APPENDIX

1. Comparative figures

Figures A.1-A.6 compare our current attainment estimates with those of Cohen and Soto (2001) and Barro and Lee (2000). Figures A.1 and A.2 show pairwise scatter diagrams in levels and in growth rates for the entire sample. Figures A.3-A.6 show comparative time profiles for each country. Figure A.3 plots the three series of average years of schooling, and Figures A.4-A.6 compare this paper and Barro and Lee (2000) in terms of the evolution of primary, secondary and university attainment in each country. This last set of figures does not include Cohen and Soto's estimates because they do not provide the necessary information. The data on years of schooling refers to the population aged 25 and over in B&L and in our data, and to the population aged 15 to 64 in Cohen and Soto.

Figures A.7-A.9 compare our current estimates of average years of schooling and secondary and higher attainment rates with those from the previous version of this data set (D&D, 2000). The use of the newly available national data has resulted in significant changes in our estimates for average years of schooling in Canada, Switzerland, Germany, Finland, Denmark and Norway. In the last two cases the change is due mostly to the important reduction in our estimate of primary attainment. Our estimate of years of schooling in the US changes because we have changed the assumed duration of *L1* and *L2.1* to make it compatible with our cutoffs for these levels.

2. Estimation of the stock of physical capital

We construct series of physical capital stocks in the OECD for the period 1950-97 using a perpetual inventory procedure with an assumed annual depreciation rate of 5%. To estimate the initial capital stock we modify the procedure proposed by Griliches (1980) to take into account the fact that the economies in our sample may be away from their steady states.

The growth rate of the stock of capital, g_k, can be written in the form

$$g_k = \frac{I}{K} - \delta$$

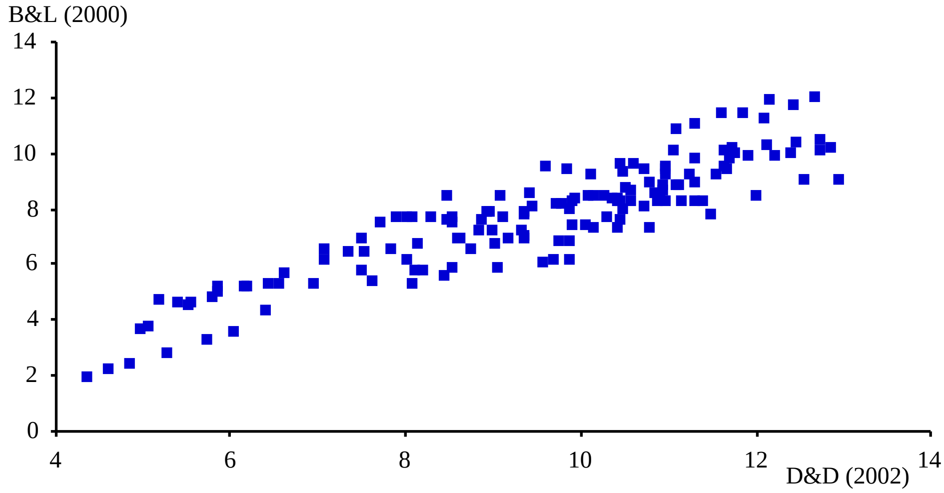
where I is investment, δ the depreciation rate and K the stock of physical capital. Solving this expression for K and assuming that the growth rate of investment is a good approximation to the growth rate of the capital stock (i.e. $g_I \approx g_k$), we obtain an expression that can be used to estimate the initial capital stock using data on investment flows:

$$(A.1)\ K = \frac{I}{g_k + \delta} \approx \frac{I}{g_I + \delta}.$$

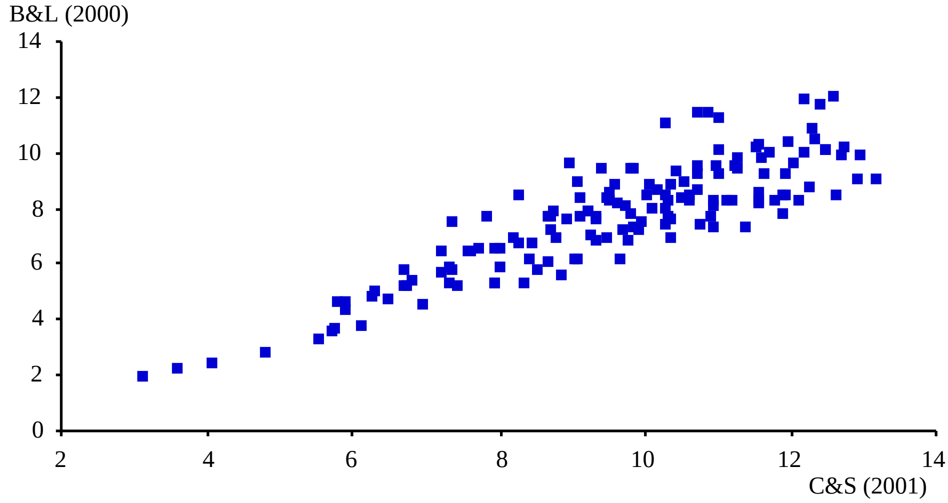
When implementing this approach, it is common to use the level of investment in the first year in the sample period and the growth rate of the same variable over the entire period. In our case, however, this does not seem to be the best way to proceed because i) investment may be subject to transitory disturbances that make it dangerous to rely on a single observation and ii) rates of

Figure A.1: Average years of schooling

a. B&L (2000) vs. D&D (2002)



b. B&L (2000) vs. C&S (2001)



c. C&S (2001) vs D&D (2002)

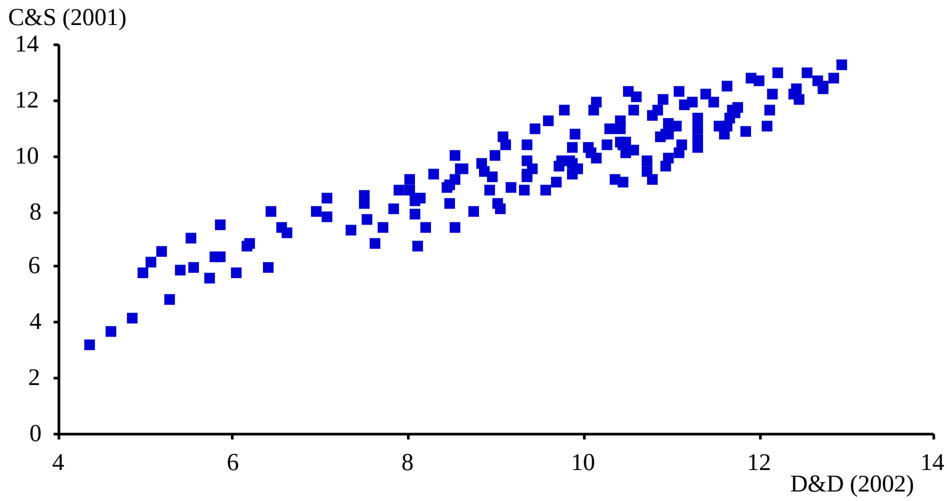
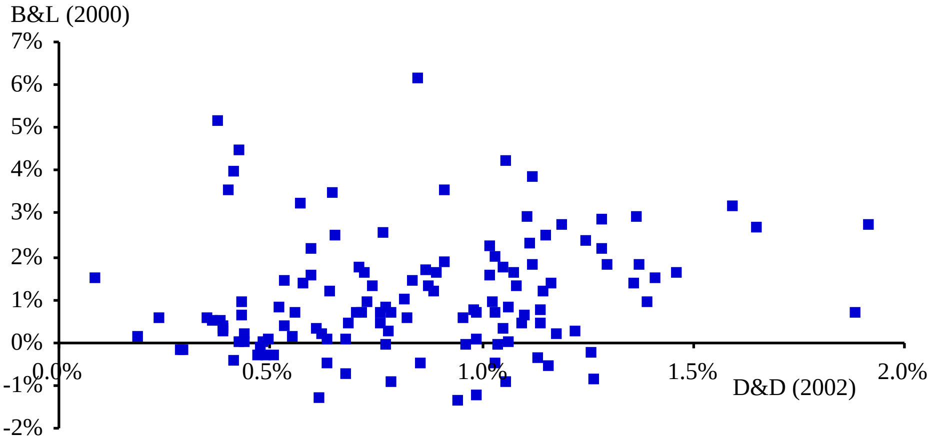
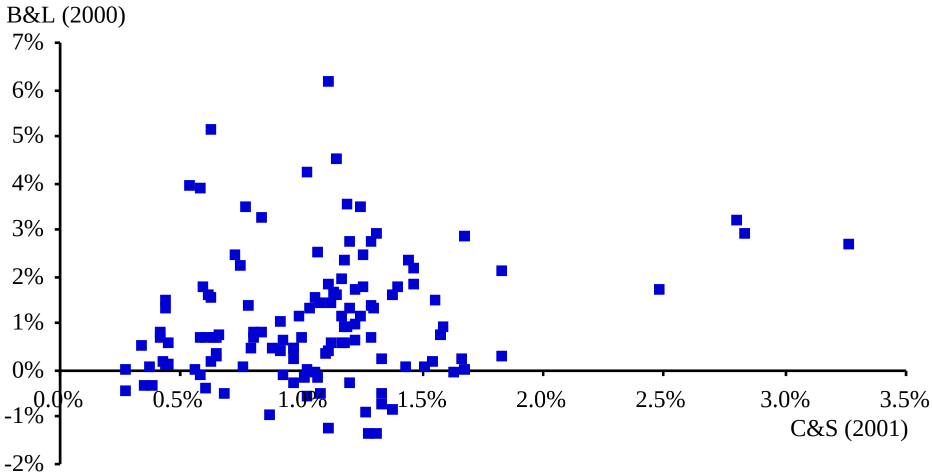


Figure A.2: Annual growth rate of average years of schooling

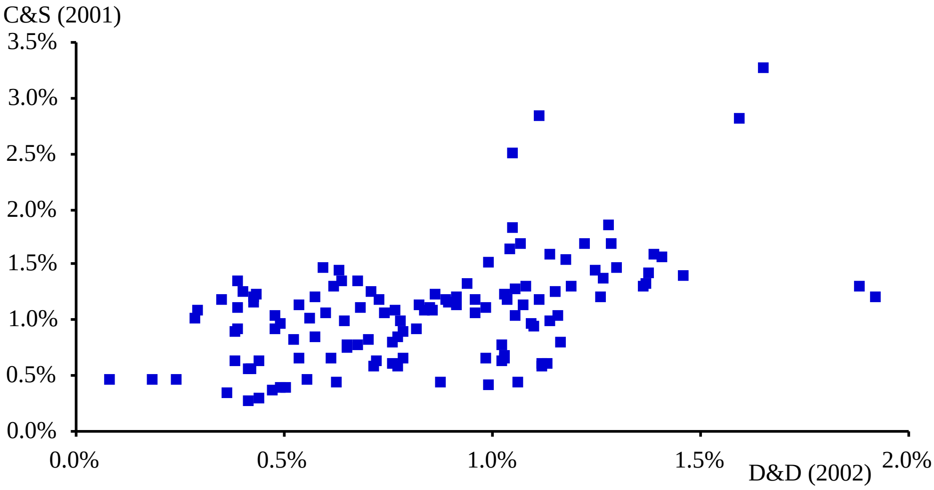
a. B&L (2000) vs. D&D (2002)



b. B&L (2000) vs. C&S (2001)



c. C&S (2001) vs D&D (2002)



investment and factor accumulation will tend to vary over time in a systematic way as countries approach their steady states.

To try to control for these factors, we use the growth rate of investment over the period 1950-60 and the HP-filtered level of investment in 1955. Hence, our version of equation (A.1) is of the form:

$$(A.2)\ K_{55} \quad \frac{I_{hp55}}{g_{I,50\text{-}60} + 0.05} \quad ,$$

where I_{hp} is the Hodrick-Prescott trend of investment (with a smoothing parameter $\lambda = 10$). We use 1955 as the base year instead of 1950 because it is known that this filter may displays anomalies at sample endpoints.[1] Our investment data are corrected for differences in PPP and are taken from the OECD's *National Accounts* and *Economic Outlook* for the period starting in 1960. Prior to that date, we use IMF data and price deflators and, for some countries where no information is available, we extrapolate investment backward using the growth rate of the capital stock provided in Summers and Heston's PWT 5.6.

3. Miscellaneous results

Tables A.1 and A.2 give further details on some results that are mentioned in the text. Table A.1 replicates Table 9 in the text using a Mincerian specification. As noted, this involves replacing logs of H by their levels, so that equation (10) in the text, for instance, becomes

$$q_{it} = \Gamma_1 + \gamma_i + \eta_{1t} + \alpha z_{it} + \rho H_{it} - \varphi e_{it} + \varepsilon_{1it} \ .$$

The coefficient of years of schooling in this modified production function, which is denoted by ρ, is sometimes called the Mincerian return to schooling. This parameter measures the percentage increase in output that would follow from an increase of one year in average attainment. The results given in Table A.1 are somewhat worse than those reported in the text for a standard Cobb-Douglas specification but continue to display a clear positive correlation with the relevant reliability ratios.

Table A.2 gives the results obtained with some selected specifications of the growth equation when the growth rate of years of schooling is instrumented by the initial (log) stock of the same variable. Comparing these results with those in panels *c* and *e* of Table 9, we see that the use of instruments considerably increases the size of the estimated human capital coefficient for most equations in growth rates and for those catch-up specifications that are estimated with early data sets. In this second case, however, the estimates obtained with our series or with Cohen and Soto's is not very sensitive to the change in the estimation procedure. One possible interpretation is that instrumenting serves to mitigate the measurement error problem in the earlier series. In addition, the pattern of results suggests that there is little danger of an upward bias arising from reverse causation.

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[1] Due to data limitations and other anomalies we have used a different base year for some countries. In particular, we use 1953 for Canada and Norway and 1960 for the UK, Greece and Ireland.

Table A.1: Alternative Mincerian specifications

a. Log levels (without fixed country effects)

	[a1]	*[a2]*	*[a3]*	*[a4]*	*[a5]*	*[a6]*	*[a7]*	*[a8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.581	0.585	0.528	0.530	0.496	0.454	0.465	0.459
	(19.82)	(15.44)	(15.58)	(17.49)	(16.57)	(14.98)	(16.69)	(14.30)
ρ	*0.007*	*0.026*	*0.021*	*0.023*	*0.036*	*0.051*	*0.047*	*0.044*
	(1.18)	(2.60)	(3.34)	(4.06)	(5.75)	(7.27)	(8.03)	(6.38)
φ	-0.255	-0.230	-0.330	-0.374	-0.452	-0.636	-0.551	-0.602
	(2.63)	(2.65)	(3.45)	(4.36)	(5.40)	(7.21)	(6.93)	(6.62)
adj. R^2	0.879	0.878	0.888	0.900	0.909	0.919	0.924	0.913
std. error reg.	0.1300	0.1027	0.1249	0.1203	0.1143	0.1082	0.1050	0.1118
no. of observ.	126	95	126	147	147	147	147	147

b. Log levels with fixed country effects

	[b1]	*[b2]*	*[b3]*	*[b4]*	*[b5]*	*[b6]*	*[b7]*	*[b8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.542	0.538	0.553	0.552	0.556	0.562	0.553	0.551
	(19.03)	(15.72)	(19.47)	(20.18)	(20.33)	(21.18)	(20.05)	(20.90)
ρ	*0.005*	*0.009*	*0.020*	*0.001*	*0.013*	*0.072*	*-0.002*	*0.069*
	(0.45)	(1.03)	(2.11)	(0.00)	(1.18)	(3.01)	(0.10)	(3.01)
adj. R^2	0.978	0.980	0.979	0.976	0.976	0.978	0.976	0.978
std. error reg.	0.0558	0.0427	0.0547	0.0587	0.0584	0.0566	0.0587	0.0566
no. of observ.	126	95	126	147	147	147	147	147

c. Growth rates

	[c1]	*[c2]*	*[c3]*	*[c4]*	*[c5]*	*[c6]*	*[c7]*	*[c8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.509	0.387	0.513	0.492	0.495	0.493	0.489	0.493
	(10.34)	(6.43)	(10.40)	(10.48)	(10.60)	(10.47)	(10.44)	(10.51)
ρ	*-0.006*	*0.016*	*0.007*	*0.006*	*0.013*	*0.017*	*-0.026*	*0.026*
	(0.44)	(1.83)	(0.87)	(0.59)	(1.36)	(0.47)	(0.86)	(0.77)
adj. R^2	0.656	0.446	0.658	0.630	0.634	0.629	0.631	0.629
std. error reg.	0.0093	0.0086	0.0093	0.0096	0.0095	0.0096	0.0096	0.0096
no. of observ.	105	74	105	126	126	126	126	126

Table A.1: Alternative Mincerian specifications (continued)

d. Growth rates with technological diffusion and fixed country effects

	[d1]	*[d2]*	*[d3]*	*[d4]*	*[d5]*	*[d6]*	*[d7]*	*[d8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.387	0.398	0.430	0.328	0.329	0.361	0.374	0.366
	(5.23)	(4.87)	(6.21)	(3.89)	(3.92)	(4.40)	(4.62)	(4.50)
ρ	*-0.030*	*0.001*	*0.006*	*-0.007*	*-0.005*	*0.036*	*0.047*	*0.047*
	(2.45)	(0.21)	(0.92)	(0.63)	(0.62)	(1.09)	(2.27)	(1.67)
λ	0.094	0.144	0.096	0.069	0.069	0.076	0.076	0.078
	(6.83)	(9.66)	(6.28)	(5.60)	(5.52)	(5.41)	(5.77)	(5.74)
adj. R^2	0.839	0.834	0.833	0.814	0.814	0.815	0.818	0.816
std. error reg.	0.0072	0.0068	0.0074	0.0076	0.0076	0.0076	0.0076	0.0076
no. of observ.	105	74	105	126	126	126	126	126

e. Growth rates with technological diffusion and significant country dummies

	[e1]	*[e2]*	*[e3]*	*[e4]*	*[e5]*	*[e6]*	*[e7]*	*[e8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.470	0.433	0.499	0.350	0.322	0.411	0.476	0.470
	(7.85)	(6.52)	(8.68)	(5.44)	(5.68)	(6.58)	(9.68)	9.78
ρ	*-0.011*	*-0.002*	*0.008*	*-0.009*	*-0.009*	*0.065*	*0.054*	*0.058*
	(2.11)	(0.27)	(1.27)	(0.98)	(1.19)	(7.52)	(8.45)	(8.23)
λ	0.106	0.144	0.092	0.064	0.060	0.085	0.088	0.093
	(7.78)	(7.37)	(7.50)	(10.36)	(10.35)	(7.07)	(6.88)	(6.93)
adj. R^2	0.842	0.845	0.840	0.821	0.819	0.821	0.823	0.822
std. error reg.	0.0072	0.0066	0.0072	0.0075	0.0075	0.0075	0.0075	0.0075
no. of observ.	105	74	105	126	126	126	126	126

f. Averages across specifications

	[f1]	*[f2]*	*[f3]*	*[f4]*	*[f5]*	*[f6]*	*[f7]*	*[f8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
ρ	-0.007	0.010	0.012	0.003	0.010	0.048	0.024	0.049
(t)	(-0.67)	(1.08)	(1.70)	(0.61)	(1.30)	(3.87)	(3.56)	(4.01)

Notes:
- All equations include period dummies.
- White's heteroscedasticity-consistent *t* ratios in parentheses below each coefficient.
- The average value of *t* shown in block *f* is computed respecting the sign of the *t* ratios obtained for the different specifications; i.e. for this computation we assign to each *t* ratio the same sign as the corresponding coefficient estimate.
- ρ is the coefficient of the human capital variable.
- *Key: NSD* = Nehru et al (1995); *KYR* = Kyriacou (1991); *B&L* = Barro and Lee (various years); *C&S* = Cohen and Soto (2001); *D&D* = de la Fuente and Doménech (various years), *D&D02* refers to this paper.

Table A.2: Instrumental variable estimates for selected specifications

a. Growth rates

	[c1]	*[c2]*	*[c3]*	*[c4]*	*[c5]*	*[c6]*	*[c7]*	*[c8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.516	0.566	0.521	0.493	0.498	0.464	0.446	0.463
	(7.82)	(5.93)	(7.52)	(5.90)	(6.00)	(6.24)	(8.27)	(6.56)
β	*0.223*	*-0.040*	*0.247*	*0.439*	*0.536*	*1.069*	*1.217*	*1.558*
	(1.24)	(0.33)	(1.60)	(2.05)	(2.77)	(3.02)	(2.71)	(3.35)
adj. R^2	0.703	0.672	0.679	0.637	0.597	0.703	0.696	0.699
std. error reg.	0.0098	0.0096	0.0102	0.0107	0.0112	0.0096	0.0098	0.0097
no. of observ.	105	74	105	126	126	126	126	126

b. Growth rates with technological diffusion and significant country dummies

	[e1]	*[e2]*	*[e3]*	*[e4]*	*[e5]*	*[e6]*	*[e7]*	*[e8]*
H data from:	*NSD*	*KYR*	*B&L93*	*B&L96*	*B&L00*	*C&S*	*D&D00*	*D&D02*
α	0.669	0.650	0.571	0.580	0.552	0.388	0.640	0.538
	(16.28)	(10.00)	(10.14)	(7.58)	(6.71)	(6.43)	(13.20)	(11.98)
β	*0.091*	*0.384*	*0.111*	*0.087*	*0.121*	*0.292*	*0.337*	*0.471*
	(1.94)	(2.91)	(2.47)	(1.55)	(1.77)	(4.30)	(5.60)	(7.31)
λ	0.094	0.120	0.102	0.093	0.098	0.092	0.090	0.103
	(6.41)	(5.12)	(7.75)	(7.08)	(7.51)	(7.40)	(8.13)	(7.64)
adj. R^2	0.807	0.679	0.834	0.810	0.812	0.821	0.808	0.824
std. error reg.	0.0079	0.0095	0.0073	0.0077	0.0077	0.0075	0.0078	0.0074
no. of observ.	105	74	105	126	126	126	126	126

Notes:
- All equations include period dummies.
- White's heteroscedasticity-consistent *t* ratios in parentheses below each coefficient.
- The instrument for Δh_{it} is the value of h_{it} at the beginning of the current subperiod, with both variables taken from the same source.

4. Correcting for correlated measurement error

In this section we develop an extension of the classical errors-in-variables model that will be used to construct refined estimates of reliability ratios for the different schooling series (allowing for measurement error to be correlated across data sets and with the remaining regressors of the growth model) and to obtain a meta-estimate of β corrected for attenuation bias. To simplify the notation, we will assume that the distributions of the variables of interest are known, so that we can work directly with population moments. The results obtained in this manner will then apply to finite samples as probability limits. It will also be assumed throughout that all the variables have zero means, so that regression constants vanish. This assumption involves no loss of generality and is, in any event, satisfied in our case, as the inclusion of time dummies in all our growth specifications is equivalent to removing period means.

We will write the model we want to estimate (the different versions of the growth equation) in the generic form

$$(1)\ \ Q = H\beta + \boldsymbol{X}\alpha + u_1$$

where H is the true stock of human capital, $\boldsymbol{X} = (X_1, X_2, \ldots.. X_N)$ a row vector of other regressors and α a column vector of coefficients.

It will be assumed that the error term u_1 satisfies all the standard assumptions of the linear regression model (and is, in particular, uncorrelated with the regressors) so that the estimation of (1) by OLS with the correctly measured stock of human capital will be consistent. Hence, the probability limit of the OLS estimator of β will be equal to the true value of the coefficient when H is correctly measured, i.e.

$$(2)\ plim\ \hat{\beta}_H = \frac{EH'Q - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H} = \beta.$$

In practice, of course, we do not observe H but only a number of noisy proxies for it,

$$(3)\ P_j = H + \varepsilon_j$$

with $j = 1,\ldots, J$ where ε_j is a measurement error term. We want to calculate the bias in β that arises when equation (1) is estimated using P_j instead of H, and to estimate the reliability ratio of P_j, which is defined as

$$(4)\ r_j \equiv \frac{EH^2}{EP_j^2}\ .$$

We will assume that the measurement error terms, ε_j, have the following structure:

$$(5)\ \varepsilon_j = \omega_j + \rho_j\varepsilon + \boldsymbol{X}\delta_{\boldsymbol{j}}$$

where ω_j is an idyosincratic error component and ρ_j a coefficient that measures the extent to which data set j amplifies or dampens a common source of error which is captured by an iid disturbance, ε. We also allow the error term to be correlated with the components of $\boldsymbol{X}$, as indicated by the last term of (5), where $\delta_{\boldsymbol{j}}$ is a column vector of coefficients. Finally, it will be assumed that both the common and the idyosincratic components of measurement error are uncorrelated with each other and with H and $\boldsymbol{X}$, i.e. that

$$(6)\ EH\varepsilon = EH\omega_j = E\omega_j\varepsilon = E\omega_j\omega_k = EX_n\varepsilon = EX_n\omega_j = 0$$

for all j and $k \neq j$ and for all components X_n of $\boldsymbol{X}$.

a. Some preliminary calculations

In this section we will gather a number of results and calculations that will be useful below.

i. Assume for the time being that H can be observed and consider the following ("forward" and "backward") regressions

$$(7a)\ H = \boldsymbol{X}\phi + u_2$$

$$(7b)\ X_n = \mu_n H + u_{n3} \quad for\ \ n = 1,\ldots, N$$

where the disturbances u_2 and u_{n3} are assumed to satisfy the assumptions required for OLS to yield consistent estimates. It is easy to show that the probability limits of the OLS estimators of ϕ and $\mu = (\mu_1, \mu_2, ...)'$ (which will be equal to the true parameter values) will be given by:

$$(8)\ \phi = (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H \quad \text{and}$$

$$(9)\ \mu = \frac{1}{EH^2}\ E\boldsymbol{X}'H.$$

The plim of the R^2 of equation (7a) will be given by

$$ER_H{}^2 \equiv plim\ R^2(H|\boldsymbol{X}) \equiv plim\frac{Explained\ SS}{Total\ SS} = \frac{E(\boldsymbol{X}\phi)'(\boldsymbol{X}\phi)}{EH'H} = \frac{\phi'(E\boldsymbol{X}'\boldsymbol{X})\phi}{EH'H}\ .$$

Using (8) in the numerator of this expression, we have

$$\phi'(E\boldsymbol{X}'\boldsymbol{X})\phi = [(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H]'(E\boldsymbol{X}'\boldsymbol{X})\ (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H =$$

$$= EH'\boldsymbol{X}[(E\boldsymbol{X}'\boldsymbol{X})']^{-1}E\boldsymbol{X}'H = EH'\boldsymbol{X}\ (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H$$

(where we have made use of the fact that $E\boldsymbol{X}'\boldsymbol{X}$ is a symmetric matrix) and therefore

$$(10)\ ER_H{}^2 = \frac{EH'\boldsymbol{X}\ (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}{EH'H}\ .$$

Using (8) and (9), this becomes

$$(10')\ ER_H{}^2 = \frac{EH'\boldsymbol{X}\ (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}{EH'H} = \frac{EH'\boldsymbol{X}\ \phi}{EH'H} = \mu'\phi\ .$$

ii. Assumptions (5) and (6) above imply

$$(11)\ E\varepsilon_j{}'\varepsilon_k = E(\rho_j\varepsilon + \omega_j + \boldsymbol{\delta_j}'\boldsymbol{X}')\ (\rho_k\varepsilon + \omega_k + \boldsymbol{X\delta_k}) = \rho_j\rho_k E\varepsilon^2 + \delta_j{}'(E\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\delta_k}$$

$$(12)\ E\varepsilon_j{}'\varepsilon_j = E(\rho_j\varepsilon + \omega_j + \boldsymbol{\delta_j}'\boldsymbol{X}')\ (\rho_j\varepsilon + \omega_j + \boldsymbol{X\delta_j}) = \rho_j{}^2E\varepsilon^2 + E\omega_j{}^2 + \delta_j{}'(E\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\delta_j}$$

$$(13)\ E\varepsilon_j{}'H = E(\rho_j\varepsilon + \omega_j + \boldsymbol{\delta_j}'\boldsymbol{X}')H = \boldsymbol{\delta_j}'E\boldsymbol{X}'H$$

$$(14)\ E\varepsilon_j{}'\boldsymbol{X} = E(\rho_j\varepsilon + \omega_j + \boldsymbol{\delta_j}'\boldsymbol{X}')\boldsymbol{X} = \boldsymbol{\delta_j}'E\boldsymbol{X}'\boldsymbol{X}$$

Using these results, we have:

$$(15)\ EP_j{}'P_k = E(H' + \varepsilon_j{}')(H + \varepsilon_k) = EH'H + EH'\varepsilon_k + E\varepsilon_j{}'H + E\varepsilon_j{}'\varepsilon_k =$$

$$= EH'H + \boldsymbol{\delta_k}'E\boldsymbol{X}'H + \boldsymbol{\delta_j}'E\boldsymbol{X}'H + \rho_j\rho_k E\varepsilon^2 + \delta_j{}'(E\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\delta_k}$$

$$= EH'H + \rho_j\rho_k E\varepsilon^2 + (\boldsymbol{\delta_k}+\boldsymbol{\delta_j})'E\boldsymbol{X}'H + \delta_j{}'(E\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\delta_k}$$

$$(16)\ EP_j{}'P_j = E(H' + \varepsilon_j{}')(H + \varepsilon_j) = EH'H + EH'\varepsilon_j + E\varepsilon_j{}'H + E\varepsilon_j{}'\varepsilon_j = EH'H + 2EH'\varepsilon_j + E\varepsilon_j{}'\varepsilon_j$$

$$= EH'H + \rho_j{}^2E\varepsilon^2 + E\omega_j{}^2 + 2\boldsymbol{\delta_j}'E\boldsymbol{X}'H + \delta_j{}'(E\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\delta_j}$$

$$(17)\ EP_j{}'X_n = E(H' + \varepsilon_j{}')X_n = EH'X_n + E\varepsilon_j{}'X_n = EH'X_n + \boldsymbol{\delta_j}'E\boldsymbol{X}'X_n$$

$$(18)\ EP_j{}'\boldsymbol{X} = E(H' + \varepsilon_j{}')\boldsymbol{X} = EH'\boldsymbol{X} + E\varepsilon_j{}'\boldsymbol{X} = EH'\boldsymbol{X} + \boldsymbol{\delta_j}'E\boldsymbol{X}'\boldsymbol{X}$$

$$(19)\ EP_j{}'Q = E(H + \varepsilon_j)'Q = EH'Q + E\varepsilon_j{}'Q = EH'Q + E(\omega_j + \rho_j\varepsilon + \boldsymbol{\delta_j}'\boldsymbol{X}')Q =$$

$$= EH'Q + \boldsymbol{\delta_j}'E\boldsymbol{X}'Q$$

To rewrite some of these expressions in a way that will be convenient below, we define

$$(20)\ e_j \equiv \rho_j\sqrt{\frac{E\varepsilon^2}{EH^2}}$$

$$(21)\ U_j{}^2 \equiv \frac{E\omega_j{}^2}{EH^2}$$

and

$$(22)\ C_{jk} \equiv \frac{\delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{k}}}{EP_j^2}.$$

Factoring out EH^2 $(= EH'H)$ in (15) and (16) and recalling (4) and (9), we have:

$$(23)\ EP_j'P_k = EH'H + \rho_j\rho_k E\varepsilon^2 + (\delta_{\mathbf{k}}+\delta_{\mathbf{j}})'E\mathbf{X}'H + \delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{k}}$$

$$= EH^2\left(1 + e_je_k + (\delta_{\mathbf{k}}+\delta_{\mathbf{j}})'\frac{E\mathbf{X}'H}{EH^2} + \frac{\delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{k}}}{EP_j^2}\frac{EP_j^2}{EH^2}\right)$$

$$= EH^2\left(1 + e_je_k + (\delta_{\mathbf{k}}+\delta_{\mathbf{j}})'\mu + C_{jk}\frac{1}{r_j}\right)$$

$$(24)\ EP_j'P_j = EH'H + \rho_j^2E\varepsilon^2 + E\omega_j^2 + 2\delta_{\mathbf{j}}'E\mathbf{X}'H + \delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{j}}$$

$$= EH^2\left(1 + e_j^2 + U_j^2 + 2\delta_{\mathbf{j}}'\frac{E\mathbf{X}'H}{EH^2} + \frac{\delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{j}}}{EP_j^2}\frac{EP_j^2}{EH^2}\right)$$

$$= EH^2\left(1 + e_j^2 + U_j^2 + 2\delta_{\mathbf{j}}'\mu + C_{jj}\frac{1}{r_j}\right)$$

iii. Let us define ER_j^2 as the probability limit of the coefficient of determination of a regression of P_j on the vector $\mathbf{X}$, $R^2(P_j|\mathbf{X})$. By analogy with (10), we have

$$(25)\ ER_j^2 \equiv plim\ R^2(P_j|\mathbf{X}) = \frac{EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'P_j}{EP_j'P_j}.$$

Notice that, using (18), the numerator of this expression can be written

$$EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'P_j = (EH'\mathbf{X} + \delta_{\mathbf{j}}'E\mathbf{X}'\mathbf{X})\,(E\mathbf{X}'\mathbf{X})^{-1}\,(E\mathbf{X}'H + E\mathbf{X}'\mathbf{X}\delta_{\mathbf{j}}) =$$

$$= EH'\mathbf{X}\,(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H + EH'\mathbf{X}\,(E\mathbf{X}'\mathbf{X})^{-1}\,E\mathbf{X}'\mathbf{X}\delta_{\mathbf{j}} + \delta_{\mathbf{j}}'E\mathbf{X}'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H + \delta_{\mathbf{j}}'E\mathbf{X}'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}\,E\mathbf{X}'\mathbf{X}\delta_{\mathbf{j}}$$

$$= EH'\mathbf{X}\,(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H + EH'\mathbf{X}\delta_{\mathbf{j}} + \delta_{\mathbf{j}}'E\mathbf{X}'H + \delta_{\mathbf{j}}'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{j}}$$

$$= EH'\mathbf{X}\,(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H + 2\delta_{\mathbf{j}}'E\mathbf{X}'H + \delta_{\mathbf{j}}'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{j}}$$

Substituting this expression into (25), and using (4) , (9), (10) and (22), we have:

$$ER_j^2 = \frac{EH'\mathbf{X}\,(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H}{EH'H}\,\frac{EH'H}{EP_j'P_j} + \frac{2\delta_{\mathbf{j}}'E\mathbf{X}'H}{EH'H}\,\frac{EH'H}{EP_j'P_j} + \frac{\delta_{\mathbf{j}}'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{j}}}{EP_j'P_j}$$

$$= ER_H^2r_j + 2\delta_{\mathbf{j}}'\mu r_j + C_{jj}$$

or

$$(26)\ ER_j^2 = r_j\,(ER_H^2 + 2\delta_{\mathbf{j}}'\mu) + C_{jj}.$$

b. Measurement error bias

Consider now what happens when we estimate the growth equation given in (1) using an imperfect proxy P_j for the stock of human capital. By analogy with (2), the probability limit of the resulting OLS estimator, $\hat{\beta}_j$, is given by

$$(27)\ plim\,\hat{\beta}_j = \frac{EP_j'Q - EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'Q}{EP_j'P_j - EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'P_j}$$

$$= \frac{EP_j'Q - EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'Q}{EH'H - EH'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H_j} * \frac{EH'H - EH'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'H}{EP_j'P_j - EP_j'\mathbf{X}(E\mathbf{X}'\mathbf{X})^{-1}E\mathbf{X}'P_j} \equiv A*B.$$

We will now consider in turn each of the two factors in the last expression. Using (19) and (2) in the first term, we have:

$$A = \frac{EP_j'Q - EP_j'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H} = \frac{(EH'Q + \delta_{\boldsymbol{j}}'E\boldsymbol{X}'Q) - (EH'\boldsymbol{X} + \delta_{\boldsymbol{j}}'E\boldsymbol{X}'\boldsymbol{X})(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}$$

$$= \frac{EH'Q + \delta_{\boldsymbol{j}}'E\boldsymbol{X}'Q - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q - \delta_{\boldsymbol{j}}'E\boldsymbol{X}'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}$$

$$= \frac{EH'Q + \delta_{\boldsymbol{j}}'E\boldsymbol{X}'Q - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q - \delta_{\boldsymbol{j}}'E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}$$

$$= \frac{EH'Q - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'Q}{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H} = \beta.$$

Next, we divide all terms of B by $EP_j'P_j$ and use the definition of r_j given in (4), the expression for ER_H^2 given in equation (10), and the analogous expression for ER_j^2 given in (25) to obtain

$$B = \frac{EH'H - EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}{EP_j'P_j - EP_j'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'P_j} = \frac{\dfrac{EH'H}{EP_j'P_j} - \dfrac{EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}{EP_j'P_j}}{1 - \dfrac{EP_j'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'P_j}{EP_j'P_j}} =$$

$$= \frac{r_j - \dfrac{EH'\boldsymbol{X}(E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H}{EH'H} * \dfrac{EH'H}{EP_j'P}}{1 - ER_j^2} = \frac{r_j - ER_H^2 r_j}{1 - ER_j^2} = \frac{r_j(1 - ER_H^2)}{1 - ER_j^2}.$$

Collecting results, we arrive at the following formula, which shows the attenuation effect as a function of P_j's reliability ratio, r_j, ER_H^2 and ER_j^2 .

$$\text{(28)}\quad plim\,\hat{\beta}_j = \beta\frac{r_j(1 - ER_H^2)}{1 - ER_j^2} \equiv a_j\beta$$

where a_j is the *attenuation coefficient* for series P_j.

This expression can be used to obtain a meta-estimate of β that will be clean of measurement error bias. For this, we need a consistent estimate of a_j or, equivalently, of r_j, ER_H^2 and ER_j^2. We will see below how these can be obtained. Before doing so, however, we will reformulate equation (28) in an equivalent way that is written in terms of an *adjusted reliability ratio* which is somewhat more convenient than the one we have been using so far.

Adjusted reliability ratios and an alternative bias formula

The reliability ratio for the series P_j has been defined in (4). Using equation (24), this definition implies that

$$r_j \equiv \frac{EH^2}{EP_j^2} = \frac{EH^2}{EH^2\left(+ e_j^2 + U_j^2 + 2\delta_{\boldsymbol{j}}'\mu + C_{jj}\frac{1}{r_j}\right)}$$

from where

$$r_j\left(+ e_j^2 + U_j^2 + 2\delta_{\boldsymbol{j}}'\mu + C_{jj}\frac{1}{r_j}\right) = 1$$

or

$$(29)\ r_j = \frac{1 - C_{jj}}{1 + e_j^2 + U_j^2 + 2\boldsymbol{\delta_j}'\mu}.$$

Notice that, under the classical asumption that measurement error is uncorrelated with $\boldsymbol{X}$ (i.e. when $\boldsymbol{\delta_j} = \boldsymbol{0}$ which in turn implies $C_{jj} = 0$), r_j must be a number between zero and one. When this assumption is relaxed, however, this need no longer be the case as r_j may exceed one if $\boldsymbol{\delta_j}'\mu$ is negative and sufficiently large. In addition, the value of r_j may be a misleading indicator of the information content of the series and can be difficult to compare across data sets because it depends on their correlation with $\boldsymbol{X}$, which as we will show below can be "cleaned off" and does not therefore necessarily raise a serious problem.

In view of this, we define for each series P_j an *adjusted reliability ratio*, r_j', as the (standard) reliability ratio of the series $P_j' = P_j - \boldsymbol{X}\boldsymbol{\delta_j}$ that is obtained by removing the component of measurement error that is correlated with $\boldsymbol{X}$. That is,

$$(30)\ r_j' = \frac{EH^2}{E(P_j - \boldsymbol{X}\boldsymbol{\delta_j})'(P_j - \boldsymbol{X}\boldsymbol{\delta_j})} = \frac{EH^2}{EH^2\left(1 + e_j^2 + U_j^2\right)} = \frac{1}{1 + e_j^2 + U_j^2}$$

which will always lie between zero and one.

To relate r_j' to r_j, notice that

$$r_j = \frac{1 - C_{jj}}{1 + e_j^2 + U_j^2 + 2\boldsymbol{\delta_j}'\mu} = \frac{1 - C_{jj}}{\frac{1}{r_j'} + 2\boldsymbol{\delta_j}'\mu}$$

from where

$$(31)\ \frac{1}{r_j'} + 2\boldsymbol{\delta_j}'\mu = \frac{1 - C_{jj}}{r_j}$$

or

$$(32)\ r_j' = \frac{1}{\frac{1 - C_{jj}}{r_j} - 2\boldsymbol{\delta_j}'\mu} = \frac{r_j}{(1 - C_{jj}) - 2\boldsymbol{\delta_j}'\mu r_j}.$$

We can now rewrite the attenuation coefficient that appears in (28) in terms of r_j'. Using (26) and (31), we have

$$a_j = \frac{r_j\,(1 - ER_H^2)}{1 - ER_j^2} = \frac{r_j\,(1 - ER_H^2)}{1 - C_{jj} - r_j\,(ER_H^2 + 2\boldsymbol{\delta_j}'\mu)} = \frac{(1 - ER_H^2)}{\frac{1 - C_{jj}}{r_j} - (ER_H^2 + 2\boldsymbol{\delta_j}'\mu)}$$

$$= \frac{(1 - ER_H^2)}{\frac{1}{r_j'} + 2\boldsymbol{\delta_j}'\mu - (ER_H^2 + 2\boldsymbol{\delta_j}'\mu)} = \frac{(1 - ER_H^2)}{\frac{1}{r_j'} - ER_H^2} = \frac{(1 - ER_H^2)r_j'}{1 - ER_H^2 r_j'}$$

or

$$(33)\ plim\,\hat{\beta}_j = a_j\beta = \frac{(1 - ER_H^2)r_j'}{1 - ER_H^2 r_j'}\,\beta.$$

which is equation (17) in the text.

c. Estimating reliability ratios with correlated errors

This section discusses the two-stage procedure used to obtain our "consistent" estimates of the reliability ratio.

First-stage OLS regressions

The first step involves regressing the different schooling series on each other and on the remaining explanatory variables of the growth model. First, we fix some data set Pj and use it to try to explain the remaining data sets $k \neq j$ as well as the other growth regressors contained in the vector $\boldsymbol{X}$. Hence, for each j we estimate by OLS the following set of equations:

(34) $P_k = r_{jk} P_j + u_{jk}$ for $k = 1..., J$ with $k \neq j$ and

(35) $X_n = \mu_{jn} P_j + u_{jn}$ for $n = 1,..., N$

where the u's are disturbance terms, J the number of alternative proxies for H that are available and N the number of explanatory variables of the growth model, excluding the stock of human capital. This yields (inconsistent) estimates of r_j and μ_n that we will denote by $\hat{r}_{jk}$ and $\hat{\mu}_{jn}$ (hats will be used throughout to indicate first-stage OLS estimates and tildes will be reserved for consistent estimates of various quantities). In addition to the J systems of the form given in (34)-(35), we also estimate by OLS all the "reverse" regressions of P_j on $\boldsymbol{X}$,

(36) $P_j = \boldsymbol{X}\phi_{\boldsymbol{j}} + u_{xj}$

to obtain coefficient estimates we will denote by $\hat{\phi}_{\boldsymbol{j}}$. In this way we obtain $J*(J-1) + 2N*J$ first-stage OLS estimates that will be functions of $J+2N$ true parameters (r_j, μ and ϕ) and the coefficients that describe the structure of the error terms (e_j, $\delta_{\boldsymbol{j}}$ and the variances of ω_j and ε). If J is sufficiently large, we will have enough degrees of freedom to estimate all the parameters of interest.

We will now compute the probability limits of the first-stage OLS estimators. Starting with equation (36), equation (8) with P_j replacing H yields

$$plim\ \hat{\phi}_{\boldsymbol{j}} = (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'P_j = (E\boldsymbol{X}'\boldsymbol{X})^{-1}[E\boldsymbol{X}'H + (E\boldsymbol{X}'\boldsymbol{X})\delta_{\boldsymbol{j}}] = (E\boldsymbol{X}'\boldsymbol{X})^{-1}E\boldsymbol{X}'H + (E\boldsymbol{X}'\boldsymbol{X})^{-1}(E\boldsymbol{X}'\boldsymbol{X})\delta_{\boldsymbol{j}}$$

where the second equality makes use of the transpose of (18). By (8), this reduces to

(37) $plim\ \hat{\phi}_{\boldsymbol{j}} = \phi + \delta_{\boldsymbol{j}}$.

Turning to (34), the plim of the pairwise estimator of r_j using series P_k as a reference, $\hat{r}_{jk}$, is given by

$$plim\ \hat{r}_{jk} = \frac{EP_j'P_k}{EP_j^2} = \frac{EH^2\left(1 + e_je_k + (\delta_{\boldsymbol{k}}+\delta_{\boldsymbol{j}})'\mu + C_{jk}\frac{1}{r_j}\right)}{EP_j^2} = r_j\left(1 + e_je_k + (\delta_{\boldsymbol{k}}+\delta_{\boldsymbol{j}})'\mu + C_{jk}\frac{1}{r_j}\right)$$

where the second equality makes use of (23), or

(38) $plim\ \hat{r}_{jk} = r_j[1 + e_je_k + (\delta_{\boldsymbol{k}}+\delta_{\boldsymbol{j}})'\mu] + C_{jk}$.

Finally, for equation (35), equation (9) with P_j replacing H implies, using (17) and (4), that

$$plim\ \hat{\mu}_{jn} = \frac{EP_j'X_n}{EP_j^2} = \frac{EH^2}{EP_j^2}\frac{EH'X_n}{EH^2} + \frac{\delta_{\boldsymbol{j}}'E\boldsymbol{X}'X_n}{EP_j^2}$$

or, by (9),

$(39)\ plim\ \hat{\mu}_{jn} = r_j\, \mu_n + C_{jn}$

where

$$(40)\ C_{jn} \equiv \frac{\delta_{\boldsymbol{j}}' E\boldsymbol{X}' X_n}{EP_j^2}\ .$$

Second-stage equations

The second-stage equations we estimate to recover the parameters of interest are obtained from equations (38) and (39) by replacing the probability limits on the left-hand side and any population moments that appear in the equation by their corresponding sample estimates (denoted by tildes) and adding a disturbance term (η) to capture random deviations from the expected asymptotic relations. Hence, for each series j we have two sets of equations of the form:

$(41)\ \hat{r}_{jk} = r_j[1 + e_j e_k + (\delta_{\boldsymbol{k}}+\delta_{\boldsymbol{j}})'\mu] + \tilde{C}_{jk} + \eta_{jk}$ for $k \neq j,\ k = 1 \ldots J$

$(42)\ \hat{\mu}_{jn} = r_j\, \mu_n + \tilde{C}_{jn} + \eta_{jn}$ for $n = 1, \ldots, N$

as well as the relation derived above

$(37)\ plim\ \hat{\phi}_{\boldsymbol{j}} = \phi + \delta_{\boldsymbol{j}}$.

To construct consistent estimators of r_j, e_j, μ_n, ϕ and $\delta_{\boldsymbol{j}}$, we proceed as follows.

1) As noted in the text, equation (37) implies that the estimation of equation (36) yields consistent estimates of $\delta_{\boldsymbol{j}}$ "up to a constant." We can use this information to significantly reduce the number of parameters to be estimated. For this, we take a specific data set (D&D02) as a reference, denote the corresponding value of $\delta_{\boldsymbol{j}}$ by δ, define $\Delta_{\boldsymbol{j}}$ by

$$\Delta_{\boldsymbol{j}} \equiv \delta_{\boldsymbol{j}} - \delta$$

and obtain a consistent estimate of its value as

$$(43)\ \tilde{\Delta}_{\boldsymbol{j}} = \hat{\phi}_j - \hat{\phi}_{\boldsymbol{DD02}}\ .$$

This leaves us with only the N components of δ to be estimated in the second stage. Once this has been done, we can go back to (37) and obtain an estimate of ϕ as

$$(44)\ \tilde{\phi} = \hat{\phi}_{\boldsymbol{DD02}} - \tilde{\delta}$$

where $\tilde{\delta}$ is the second-stage estimate of δ whose construction will be discussed below.

2) Next, we use equations (41) and (42) to obtain estimates of r_j, e_j, μ_n, and δ. For this, we use the estimated values of $\tilde{\Delta}_{\boldsymbol{j}}$ and rewrite (41) and (42) as functions of δ.

To proceed with the necessary calculations, we will make our notation a bit more specific. In all the cases we consider, $\boldsymbol{X}$ is a vector of two variables: the capital labour ratio *(z)* and the employment ratio *(e)*. Hence, δ and μ are 2-vectors:

$$(45)\ \delta = (\delta_z, \delta_e)' \quad \text{and} \quad \mu = (\mu_z, \mu_e)'$$

and $\boldsymbol{V} = E\boldsymbol{X}'\boldsymbol{X}$ is a (symmetric) 2 by 2 matrix which can be consistently estimated by the sample variance-covariance matrix of the components of $\boldsymbol{X}$, which will be denoted by $\tilde{\boldsymbol{V}}$ and writen it in the form

(46) $\tilde{\mathbf{V}} = \begin{bmatrix} \tilde{V}_{zz} & \tilde{V}_{ze} \\ \tilde{V}_{ze} & \tilde{V}_{ee} \end{bmatrix} = \left(\tilde{\mathbf{V}}_z, \tilde{\mathbf{V}}_e\right)$

where $\tilde{\mathbf{V}}_{\mathbf{n}}$ is its n-th colum. Finally, the terms $\tilde{d}_{jk}$ and $\tilde{d}_{jn}$ are defined as

(47) $\tilde{d}_{jk} \equiv \tilde{\Delta}_j'\tilde{\mathbf{V}}\tilde{\Delta}_k$ and $\tilde{d}_{jn} \equiv \tilde{\Delta}_j'\tilde{\mathbf{V}}_{\mathbf{n}}$

and can be computed using information known at this stage.

Using this notation, we can now write the terms $\tilde{C}_{jk}$, $\tilde{C}_{jn}$ and $(\delta_{\mathbf{k}}+\delta_{\mathbf{j}})'\mu$ that appear in equations (41) and (42) as explicit functions of the parameters to be estimated and of quantities that can be consistently estimated using previous results.

Using (43), we can write

$$\delta_{\mathbf{j}}'\mu = (\delta+\Delta_{\mathbf{j}})'\mu = (\delta_z+\Delta_{jz}, \delta_e+\Delta_{je})\begin{pmatrix}\mu_z \\ \mu_e\end{pmatrix} = (\delta_z+\Delta_{jz})\,\mu_z + (\delta_e+\Delta_{je})\mu_e$$

from where

(48) $$(\delta_{\mathbf{k}}+\delta_{\mathbf{j}})'\mu = (\delta_z\mu_z + \delta_e\mu_e + \Delta_{kz}\mu_z + \Delta_{ke}\mu_e) + (\delta_z\mu_z + \delta_e\mu_e + \Delta_{jz}\mu_z + \Delta_{je}\mu_e)$$
$$= 2(\delta_z\mu_z + \delta_e\mu_e) + (\Delta_{kz} + \Delta_{jz})\mu_z + (\Delta_{ke} + \Delta_{je})\mu_e\,.$$

Recalling (22), C_{jk} can be written in the form

(49) $$C_{jk} \equiv \frac{\delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{k}}}{EP_j^2} \equiv \frac{1}{EP_j^2}c_{jk}$$

where

(50) $$c_{jk} = \delta_j'(E\mathbf{X}'\mathbf{X})\delta_{\mathbf{k}} = \delta_j'\mathbf{V}\delta_{\mathbf{k}} = (\delta'+\Delta_j')\mathbf{V}(\delta+\Delta_k) = \delta'\mathbf{V}\delta + \delta'\mathbf{V}\Delta_k + \Delta_j'\mathbf{V}\delta + \Delta_j'V\Delta_k$$
$$= \delta'\mathbf{V}\delta + (\Delta_j+\Delta_k)'\mathbf{V}\delta + d_{jk}$$

Consider now the first two terms in this expression. The first one can be written

(51) $$\delta'\mathbf{V}\delta = (\delta_z, \delta_e)\begin{pmatrix}V_{zz} & V_{ze} \\ V_{ze} & V_{ee}\end{pmatrix}\begin{pmatrix}\delta_z \\ \delta_e\end{pmatrix} = (\delta_z, \delta_e)\begin{pmatrix}V_{zz}\delta_z + V_{ze}\delta_e \\ V_{ze}\delta_z + V_{ee}\delta_e\end{pmatrix} =$$
$$= \delta_z(V_{zz}\delta_z+V_{ze}\delta_e) + \delta_e(V_{ze}\delta_z+V_{ee}\delta_e) = V_{zz}\delta_z^2 + 2V_{ze}\delta_z\delta_e + V_{ee}\delta_e^2$$

As for the second one,

$$(\Delta_j+\Delta_k)'\mathbf{V}\delta = \Delta_j'\mathbf{V}\delta + \Delta_k'\mathbf{V}\delta\,,$$

notice that we can write

$$\Delta_j'\mathbf{V}\delta = \Delta_j'(\mathbf{V_z}, \mathbf{V_e})\delta = (\Delta_j'\mathbf{V_z}, \Delta_j'\mathbf{V_e})\begin{pmatrix}\delta_z \\ \delta_e\end{pmatrix} =$$
$$= \Delta_j'\mathbf{V_z}\delta_z + \Delta_j'\mathbf{V_e}\delta_e = d_{jz}\delta_z + d_{je}\delta_e$$

Hence

(52) $$(\Delta_j+\Delta_k)'\mathbf{V}\delta = (d_{jz} + d_{kz})\delta_z + (d_{je} + d_{ke})\delta_e$$

Substituting (51) and (52) back into (50), we have

$$c_{jk} = \delta'\mathbf{V}\delta + (\Delta_j+\Delta_k)'\mathbf{V}\delta + d_{jk}$$
$$= V_{zz}\delta_z^2 + 2V_{ze}\delta_z\delta_e + V_{ee}\delta_e^2 + (d_{jz} + d_{kz})\delta_z + (d_{je} + d_{ke})\delta_e + d_{jk}$$

and therefore

(53) $$C_{jk} = \frac{1}{EP_j^2}\{d_{jk} + V_{zz}\delta_z^2 + 2V_{ze}\delta_z\delta_e + V_{ee}\delta_e^2 + (d_{jz} + d_{kz})\delta_z + (d_{je} + d_{ke})\delta_e\}$$

Finally, recalling (40), C_{jn} can be written in the form

$$(54)\ C_{jn} \equiv \frac{\delta_{\boldsymbol{j}}' E\boldsymbol{X}' X_n}{EP_j^2} \equiv \frac{1}{EP_j^2} c_{jn}$$

where

$$c_{jn} = \delta_{\boldsymbol{j}}' E\boldsymbol{X}' X_n$$

for $n = z, e$. Notice that

$$E\boldsymbol{X}'X_z = E\begin{pmatrix} X_z \\ X_e \end{pmatrix} X_z = \begin{pmatrix} EX_z^2 \\ EX_e X_z \end{pmatrix} = \begin{pmatrix} V_{zz} \\ V_{ze} \end{pmatrix} = \boldsymbol{V_z} \quad \text{and}$$

$$E\boldsymbol{X}'X_e = \begin{pmatrix} V_{ze} \\ V_{ee} \end{pmatrix} = \boldsymbol{V_e},$$

where the subindex in $\boldsymbol{V_n}$ indicates the column of matrix $\boldsymbol{V}$ we are taking. Hence,

$$c_{jn} = \delta_{\boldsymbol{j}}'\boldsymbol{V_n} = (\delta' + \Delta_j')\boldsymbol{V_n} = \delta'\boldsymbol{V_n} + \Delta_j'\boldsymbol{V_n} = \delta'\boldsymbol{V_n} + d_{jn}$$

$$= (\delta_z, \delta_e)\begin{pmatrix} V_{nz} \\ V_{ne} \end{pmatrix} + d_{jn} = V_{nz}\,\delta_z + V_{ne}\delta_e + d_{jn}.$$

and therefore

$$(55)\ C_{jn} = \frac{1}{EP_j^2}(V_{nz}\,\delta_z + V_{ne}\delta_e + d_{jn}).$$

Using (48), (53) and (55), equations (38) and (39) can be rewritten in the form

$$(56)\ \ plim\ \hat{r}_{jk} = r_j[1 + e_j e_k + 2(\delta_z\mu_z + \delta_e\mu_e) + (\Delta_{kz} + \Delta_{jz})\mu_z + (\Delta_{ke} + \Delta_{je})\mu_e] +$$

$$+ \frac{1}{EP_j^2}\{d_{jk} + V_{zz}\delta_z^2 + 2V_{ze}\delta_z\delta_e + V_{ee}\delta_e^2 + (d_{jz} + d_{kz})\delta_z + (d_{je} + d_{ke})\delta_e\}$$

$$(57)\ plim\ \hat{\mu}_{jn} = r_j\mu_n + \frac{1}{EP_j^2}(V_{nz}\,\delta_z + V_{ne}\delta_e + d_{jn})\ .$$

Finally, we proceed as above to obtain the appropriate small sample expressions to be estimated. Using the sample variance of schooling series j (denoted by $svar\ P_j$) to estimate EP_j^2 we have

$$(56')\ \ \hat{r}_{jk} = r_j[1 + e_j e_k + 2(\delta_z\mu_z + \delta_e\mu_e) + (\tilde{\Delta}_{jz} + \tilde{\Delta}_{kz})\,\mu_z + (\tilde{\Delta}_{je} + \tilde{\Delta}_{ke})\,\mu_e] +$$

$$+ \frac{1}{svar\ P_j}\left\{\tilde{d}_{jk} + \tilde{V}_{zz}\delta_z^2 + 2\tilde{V}_{ze}\delta_z\delta_e + \tilde{V}_{ee}\delta_e^2 + \left(\tilde{d}_{jz} + \tilde{d}_{kz}\right)\delta_z + \left(\tilde{d}_{je} + \tilde{d}_{ke}\right)\delta_e\right\} + \eta_{jk}$$

$$(57')\ \hat{\mu}_{jn} = r_j\mu_n + \frac{1}{svar\ P_j}\left\{\tilde{d}_{jn} + \tilde{V}_{nz}\delta_z + \tilde{V}_{ne}\delta_e\right\} + \eta_{jn}$$

where tildes denote consistent sample estimates of different variables. Notice that the only unknown quantities in these expressions are the coefficients to be estimated: r_j and e_j for j = $1...J$, μ_z, μ_e, δ_z and δ_e.

We estimate (56') and (57') jointly by "stacking them" so that, for each j, the first J observations of the dependent variable (one of which will be missing as k must be different from j) correspond to the first-stage pairwise estimates of the reliability ratio of P_j, and the last two to the first-stage estimates, $\hat{\mu}_{jn}$. Notice that the resulting system is non-linear and requires heavy use of dummy variables for its estimation. The following section contains a detailed discussion of the estimation procedure and can be skipped without great loss.

d. Further details on the estimation of the second-stage equations

This section describes how equations (56') and (57') are estimated using *Eviews*. We have eight different schooling data sets, so $J = 8$ and, as noted above $N = 2$. We will estimate a system of 8 equations (one for each data set) with 10 observations, one of which will be missing.

We begin by reading the first-stage estimates $\hat{r}_{jk}$ and $\hat{\mu}_{jn}$ and $\tilde{\Delta}_j$ into a spreadsheet along with other quantities of interest, such as the variances and covariances of the components of $\boldsymbol{X}$ (i.e. the entries of the matrix $\tilde{\boldsymbol{V}}$) and the (inverses of the) variances of the (appropriately transformed) schooling series, which we denote by $INVVAR_j$. We then construct in the same spreadsheet the dependent variable y_j and a set of dummy variables for the "reference variable" used in each case (R_i for i = 1 to 8, R_z, R_e and R_p) as follows:

$$(58)\quad y_{jk} = \begin{cases} \hat{r}_{jk} & \text{for } k = 1 \text{ to } 8,\ k \neq j \\ \text{n.a.} & \text{for } k = 1 \text{ to } 8,\ k = j \\ \hat{\mu}_{jz} & \text{for } k = 9 \\ \hat{\mu}_{je} & \text{for } k = 10 \end{cases}$$

$$(59)\quad R_{ik} = \begin{cases} 1 & \text{for } k = 1 \text{ to } 8,\ k = i \\ 0 & \text{otherwise} \end{cases}$$

$$(60)\quad R_{zk} = \begin{cases} 1 & \text{for } k = 9 \\ 0 & \text{otherwise} \end{cases}$$

$$(61)\quad R_{ek} = \begin{cases} 1 & \text{for } k = 10 \\ 0 & \text{otherwise} \end{cases}$$

$$(62)\quad R_{pk} = \sum_{i=1}^{8} R_{ik}$$

Hence, y_j "stacks" $\hat{r}_{jk}$ and $\hat{\mu}_{jn}$ into a single dependent variable, R_i identifies those pairwise reliability ratio estimates where the schooling series P_i is the reference variable, R_z and R_e identify the $\hat{\mu}_{jz}$ and $\hat{\mu}_{je}$ observations, and R_p all the $\hat{r}_{jk}$ observations.

We copy all these variables into an *Eviews* workfile and place a "matrix" whose columns are the estimated vectors $\tilde{\Delta}_j$ into a group called *GROUPDELTA*. The *Eviews* program shown in Box 1 then calculates

$$(47)\quad \tilde{d}_{jk} \equiv \tilde{\Delta}_j{}'\tilde{\boldsymbol{V}}\tilde{\Delta}_k \quad \text{and} \quad \tilde{d}_{jn} \equiv \tilde{\Delta}_j{}'\tilde{\boldsymbol{V}}_{\boldsymbol{n}}$$

and constructs a set of artificial variables that correspond to the coefficients of μ_z, μ_e, δ_z and δ_e in equations (56') and (57') and to the constants of the form $\tilde{d}_{jk}$ and $\tilde{d}_{jn}$ that enter the second term of each equation. These variables are called $Xmuz_j$, $Xmue_j$, $Xdeltaz_j$, $Xdeltae_j$ *and* Xd_j and are defined as follows:

$$(63)\quad Xmuz_{jk} = \begin{cases} \tilde{\Delta}_{jz} + \tilde{\Delta}_{kz} & \text{for } k = 1 \text{ to } 8 \\ 1 & \text{for } k = 9 \\ 0 & \text{for } k = 10 \end{cases}$$

$$(64)\quad Xmue_{jk} = \begin{cases} \tilde{\Delta}_{je} + \tilde{\Delta}_{ke} & \text{for } k = 1 \text{ to } 8 \\ 0 & \text{for } k = 9 \\ 1 & \text{for } k = 10 \end{cases}$$

$$(65)\quad Xdeltaz_{jk} = \begin{cases} \tilde{d}_{jz} + \tilde{d}_{kz} & \text{for } k = 1 \text{ to } 8 \\ \tilde{V}_{zz} & \text{for } k = 9 \\ \tilde{V}_{ez} & \text{for } k = 10 \end{cases}$$

$$(66)\quad Xdeltae_{jk} = \begin{cases} \tilde{d}_{je} + \tilde{d}_{ke} & \text{for } k = 1 \text{ to } 8 \\ \tilde{V}_{ze} & \text{for } k = 9 \\ \tilde{V}_{ee} & \text{for } k = 10 \end{cases}$$

$$(67)\quad Xd_{jk} = \begin{cases} \tilde{d}_{jk} & \text{for } k = 1 \text{ to } 8 \\ \tilde{d}_{jz} & \text{for } k = 9 \\ \tilde{d}_{je} & \text{for } k = 10 \end{cases}$$

Next, we estimate the system of second-stage equations. It will be formed by 8 equations, one for each data set. In the notation of this section, the equation for data set j will be of the form:

$$(68)\quad y_j = r_j\left(1*RP + e_j*\left(\sum_{i=1}^{8} R_i e_i\right) + 2*RP*(\delta_z\mu_z + \delta_e\mu_e) + \mu_z*Xmuz_j + \mu_e*Xmue_j\right)$$

$$+\ INVVAR_j*\{Xd_j + RP*\left(\tilde{V}_{zz}\delta_z^2 + 2\tilde{V}_{ze}\delta_z\delta_e + \tilde{V}_{ee}\delta_e^2\right) + \delta_z*Xdeltaz_j + \delta_e*Xdeltae_j\}$$

We estimate equation (68) and a restricted version of the same equation where we impose the assumption that measurement error is not correlated with the components of $\boldsymbol{X}$ (but continue to allow for correlation across data sets). For the *Eviews* NLS algorithm to start iterating, non-zero initial values must be assigned to at least some of the parameters. We set initial values by estimating a log-linear approximation to the restricted version of equation (68). We have also repeated the estimation in *RATS* and obtained very similar results.

Box 1: *Eviews* program for constructing variables (63) to (67)

```
' The matrix MDELTACAPnj will contain the Δj's estimated in the first stage
' each Δj vector is a column of the matrix.
' we read it from a preexisting group imported from Excel called GROUPDELTA
MATRIX(2,8) MDELTACAPnj
MDELTACAPnj=@CONVERT(GROUPDELTA)

'Read in V = EX'X (note: these values are for the data in levels)
MATRIX(2,2) V
V(1,1)=0.1413
V(1,2)=3.44158E-04
V(2,1)=V(1,2)
V(2,2)=0.01418

' Declare auxiliary vectors that will be used in the computations
VECTOR(2) DELTAj
VECTOR(2) DELTAk
VECTOR(2) Vn

'The matrix MDjk will contain the djk and djn terms to be calculated below
MATRIX(8,10) MDjk

'Compute djk
FOR !J=1 TO 8
  DELTAj=@COLUMNEXTRACT(MDELTACAPnj,!J)
  FOR !K=1 TO 8
    DELTAk=@COLUMNEXTRACT(MDELTACAPnj,!K)
    MATRIX MTEMP1=@TRANSPOSE(DELTAj)*V*DELTAk
    MDjk(!J,!K)=MTEMP1(1,1)
  NEXT
NEXT
```

Box 1: *Eviews* program for constructing variables (63) to (67), continued

```
'Compute d_jn
FOR !J=1 TO 8
  DELTAj=@COLUMNEXTRACT(MDELTACAPnj,!J)
  FOR !N=1 TO 2
    Vn=@COLUMNEXTRACT(V,!N)
    MATRIX MTEMP2=@TRANSPOSE(DELTAj)*Vn
    MDjk(!J,8+!N)=MTEMP2(1,1)
  NEXT
NEXT

' We now construct the regressors given in (63) to (67). Notice that they are constructed so as to reduce
' the need for dummies in the equation to be estimated.

'Construct variables Xd_j(k) containing the terms d_jk and d_jn
FOR !J=1 TO 8
   SERIES XD{!J}
   FOR !K=1 TO 10
       XD{!J}(!K)=MDjk(!J,!K)
   NEXT
NEXT

'Construct variables Xmuz_j and Xmue_j
FOR !J=1 TO 8
    SERIES XMUz{!J}
    SERIES XMUe{!J}
    FOR !K=1 TO 8
         XMUz{!J}(!K)=MDELTACAPnj(1,!J)+MDELTACAPnj(1,!K)
         XMUe{!J}(!K)=MDELTACAPnj(2,!J)+MDELTACAPnj(2,!K)
    NEXT
    XMUz{!J}(9)=1
    XMUz{!J}(10)=0
    XMUe{!J}(9)=0
    XMUe{!J}(10)=1
NEXT

'Construct variables Xdeltaz_j and Xdeltae_j
FOR !J=1 TO 8
    SERIES XDELTAz{!J}
    SERIES XDELTAe{!J}
    FOR !K=1 TO 8
         XDELTAz{!J}(!K)=MDjk(!J,9)+MDjk(!K,9)
         XDELTAe{!J}(!K)=MDjk(!J,10)+MDjk(!K,10)
    NEXT
    XDELTAz{!J}(9)=V(1,1)
    XDELTAz{!J}(10)=V(1,2)
    XDELTAe{!J}(9)=V(2,1)
    XDELTAe{!J}(10)=V(2,2)
NEXT
```

e. Adjusted reliability rates and attenuation coefficients

Once we have estimated the system, we recover the adjusted reliability ratios, r_j', and construct estimates of ϕ and ER_H^2 using equations (32), (44) and (10') respectively.[2] With these variables, we construct the attenuation coefficients given in equation (33). The *Eviews* programshown in Box 2 performs these calculations.

Box 2: *Eviews* program for constructing the adjusted reliability ratios and attenuation coefficients

```
' Execute this program after having estimated the system given in (68).

' Read estimated coefficients into vectors
' Read μz and μe into vector MU
VECTOR(2) MU
MU(1)=C(9)
MU(2)=C(10)

'Read δz and δe into vector VDELTAMIN
VECTOR(2) VDELTAMIN
VDELTAMIN(1)=C(29)
VDELTAMIN(2)=C(30)

'Read ϕz  and ϕe into vector PHI
' Note: these values are for the levels specification
VECTOR(2) PHI
PHI(1)=0.574
PHI(2)=1.470

'Calculate ERH2
VECTOR(1) VTEMP5=@TRANSPOSE(MU)*PHI
SCALAR ER2H=VTEMP5(1)

'Read reliability ratios (rj) into coefficient vector RR
COEFFICIENT(8) RR
FOR !J=1 TO 8
   RR(!J)=C(20+!J)
NEXT

'Construct matrix containing vectors δj =  δ + Δj
MATRIX(2,8) MDELTAMINnj
FOR !J=1 TO 8
  MDELTAMINnj(1,!J)=MDELTACAPnj(1,!J)+VDELTAMIN(1)
  MDELTAMINnj(2,!J)=MDELTACAPnj(2,!J)+VDELTAMIN(2)
NEXT
```

[2] In the case of the fixed effects specification, the estimated value of ER_H^2 is a (very small) negative number. We assume $ER_H^2 = 0$ to construct the attenuation coefficients.

Box 2: continued

```
'Construct matrix containing the adjustment factors Cjk
MATRIX(8,8) Cjk
FOR !J=1 TO 8
   DELTAj=@COLUMNEXTRACT(MDELTAMINnj,!J)
   FOR !K=1 TO 8
      DELTAk=@COLUMNEXTRACT(MDELTAMINnj,!K)
      MATRIX TEMP1=@TRANSPOSE(DELTAj)*V*DELTAk
     Cjk(!J,!K)=TEMP1(1,1)*INVVAR{!J}(1)
   NEXT
NEXT

'Compute the adjusted reliability ratios, rj ' and attenuation coefficients, aj
' put them into coefficient vectors called RRPRIME and ATT

COEFFICIENT(8) RRPRIME
COEFFICIENT(8) ATT
FOR !J=1 TO 8
   DELTAj=@COLUMNEXTRACT(MDELTAMINnj,!J)
   VECTOR TEMPV6=@TRANSPOSE(DELTAj)*MU
    SCALAR DENOM=(1-Cjk(!J,!J))-2*TEMPV6(1)*RR(!J)
    RRPRIME(!J)=RR(!J) / DENOM
    ATT(!J)=RRPRIME(!J)*(1-ER2H) / (1-ER2H*RRPRIME(!J))
NEXT
```

f. Detailed results

Tables A.3 and A.4 show the detailed results of the second-stage estimation. Table A.3 gives the results of equation (36) which yields estimates of $\phi+\delta_j$. Table A.4 shows the estimated values of the raw ("consistent") reliability ratios, r_j, and the coefficients e_j, μ, δ , ϕ and $ER_H{}^2$. For each growth specification (levels, fixed effects and differences), we show results for a restricted model where we impose the assumption that $\delta_j = \mathbf{0}$ and for the full model developed above where we estimate δ_j. In the first case, the estimates of ϕ are also obtained from a restricted version of equation (36), where we impose a common coefficient for all data sets. See the discussion in the text about the estimation of $ER_H{}^2$, which is generally based on (10').

Notice that the hypothesis that $\delta = \mathbf{0}$ cannot be rejected for the data in differences (see the first panel of Table A.4). For the data in levels, however, δ_e is significantly lower than zero, indicating a negative correlation between measurement error and the employment ratio for the D&D02 data set. Since the estimated values of $\phi+\delta_j$ are considerably lower for manyof the remaining data sets (see the first panel of Table A.3), the correlation appears to be even stronger in these cases.

Table A.3: Estimates of equation (36)

a. levels

	$\phi+\delta_1$ *NSD*	$\phi+\delta_2$ *Kyr*	$\phi+\delta_3$ *B&L93*	$\phi+\delta_4$ *B&L96*	$\phi+\delta_5$ *B&L00*	$\phi+\delta_6$ *C&S*	$\phi+\delta_7$ *D&D00*	$\phi+\delta$ *D&D02*
Z	0.127	0.275	0.551	0.470	0.465	0.358	0.339	0.374
	(2.21)	(6.37)	(8.07)	(7.88)	(8.71)	(9.70)	(8.74)	(10.48)
E	-0.022	-0.259	0.371	0.365	0.547	0.656	0.633	0.778
	(0.12)	(2.26)	(1.66)	(1.94)	(3.25)	(5.63)	(5.17)	(6.90)
R_j^2	0.0397	0.3306	0.3720	0.3317	0.3982	0.4944	0.4450	0.5511

b. fixed effects

	$\phi+\delta_1$ *NSD*	$\phi+\delta_2$ *Kyr*	$\phi+\delta_3$ *B&L93*	$\phi+\delta_4$ *B&L96*	$\phi+\delta_5$ *B&L00*	$\phi+\delta_6$ *C&S*	$\phi+\delta_7$ *D&D00*	$\phi+\delta$ *D&D02*
Z	0.010	0.139	0.0898	0.114	0.079	0.047	0.072	0.055
	(0.28)	(2.36)	(1.65)	(3.40)	(2.33)	(2.67)	(5.45)	(4.11)
E	-0.092	-0.881	0.195	-0.011	-0.007	-0.213	-0.027	-0.139
	(0.63)	(4.15)	(0.84)	(0.07)	(0.05)	(2.71)	(0.46)	(2.32)
R_j^2	0.0397	0.2767	0.0231	0.0841	0.0413	0.1271	0.1972	0.1784

c. differences

	$\phi+\delta_1$ *NSD*	$\phi+\delta_2$ *Kyr*	$\phi+\delta_3$ *B&L93*	$\phi+\delta_4$ *B&L96*	$\phi+\delta_5$ *B&L00*	$\phi+\delta_6$ *C&S*	$\phi+\delta_7$ *D&D00*	$\phi+\delta$ *D&D02*
Z	0.029	0.222	-0.004	0.041	0.008	0.037	0.034	0.023
	(0.63)	(1.89)	(0.03)	(0.55)	(0.13)	(1.57)	(1.80)	(1.27)
E	-0.102	-0.115	-0.042	-0.226	-0.181	-0.026	0.053	0.027
	(0.98)	(0.47)	(0.15)	(1.31)	(1.19)	(0.48)	(1.25)	(0.64)
R_j^2	0.0165	0.0542	0.0002	0.0237	0.0153	0.0304	0.0402	0.0178

- *Note: t* ratios below each coefficient.

Table A.4: "Consistent" reliability ratio estimates. Detailed results

	levels		*fixed effects*		*differences*	
	restricted	*unrestr.*	*restricted*	*unrestr.*	*restricted*	*unrestr.*
r_j						
NSD	0.186	1.407	0.117	0.135	0.086	0.142
	[0.098]	[0.304]	[0.053]	[0.051]	[0.062]	[0.063]
KYR	0.379	2.934	0.197	0.032	0.056	0.028
	[0.196]	[0.710]	[0.072]	[0.030]	[0.070]	[0.050]
B&L93	0.116	0.730	0.095	0.051	0.057	0.046
	[0.059]	[0.151]	[0.041]	[0.022]	[0.035]	[0.028]
B&L96	0.143	0.902	0.221	0.112	0.150	0.119
	[0.073]	[0.188]	[0.061]	[0.041]	[0.068]	[0.059]
B&L00	0.157	0.994	0.240	0.135	0.177	0.155
	[0.080]	[0.208]	[0.063]	[0.048]	[0.073]	[0.069]
C&S	0.272	1.752	0.591	0.403	0.729	0.633
	[0.140]	[0.365]	[0.079]	[0.076]	[0.079]	[0.068]
D&D00	0.263	1.752	0.914	0.332	0.570	0.432
	[0.135]	[0.362]	[0.094]	0.125	[0.085]	[0.083]
D&D02	0.255	1.669	0.959	0.643	0.818	0.772
	[0.131]	[0.347]	[0.095]	[0.099]	[0.087]	[0.078]
e_j						
NSD	0.912	-0.381	0.741	0.383	0.341	-0.149
	[0.470]	[0.049]	[0.399]	[0.297]	[0.402]	[0.253]
KYR	0.633	-0.039	-0.033	0.502	-0.998	-0.967
	[0.386]	[0.027]	[0.286]	[0.442]	[0.400]	[0.409]
B&L93	2.590	-0.813	2.020	3.148	2.250	2.830
	[0.720]	[0.087]	[0.550]	[0.729]	[0.706]	[0.892]
B&L96	2.301	-0.737	1.657	2.341	2.306	2.622
	[0.690]	[0.079]	[0.493]	[0.648]	[0.748]	[0.902]
B&L00	2.322	-0.706	1.152	1.654	1.209	1.274
	[0.693]	[0.076]	[0.388]	[0.496]	[0.454]	[0.475]
C&S	1.479	-0.371	0.209	0.214	-0.028	0.018
	[0.564]	[0.043]	[0.157]	0.127	[0.082]	[0.071]
D&D00	1.495	-0.374	-0.288	-0.061	-0.378	-0.240
	[0.559]	[0.042]	[0.110]	[0.132]	[0.135]	[0.115]
D&D02	1.540	-0.348	-0.200	-0.186	-0.257	-0.194
	[0.575]	[0.041]	[0.101]	[0.075]	[0.092]	[0.075]

Table A.11: "Consistent" reliability ratio estimates. Detailed results (continued)

	levels		*fixed effects*		*differences*	
	restricted	*unrestr.*	*restricted*	*unrestr.*	*restricted*	*unrestr.*
δ_z		-0.200		0.049		0.014
		[0.165]		[0.010]		[0.015]
δ_e		-0.691		-0.212		0.038
		[0.295]		[0.166]		[0.069]
μ_z	3.456	0.910	2.580	0.801	1.301	0.755
	[1.774]	[0.132]	[0.258]	[0.653]	[0.184]	[0.583]
μ_e	0.340	0.249	-0.353	-0.340	0.037	-0.216
	[0.249]	[0.084]	[0.160]	[0.517]	[0.160]	[0.530]
ϕ_z	0.278	0.574	0.052	0.00622	0.029	0.009
ϕ_e	0.319	1.469	-0.081	0.07336	0.033	-0.011
$ER_H{}^2$	1.069	0.887	0.163	-0.020	0.039	0.009

Notes:

- Standard errors in brackets below each coefficient
- The restricted model assumes $\delta_j = 0$.