation problem designed in two-stages. Provided the selected quantities of present and future consumption previously determined in the preceding section, we will consider here a consumer that decides how to distribute his optimal aggregated consumption into goods of tradable and non-tradable nature.

[7] The question mark is introduced as a superscript of the inequality sign to remark the theoretical ambiguity of the result. If, for instance, following the empirical evidence the prospects of α being greater than β are high, then our terminological convention is to express that as $\alpha >^? \beta$.

Under this approach the following identity will be therefore true,

$$P_T\,C_T + P_N\,C_N = P\,C \tag{76}$$

where the sub-index T and N means tradable and non-tradable respectively, P is a price index and C is consumption. The price index P accompanying the global consumption C is considered as a composite index of the respective P_T and P_N ones. Particularly, it is defined as follows,

$$P = \theta P_T + (1-\theta)P_N \tag{77}$$

So we can make use of (77) to reformulate equation (76) as in (78), where it is expressed in terms of the tradable goods,

$$C_T + R\,C_N = [\,\theta + (1-\theta)\,R]C \tag{78}$$

In the same way, a standard intra-temporal utility function should be specified here. Following the usual practise, we assume next a simple Cobb-Douglas utility function that represents the consumer preferences on tradable and non-tradable goods,

$$U(C_T, C_N) = C_T^{\alpha}\,C_N^{\beta} \tag{79}$$

From (78) and (79), the optimisation problem will remain as follows,

$$\ell(C_T, C_N) = C_T^{\alpha}\,C_N^{\beta} - \lambda\,[\,C_T + R\,C_N - \theta\,C - (1-\theta)\,R\,C] \tag{80}$$

Expressions (81) and (82) show the first order conditions,

$$\frac{\partial\,\ell}{\partial\,C_T} = \frac{\alpha\,C_N^{\beta}}{C_T^{1-\alpha}} - \lambda = 0 \tag{81}$$

$$\frac{\partial\,\ell}{\partial\,C_N} = \frac{\beta C_T^{\alpha}}{C_N^{1-\beta}} - \lambda\,R = 0 \tag{82}$$

From the combination of (81) and (82), it is obtained the equilibrium relationship in (83) between the C_T and C_N variables,

$$\frac{C_T}{C_N} = R\frac{\alpha}{\beta} \tag{83}$$

So, we can make use of (83) to solve finally equation (78) for the consumption of tradables,

$$C_T = \frac{\alpha}{\alpha+\beta}\left[\theta+(1-\theta)R\right]C \tag{84}$$

For simplifying reasons, we can assume that a fix proportion, for instance κ, of the desired consumption of tradables is from abroad, so that the volume of imports can be easily expressed as in (85),

$$M = \kappa\frac{\alpha}{\alpha+\beta}\left[\theta+(1-\theta)R\right]C \tag{85}$$

Or, alternatively, as in (86) in terms of GDP,

$$\frac{M}{Q} = \kappa\frac{\alpha}{\alpha+\beta}\left[\theta+(1-\theta)R\right]\frac{C}{Q} \tag{86}$$

Following equation (86), we can underline the real exchange rate and the consumption ratio as the main explanatory variables of the desired volume of imports. That is the following,

$$\frac{M}{Q} = F_M\left(R, \frac{C}{Q}\right) \tag{87}$$

where $(F_M)'_R > 0$ and $(F_M)'_{C/Q} > 0$.

In the same way, the reasoning developed for the analysis of imports can be extended to the case of exports, so that equation (88) would eventually represent the behaviour of exports in this economy,

$$\frac{X}{Q^*} = F_X\left(R, \frac{C^*}{Q^*}\right) \tag{88}$$

where $(F_X)'_R < 0$ and $(F_X)'_{C^*/Q^*} > 0$ in this case.

Provided now with equations (87) and (88), we can finally formulate the trade balance ratio as an unambiguous function of the real exchange rate, the national consumption ratio and the foreign consumption ratio. In the case of a stationary relationship between the national and foreign GDP's, the expression (89) will represent the desired trade balance equation,

$$\frac{TB}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) \tag{89}$$

where $(F_{TB})'_R < 0$, $(F_{TB})'_{C^*/Q^*} > 0$ and $(F_{TB})'_{C/Q} < 0$. See appendix 6 for details.

Regarding the current account behaviour, let us define this variable as follows,

$$\frac{CA}{Q} = \frac{TB}{Q} - \frac{DS}{Q} \tag{90}$$

Where CA is referred to the current account and DS to the total amount of debt service. The payments derived from the possession of foreign debt, the so-called DS, are assumed in this model to depend exclusively up on the accumulated stock of assets. In that case, a function specifying the current account behaviour can be therefore formulated as follows,

$$\frac{CA}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) - F_{DS}\left(\frac{D}{Q}\right) \tag{91}$$

where $(F_{DS})'_{D/Q} > 0$.

Moreover, taking into account expression (73), which explains the consumption ratio, equation (91) can be easily reformulated in terms of the predetermined and exogenous variables of the model,

$$CA = F_{CA}\left(R, \frac{C^*}{Q^*}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{92}$$

where $(F_{CA})'_R < 0$, $(F_{CA})'_{C^*/Q^*} > 0$, $(F_{CA})'_{K/Q} < 0$, $(F_{CA})'_{D/Q} \overset{?}{<} 0$ $(F_{CA})'_r > 0$, $(F_{CA})'_\varphi < 0$.

The sign of all variables, with the exception of foreign debt, can be unambiguously deduced from the behavioural equations of the consumption and trade balance ratios to GDP. Regarding the relationship between foreign debt and current account, there is an ambiguous effect derived from the opposite influences of debt on consumption and on the service of the net foreign debt. Nevertheless, the more intuitive relationship between the two variables is a negative response of the current account to variations in foreign debt. The reason is that in an indebted economy, the introduction of risk premiums on behalf of the foreign sector will end up causing that the interest payment exceeds the improvement that the increment in debt has on the trade balance. Appendix 7 shows the details of this analysis.

4. The Model

In the previous section the main behavioural equations conforming the Natrex model, that is, the investment, consumption and trade balance equations, have been sufficiently founded.

Provided then with the necessary aggregated macroeconomic functions, we develop in this section the complete set of equations that define both the medium and long run equilibrium that characterise the Natrex approach.

4.1 The Analytical Framework of Equilibrium

As section 2 details, the Natrex is understood as the real exchange rate that prevails once speculative and cyclical factors are absent and the unemployment is at its natural rate. This is the case when, in a non-inflationary context, there exists a balance between the international flow of capitals. Or, in analytical terms, when the desired social savings (S) minus social investment (I) equal the current account (CA) as expression (93), with variables given in ratios to GDP, particularly shows,

$$\frac{CA}{Q}=\frac{S}{Q}-\frac{I}{Q} \tag{93}$$

Alternatively, given the definitions of savings and current account in terms of the net payment of services, equation (93) can be also reformulated as follows,

$$\frac{TB}{Q}=1-\left(\frac{I}{Q}+\frac{C}{Q}\right) \tag{94}$$

Both, equations (93) and (94), alternatively represent the national account equilibrium condition of the economy under analysis.

In our model, we are going to consider that, given the decisions on investment and consumption, it is the trade balance -through the adjustments in the real exchange rate- which will accommodate to obtain the external equilibrium of the system. Under this assumption, the equation determining the real exchange rate is obtained from the trade balance equation (89), while the trade balance account can be determined by means of equation (94). The system in this approach would be then conformed by equations (42), (73) and (89), solved for the respective endogenous variables of the model, plus equation (94) describing the trade balance behaviour. In that case, expressions (42), (73) and (89) would remain as follows,

$$\frac{I}{Q}=\tilde{F}_I\,(\hat{A},\,r)+(F_I)'_{K/Q}\,\frac{K}{Q}+(F_I)'_R\,R \tag{95}$$

$$\frac{C}{Q}=(F_C)'_{K/Q}\,\frac{K}{Q}+(F_C)'_{D/Q}\,\frac{D}{Q}+\tilde{F}_C\,(r,\,\varphi) \tag{96}$$

$$R=\frac{1}{(F_{TB})'_R}\left[\frac{TB}{Q}-(F_{TB})'_{C/Q}\,\frac{C}{Q}-\tilde{F}_{TB}\left(\frac{C^*}{Q^*}\right)\right] \tag{97}$$

However, to define the complete model it is also necessary to include the uncovered interest rate parity condition, the Fisher equation and the dynamic equations for the capital stock and the stock of foreign debt. These are expressions from (98) to (101) respectively.

$$R - R_{LR}^{eq} = r - r^* \tag{98}$$

$$i - i^* = (\pi - \pi^*) + (r - r^*) \tag{99}$$

$$K = (1-\delta)K_{-1} + I \tag{100}$$

$$D - D_{-1} = I - S \tag{101}$$

Where the superscript eq means equilibrium level and the subscript LR means long run, π is the inflation rate and δ the depreciation rate of the capital stock.

The model is finally characterised by equations from (94) to (101).

4.2 Characterisation of the Medium Run Equilibrium

Provided the behavioural equations of the model, the real exchange rate derived from (93) or (94) is in fact a medium run market clearing condition given that, as the capital stock and the net foreign debt move only slowly over time, in the short run they can be treated as exogenous. Then, in the medium run equilibrium, conditions (94), (95), (96) and (97) will apply, so that equations from (102) to (105) will be the ones describing the equilibrium,

$$\left(\frac{I}{Q}\right)^{MR} = \tilde{F}_I(\hat{A}, r) + (F_I)'_{K/Q}\frac{K}{Q} + (F_I)'_R R^{MR} \tag{102}$$

$$\left(\frac{C}{Q}\right)^{MR} = (F_C)'_{K/Q}\frac{K}{Q} + (F_C)'_{D/Q}\frac{D}{Q} + \tilde{F}_C(r, \varphi) \tag{103}$$

$$\left(\frac{TB}{Q}\right)^{MR} = 1 - \left[\left(\frac{I}{Q}\right)^{MR} + \left(\frac{C}{Q}\right)^{MR}\right] \tag{104}$$

$$R^{MR} = \frac{1}{(F_{TB})'_R}\left[\left(\frac{TB}{Q}\right)^{MR} - (F_{TB})'_{C/Q}\left(\frac{C}{Q}\right)^{MR} - \tilde{F}_{TB}\left(\frac{C^*}{Q^*}\right)\right] \tag{105}$$

In this case, $\hat{A}$, K/Q, r, $\hat{Q}$, D/Q, φ, C*/Q* are the exogenous variables of the system, while the investment, consumption and trade balance ratios, together with the real exchange rate, are the endogenous ones to be solved in the context of the model.

This medium run equilibrium can be also specified in graphical terms taking into account the market clearing condition (94). From this equation, the medium run equilibrium can be understood as the level that would balance both, the trade balance economic function on one hand and, on the other, a new function defined by the difference between the unity and the sum of the investment and consumption ratios in terms of GDP. In that case, the national account identity (94) can be therefore reformulated as follows,

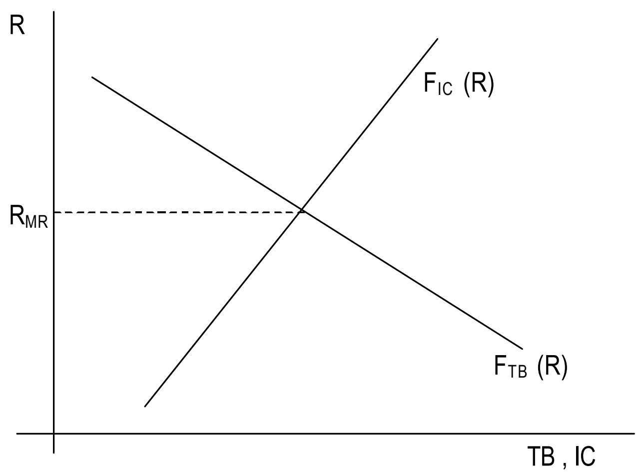
$$F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) = F_{IC}\left(R, \hat{A}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{106}$$

with $(F_{TB})'_R < 0,\ (F_{TB})'_{C^*/Q^*} > 0,\ (F_{TB})'_{K/Q} < 0,\ (F_{TB})'_{D/Q} > 0,\ (F_{TB})'_r > 0,\ (F_{TB})'_\varphi < 0$ and $(F_{IC})'_R > 0,\ (F_{IC})'_{\hat{A}} < 0,\ (F_{IC})'_{K/Q} \overset{?}{>} 0,\ (F_{IC})'_{D/Q} > 0,\ (F_{IC})'_r > 0,\ (F_{IC})'_\varphi < 0$. The only ambiguous relationship in this analysis is that of the derivative $(F_{IC})'_{K/Q}$. The reason is that the accumulation of capital stock influences oppositely investment and consumption decisions. While an increment in the stock of capital disincentives investment; it conversely boosts the level of consumption. Regarding this relationship, the empirical evidence seems to support a higher sensitivity of response in investment than in consumption to changes in the capital stock, so that the most probable result can be $(F_{IC})'_{K/Q}$ being higher than zero.

From (106) it is obtained figure 1, that shows the medium run equilibrium of the real exchange rate,

Figure 1

The Real Exchange Rate Medium Run Equilibrium



Moreover, from the resolution of the system formed by equations from (102) to (105) it is also possible to detail the exogenous fundamentals of this medium run equilibrium,

$$R^{MR} = \frac{\tilde{F}_R\left(\hat{A},\ r,\varphi,\dfrac{C^*}{Q^*}\right) + \alpha^{MR}_{K/Q}\dfrac{K}{Q} + \alpha^{MR}_{D/Q}\dfrac{D}{Q}}{-\left[(F_{TB})'_R + (F_I)'_R\right]} \tag{107}$$

where $\tilde{F}_R$, $\alpha^{MR}_{K/Q}$ and $\alpha^{MR}_{D/Q}$ are defined as follows,

$$\tilde{F}_R\left(\hat{A},\ r,\varphi,\frac{C^*}{Q^*}\right) = \tilde{F}_I(\hat{A},\ r) + \left[1 + (F_{TB})'_{C/Q}\right]\tilde{F}_C(r,\ \varphi) + \tilde{F}_{TB}\left(\frac{C^*}{Q^*}\right) - 1 \tag{108}$$

$$\alpha^{MR}_{K/Q} = (F_I)'_{K/Q} + (F_C)'_{K/Q}\left[1 + (F_{TB})'_{C/Q}\right] \tag{109}$$

$$\alpha^{MR}_{D/Q} = (F_C)'_{D/Q}\left[1 + (F_{TB})'_{C/Q}\right] \tag{110}$$

with $(\tilde{F}_R)'_{\hat{A}} > 0$, $(\tilde{F}_R)'_r < 0$, $(\tilde{F}_R)'_\varphi > 0$, $(\tilde{F}_R)'_{C^*/Q^*} > 0$; $\alpha^{MR}_{K/Q} \overset{?}{<} 0$ and $\alpha^{MR}_{D/Q} < 0$.

Summarising then these results, we can specify a simplified functional relationship between the real exchange rate and its medium run determinants as follows,

$$R^{MR} = F_{R_{MR}}\left(\hat{A},\ r,\ \varphi,\ \frac{C^*}{Q^*},\ \frac{K}{Q},\ \frac{D}{Q}\right) \tag{111}$$

with $(F_{R_{MR}})'_{\hat{A}} > 0$, $(F_{R_{MR}})'_r < 0$, $(F_{R_{MR}})'_\varphi > 0$, $(F_{R_{MR}})'_{C^*/Q^*} > 0$, $(F_{R_{MR}})'_{K/Q} \overset{?}{<} 0$ and $(F_{R_{MR}})'_{D/Q} < 0$.

4.3 Stock Variables and The Long Run Equilibrium

As the stock variables of the model move to their steady state positions, the medium run equilibrium will evolve to equilibrium situations of a longer horizon. The Natrex equilibrium is in fact a sequence of medium run equilibriums evolving to a long run reference.

In the long run, the steady state conditions for the capital stock and the net foreign asset position need also to be implemented. To determine those conditions, we can rewrite the dynamic equations (100) and (101) as ratios to GDP,

$$\frac{K}{Q} = \frac{1-\delta}{1+\hat{Q}}\left(\frac{K}{Q}\right)_{-1} + \frac{I}{Q} \tag{112}$$

$$\frac{D}{Q}=\frac{1}{1+\hat{Q}}\left(\frac{D}{Q}\right)_{-1}+\frac{I}{Q}-\frac{S}{Q} \qquad (113)$$

Given the dynamic equations (112) and (113), the steady state conditions (114) and (115) can be easily obtained,

$$\frac{K}{Q}=\gamma\frac{I}{Q} \qquad (114)$$

$$\frac{D}{Q}=\rho\left(\frac{I}{Q}-\frac{S}{Q}\right) \qquad (115)$$

where γ and ρ are defined as follows,

$$\gamma=\frac{1+\hat{Q}}{\delta+\hat{Q}} \qquad (116)$$

$$\rho=\frac{1+\hat{Q}}{\hat{Q}} \qquad (117)$$

Regarding the determination of the real exchange rate long run equilibrium, besides the medium run condition (107), it should be also taken into account the steady state equations (114) and (115), plus the corresponding behavioural equations defining the investment and saving decisions. Specifying these ones solved for the endogenous variables of the model, the useful equation for investment would be (95), while for savings is (74) but reformulated in the following terms,

$$\frac{S}{Q}=\tilde{F}_S(r,\ \varphi)+(F_S)'_{K/Q}\,\frac{K}{Q}+(F_S)'_{D/Q}\,\frac{D}{Q} \qquad (118)$$

Then, equations from (119) to (123) will describe in this case the desired long run equilibrium,

$$R^{LR}=\frac{\tilde{F}_R\left(\hat{A},\, r^*,\varphi,\frac{C^*}{Q^*}\right)+\alpha^{MR}_{K/Q}\left(\frac{K}{Q}\right)^S+\alpha^{MR}_{D/Q}\left(\frac{D}{Q}\right)^S}{-\left[(F_{TB})'_R+(F_I)'_R\right]} \qquad (119)$$

$$\left(\frac{K}{Q}\right)^S=\gamma\left(\frac{I}{Q}\right)^{LR} \qquad (120)$$

$$\left(\frac{D}{Q}\right)^S=\rho\left[\left(\frac{I}{Q}\right)^{LR}-\left(\frac{S}{Q}\right)^{LR}\right] \qquad (121)$$

$$\left(\frac{I}{Q}\right)^{LR} = \tilde{F}_I\,(\hat{A},\, r^*) + (F_I)'_{K/Q} \left(\frac{K}{Q}\right)^S + (F_I)'_R\, R^{LR} \tag{122}$$

$$\left(\frac{S}{Q}\right)^{LR} = \tilde{F}_S\left(r^*,\ \varphi\right) + (F_S)'_{K/Q} \left(\frac{K}{Q}\right)^S + (F_S)'_{D/Q} \left(\frac{D}{Q}\right)^S \tag{123}$$

Or, alternatively, equations from (124) to (126) if it is solved for the investment and saving ratios,

$$R^{LR} = \frac{\tilde{F}_R\left(\hat{A},\, r\,,\varphi,\frac{C^*}{Q^*}\right) + \alpha^{MR}_{K/Q}\left(\frac{K}{Q}\right)^S + \alpha^{MR}_{D/Q}\left(\frac{D}{Q}\right)^S}{-\left[\,(F_{TB})'_R + (F_I)'_R\,\right]} \tag{124}$$

$$\left(\frac{K}{Q}\right)^S = \frac{\tilde{F}_I\,(\hat{A},\, r^*)}{\theta_1} + \frac{(F_I)'_R}{\theta_1} R^{LR} \tag{125}$$

$$\left(\frac{D}{Q}\right)^S = \frac{\theta_3\,\tilde{F}_I\,(\hat{A},\, r^*) - \theta_1\,\tilde{F}_S\left(r^*,\ \varphi\right)}{\theta_1\,\theta_2} + \frac{\theta_3\,(F_I)'_R}{\theta_1\,\theta_2} R^{LR} \tag{126}$$

with θ_1, θ_2 and θ_3 defined as follows,

$$\theta_1 = 1/\gamma - (F_I)'_{K/Q} \tag{127}$$

$$\theta_2 = 1/\rho + (F_S)'_{D/Q} \tag{128}$$

$$\theta_3 = 1/\gamma - (F_S)'_{K/Q} \tag{129}$$

being the three parameters positive values.

Finally, solving the system formed by equations from (124) to (126), we can obtain the real exchange rate long run equilibrium as a definitive function of its exogenous variables,

$$R^{LR} = \frac{\theta_1\,\theta_2\,\tilde{F}_R\left(\hat{A},\, r\,,\varphi,\frac{C^*}{Q^*}\right) + \left[\theta_2\,\alpha^{MR}_{K/Q} + \theta_3\,\alpha^{MR}_{D/Q}\right]\tilde{F}_I\,(\hat{A},\, r^*) - \theta_1\,\alpha^{MR}_{D/Q}\,\tilde{F}_S\left(r^*,\ \varphi\right)}{\theta_1\,\theta_2\,(\text{-}\,F_{TB})'_R + \left[\theta_1\,\theta_2 + \theta_2\,\alpha^{MR}_{K/Q} + \theta_3\,\alpha^{MR}_{D/Q}\right](\text{-}\,F_I)'_R} \tag{130}$$

4.4 Equilibrium and Stability Conditions

Regarding the medium run stability condition we should take into account the following dynamic equation obtained from the external equilibrium condition (94),

$$(F_{TB})'_R\,(dR) = \text{-}(F_I)'_R\,(dR_{-1}) \tag{131}$$

This equation is justified by the fact that the real exchange rate is the variable that accommodates the trade balance to the levels determined by the F_{IC} function.

From (131), the dynamic equation can be also specified as follows,

$$dR = -\frac{(F_I)'_R}{(F_{TB})'_R}\, dR_{-1} \tag{132}$$

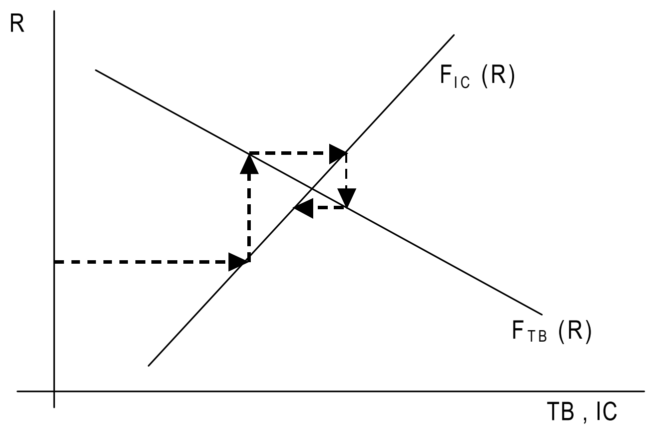
Equation (132) suggests an oscillatory process of convergence to equilibrium that will be stable only if the sensitivity of investment to the real exchange rate is in absolute values smaller than the corresponding one of the trade balance function. That is to say,

$$\left|(F_I)'_R\right| < \left|(F_{TB})'_R\right| \tag{133}$$

The intuition of this medium run condition can be easily observed in graphical terms as follows,

Figure 2

The Dynamics of the Medium Run Convergence



Regarding the long run equilibrium, let us start specifying the following dynamic equations for the stock variables of the system, which have been obtained from (112) and (113) respectively,

$$d\left(\frac{K}{Q}\right) = -\frac{\delta+\hat{Q}}{1+\hat{Q}}\frac{K}{Q} + F_I\left(\hat{A}, \frac{K}{Q}, r, R\right) \tag{134}$$

$$d\left(\frac{D}{Q}\right)=-\frac{\hat{Q}}{1+\hat{Q}}\left(\frac{D}{Q}\right)+F_I\left(\hat{A},\frac{K}{Q},r,R\right)-F_S\left(\frac{K}{Q},\frac{D}{Q},r,\varphi\right) \tag{135}$$

Making use of equations (95) and (118), the previous (134) and (135) can be solved by its corresponding endogenous variables,

$$d\left(\frac{K}{Q}\right)=\left[-\frac{\delta+\hat{Q}}{1+\hat{Q}}+(F_I)'_{K/Q}\right]\frac{K}{Q}+(F_I)'_R\,R+\tilde{F}_I\,(\hat{A},r) \tag{136}$$

$$d\left(\frac{D}{Q}\right)=-\left[\frac{\hat{Q}}{1+\hat{Q}}+(F_S)'_{D/Q}\right]\frac{D}{Q}+F_I\left(\hat{A},\frac{K}{Q},r,R\right)-(F_S)'_{K/Q}\frac{K}{Q}-\tilde{F}_S(r,\varphi) \tag{137}$$

From (136) and (137) we can check that the fact of $(F_I)'_{K/Q}$ being negative and $(F_S)'_{D/Q}$ being positive are both sufficient conditions for the convergence of the stock variables in the long run equilibrium.

4.5 The Real Exchange Rate and its Fundamental Determinants

Finally, we analyse in this section the basic Natrex predictions about the medium and long run consequences of a shock over the productivity factor and the preferences for consumption.

Regarding the medium run effects we can obtain straightforward conclusions from equation (107), where the medium run equilibrium has been previously obtained. Given the results of $(F_{R_{MR}})'_{\hat{A}}>0$ and $(F_{R_{MR}})'_{\varphi}>0$ it is easy to conclude that, according with the Natrex suggestion, in the medium run a positive shock on productivity will appreciate the currency, while a lower preference for consumption will depreciate it.

However in the long run, it is an interesting conclusion of this research our result of an ambiguous effect after a shock affecting both, preferences for consumption and productivity. This contrasts strongly with Stein's prediction of a long run depreciating effect after a positive shock over the preference for consumption.

From (130) we obtain the derivatives of R^{LR} with respect to φ and $\hat{A}$,

$$\frac{\partial R^{LR}}{\partial\varphi}=\frac{\theta_1\left[\theta_2\,(\tilde{F}_R)'_{\varphi}-\alpha^{MR}_{D/Q}\,(\tilde{F}_S)'_{\varphi}\right]}{\theta_1\,\theta_2\,(-F_{TB})'_R+\left[\theta_2\left[\theta_1+\alpha^{MR}_{K/Q}\right]+\theta_3\,\alpha^{MR}_{D/Q}\right](-F_I)'_R} \tag{138}$$

$$\frac{\partial R^{LR}}{\partial \hat{A}} = \frac{\theta_1\,\theta_2\,(\tilde{F}_R)'_{\hat{A}} + \left[\theta_2\,\alpha^{MR}_{K/Q} + \theta_3\,\alpha^{MR}_{D/Q}\right](\tilde{F}_I)'_{\hat{A}}}{\theta_1\,\theta_2\,(-F_{TB})'_R + \left[\theta_2\left[\theta_1 + \alpha^{MR}_{K/Q}\right] + \theta_3\,\alpha^{MR}_{D/Q}\right](-F_I)'_R} \tag{139}$$

Given θ_1, θ_2, θ_3, defined in expressions from (127) to (129), $(\tilde{F}_R)'_\varphi$ and $(\tilde{F}_R)'_{\hat{A}}$, obtained from equation (108), and the conditions saying that $(\tilde{F}_S)'_\varphi = -(\tilde{F}_C)'_\varphi$, $(\tilde{F}_S)'_{K/Q} = -(\tilde{F}_C)'_{K/Q}$ and $(\tilde{F}_S)'_{D/Q} + (\tilde{F}_C)'_{D/Q} = -(\tilde{F}_{DS})'_{D/Q}$, the previous (138) and (139) equations can be reformulated as follows,

$$\frac{\partial R^{LR}}{\partial \varphi} = \frac{\alpha\,\theta_1\,(\tilde{F}_C)'_\varphi\left[1 + (F_{TB})'_{C/Q}\right]}{\theta_1\,\theta_2\,(-F_{TB})'_R + \left[\alpha\left[\frac{1}{\gamma} + (F_C)'_{K/Q}\left[1 + (F_{TB})'_{C/Q}\right]\right] + \frac{(F_{TB})'_{C/Q}\,(F_C)'_{D/Q}}{\gamma}\right](-F_I)'_R} \tag{140}$$

$$\frac{\partial R^{LR}}{\partial \hat{A}} = \frac{\left[\alpha\left[\frac{1}{\gamma} + (F_C)'_{K/Q}\left[1 + (F_{TB})'_{C/Q}\right]\right] + \frac{(F_{TB})'_{C/Q}\,(F_C)'_{D/Q}}{\gamma}\right](\tilde{F}_I)'_{\hat{A}}}{\theta_1\,\theta_2\,(-F_{TB})'_R + \left[\alpha\left[\frac{1}{\gamma} + (F_C)'_{K/Q}\left[1 + (F_{TB})'_{C/Q}\right]\right] + \frac{(F_{TB})'_{C/Q}\,(F_C)'_{D/Q}}{\gamma}\right](-F_I)'_R} \tag{141}$$

where

$$\alpha = 1/\rho - (F_{DS})'_{D/Q} \tag{142}$$

From (140) it is easy to conclude that after a positive shock on the preference for consumption, only in the case of α being negative, there exists an opportunity for the real exchange rate to depreciate in the long run, as it is the prediction of the Natrex approach. Rather on the contrary, with a positive α it is sure that both a medium and long run real appreciation will follow a decrease in thriftiness.

Alternatively, when a positive shock on productivity occurs, the evidence of $\alpha > 0$ will also assure the result of a long run appreciation in the real exchange rate. Nevertheless, from the parameters of the model it is deduced that, in contrast with the previous result, in the particular case of an increment in productivity, even with the alpha parameter being lower than zero, it still exists an opportunity for the real exchange rate to appreciate.

From these results, it is clear that some standard Natrex predictions about shocks on fundamentals are clearly broken down by the conclusions of this research.

5. Conclusions

This paper aims to contribute to the theoretical literature on the structural macroeconomic approach analysing the real exchange rate from a dynamic general equilibrium perspective. In particular, the research has followed the philosophy of the Natrex approach due to its, at least from our point of view, interesting and accurate dynamic notion of the concept of equilibrium. The main peculiarities of a Natrex model within the context of the structural approach are related to its singular stock-flow interaction, and to the considered distinction between the medium and long-run equilibrium, differentiated both by the possibility that the stock variables have reached their steady state positions.

However, despite the theoretical spirit of Natrex, presented in origin as a set of solidly based models, the reality is that the approach has ended up circumscribed to the empirical estimation of reduced models of equilibrium. The unfortunate result is that the approach lacks of well-integrated structures, not to mention that it has completely omitted some important theoretical restrictions that are in fact determinant for the structural characterisation of the model. Moreover, and not less important, there is the fact of the absence of a formal discussion about the convergence and stability of the systems under analysis. In that sense, the main contribution of this paper is the development of a solid theoretical framework that analyses in depth the basis of the real exchange rate, as well as the details of the equilibrium dynamics after any shock influencing steady state positions.

The main results of the paper are the following. In first place, a complete well-integrated structural model not known up to the moment for the long-run real exchange rate determination is developed from first principles. Moreover, within the concrete dynamics of the model we find that, for convergence reasons, there are some restrictions that the model will necessarily need to satisfy. For the medium run convergence, the sensitivity of the trade balance to changes in the real exchange rate should be higher in absolute values than the correspondent ones in investment decisions. Regarding the long-run, it is also necessary both that there exists a negative relationship between investment and capital stock accumulation and that the global savings of the economy (integrating public and private sectors) depends positively on net foreign debt accumulation.

In addition, there are also interesting conclusions about the results that certain shocks over the exogenous variables of the model have on real exchange rates. Being more specific, the Natrex approach predicts that in the medium run a real appreciation will follow both a positive shock on the preference for consumption and on productivity. However, the prediction changes dramatically in the long run. Following the Natrex suggestion, we expect a long run real depreciation in the case of a higher preference for consumption and an ambiguous result, although most probably a real appreciation, in the case of a better productivity factor. Our research, alternatively, breaks down these predictions and prognosticates that in the particular case of a shock on the thriftiness parameter, the mentioned long run real depreciation can be only assured when a particular restriction over the estimated parameters of the model occurs.

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APPENDIX

Appendix 1: The Real Exchange Rate Definition

Consider an economy where the produced and consumed goods are of tradable (T) and non-tradable (N) nature. Without loss of generality, tradables can be assumed homogeneous across countries, so that their price is therefore determined in international markets while, on the contrary, non-tradables will have to be dealt -produced and consumed- inside the country. In this case the terms of trade will collapse to one[8].

Under the previous assumption, the national and foreign price indexes –P and P* respectively– can be simply defined as follows,

$$P = P_N^{\alpha} P_T^{1-\alpha} \tag{A.1}$$

$$P^{*} = \left(P_N^{*}\right)^{\beta} \left(P_T^{*}\right)^{1-\beta} \tag{A.2}$$

The star indicates foreign sector.

Given next the definition in (A.3) for the real exchange rate (R), where an increment means a real appreciation, price indexes (A.1) and (A.2) can be used to obtain expression (A.4),

$$R = \frac{EP}{P^{*}} \tag{A.3}$$

$$R = \left(\frac{P_N}{P_T}\right)^{\alpha} \left(\frac{P_T^{*}}{P_N^{*}}\right)^{\beta} \frac{EP_T}{P_T^{*}} \tag{A.4}$$

where E is the nominal exchange rate.

Taking into account the following (A.5) and (A.6) definitions referred to the relative price of non-tradables with respect to tradables for the national and foreign economies respectively,

$$R_n = \frac{P_N}{P_T} \tag{A.5}$$

$$R_n^{*} = \frac{P_N^{*}}{P_T^{*}} \tag{A.6}$$

[8] This simplifying assumption has not to be a problem in the case of an empirical estimation, given that R can be introduced as a global factor.

expression (A.4) can simplify to the next one,

$$R=\frac{R_n^{\alpha}}{\left(R_n^{*}\right)^{\beta}}\frac{EP_T}{P_T^{*}} \tag{A.7}$$

Moreover, if the law of the single price is also into operation, the following (A.8) condition should satisfy,

$$P_T^{*}=EP_T \tag{A.8}$$

In that case, the real exchange rate can be finally reduced to expression (A.9), where R^*_n is assumed an exogenous variable for the national country.

$$R=\frac{R_n^{\alpha}}{\left(R_n^{*}\right)^{\beta}} \tag{A.9}$$

We then see a real exchange rate defined as a fixed proportion, δ, of R_n.

$$R=\delta R_n^{\alpha} \tag{A.10}$$

where δ is defined as follows,

$$\delta=\frac{T^{\alpha+\beta-1}}{\left(R_n^{*}\right)^{\beta}} \tag{A.11}$$

Appendix 2: Response of the Investment Ratio to its Explanatory Variables

Given the investment ratio obtained in (41) and reproduced here in (A.12), the partial derivatives of this endogenous variable with respect to its explanatory variables are determined next,

$$\frac{I}{Q}=\frac{\hat{A}}{(K/Q)^{\frac{\alpha}{1-\alpha}}}\frac{\beta}{\alpha}\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\frac{\alpha}{\beta}+R^{1/\beta}\,\frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\frac{K_N}{K_T}}{1+R^{1/\beta}\,\frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\frac{K_N}{K_T}}\right)^{\frac{1}{1-\alpha}} \tag{A.12}$$

Derivative with respect to the productivity index $\hat{A}$,

$$(F_I)'_{\hat{A}} = \frac{I/Q}{\hat{A}} > 0 \tag{A.13}$$

Derivative with respect to the capital stock ratio K/Q,

$$(F_I)'_{K/Q} = -\frac{\alpha}{1-\alpha}\frac{I}{K} < 0 \tag{A.14}$$

Derivative with respect to the real interest rate r,

$$(F_I)'_r = -\frac{1}{1-\alpha}\frac{1}{r}\frac{I}{Q} < 0 \tag{A.15}$$

Derivative with respect to the real exchange rate R,

$$(F_I)'_R = \frac{-(\alpha-\beta)\dfrac{R^{1-\beta/\beta}}{\beta^2}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\dfrac{I}{Q}}{\left(\dfrac{\alpha}{\beta}+R^{1/\beta}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\right)\left(1+R^{1/\beta}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\right)} \tag{A.16}$$

Expression (A.16) would be negative whenever α is assumed greater than β, which is in fact commonly corroborated by the empirical evidence given that it implies the realistic assumption that the tradable sector is intensive in capital while the non-tradable is in labour.

Appendix 3: Elasticity of Substitution of a CES

In appendix 3 we are going to determine the elasticity of substitution of the following utility function,

$$U(c_1, c_2) = c_1^{\gamma} + c_2^{\gamma} \tag{A.17}$$

Let us start specifying in expression (A.18) the concept of elasticity of substitution for the case of a generic utility function,

$$\varepsilon_{12} = \frac{\partial(c_2/c_1)}{\partial \mathrm{MRS}}\frac{\mathrm{MRS}}{(c_2/c_1)} \tag{A.18}$$

where MRS is the marginal rate of substitution, which in its turn is defined as follows,

$$\mathrm{MRS} = -\frac{\partial c_2}{\partial c_1} = -\frac{\partial U/\partial c_1}{\partial U/\partial c_2} \tag{A.19}$$

The marginal rate of substitution for the utility function (A.17) is in this particular case the following,

$$MRS = -(1+r_1)\left(\frac{c_2}{c_1}\right)^{1-\gamma} \qquad \text{(A.20)}$$

Finally, making use of expression (A.20), it is easily deduced the desired elasticity of substitution we are looking for,

$$\varepsilon_{12} = \frac{1}{1-\gamma} \qquad \text{(A.21)}$$

Appendix 4: Response of the Consumption Ratio to its Explanatory Variables

Given the consumption ratio obtained in (71) and written again here in equation (A.22), in this appendix 4 it will be determined the partial derivatives of this consumption ratio with respect to each explanatory variable.

$$\frac{C_1}{Q_1} = \frac{1-(Q_0/Q_1)(1+r)\dfrac{D_0}{Q_0} + \dfrac{f_q}{1+r}\left(1+\varphi_1 - \alpha\dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \qquad \text{(A.22)}$$

Derivative with respect to the capital ratio K/Q,

$$(F_C)'_{K/Q} = \frac{(f_q)'_{K/Q}\left(1+\varphi_1 - \alpha\dfrac{D_0}{Q_0}\right)}{1+r+(1+r)^{1/1-\gamma}} > 0 \qquad \text{(A.23)}$$

Derivative with respect to the ratio of the foreign debt D/Q,

$$(F_C)'_{D/Q} = -\frac{(Q_0/Q_1)(1+r)+\dfrac{\alpha f_q}{1+r}}{1+(1+r)^{\gamma/1-\gamma}} < 0 \qquad \text{(A.24)}$$

Derivative with respect to the real interest rate r,

$$(F_C)'_r = -\frac{(Q_0/Q_1)(1+r)+\dfrac{f_q}{1+r}\left(1+\varphi_1 - \alpha\dfrac{D_0}{Q_0}\right)+\dfrac{\gamma}{1-\gamma}\dfrac{C_1}{Q_1}(1+r)^{\gamma/1-\gamma}}{1+r+(1+r)^{1/1-\gamma}} < 0 \qquad \text{(A.25)}$$

Finally, we obtain the derivative with respect to the parameter φ_1 measuring the preference for consumption,

$$(F_C)'_{\varphi} = \frac{f_q}{1+r_1+(1+r_1)^{1/1-\gamma}} > 0 \tag{A.26}$$

Appendix 5: Response of the Saving Ratio to its Explanatory Variables

In order to determine the partial derivatives of the saving ratio, obtained in (72), with respect to its explanatory variables, this (72) equation can be rewritten as follows,

$$\frac{S_1}{Q_1} = 1-(Q_0/Q_1)\,r\,\frac{D_0}{Q_0} - \frac{C_1}{Q_1} \tag{A.27}$$

From (A.27) it is clear that in this case the derivatives with respect to K/Q and φ_1 will have the opposite sign that in the case of the consumption ratio. That is the following,

$$(F_S)'_{\upsilon} = -(F_C)'_{\upsilon} \qquad \upsilon \neq \frac{D_0}{Q_0},\ r \tag{A.28}$$

However in the case of r and D_0/Q_0, it is necessary to perform separate derivatives,

$$(F_S)'_r = -(Q_0/Q_1)\frac{D_0}{Q_0} - (F_C)'_r \tag{A.29}$$

$$(F_S)'_{D/Q} = -(Q_0/Q_1)\,r - (F_C)'_{D/Q} \tag{A.30}$$

Being $(F_C)'_r$ and $(F_C)'_{D/Q}$ both lower than zero, the signs of (A.29) and (A.30) will result necessarily ambiguous.

Appendix 6: Response of the Trade Balance Ratio to its Explanatory Variables

In order to determined the partial derivatives of the trade balance ratio let us show here the obtained (86) equation for the imports ratio,

$$\frac{M}{Q} = \kappa\frac{\alpha}{\alpha+\beta}[\theta+(1-\theta)R]\frac{C}{Q} \tag{A.31}$$

In the same way, the exports ratio can be formulated as a symmetric function of (A.31),

$$\frac{X}{Q^*} = \kappa^* \frac{\alpha^*}{\alpha^* + \beta^*}\left(\theta^* + \frac{1-\theta^*}{R}\right)\frac{C^*}{Q^*} \tag{A.32}$$

So that the trade balance equation will remain as follows,

$$\frac{TB}{Q} = \kappa^* \frac{\alpha^*(Q^*/Q)}{\alpha^* + \beta^*}\left(\theta^* + \frac{1-\theta^*}{R}\right)\frac{C^*}{Q^*} - \kappa\frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right]\frac{C}{Q} \tag{A.33}$$

Next the first derivatives of (A.33) with respect to its main explanatory variables are obtained.

Derivative with respect to the real exchange rate R,

$$(F_{TB})'_R = -\left[\kappa^* \frac{\alpha^*(Q^*/Q)}{\alpha^* + \beta^*}\frac{C^*}{Q^*}\frac{1-\theta^*}{R^2} + \kappa\frac{\alpha(1-\theta)}{\alpha+\beta}\frac{C}{Q}\right] < 0 \tag{A.34}$$

Derivative with respect to the foreign consumption ratio C*/Q*,

$$(F_{TB})'_{C^*/Q^*} = \kappa^* \frac{\alpha^*(Q^*/Q)}{\alpha^* + \beta^*}\left(\theta^* + \frac{1-\theta^*}{R}\right) > 0 \tag{A.35}$$

Derivative with respect to the national consumption ratio C/Q,

$$(F_{TB})'_{C/Q} = -\kappa\frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right] < 0 \tag{A.36}$$

Appendix 7: Response of the Current Account Ratio to its Explanatory Variables

To determine the partial derivatives of the current account ratio with respect to its explanatory variables, let us write here equations (91) and (73) again,

$$\frac{CA}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) - F_{DS}\left(\frac{D}{Q}\right) \tag{A.37}$$

$$\frac{C}{Q} = F_C\left(\hat{Q}, \frac{D}{Q}, r, \varphi\right) \tag{A.38}$$

From (A.37) and (A.38) it is clear that in this case the derivatives with respect to R, C*/Q*, $\hat{Q}$, r and φ will have the same sign that in the case of the trade balance ratio, that is to say,

$$(F_{CA})'_{\upsilon} = (F_{TB})'_{\upsilon} \frac{\partial}{\partial \upsilon}\left(\frac{TB}{Q}\right) \quad \upsilon \neq \frac{D}{Q} \tag{A.39}$$

However, in the case of D/Q, it is necessary to perform a separate analysis,

$$(F_{CA})'_{D/Q} = (F_{TB})'_{C/Q}\,(F_C)'_{D/Q} - (F_{DS})'_{D/Q} \tag{A.40}$$

Taking into account that the consumer distributes his disposable income (Q_D)[9] between consumption and savings, the following relationship will necessarily occur,

$$(F_C)'_{D/Q} = -\left[(F_{DS})'_{D/Q} + (F_S)'_{D/Q}\right] \tag{A.41}$$

So, substituting expression (A.41) into the previous (A.40), it is obtained the following,

$$(F_{CA})'_{D/Q} = -\left[(F_{TB})'_{C/Q}\,(F_S)'_{D/Q} + (F_{DS})'_{D/Q}\left[1 + (F_{TB})'_{C/Q}\right]\right] \tag{A.42}$$

Or renaming $(F_{TB})'_{C/Q}$ as $-\alpha$ in order to specify the variable in positive terms, (A.42) can be reformulated as follows,

$$(F_{CA})'_{D/Q} = \alpha\,(F_S)'_{D/Q} - (1-\alpha)\,(F_{DS})'_{D/Q} \tag{A.43}$$

The necessary condition for $(F_{CA})'_{D/Q}$ to be negative is shown in equation (A.44),

$$(F_{DS})'_{D/Q} > \frac{\alpha}{1-\alpha}\,(F_S)'_{D/Q} \tag{A.44}$$

As it was discussed in the main body of the paper the condition for the negative relationship between the current account and the stock of foreign debt is that the interest payment exceeds the improvements that a higher debt causes in the volume of national savings.

[9] Being this determined as follows, Q_D=Q-DS