On the Choice of an Exchange Regime: Target Zones Revisited

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Abstract

From the classical gold standard up to the current ERM2 arrangement of the European Union, target zones have been a widely used exchange regime in contemporary history. This paper presents a benchmark model that rationalizes the choice of target zones over the rest of regimes: the fixed rate, the free float and the managed float. It is shown that the monetary authority may gain efficiency by reducing volatility of both the exchange rate and the interest rate at the same time. Furthermore, the model is consistent with some known stylized facts in the empirical literature that previous models were not able to generate, namely, the positive relation between the exchange rate and the interest rate differential, the degree of non-linearity of the function linking the exchage rate to fundamentals and the shape of the exchange rate stochastic distribution. **Keywords**: Target zones, exchange rate agreements, monetary policy, time consistency. **JEL classification**: E52, F31, F33.

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1. Introduction

This paper tackles the question of the choice of an exchange regime, with a special focus on target zones. During the nineties, a large amount of literature analyzed a variety of issues regarding target zones, both from a theoretical perspective (i.e. mainly the properties of the model proposed by Krugman [15] and of its variants) as well as from an applied point of view (i.e. mainly the crisis of the European Monetary System). The creation of the European Monetary Union led exchange rate researchers to move their interests to other questions apart from the ones proposed by the target zone literature. Yet, we believe there are still useful insights to be learnt on target zones. The reasons why we persist are threefold:

1. *Target zones have been the predominant exchange rate agreement in the world throughout contemporary history.* Some recent analyses have reclassified exchange rate regimes focusing on what countries actually do rather than on what they say they do (see Calvo and Reinhart [6], Reinhart [23], Reinhart and Rogoff [24], Fischer [8] and Levy-Yeyati and Sturzenegger [17]). This implies a distinction between *de jure* and *de facto* typologies. *De jure* examples of target zones are numerous: the interwar Gold Standard, the postwar Bretton Woods regime, the European *monetary snake* of 1971, the European Monetary System (EMS) from 1978 to 1998, or the unilateral target zone of Sweden from August 1977, among others. On the other hand, the number of *de facto* examples of target zones is much larger. The aforementioned literature has stressed the fact that many of the *de jure* free floaters strongly intervene to soften the fluctuations of the nominal exchange rate. This has been labeled as *fear of floating.* The recent *de facto* classification of Reinhart and Rogoff [24] confirms that most of the major currencies are traded in an international exchange market characterized by bands of fluctuations (see their Appendix III). According to these authors, "...the most popular exchange rate regime over modern history has been the crawling peg or narrow crawling band, which accounts for over 26 percent of the observations". In the Western Hemisphere, this accounts for about a 42 percent of the observations. Their sample consists on market-determined paralell exchange rates observations of 153 countries from 1946 through 2001.

2. *Target zones shall probably be an important exchange rate agreement at least in the nearby future.* As an example, consider the ERM2 which, from Jan-

uary 1st 1999, is working as a "hub-and-spokes" zone between the euro and those currencies of the European Union countries not participating in the Monetary Union (United Kingdom, Denmark and Sweden). Also, accession countries to the European Union shall commit to a target zone arrangement during the transition period. Again, many other *de facto* examples could be quoted.

3. *Any exchange rate regime can be seen as an special case of a target zone.* Pegged rates can be seen as target zones with band widths equal to zero and pure floating rates are equivalent to fluctuation bands with an infinite width. In between, dirty floats can be seen as implicit target zones with finite bands while target zone regimes impose those bands explicitly. The choice of an exchange regime then consists on deciding how wide should the bandwidth be. It must be clear that we will not try to give a typology of exchange regimes alternative to the ones given by Reinhart and Rogoff [24] or by Levy-Yeyati and Sturzenegger [17]. Instead, what we do is to construct a general model able to represent any exchange agreement. By doing so, we can rationalize such a choice.

The need of a more formal target zone model becomes more important given that the extreme regimes (i.e. the fixed rate and the flexible rate) are only standard textbook cases. While Obstfeld and Rogoff [22] predicted a world of widely floating exchange rates, given the removal of controls to international capital mobility, the *fear of floating* literature has pointed out that countries reveal a preference towards smoothing the dynamics of the exchange rate. Intermediate regimes seem to be defining the current world so that completely fixed or fully flexible rates (see Fischer [8]) are seldom observed. Pure fixed exchange rate regimes are rarely used since they compromise a large amount of monetary independence. A monetary commitment that pegs the exchange rate to a low inflation foreign currency can establish a disciplining effect that motivates a higher degree of credibility and price stability (see Giavazzi and Pagano [12]). However, given the evidence on the choice of currency regimes by central banks, it seems that the gains from stability of fixed exchange rates have not been big enough to dampen the losses from less monetary independence (see Svensson [30] for a detailed discussion). On the other hand, pure free floating regimes are not observed either. Monetary authorities use to play *leaning against the wind* policies in an attempt to control the exchange rate around some target level, official or unofficially. Thus, it seems that hybrid exchange regimes based on target zones have been widely used on the basis that

it apparently reaps the benefits of both flexible and fixed exchange rates.

Krugman [15] presented the model that has become the standard tool to study target zones regimes. The dynamics of the exchange rate were derived from a linear asset pricing relation and an arbitrage condition given by the uncovered interest rate parity. His model assumed that the central bank intervenes so as to maintain the exchange rate within the band. In this paper, we look at two issues. First, we extend Krugman's model in two directions: (i) we allow the central bank to perform intramarginal interventions, that is, to use its monetary policy to affect the exchange rate before it hits the limits of the band and, (ii) we introduce lack of credibility of the target zone, that is, the perception by market participants of the possibility that the central bank will not defend the band when it has to. Unlike the literature following Krugman's work that has concentrated on one of these extensions at a time, we show that both features are needed in order to reconcile the model with data. As a second issue, we use the model to rationalize the choice of target zones over the rest of regimes: the fixed rate, the free float and the managed float. It is shown that, by imposing a target zone, the monetary authority may gain efficiency through reducing volatility of both the exchange rate and the interest rate simultaneously.

The paper is organized as follows. Section 2 sets up the general model. Subsections 2.2 to 2.4 show how the fixed rate, the managed float and the free float can be seen as extreme cases of a target zone. Section 2.5 develops the target zone solution. Section 3 deals with the two questions outlined in the previous paragraph. The last section concludes.

2. Set up of the Model

2.1. General assumptions

The model represents a highly stylized dynamic stochastic general equilibrium economy. It can be thought of as a reduced form version of more complicated fully optimizing models. Time is discrete. First, consider an equation specifying equilibrium in the money market:

$$m_t - p_t = \varphi y_t - \gamma i_t + \xi_t, \qquad (2.1)$$

where m_t is money supply, p_t is the domestic price level of y_t, a tradable good, i_t is the domestic interest rate of a one period of maturity bond, and ξ_t is some shock to money demand. They are all expressed in logs, with the exception of

the interest rate. Parameters φ and γ are both positive: money demand increases with output because of a transaction motive and there is an implicit liquidity preference behavior, meaning that money can be a substitute for a bond that returns a nominal interest i_t.

Let x_t be the log of the nominal exchange rate, expressing the price of one unit of foreign currency in terms of domestic currency. The (log) real exchange rate is given by

$$q_t = x_t - p_t + p_t^*, \tag{2.2}$$

where p_t^* is the foreign price (variables with star will denote the foreign analogue).

Call $d_t \equiv i_t - i^*$, the interest rate differential, where i^* is the foreign (constant) rate, and assume perfect capital mobility, risk aversion and the uncovered interest rate parity condition (UIP)

$$d_t = E_t \{x_{t+1} - x_t\} + r_t, \tag{2.3}$$

where E_t is the expectation operator conditional on information available at time t. Thus, the expected rate of depreciation must compensate for the interest rate differential plus the foreign premium, r_t. We assume the variable r_t to be exogenous and governed by a first order Markov process

$$r_t = r_{t-1} + \varepsilon_t. \tag{2.4}$$

The white noise $\{\varepsilon_t\}$ is supposed to be Gaussian, $\varepsilon_t \sim N(0, \sigma^2)$, for convenience.[1]

Using (2.2), (2.3) and (2.1) one obtains

$$
\begin{aligned}
x_t &= f_t + \gamma E_t \{x_{t+1} - x_t\}, \tag{2.5}\\
f_t &= m_t + v_t,\\
v_t &= \theta_t + \gamma r_t,\\
\theta_t &= q_t - p_t^* - \varphi y_t + \gamma i^* - \xi_t.
\end{aligned}
$$

[1]The foreign risk premium plays a key role in the current paper. In general, the UIP does not hold (see Ayuso and Restoy [1]). Conventional target zone models consider that deviations from UIP are negligible in target zones (see Svensson [28]). A common practice in some credibility tests of target zones relies in this idea (*v.g.* the simplest test, see Svensson [25], and the drift adjustment method of Bertola and Svensson [5]). However, Bekaert and Gray [3] find that the risk premia in a target zone are sizable and should not be ignored. They argue that this might be the reason of why the credibility tests run on EMS at the beginning of the nineties failed in anticipating the 1992-93 turbulences.

Total fundamentals f_t amount to an endogenous process, m_t, plus an exogenous process, v_t. By iterating forward on x_t we have that

$$x_t = (1 - \nu) \sum_{\tau=0}^{\infty} \nu^{\tau} E_t f_{t+\tau} = (1 - \nu) \sum_{\tau=0}^{\infty} \nu^{\tau} E_t \left\{ m_{t+\tau} + v_{t+\tau} \right\}, \qquad (2.6)$$

with $\nu \equiv \gamma \left(1 + \gamma\right)^{-1}$. The forward looking behavior from UIP implies that the value of asset x_t is the present discounted value of the future stream of fundamentals. Notice that $(1 - \nu) \sum_{\tau=0}^{\infty} \nu^{\tau} = 1$, thus, if $m_{t+\tau}$ were orthogonal to $v_{t+\tau}$ for all t and τ, the long run effect of an increase in m_t is to impulse x_t by an equal amount. We assume that both the central bank and traders can observe the realization of θ_t and r_t.

The monetary authority must choose the path for money $\{m_t\}$ so to maximize its preferences subject to the evolution of $\{v_t\}$ and the exchange rate regime. The objective is to set up these preferences to capture the degree of monetary independence that arises under alternative exchange rate regimes. As in Svensson [29], the concept of monetary independence is associated with the interest rate variability. At one extreme, and in the absence of realignments, a fixed rate eliminates monetary independence. At the other extreme, a managed float regime provides the highest degree of independence. In between, a target zone gives some scope to focus the monetary policy on domestic problems. To capture this idea, the central bank (henceforth, CB) preferences are modeled to evaluate the trade-off between interest rate variability *versus* exchange rate variability

$$J = \frac{1}{2} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left[d_{t+\tau}^2 + \lambda \left(x_{t+\tau} - c_t \right)^2 \right] \right\}. \qquad (2.7)$$

The long run desirable target for the interest rate differential is zero. The target for the exchange rate is the central parity c_t around which deviations are punished by the relative penalty λ. The factor $\beta \in (0, 1)$ is a time discount rate.

We think of a very short maturity term for the bond in the UIP, say a few days, a week or a month at most. The idea is that the CB controls some monetary aggregate $\{m_t\}$ to target $\{d_t, x_t\}$. Output realizations and real fluctuations are observed with some delay, and not available by the time monetary policy is decided so the only available information at any period is $\{\theta_t, r_t\}$. From (2.3) and (2.5) it is easy to show that m_t will respond one to one to the shock θ_t and the problem reduces to deciding how to split the shock r_t between d_t and x_t. Thus, the CB just needs to choose d_t and x_t every period given the value of r_t to minimize

(2.7) subject to (2.3), (2.4) and the exchange regime which restricts the policies available to the CB.

In what follows, any of the exchange rate regimes can be characterized by the triple $\{\lambda, w, \alpha\}$. The first element is related to the preferences. The number w is the band width. The last term α is the probability that the CB defends the currency when it is outside the band and measures the credibility of the exchange regime. This triple will adopt particular values for any of the exchange regimes considered.

2.2. The fixed exchange rate

Consider first a CB that can credibly commit to fix the exchange rate to $c_t = c$. This regime appears as a particular case of the target zone when $w = 0$ and $\alpha = 1$. Given the forward looking restriction of (2.3) and the pure random walk exhibited by r_t, the solution for d_t and x_t is

$$d_t = r_t \text{ and } x_t = c. \tag{2.8}$$

That is, under a fixed exchange rate regime the nominal interest rate absorbs the whole variability of r_t. The expected value of the game under this regime is

$$J^c(r_t) = \frac{1}{2}\left(\frac{1}{1-\beta}\right)\left[r_t^2 + \frac{\beta}{1-\beta}\sigma^2\right]. \tag{2.9}$$

The variability of the interest rate is a source of time inconsistency. If the CB can commit to a rule like (2.8), it must exhibit a stronger response to the risk premium shock in an attempt to content the exchange rate at the central parity. However, there arises the possibility of deviating from this simple rule in order to reduce interest rate variability. If this temptation is captured by market participants, the simple rule will no longer be credible. Hence, (2.8) would be time inconsistent.

2.3. The managed float

This regime is a particular case of the target zone when $\{\lambda > 0, w \to \infty, \alpha = 1\}$. The solution is related to a managed or dirty floating rate where the CB exploits the trade-off between the variability of the exchange rate and the interest rate every period. After the shock r_t is realized and agents have formed their expectations on future values for the exchange rate, the CB must set the two target

variables $\{d_{t+\tau}, x_{t+\tau}\}_{\tau=0}^{\infty}$ with one instrument $\{m_{t+\tau}\}_{\tau=0}^{\infty}$, to minimize the loss (2.7) subject to the arbitrage condition (2.3) and the relation in (2.4). Notice the exchange regime does not impose additional restrictions.

The optimality condition is given by the difference equation

$$x_t = c + \frac{r_t}{\lambda} + \frac{1}{\lambda} E_t \{x_{t+1} - x_t\}. \tag{2.10}$$

The central parity is constant here since, with an infinitely-wide band, there will be no realignments. Forward iteration yields

$$x_t = c + \frac{1}{1+\lambda} E_t \sum_{\tau=0}^{\infty} \frac{r_{t+\tau}}{(1+\lambda)^{\tau}}. \tag{2.11}$$

It is useful to compare expression (2.11) to (2.6): the optimal exchange rate is equal to the central parity plus the present discounted value of the future stream of foreign risk premia which is the only fundamental shock that affects the exchange rate. From (2.4), a linear closed form solution of (2.11) is

$$x_t = c + \frac{r_t}{\lambda}. \tag{2.12}$$

The discretionary rate is then equal to the fixed rate plus a depreciation bias. Forming a one-period ahead expectation yields

$$E_t x_{t+1} = c + \frac{r_t}{\lambda} = x_t, \tag{2.13}$$

so, from (2.3), the target variable d_t is

$$d_t = r_t. \tag{2.14}$$

A shock to the risk premium impulses exchange and interest rates in the same direction. Combining (2.12) and (2.14), a linear control condition is given by

$$d_t = \lambda (x_t - c), \tag{2.15}$$

meaning that the interest rate differential is proportional to the depreciation (or appreciation) bias with respect to the central parity. This is just a *leaning against the wind* policy, where the interest rate differential is positive when the currency is above the target and negative below. Under the uncovered interest rate parity,

conditional on x_t, the risk premium has a positive relation with d_t. Similarly, conditional on d_t, r_t has also a positive relation with x_t. Therefore, a central bank that is concerned about the volatility of interest rate differentials and of exchange rates will optimally split the variation of r_t between d_t and x_t. This results in a regime of intra-marginal interventions or managed float. The higher the value of λ, the smaller the variance for the exchange rate is.

The explicit form for the expected value of the game under the discretionary managed float is calculated as

$$J^{mf}(r_t) = \frac{1+\lambda}{2\lambda}\left(\frac{1}{1-\beta}\right)\left[r_t^2 + \frac{\beta}{1-\beta}\sigma^2\right]. \tag{2.16}$$

It is straightforward to see that the previous fixed rate regime is a first best. There is no trade-off between the exchange rate and the interest rate. The attempt of the CB in giving the interest rate a lower volatility by trading with the exchange rate variance is a vain effort. It results in a discretionary time consistent solution, where the volatility of the interest rate is unaffected and the exchange rate begins to float. The CB is worse off.

Finally, once the target variables have been determined, equation (2.5) gives us the optimal level for the monetary instrument. From (2.5), the CB will expand or absorb m_t, according to

$$m_t = c + (1+\gamma\lambda)\frac{r_t}{\lambda} - \theta_t . \tag{2.17}$$

2.4. The pure free float

Consider now the case where the CB announces that the risk premium shocks will not be dampened over the exchange rate, whatever the preference parameter λ is. That means

$$x_t = c + r_t. \tag{2.18}$$

This is a *laissez faire* solution. The slope of the exchange rate function with respect to the fundamental is one. According to the UIP (2.3), the interest rate differential is again given by

$$d_t = x_t - c = r_t.$$

Is it a time consistent result? As with the fixed rate, the CB is not deciding its monetary policy rule to optimize (2.7) and this may be a source of time inconsistency. The result in this regime, depends on the value of λ, though. For

the particular case $\lambda = 1$, free and managed float solutions are identical and consistency over time is therefore preserved. However, for values of $\lambda \neq 1$, the market will perceive that CB's incentives to trade volatility of the exchange rate for volatility of the interest rate are different than 1 and the announcement of the free float will be no longer credible. Notice, that the relevant regime to be compared with the target zone is the managed float.

For the sake of completeness, we can compute the indirect value function for the free float:

$$J^{ff}(r_t) = \frac{1}{2} \left(\frac{1+\lambda}{1-\beta} \right) \left[r_t^2 + \frac{\beta\sigma^2}{1-\beta} \right]. \tag{2.19}$$

The relation between the values for the fixed, the managed and the free float rates, can be written as

$$J^{ff} = \lambda J^{mf} = (1 + \lambda) J^c,$$

for any r_t, see expressions (2.9), (2.16) and (2.19), respectively. Differences among regimes will have to be found in the exchange rate dynamics since they imply the same process for the interest rate differential. Clearly, the *fixed rate* is preferred over the two other regimes, although it is time inconsistent. The ordering of the losses for the *free float* and the *managed float* will depend upon the value of λ with respect to 1. If $\lambda < 1$, the CB is better off with the free float. Otherwise, for $\lambda \geq 1$, managed float is a better regime.

2.5. The target zone

In this case, the regime is characterized by $\{\lambda \geq 0, w \geq 0, \alpha \in [0,1]\}$. The first element is related to the preferences in (2.7). The number w is the band width. The last term α is the probability that the CB defends the currency when it is outside the band and measures the credibility of the exchange regime. We suppose that, within the bands, the CB defends the currency with probability 1. As stated in the introduction, the purpose is to construct a model able to generate a consistent target zone solution, in the sense that the CB finds it optimal to defend the target zone at the margins by probability α. There are not further incentives to renege from the target zone ex-post.

From the previous discussion, both the exchange rate and the interest rate differential will be functions of the shock r_t. So, the timing of events at any time t will be as follows:

1. The shock r_t is realized.

2. If the realization of the shock makes the exchange rate be within the target zone, the CB solves a minimization problem. The process is as follows:

   - The central parity from last period is left unaltered: $c_t = c_{t-1}$.
   - Forward looking agents form expectations $E_t x_{t+1}$.
   - For given $\{E_t x_{t+1}, r_t, c_t\}$, the CB chooses $\{x_t, d_t\}$ by minimizing the loss (2.7) subject to the arbitrage condition (2.3), the relation in (2.4) and the additional restriction imposed by the target zone system.

3. If for that value of the shock the exchange rate should be outside a band $[c_t - w, c_t + w]$, for instance, say it is above the upper limit $c_t + w$, the CB can do any of two actions:

   - It defends the currency with probability α. This means that the exchange rate is pegged at the edge of the band, $x_t = c_t + w$, and the central parity is not altered, $c_t = c_{t-1}$.
   - It realigns the currency with probability $(1 - \alpha)$. In this case, the central bank devalues the central parity by $\mu \geq w$, i.e. $c_t = c_{t-1} + \mu$, and situates the exchange rate at $x_t = c_t$.

4. Finally, for a given shock θ_t in the money demand equation (2.5), the CB supplies the optimal quantity of money m_t supporting the pair $\{x_t, d_t\}$.

To understand the way this economy works consider the following. Assume the risk premium starts from $r_0 = 0$. The CB can set the exchange rate at $x_0 = 0$ and a target zone of width w around $c_0 = 0$. Because of the symmetry of the forcing process, the interest rate differential is $d_0 = 0$, which equals the target value for that variable. As time progresses, the risk premium wanders around and the exchange rate moves away from its center $c_0 = 0$. Given that the exchange rate is a function of the shock r_t, the foreign risk premium must also be fluctuating within a symmetric zone with center $\rho_0 = 0$. This zone is denoted as $[\rho_0 - \bar{r}, \rho_0 + \bar{r}]$. How far the exchange rate wanders from the center of its band should be a function of the distance between the risk premium and ρ_0. So, for the periods before the first realignment, we could write

$$x_t = c_0 + u(r_t - \rho_0),$$

where $u(r_t - \rho_0)$ is the function linking the exchange rate to the fundamental process r_t *within the band.*

Imagine that at time $t = \tau$, one of the limits of the exchange rate band is reached. This means that $r_\tau = \rho_0 \pm \overline{r}$ two level conditions must be satisfied

$$u\left(-\overline{r}\right) = -w, \qquad (2.20)$$

and

$$u\left(\overline{r}\right) = w. \qquad (2.21)$$

When exchange rate is at the boundaries of the target zone, the CB could keep x_t at the edge of its band. In such a case, all movements in r_t will be transferred to the interest rate differential. The other possibility implies the central bank realigning the target zone. In such a case, the exchange rate jumps to $c_\tau = c_{\tau-1} + \mu$, and the center of the band for the risk premium moves to $\rho_\tau = r_\tau$. After the realignment takes place, the behavior of the exchange rate band within the target zone is again governed by the function u so we can write in general

$$x_t = c_t + u(r_t - \rho_t).$$

Let $x(r_t)$ be the function relating the exchange rate to the risk premium at any time, that is, within as well as outside the target zone. This function satisfies

$$x\left(r_t\right) = c_t \begin{cases} +\mu & \text{if } r_t > \rho_t + \overline{r} \text{ with probability } 1 - \alpha \\ +w & \text{if } r_t > \rho_t + \overline{r} \text{ with probability } \alpha \\ +u\left(r_t - \rho_t\right) & \text{if } r_t \in [\rho_t - \overline{r}, \rho_t + \overline{r}] \\ -w & \text{if } r_t < \rho_t - \overline{r} \text{ with probability } \alpha \\ -\mu & \text{if } r_t < \rho_t - \overline{r} \text{ with probability } 1 - \alpha. \end{cases} \qquad (2.22)$$

The unknown continuous function $u\left(r_t - \rho_t\right)$ represents the CB's best response when the risk premium lies within its band $[\rho_t - \overline{r}, \rho_t + \overline{r}]$. To find it, we use the first order condition for values of $r_t \in [\rho_t - \overline{r}, \rho_t + \overline{r}]$,

$$\left(1 + \lambda\right)\left[u\left(r_t - \rho_t\right) - c_t\right] = r_t + E_t\left[x\left(r_{t+1}\right)\right]. \qquad (2.23)$$

Outside of the band, condition (2.23) does no longer hold and the trade-off is not optimal. In order to find out the particular form of the function $u\left(r_t - \rho_t\right)$, the band for the fundamental $[-\overline{r}, \overline{r}]$ must be computed, given the level conditions (2.20) and (2.21). Forward recursion is used to solve for $u\left(r_t\right)$ with the first

order condition (2.23) subject to (2.22). Appendix A provides all the details for computation of the solution.

From condition (2.23) and (2.3), it is straightforward to see that the model produces a positive relation between the exchange rate of depreciation within the band and the interest rate differential:

$$d_t = \lambda \left[u \left(r_t - \rho_t \right) - c_t \right] + \rho_t, \tag{2.24}$$

outside the bands, the trade-off is not optimal and expression (2.24) does not hold.

3. Results

Before we start with the computations, let us first remind the reader about the implications derived from the main features of Krugman's model: the *honey moon* and the *smooth pasting* effects (see Svensson [27] and Krugman and Miller [16]). The first means that the response of the exchange rate to changes in fundamentals is smaller than the response under the free floating regime. That is, as compared to a free floating, the band works as an exchange rate stabilizer. The smooth pasting implies that the response of the exchange rate tends to zero as it approaches the edges of the band. This is because agents are forward looking and anticipate the intervention by the central bank when the exchange rate gets close to the limit of the band.

Svensson [27] has summarized the testable implications of Krugman's model. First, the distributions of both the exchange rate and the interest rate are U-shaped. That is, the exchange rate tends to live close to the limits of the band. Second, there is a negative and deterministic relation between the exchange rate and the interest rate differential. This relation is given by the uncovered interest rate parity. Finally, the exchange rate exhibits a non-linearity in its univariate forecasting equation, which is a consequence of the smooth pasting effect. However, empirical tests have challenged these predictions. The distribution of the exchange rate has been observed to be *hump*-shaped rather than U-shaped, so the exchange rate accumulates probability around the center of the band. Secondly, the relationship between the exchange rate and the interest rate is positive rather than negative. Finally, it seems that the effects of the smooth pasting condition are not as relevant as predicted by theory and exchange rates are linear functions of the fundamentals.

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In what follows we calibrate our model and compare its predictions with the data. Furthermore, we compute the losses (2.7) under different regimes and provide a rationale as of why target zones may be widely used as an exchange rate system.

3.1. Parameters

The following values for the parameters are used in the numerical exercises of this section. We think of a week as the time frequency. As in Svensson [29], the time discount factor is set to $\beta = 0.90^{\frac{1}{52}}$. We use this value to ease the calculations but the main results of the paper do not hinge on it.

The selection of a standard deviation for the shock to the risk premium, σ, is a troublesome task. Bekaert [2] reports an unconditional variance for a time invariant risk premium of 10.622^2, for the Dollar/Yen rate. This figure yields a weekly standard deviation of about $\sigma = 0.002$ basis points per week. Unfortunately, we have not found an estimation of a time varying risk premium governed by a random walk structure, as assumed throughout this paper. As an approximation, we have used the ARCH-in-mean estimation by Domowitz and Hakkio [7], for monthly observations 1973:6-1982:8, of the British Pound, the French Franc, the Deustche Mark, the Japanese Yen and the Swiss Franc, all against the US Dollar. A Montecarlo simulation has been run in order to calculate this moment. Table 1 reports the standard deviations for a first difference of r_t. The Swiss Franc presents a time invariant risk premium. Hence, it seems reasonable to assume an a priori value $\sigma = 0.002$, as in Bekaert [2].

Two bands are chosen, $w = \pm 2.25\%$ and $\pm 6.00\%$, the widths experienced in the ERM. The value of the constant μ is estimated depending on the band width w. Tables 2 collects data of realignment rates for the currencies participating in the ERM of the EMS, except for the Dutch Guilder. From this table it seems that realignment rates of $\pm 4.5\%$ and $\pm 6.3\%$ for widths of ± 2.25 and $\pm 6\%$, respectively, are consistent with the EMS history

Since there is no prior for the relative weight of the exchange rate variability in the preferences of the CB, we use a wide range of values within which the parameter λ is thought to be about. These values are $\lambda = 0.2, 0.5, 1.0, 2.0$ and 5.0.

Tables 1 and 2 about here

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3.2. Nonlinearities in the exchange rate function

Let us start examining the degree of nonlinearities in the exchange rate function predicted by our model.[2] Figures 1 to 3 plot the function $u(r_t)$ for probabilities of defense $\alpha = 0$, $\frac{1}{2}$, and 1, together with the managed float and the free float. The three figures differ on the value of λ which are 0.5, 1, and 2, respectively. The 45 degree line corresponds to the free float. The other linear solution is the managed float. We can see that the relative slopes of these two solutions depend on the value of λ. Among the nonlinear solutions, the flattest function corresponds to $\alpha = 1$, the target zone regime under perfect credibility. On the other extreme, the most sloping curve corresponds to the case where the currency is realigned for sure at margins, that is, $\alpha = 0$. We can see that the degree of nonlinearity of the target zone solutions to be very small. Furthermore, as the value of λ increases, the target zone solutions get closer to the managed float which is the relevant regime to be compared with. This later result is independent of the credibility of the target zone.

We also observe that the slope of the function $u(r_t)$ changes with r and this change depends on the value of α. To make this result clearer, Figure 4 plots $u'(r_t)$ for $\lambda = 1$ and for values of α equal to 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1. First, we see that for α smaller than $\frac{3}{4}$, the slope tends to increase as the exchange rate approaches the limits of its band and we obtain the inverted S-curve as in Bertola and Caballero [4]. The opposite occurs when α is above $\frac{3}{4}$. However, even in the fully credible case, the slope does not reach zero at the edges, that is, there is no smooth pasting condition. Violation of smooth pasting in the standard model implied a discontinuity jump in the expected rate of depreciation function. In our model, continuity of functions $u(r_t)$ and $E_t x_{t+1}$ is preserved under the level condition $u(\overline{r}) = w$, and the former arbitrage arguments do not apply here (see Appendix

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[2]The empirical literature finds that the effects of the non-linearities from the smooth pasting condition are not as much important as predicted. See, for example, Meese and Rose [21] for three alternative regimes, the $\pm 1\%$ band of the Bretton Woods system, the gold standard for the British Pound, the French Franc and the Deustche Mark (versus the US Dollar), and third, the EMS regime for the Dutch Guilder and the French Franc cases (versus the Deustche Mark). The null hypothesis of non linearities is rejected for the three regimes. Lewis [18] runs a variety of tests for the G-3 case (US, Germany and Japan) in order to check for possible non linearities arising from *implicit* target bands and from the intervention policy. In all the cases, the exchange rate seemed to be a linear function of the estimated fundamentals. Evidence on rejection of this hypothesis is also reported in Lindberg and Söderlind [19] for Swedish data. Flood, Rose and Mathieson [9] do not either find relevant evidence of the smooth pasting for three alternative methods (graphical examination, parametric tests and forecasting analysis).

B for a detailed explanation justifying this non smooth pasting result).

Figures 1, 2, 3, 4 about here

3.3. U-shaped *versus* hump-shaped distributions

In the standard model, the U-shaped distribution is implied by the perfect credibility assumption. Since the exchange rate function flattens near the edges, a considerable mass of probability will be concentrated at the margins. However, the empirical literature has pointed out that the distribution of the exchange rate is *hump*-shaped rather than U-shaped.[3]

Figures 5, 6 and 7 plot the ergodic probabilities of different depreciation rates within the band for $\lambda = 2$ and for three different values of α: 1, 0.8 and 0. As suggested before, figure 5 shows that for the particular case of $\alpha = 1$, the exchange rate distribution displays a U-shaped distribution. On the contrary, figures 6 and 7 show that for α different than 1, the distribution is hump-shaped. This occurs even for values of α close to 1. The explanation is twofold. The first reason is related to the value of α. For low values of α, continuous realignments move the exchange rate to new central parities, that is, to new centers of new bands. Additionally, lack of credibility implies that the slope of exchange rate function increases at margins, as we have shown in the previous subsection. Thus, stability is higher around the centers. This helps to accumulate probability mass at the interior of the bands. The second reason has to do with the value of λ. If this parameter is high, the central bank will have incentives to further stabilize the exchange rate.

Figures 5, 6, 7 here

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[3]See, for example, Bertola and Caballero [4] for the French Franc against the Deustche Mark exchange rate case during 1979-87; and Lindberg and Söderlind [19], [20], for the Swedish unilateral target zone with a vast set of daily data covering from 1982 to 90. The work of Flood, Rose and Mathieson [9] also finds *hump*-shaped histograms for EMS exchange rates. They use daily data for eight EMS currencies and the British Pound (during its pre EMS membership) over 1979-90, weekly data from the classical gold standard (UK, US, France and Germany), and monthly data from the Bretton Woods regime.

3.4. The relation between exchange rates and the interest rate differentials

Empirical observations have shown the existence of a positive relationship between the exchange rate and the interest rate differential.[4] Bertola and Svensson [5] suggest that incorporating a realignment risk premium may alter the sign of the covariance between these two variables from negative to positive.

In our model, this relation has always a positive sign, regardless the value of the parameters. This is a consequence of intramarginal interventions, summarized by the first order condition (2.15). Both the interest rate and the exchange rate are driven by the same variable, namely, the risk premium. The central bank decides at each period how to distribute the shock on r_t between x_t and d_t. Expression (2.15) tells us that it is always optimal to move both variables on the same direction with λ being the ratio between the two. This gives rise to a positive linear relationship between the two variables. Hence, the honey moon effect is also transmitted to the interest rate smoothing.

3.5. On the choice of a target zone regime

In this subsection, we estimate the loss functions for the four regimes using the set of parameters early stated. Losses for the fixed rate, the managed float and the free float are given by expressions (2.9), (2.16) and (2.19). The target zone loss is numerically computed. We consider a starting point $r_0 = 0$, and a time horizon of 2000 periods. This time length accumulates a 91.2% from the total accruing value and does not alter the ordinality of values for the exchange regimes. Calculations of the indirect costs are reported in table 3 for $w = \pm 2.25\%$ and table 4 for $w = \pm 6\%$. In each table, the first row refers to the cost of the *fixed rate regime*, J^c, and the following five rows represent the costs of a *target zone* for $\alpha = 0$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1, respectively. The seventh and eighth rows collect the *managed float* costs, J^{mf}, and the *free float* costs, J^{ff}. From these tables we highlight the following results.

First, as it was mentioned above, the fixed rate is the regime with the lowest value for the loss function.

Second, with respect to the target zone regime, the closer α is to 1, the smaller the losses are. This represents the gain from credibility of commitment to a zone. The slopes of both the exchange rate function and the interest rate differential

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[4]See Svensson [26], Flood, Rose and Mathieson [9], Lindberg and Söderlind [19], and Bertola and Caballero [4].

flatten for α approaching to 1. When market traders perceive that the CB will defend the zone with a high probability, the realignment risk will be low and the exchange rate responses to changes in the foreign risk premium will be soft. In turn, this perception will contribute to smooth the expected rate of depreciation. Hence, the outcome is that the interest rate differential will be more stable as well.

Third, for target zones very close to being credible (α close to 1), the present value of the costs of a target zone are smaller than the costs of a managed float. Notice that, unlike the previous literature that used the free floating as an alternative, the right regime to be compared with is the managed float. As with the second result, forward looking agents help stabilize the rate without pressuring the interest rate, due to the honey moon effect.

To make this point clearer, figures 8, 9 and 10 represent the contributions to the loss functions due to the variability of the exchange rate and the interest rate for all the regimes and values of λ equal to 0.5, 1, and 2, respectively. The circle corresponds to the fixed rate, the square to the managed float and the triangles are the target zones for different values of α. It is clear that there must be an interval for α close to 1 that reduces the volatility of both the exchange rate and the interest rate with respect to the managed float.

Table 4 and 5 here
Figures 8, 9 and 10 here

4. Conclusions

This paper presents a target zone model based on two extensions of Krugman's [15] model. These extensions are the introduction of intramarginal interventions and the lack of credibility of the target zone. These two features had been analyzed separately by the literature. Here, we show that both extensions are needed to reconcile the model with the data. The leaning against the wind policy derived from the time-consistent intramarginal intervention produces a positive relation between exchange rates and interest rates differentials as we observe in the data as well as is behind the almost linear relation between exchange rates and fundamentals. Furthermore, lack of credibility is responsible for the hump-shape distribution of exchange rates.

As a second application, the model provides a framework in which to evaluate why target zones seem to be the dominant exchange rate regime in contemporary

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history. First, we argue that target zones should be compared with the managed float which is the other time consistent exchange arrangement. For this comparison, we show that even not perfectly credible target zones may improve with respect to the managed float in terms of reducing simultaneously the volatility of both exchange rates and interest rates differentials. In this way, we operationalize the common view that target zones are preferred because they reap the benefits of both flexible and fixed exchange rates, by stabilizing the exchange rate without loosing monetary independence. These two benefits are explicitly included in the model and are a result of the interaction between the preferences of the central bank and the credibility of its monetary policy.

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A. Solution of the Target Zone

In this appendix we show how to make the computations for the solution of the target zone regime. Forward recursion in the first order condition (2.23) is used to solve for $u(r_t)$ subject to (2.22). Let the expected rate of depreciation out of the band be given by

$$
\begin{aligned}
\delta_{\tau,\tau-1} &= \alpha w \left[\Pr\left(r_{t+\tau} > \overline{r} | r_{t+\tau-1} \right) - \Pr\left(r_{t+\tau} < -\overline{r} | r_{t+\tau-1} \right) \right] \\
&+ (1-\alpha) \int_{-\infty}^{-\overline{r}} \mu\left(r_{t+\tau} \right) \phi\left(r_{r+\tau} | r_{t+\tau-1} \right) dr_{t+\tau} \\
&+ (1-\alpha) \int_{\overline{r}}^{\infty} \mu\left(r_{t+\tau} \right) \phi\left(r_{r+\tau} | r_{t+\tau-1} \right) dr_{t+\tau}.
\end{aligned}
$$

Forward iteration of (2.23) leads to the following general solution

$$
u(r_t) = \frac{r_t}{1+\lambda} + \frac{1}{1+\lambda} \sum_{\tau=1}^{\infty} \frac{F\left[r_{t+\tau} | \mathcal{F}_\tau \right]}{(1+\lambda)^{-\tau}} + \sum_{\tau=1}^{\infty} \frac{F\left[\delta_{\tau,\tau-1} | \mathcal{F}_{\tau-1} \right]}{(1+\lambda)^{-\tau}}, \tag{A.1}
$$

where the sequence $\mathcal{F}_{\tau\geq 0}$ represents filtered information sets of the next form:

$$
\begin{aligned}
\mathcal{F}_0 &= \{r_t\}, \tag{A.2} \\
\mathcal{F}_{\tau\geq 1} &= \left\{ \{r_{t+n} \in [-\overline{r}, \overline{r}]\}_{n=1}^{\tau}, r_t \right\}. \tag{A.3}
\end{aligned}
$$

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On the other hand, the components in (A.1) are given by:

$$F\left[\delta_{1,0} \,|\mathcal{F}_0\right] = \alpha w \left[\Pr\left(r_{t+1} > \overline{r}|r_t\right) - \Pr\left(r_{t+1} < -\overline{r}|r_t\right)\right] \qquad (A.4)$$

$$+ (1-\alpha) \int_{-\infty}^{-\overline{r}} \mu\left(r_{t+\tau}\right) \phi\left(r_{t+1}|r_t\right) dr_{t+1}$$

$$+ (1-\alpha) \int_{\overline{r}}^{\infty} \mu\left(r_{t+\tau}\right) \phi\left(r_{t+1}|r_t\right) dr_{t+1}$$

$$F\left[r_{t+1} \,|\mathcal{F}_1\right] = \int_{-\overline{r}}^{\overline{r}} r_{t+1}\phi\left(r_{t+1}|r_t\right) dr_{t+1}, \qquad (A.5)$$

for $\tau = 1$, and

$$F\left[\delta_{\tau,\tau-1} \,|\mathcal{F}_{\tau-1}\right] = \int_{-\overline{r}}^{\overline{r}} ... \int_{-\overline{r}}^{\overline{r}} \delta_{\tau,\tau-1}\phi\left(r_{t+\tau-1}, ..., r_{t+1}|r_t\right) dr_{t+\tau-1}...dr_{t+1}, \qquad (A.6)$$

$$F\left[r_{t+\tau} \,|\mathcal{F}_{\tau}\right] = \int_{-\overline{r}}^{\overline{r}} ... \int_{-\overline{r}}^{\overline{r}} r_{t+\tau}\phi\left(r_{t+\tau}, ..., r_{t+1}|r_t\right) dr_{t+\tau}...dr_{t+1}, \qquad (A.7)$$

for $\tau = 2, 3, ...$, where

$$\phi\left(r_{t+\tau}, ..., r_{t+1}|r_t\right) = \left(2\pi\sigma^2\right)^{-\tau/2} \exp\left[\frac{-1}{2\sigma^2} \sum_{n=1}^{\tau} \left(r_{t+n} - r_{t+n-1}\right)^2\right]. \qquad (A.8)$$

This general solution is consistent with (2.23) and (2.22). In order to determine a particular solution, it is necessary to identify the value of $\overline{r}$ for which $u\left(\overline{r}\right) = w$.

A.1. A numerical approximation

The general solution (A.1) involves a collection of integrals where only (A.4) and (A.5) enjoy an explicit form. Numerical solutions are requested for the remaining ones. Here, we propose a method that discretizes variable r_t on K values ($K \geq 3$ odd) within the interval $\left[-\overline{r}, \overline{r}\right]$ as

$$r = r_1, r_2, ..., r_K, \qquad (A.9)$$

$$r_1 = -\overline{r},$$

$$r_k = r_{k-1} + h, \qquad (A.10)$$

$$h = 2\overline{r}/\left(K-1\right) > 0, \qquad (A.11)$$

$$r_K = \overline{r},$$

$$r_{\frac{K+1}{2}} = 0,$$

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Let P, Q, R, S and T be row vectors $(1 \times K)$, adopting the following form

$$
\begin{aligned}
P_1 &= \Phi\left(\frac{-\bar{r}+h/2-r_t}{\sigma}\right) - \Phi\left(\frac{-\bar{r}-r_t}{\sigma}\right) \qquad \text{(A.12)}\\
P_k &= \Phi\left(\frac{r_k+h/2-r_t}{\sigma}\right) - \Phi\left(\frac{r_k-h/2-r_t}{\sigma}\right)\\
P_K &= \Phi\left(\frac{\bar{r}-r_t}{\sigma}\right) - \Phi\left(\frac{\bar{r}-h/2-r_t}{\sigma}\right)
\end{aligned}
$$

with r_t given and

$$
\begin{aligned}
Q_k &= 1 - \Phi\left(\frac{\bar{r}-r_k}{\sigma}\right) - \Phi\left(\frac{-\bar{r}-r_k}{\sigma}\right), \qquad\text{(A.13)}\\
R_k &= \left[\Phi\left(\frac{\bar{r}-r_k}{\sigma}\right) - \Phi\left(\frac{-\bar{r}-r_k}{\sigma}\right)\right] r_k + \sigma\left[\phi\left(\frac{-\bar{r}-r_k}{\sigma}\right) - \phi\left(\frac{\bar{r}-r_k}{\sigma}\right)\right] \quad\text{(A.14)}\\
S_k &= \int_{\bar{r}}^{\infty} \mu(s)\,\phi(s|r_k)\,ds + \int_{-\infty}^{-\bar{r}} \mu(s)\,\phi(s|r_k)\,ds, \qquad\text{(A.15)}
\end{aligned}
$$

for $k = 1, 2, ..., K$, and

$$
T = \alpha w Q + (1-\alpha) S, \qquad\text{(A.16)}
$$

where ϕ and Φ represent, respectively, the Gaussian pdf and cdf.

For the *first* period ahead, and only for this period, integrals (A.4) and (A.5) have explicit form:

$$
\begin{aligned}
E\left[\delta_{1,0}\,|\mathcal{F}_0\right] &= \alpha w\left[1 - \Phi\left(\frac{\bar{r}-r_t}{\sigma}\right) - \Phi\left(\frac{-\bar{r}-r_t}{\sigma}\right)\right]\\
&\quad + (1-\alpha)\int_{\bar{r}}^{\infty} \mu(r_{t+1})\,\phi(r_{t+1}|r_t)\,dr_{t+1}\\
&\quad + (1-\alpha)\int_{-\infty}^{-\bar{r}} \mu(r_{t+1})\,\phi(r_{t+1}|r_t)\,dr_{t+1},
\end{aligned}
$$

$$
\begin{aligned}
E\left[r_{t+1}\,|\mathcal{F}_1\right] &= \left[\Phi\left(\frac{\bar{r}-r_t}{\sigma}\right) - \Phi\left(\frac{-\bar{r}-r_t}{\sigma}\right)\right] r_t\\
&\quad + \sigma\left[\phi\left(\frac{-\bar{r}-r_t}{\sigma}\right) - \phi\left(\frac{\bar{r}-r_t}{\sigma}\right)\right].
\end{aligned}
$$

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For the *second* period ahead, a numerical approximation is given by

$$E\left[\delta_{2,1}\left|\mathcal{F}_1\right.\right] = TP',$$
$$E\left[r_{t+2}\left|\mathcal{F}_2\right.\right] = RP'.$$

This implies that the value of the integrals at $t+2$ is determined for any possible mean at $t+1$, $r_{t+1} \in [-\overline{r}, \overline{r}]$, and any of these means is weighted by a probability P, given a starting value r_t at t.

The loop becomes harder as the period ahead increases over three. In order to solve this problem, we develope a backward recursion algorithm, for which the last period integral is firstly solved and then proceed backward up to the first one. Thereby, consider the following *non negative* matrix $M \in \mathbb{R}^{K \times K}$

$$M_{1,l} = \Phi\left(\frac{-\overline{r} + h/2 - r_l}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_l}{\sigma}\right),$$

$$M_{k,l} = \Phi\left(\frac{r_k + h/2 - r_l}{\sigma}\right) - \Phi\left(\frac{r_k - h/2 - r_l}{\sigma}\right),$$

$$M_{K,l} = \Phi\left(\frac{\overline{r} - r_l}{\sigma}\right) - \Phi\left(\frac{\overline{r} - h/2 - r_l}{\sigma}\right),$$

for $k, l = 1, 2, ...K$, with the following properties: the sum over each column gives a row vector $m_c \in \mathbb{R}^{1 \times K}$, with all its components lying within the $(0, 1)$ interval

$$m_c(l) = \sum_{k=1}^{K} M_{kl} = \int_{-\overline{r}}^{\overline{r}} \phi(s|r_l)\, ds \in (0, 1) \tag{A.17}$$

for $l = 1, 2, ..., K$. The proof follows. This property is sufficient to verify the Hawkins-Simon condition (Brauer-Solow Theorem).

Matrix M contains the transition probabilities in the intermediate periods from τ up to $\tau + 1$, within the interval $[-\overline{r}, \overline{r}]$ and for any mean belonging to $[-\overline{r}, \overline{r}]$. Thus, the solution for the third period is given by

$$E\left[\delta_{3,2}\left|\mathcal{F}_2\right.\right] = TMP',$$
$$E\left[r_{t+3}\left|\mathcal{F}_3\right.\right] = RMP'.$$

Again, the value of the integrals are first determined at $t + 3$ for any possible mean at $t + 2$, $r_{t+2} \in [-\overline{r}, \overline{r}]$, given by the columns of M. The vector P closes the calculation for any possible mean at $t + 1$, $r_{t+1} \in [-\overline{r}, \overline{r}]$, for given a starting value r_t.

For $\tau = 2, 3, ...$, further generalization gives a sequence:

$$
\begin{aligned}
E\left[\delta_{\tau,\tau-1} \,|\mathcal{F}_{\tau-1}\right] &= TM^{\tau-2}P', \\
E\left[r_{t+\tau} \,|\mathcal{F}_\tau\right] &= RM^{\tau-2}P'.
\end{aligned}
$$

Plugging these values into (A.1), one obtains

$$
\begin{aligned}
u\left(r_t\right) \;=\; & \frac{r_t}{1+\lambda} + \frac{E\left[r_{t+1} \,|\mathcal{F}_1\right]}{\left(1+\lambda\right)^2} + \frac{E\left[\delta_{1,0} \,|\mathcal{F}_0\right]}{1+\lambda} \\
& + \frac{1}{1+\lambda} \sum_{\tau=2}^{\infty} \frac{RM^{\tau-2}P'}{\left(1+\lambda\right)^\tau} + \sum_{\tau=2}^{\infty} \frac{TM^{\tau-2}P'}{\left(1+\lambda\right)^\tau}.
\end{aligned}
$$
(A.18)

Let the vector of eigenvalues be given by $\eta\left(M\right)$, and call $\eta^*\left(M\right)$ the Frobenius root, *i.e.* the maximum eigenvalue. From (A.17) we know that $m_c\left(l\right) < 1$, this is a sufficient condition to verify the Hawkins-Simon condition (see Brauer-Solow Theorem). In turn, verification of the Hawkins-Simon condition implies that $\eta^*\left(M\right) < 1$, (see Hawkins and Simon [14] and [13]). Then, matrix $\left(I - M\right)^{-1}$ exists, it is non negative and can be written as

$$
\left(I - M\right)^{-1} = \sum_{j=0}^{\infty} M^j
$$

This gives rise to a convenient simplification

$$
\overline{M} = \sum_{\tau=2}^{\infty} \left(1+\lambda\right)^{-\tau} M^{\tau-2} = \frac{1}{1+\lambda} \left[\left(1+\lambda\right) I - M\right]^{-1}.
$$

The general solution becomes

$$
u\left(r_t\right) = \frac{\Omega\left(r_t\right)}{1+\lambda} + \Delta\left(r_t\right),
$$
(A.19)

with

$$
\Omega\left(r_t\right) \equiv r_t + \frac{E\left[r_{t+1} \,|\mathcal{F}_1\right]}{1+\lambda} + R\overline{M}P',
$$

$$
\Delta\left(r_t\right) \equiv \frac{E\left[\delta_{1,0} \,|\mathcal{F}_0\right]}{1+\lambda} + T\overline{M}P'.
$$

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Application of level conditions (2.20) and (2.21) gives the particular solution

$$\frac{\Omega(\overline{r})}{1+\lambda} + \Delta(\overline{r}) = w.$$

Finally, the exchange rate expectation (B.1) is

$$E_t x_{t+1} = \sum_{\tau=1}^{\infty} \frac{F\left[r_{t+\tau}|\mathcal{F}_\tau\right]}{(1+\lambda)^\tau} + (1+\lambda)\sum_{\tau=1}^{\infty}\frac{F\left[\delta_{\tau,\tau-1}|\mathcal{F}_{\tau-1}\right]}{(1+\lambda)^\tau},$$

or using the previous approximation

$$E_t x_{t+1} = \Omega(r_t) + (1+\lambda)\Delta(r_t) - r_t. \tag{A.20}$$

Once we know the expression for the exchange rate and the expectation, the interest rates differential is obtained from the uncovered interest parity condition as

$$d_t = \Omega(r_t) + (1+\lambda)\Delta(r_t) - x_t \tag{A.21}$$

B. Smooth Pasting Revised

In this appendix we show how the smooth pasting condition does not have to hold in this model. Consider a center $\rho_t = 0$ and parity $c = 0$. First of all, by use of (2.22) one can compute the one period ahead exchange rate expectation as

$$\begin{aligned}
E_t x_{t+1} &= \alpha w\left[\Pr\left(r_{t+1} > \overline{r}|r_t\right) - \Pr\left(r_{t+1} < -\overline{r}|r_t\right)\right] \tag{B.1}\\
&\quad - (1-\alpha)\int_{-\infty}^{-\overline{r}} \mu\phi\left(r_{t+1}|r_t\right) dr_{t+1}\\
&\quad + (1-\alpha)\int_{\overline{r}}^{\infty} \mu\phi\left(r_{t+1}|r_t\right) dr_{t+1}\\
&\quad + \int_{-\overline{r}}^{\overline{r}} u\left(r_{t+1}\right)\phi\left(r_{t+1}|r_t\right) dr_{t+1},
\end{aligned}$$

where $\phi\left(r_{t+1}|r_t\right)$ represents the Gaussian density of r_{t+1} given the current level r_t. The last term in this expression determines the expected exchange rate within the band, under an optimal trade-off.

A second order expansion of $u\left(r_{t+1}\right)$ around a value of $r_t \in [-\overline{r}, \overline{r}]$, gives

$$u\left(r_{t+1}\right) \simeq u\left(r_t\right) + u'\left(r_t\right)\left(r_{t+1} - r_t\right) + \frac{1}{2}u''\left(r_t\right)\left(r_{t+1} - r_t\right)^2 + o\left(r_{t+1}\right). \quad \text{(B.2)}$$

Weighting this expression by $\phi\left(r_{t+1}|r_t\right)$ and integrating for $r_{t+1} \in [-\overline{r}, \overline{r}]$, yields

$$\int_{-\overline{r}}^{\overline{r}} u\left(r_{t+1}\right)\phi\left(r_{t+1}|r_t\right)dr_{t+1} = u\left(r_t\right)p\left(r_t;\overline{r}\right) + u'\left(r_t\right)S_1\left(r_t;\overline{r}\right) \quad \text{(B.3)}$$

$$+ \frac{1}{2}u''\left(r_t\right)S_2\left(r_t;\overline{r}\right),$$

where

$$p\left(r_t;\overline{r}\right) = \Pr\left[r_{t+1} \in [-\overline{r}, \overline{r}]\,|r_t\right] \quad \text{(B.4)}$$

$$= \Phi\left(\frac{\overline{r} - r_t}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_t}{\sigma}\right),$$

which represents the probability the risk premium wanders between $[-\overline{r}, \overline{r}]$ for the next period, given the present level r_t, and function $S_1\left(r_t;\overline{r}\right)$ represents the expected change of the risk premium:

$$S_1\left(r_t;\overline{r}\right) = p'\left(r_t;\overline{r}\right) \quad \text{(B.5)}$$

$$= \sigma\left[\phi\left(\frac{-\overline{r} - r_t}{\sigma}\right) - \phi\left(\frac{\overline{r} - r_t}{\sigma}\right)\right]$$

$$= p\left(r_t;\overline{r}\right)E\left[\left(r_{t+1} - r_t\right)|r_{t+1} \in [-\overline{r}, \overline{r}], r_t\right].$$

Function $S_2\left(r_t;\overline{r}\right)$ represents the expected quadratic change of the risk premium:

$$S_2\left(r_t;\overline{r}\right) = p\left(r_t;\overline{r}\right)\left[1 + \frac{p''\left(r_t;\overline{r}\right)}{p\left(r_t;\overline{r}\right)}\right]\sigma^2 \quad \text{(B.6)}$$

$$= \sigma^2 p\left(r_t;\overline{r}\right) - \sigma\left[\phi\left(\frac{-\overline{r} - r_t}{\sigma}\right)\left(\overline{r} + r_t\right) + \phi\left(\frac{\overline{r} - r_t}{\sigma}\right)\left(\overline{r} - r_t\right)\right]$$

$$= p\left(r_t;\overline{r}\right)E\left[\left(r_{t+1} - r_t\right)^2|r_{t+1} \in [-\overline{r}, \overline{r}], r_t\right].$$

Functions (B.4), (B.5) and (B.6) are all continuous, a property that will result relevant to deal with the smooth pasting argument.

Plugging expressions (B.3) and (B.1) into the first order condition (2.23), for $r_t \in [-\overline{r}, \overline{r}]$ one obtains

$$
\begin{aligned}
(1 + \lambda) u(r_t) \;=\; & r_t + \alpha w \left[\Pr(r_{t+1} > \overline{r} | r_t) - \Pr(r_{t+1} < -\overline{r} | r_t) \right] \qquad \text{(B.7)} \\
& - (1 - \alpha) \int_{-\infty}^{-\overline{r}} \mu \phi(r_{t+1} | \overline{r}) \, dr_{t+1} \\
& + (1 - \alpha) \int_{\overline{r}}^{\infty} \mu \phi(r_{t+1} | \overline{r}) \, dr_{t+1} \\
& + u(r_t) p(r_t; \overline{r}) + u'(r_t) S_1(r_t; \overline{r}) \\
& + \frac{1}{2} u''(r_t) S_2(r_t; \overline{r}).
\end{aligned}
$$

Now, we will approximate the particular solutions for the extreme case $\alpha = 1$. Under perfect credibility, this model does not exhibit the S-shaped of the standard model. Let us first briefly remember the foundation of why the standard model is zero sloped at the edges and, later, see how the smooth pasting condition should be reinterpreted within our model[5].

In the standard model, the exchange rate depends linearly of the fundamental and the expected rate of depreciation. The fundamental follows a regulated Brownian motion such that, at margins, monetary policy push it back inside the band. If the slope of the exchange rate function were strictly positive at these edges, the expected rate of depreciation would display a discontinuity. Hence, a sure jump would be given towards the interior of the band, and would provide safe bets opportunities against the movement of the exchange rate. Therefore, in order to avoid such a discontinuity jump, arbitrage considerations require that the slope must be zero at margins.

In our model, expectations are given to the CB and the exchange rate is determined from the first order condition (2.23), subject to (2.22) and the level conditions (2.20) and (2.21). The risk premium r_t is a sufficient statistic to determine the position of the exchange rate inside the band. The CB cannot govern this process and must ensure the equilibrium in the money market given in (2.1) by responding to variations in fundamental. Continuity of functions $x(r_t)$ and $E_t[x_{t+1}]$ is not affected, no matter the value of slope $u'(\overline{r})$.

The aforementioned argument can be developed by proving the continuity of

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[5]The argument provided in this appendix widely follows that of Froot & Obstfeld [11].

the expectations function for $\alpha = 1$:

$$E_t x_{t+1} = w\left[\Pr\left(r_{t+1} > \overline{r}|r_t\right) - \Pr\left(r_{t+1} < -\overline{r}|r_t\right)\right] \tag{B.8}$$
$$+u\left(r_t\right)p\left(r_t;\overline{r}\right) + u'\left(r_t\right)S_1\left(r_t;\overline{r}\right) + \frac{1}{2}u''\left(r_t\right)S_2\left(r_t;\overline{r}\right).$$

This is a sum of continuous functions defined for any $r_t \in \mathbb{R}$, and therefore function $E_t\left[x_{t+1}\right]$ is also continuous. The expected conditional increase, $S_1\left(r_t;\overline{r}\right)$, and the conditional variability $S_2\left(r_t;\overline{r}\right)$, are continuous as well, even when $r_t = \pm\overline{r}$. Continuity is then exhibited by the expectation. Analogously, the expected rate of depreciation

$$E_t x_{t+1} - x_t,$$

is also a continuous function, where

$$x_t = \begin{cases} +w & \text{if } r_t > \overline{r} \\ u\left(r_t\right) & \text{if } r_t \in \left[-\overline{r}, \overline{r}\right] \\ -w & \text{if } r_t < -\overline{r} \end{cases}, \tag{B.9}$$

and $u\left(\overline{r}\right) = w$, $u\left(-\overline{r}\right) = -w$. The smooth pasting condition is not needed for continuity.

C. Tables and Figures

<div align="center">

Table 1
Estimation of standard deviation (σ)

	std $(\Delta r_t) \simeq \sigma$
British Pound	0.0005
French Franc	0.0011
Deustche Mark	0.0025
Japanese Yen	0.0012
Swiss Franc	0

Table 2
Realignment rates in the ERM[6]

Date	FF	IRP	BF	DK	ITL	SP	PE
24-Sep-1979	2.00	2.00	2.00	5.00	2.00		
30-Nov-1979			0.14	5.00			
23-Mar-1981			-0.14		6.38		
5-Oct-1981	8.76	5.50	5.50	5.50	8.76		
22-Feb-1982			9.29	3.09			
14-Jun-1982	10.61	4.25	4.25	4.25	7.20		
21-Mar-1983	8.20	9.33	3.94	2.93	8.20		
25-Jul-1985					8.51		
6-Apr-1986	6.19	3.00	1.98	1.98	3.00		
2-Aug-1986		8.70					
12-Jan-1987	3.00	3.00	0.98	3.00	3.00		
8-Jan-1990					3.82		
14-Sep-1992					7.25		
17-Sep-1992						5.26	
23-Nov-1992						6.38	6.38
14-May-1993						8.70	6.95
Average	6.46	5.86	3.10	3.84	5.54	6.78	6.67
std	3.39	3.41	3.01	1.26	2.42	1.75	0.40

</div>

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[6]In this table, FF stands for French Franc, IRP for Irish Punt, BF for Belgium Franc, DK for Danish Krona, ITL for Italian Lira, SP for Spanish Peseta and PE for Portuguese escudo. The ITL participated in the narrow $\pm 2.25\%$ band from 12th-January-1987 until 14th-September-1992, when it left the ERM.

Table 3

Loss function for $w = 0.0225$

λ	0.2	0.5	1.0	2.0	5.0
J^c	0.4444	0.4444	0.4444	0.4444	0.4444
$J^{tz}\left(\alpha = 0\right)$	15.4674	15.1098	4.3379	1.6561	0.6280
$J^{tz}\left(\alpha = \frac{1}{4}\right)$	10.0236	8.3402	3.2813	1.4845	0.6123
$J^{tz}\left(\alpha = \frac{1}{2}\right)$	5.7592	4.9529	2.5509	1.3253	0.5999
$J^{tz}\left(\alpha = \frac{3}{4}\right)$	3.0643	3.0624	2.0009	1.1846	0.5864
$J^{tz}\left(\alpha = 1\right)$	0.4655	0.4919	0.5207	0.5430	0.5244
J^{mf}	2.6665	1.3333	0.8888	0.6666	0.5333
J^{ff}	0.5333	0.6666	0.8888	1.3333	2.6665

Table 4

Loss function for $w = 0.06$

λ	0.2	0.5	1.0	2.0	5.0
J^c	0.4444	0.4444	0.4444	0.4444	0.4444
$J^{tz}\left(\alpha = 0\right)$	2.0927	1.3081	0.9428	0.6559	0.5333
$J^{tz}\left(\alpha = \frac{1}{4}\right)$	1.9960	1.2758	0.9403	0.6559	0.5333
$J^{tz}\left(\alpha = \frac{1}{2}\right)$	1.8762	1.2399	0.9301	0.6547	0.5333
$J^{tz}\left(\alpha = \frac{3}{4}\right)$	1.6704	1.1951	0.9102	0.6544	0.5333
$J^{tz}\left(\alpha = 1\right)$	0.5761	0.6778	0.7106	0.6487	0.5333
J^{mf}	2.6665	1.3333	0.8888	0.6666	0.5333
J^{ff}	0.5333	0.6666	0.8888	1.3333	2.6665

Figure 1

Function *u* [*r*] (λ = 0.5)

Figure 2
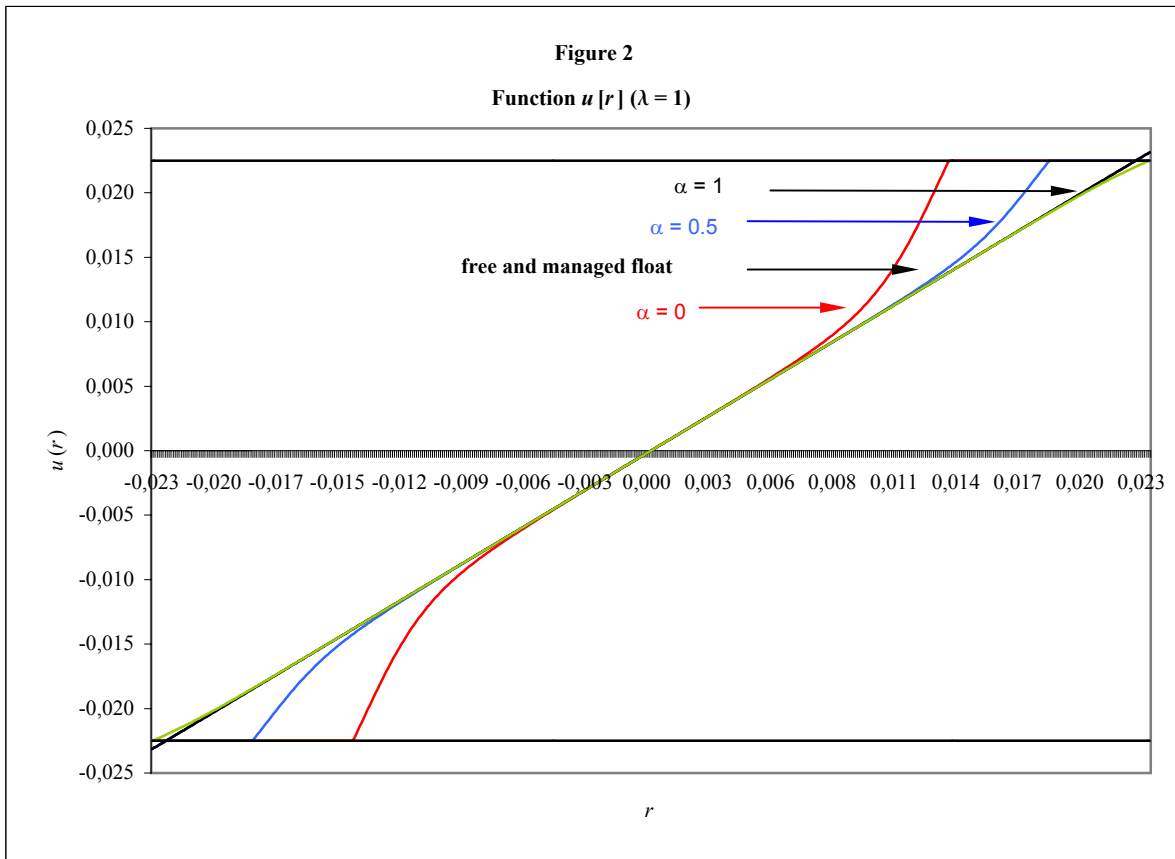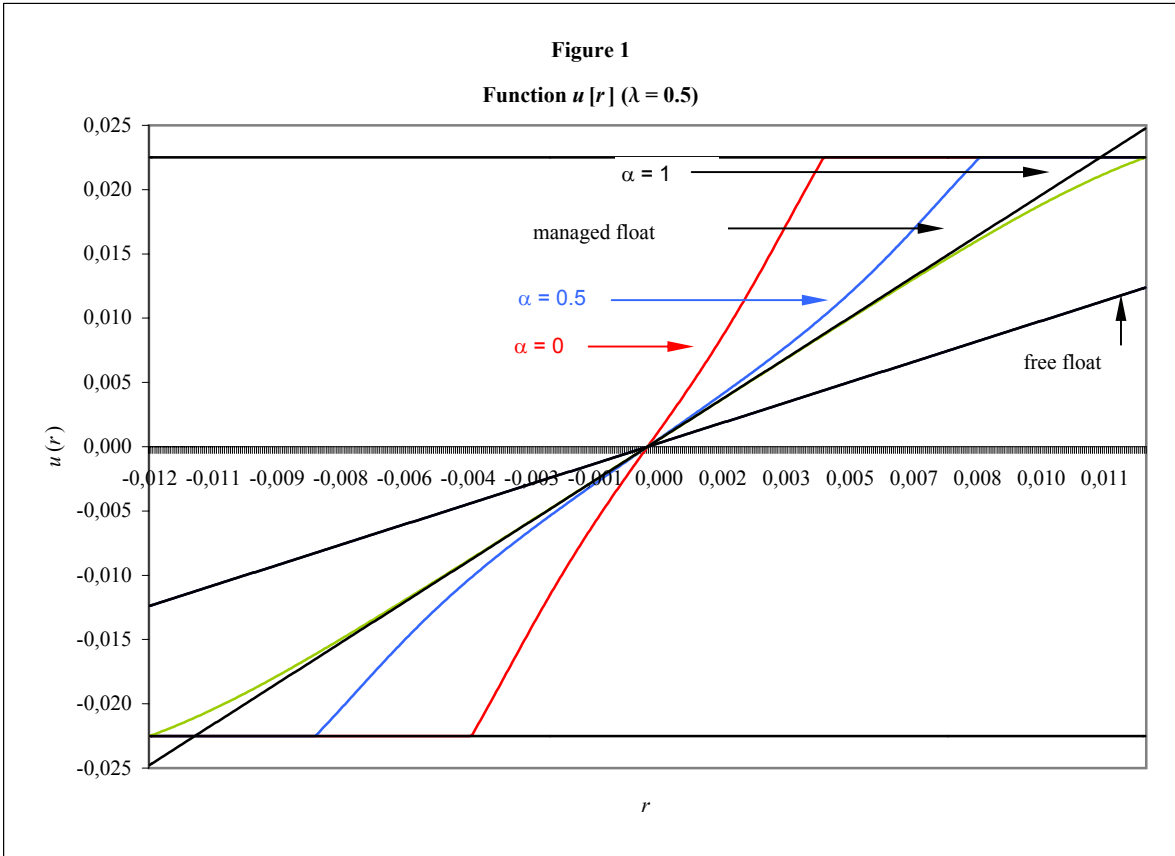
Function *u* [*r*] (λ = 1)

Figure 3
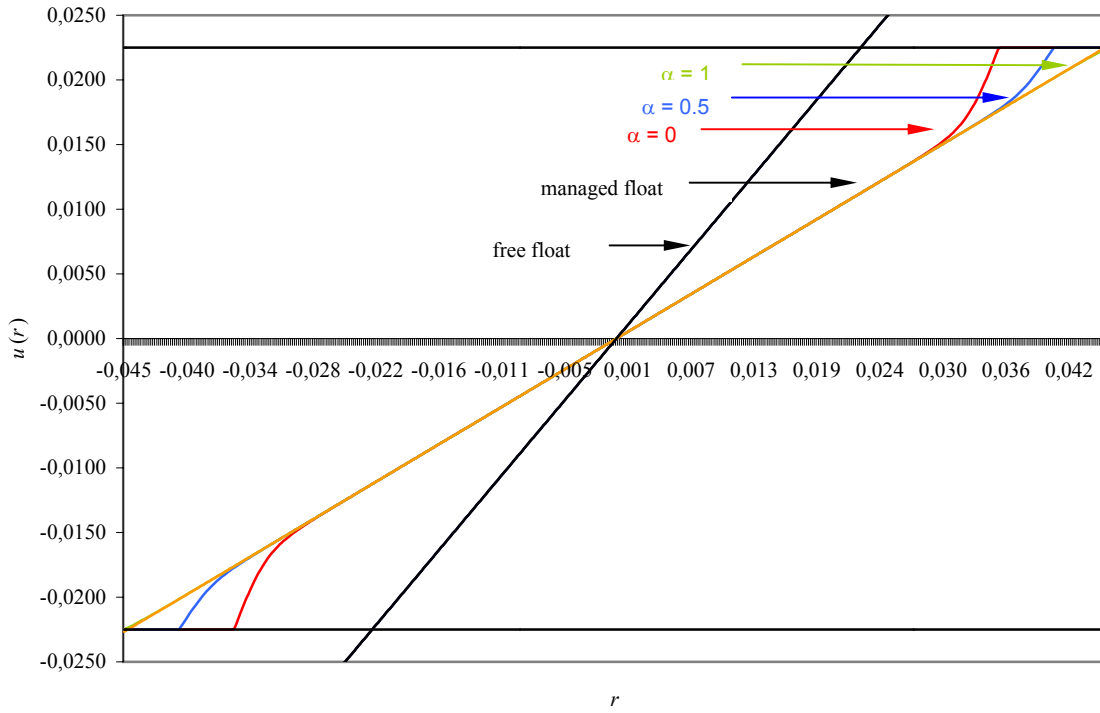
Function _u_ [_r_] (λ = 2)
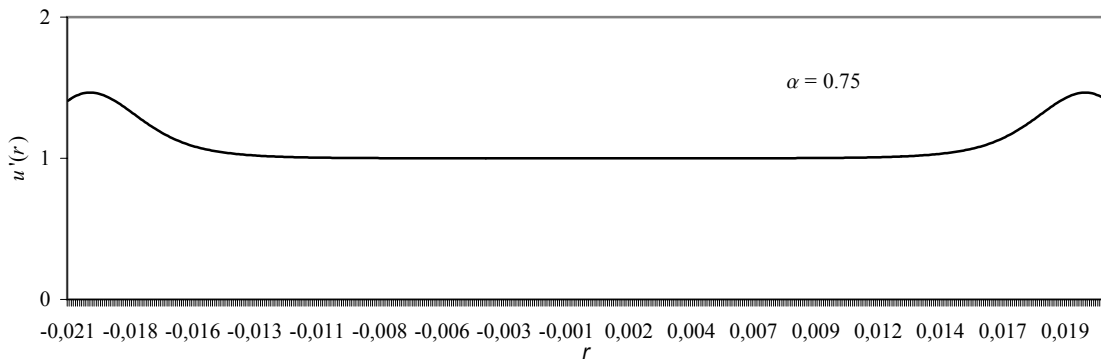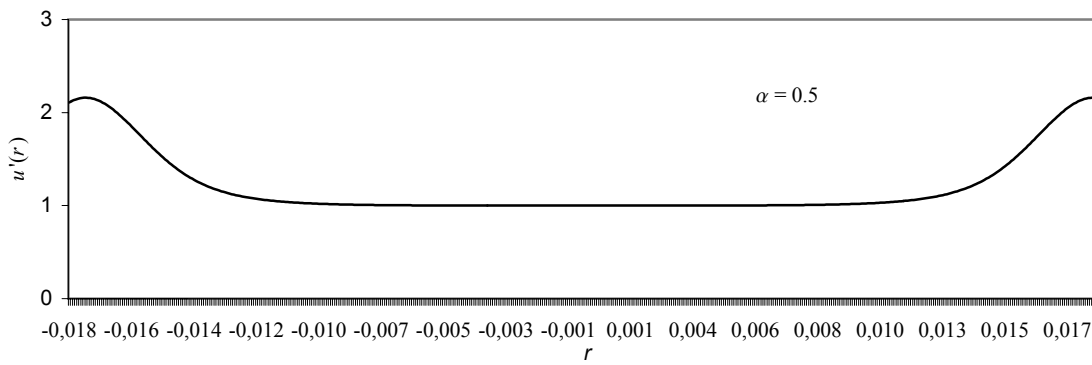
Figure 4
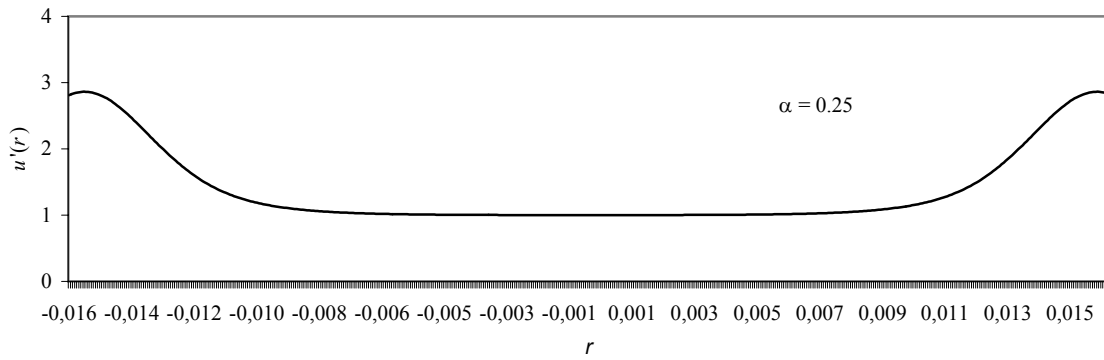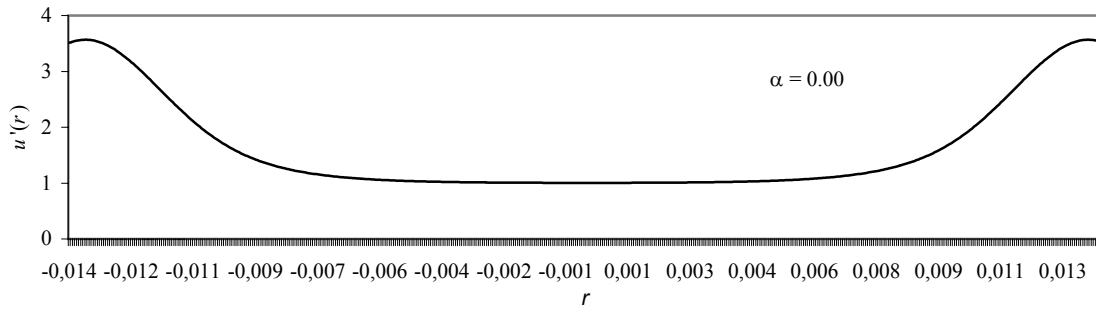
Function $u'[r]$ ($\lambda = 1$)
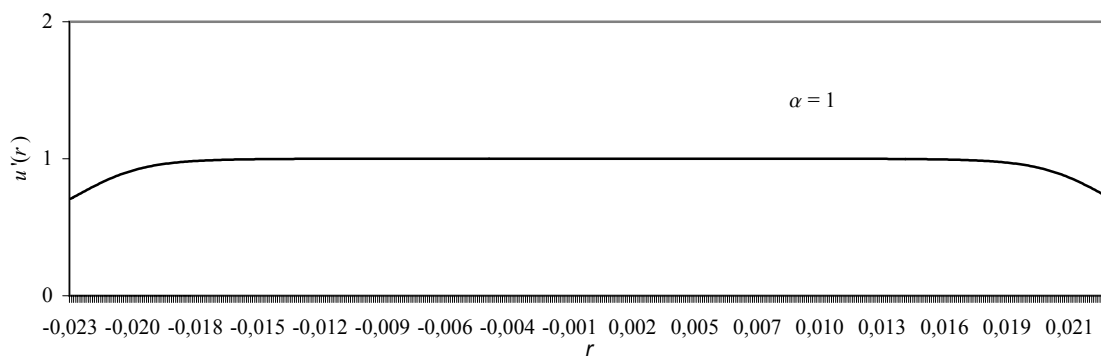
Figure 5

Stationary probability distribution ($\lambda = 2$; $\alpha = 1$)
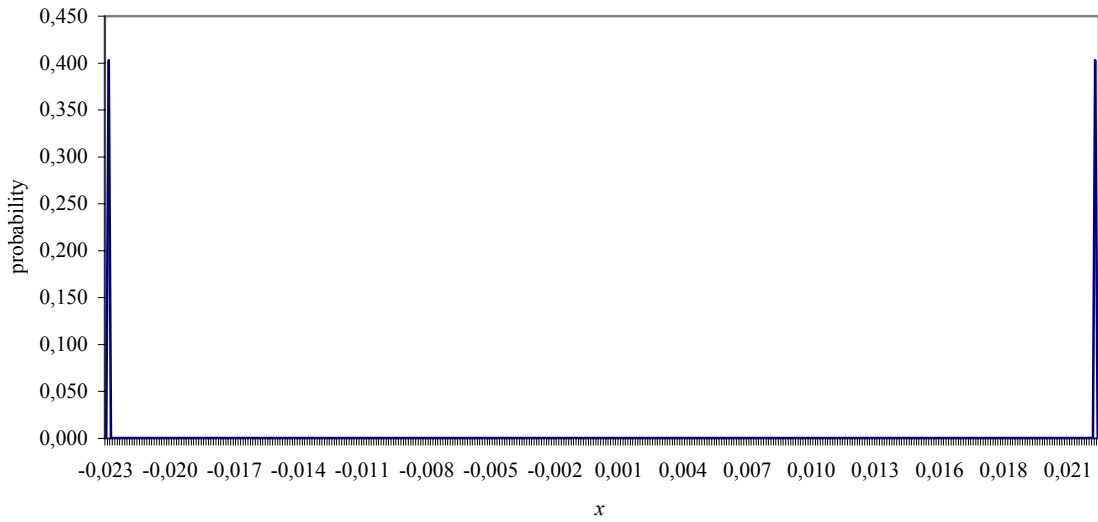

Figure 6

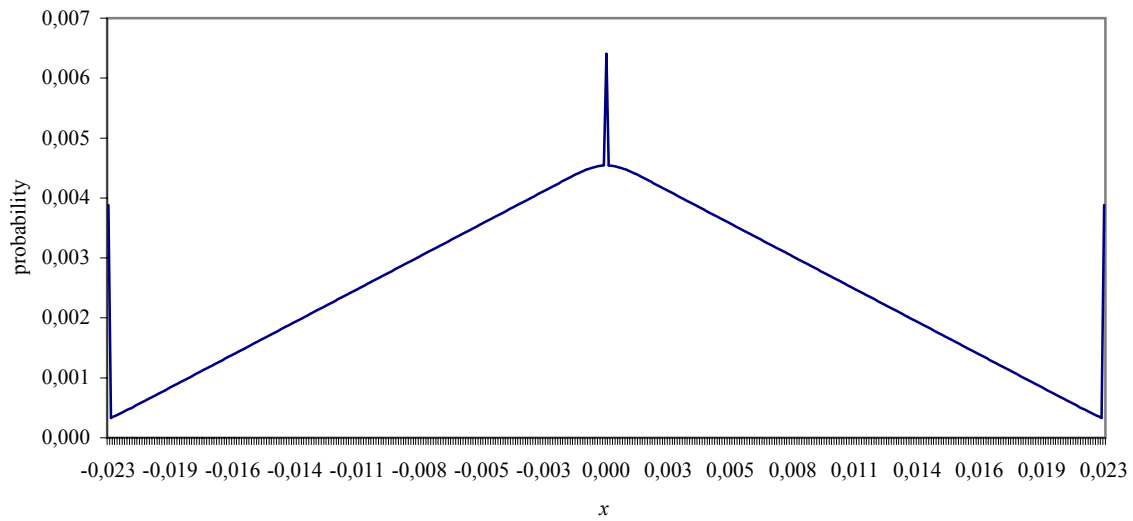**Stationary probability distribution ($\lambda = 2$; $\alpha = 0.8$)**

Figure 7

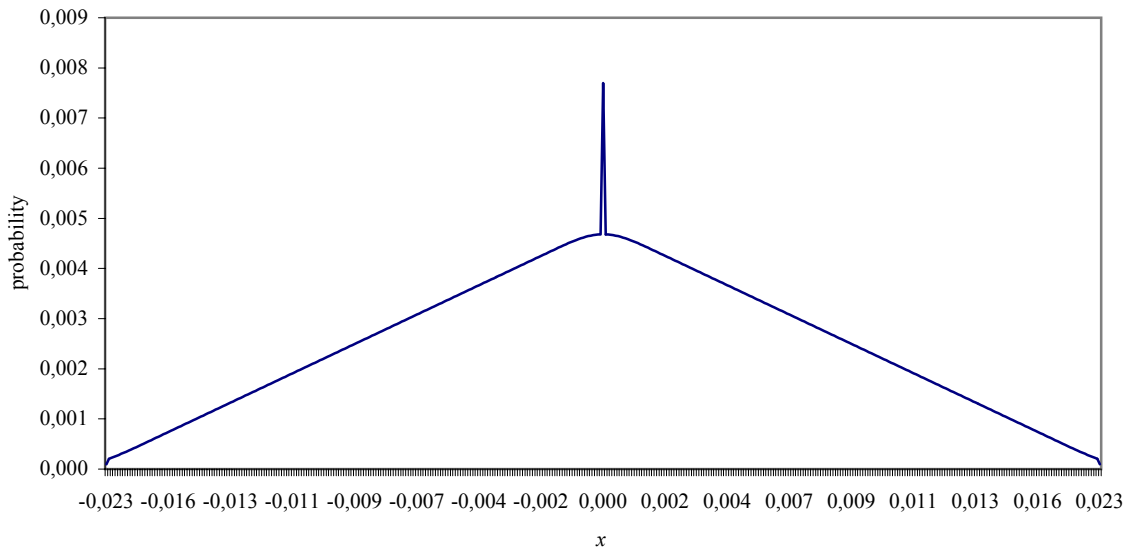**Stationary probability distribution ($\lambda = 2$; $\alpha = 0$)**

Figure 8
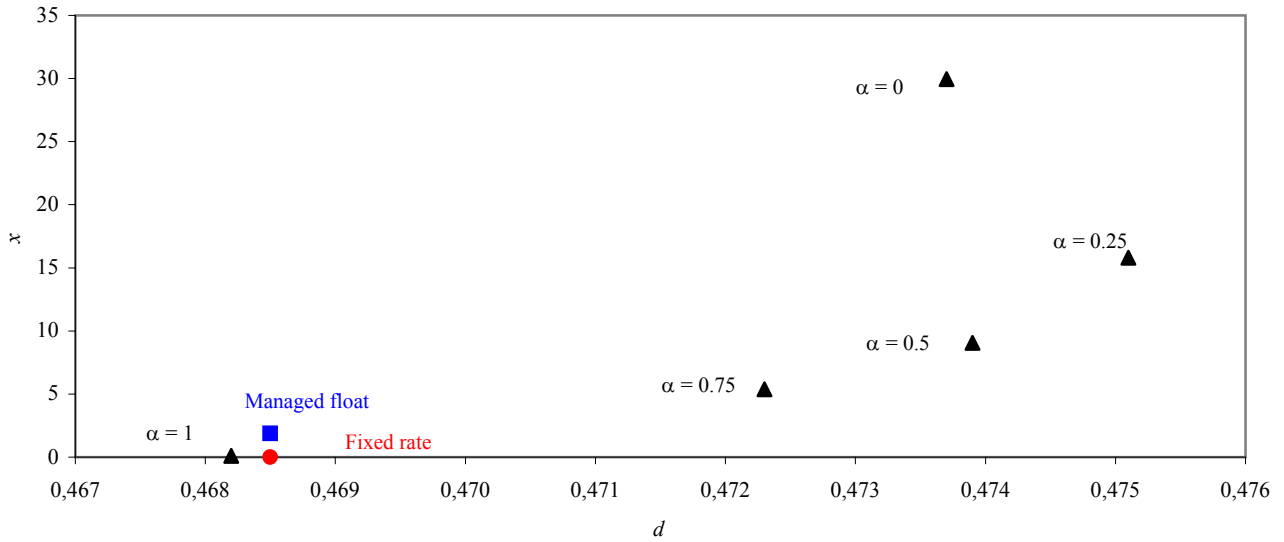**Contributions to the costs (λ = 0.2)**

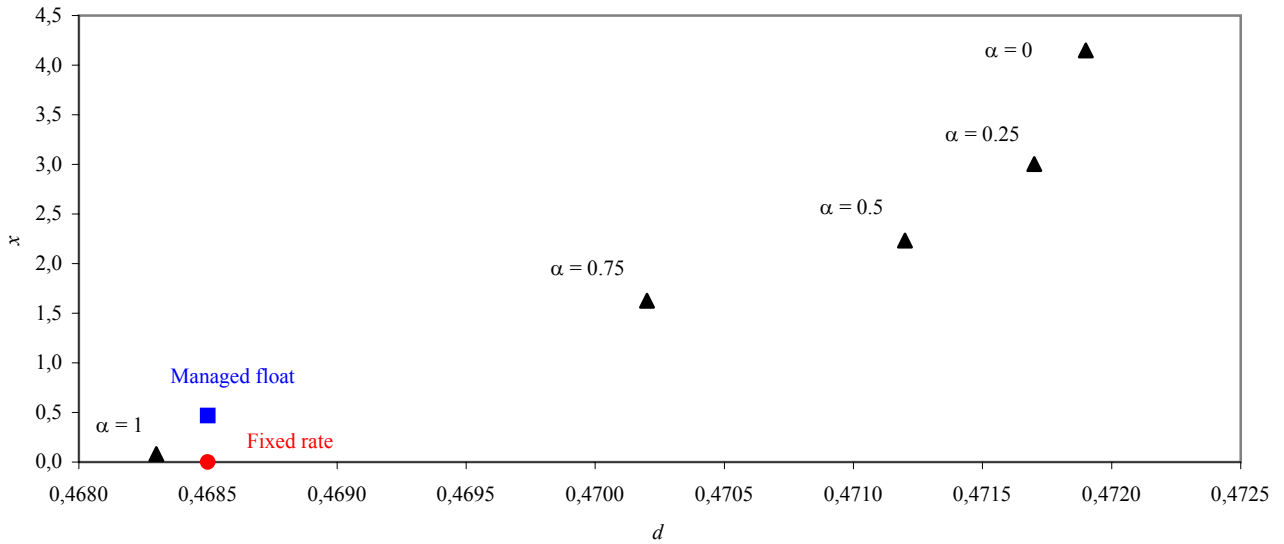
Figure 9
Contributions to the costs (λ = 1)

Figure 10
Contributions to the costs (λ = 2)