Sequential Formation of Coalitions through Bilateral Agreements*

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Abstract

We study a sequential protocol of endogenous coalition formation based on a process of bilateral agreements among the players. We apply the game to a Cournot environment with linear demand and constant average costs. We show that the final outcome of any Subgame Perfect Equilibrium of the game is the grand coalition, provided the initial number of firms is high enough and they are sufficiently patient.

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1 Introduction

The incentives that firms have to merge have recently been studied in non-cooperative games of endogenous coalition formation. The usual way of analyzing these games is by assuming that the forming of a coalition or the negotiation of a merger has no cost for the participants, in particular, many players may consider simultaneously whether to form a coalition or not.

This paper explores incentives to merge when only bilateral agreements are feasible at every point in time. This restriction does not mean, however, that only small coalitions may be formed. By sequentially meeting over time, coalitions may grow in size. In other words, once some coalitions are formed, they may decide to continue with the process and form even larger entities.

The sector of firms that provide professional services (accounting, consulting, etc.) offers a relevant set of examples of such a sequential process of mergers. Some of the major firms in this sector, (i.e., Ernst & Young, KPGM and PricewaterhouseCoopers) are the outcome of a sequential process of mergers with a small number of parties involved. In particular, since Arthur Young opened an accounting firm in Chicago (1894), and the brothers Alvin and Theodore Ernst settled their firm in Cleveland (1903), at least four bilateral mergers have taken place before the present structure of Ernst and Young was arrived at.

The banking sector provides other examples. In Spain, the bank that is now known as SCH is the outcome of a merger between the Banco de Santander and the Banco Central Hispano which, in turn, was the result of the merger between the banks Central and Hispano. Similarly, the banks of Bilbao and Vizcaya first merged to form the BBV and then the new firm merged again with the Banco Argentaria to form the BBVA.

We model the formation of coalitions as a sequential process in which, at each moment in time, only two existing coalitions can decide to merge. We study the subgame perfect equilibria of such a game. The sequential process of coalition formation that we propose can be useful in analyzing sequential formation of bilateral agreements in several economic environments where groups of agents interact, including mergers, environmental cartels, and networks.

In this paper, we consider a market in which identical firms with constant returns facing linear demand compete à la Cournot. At each period, the firms make decisions on quantity. To focus our analysis on the incentives to form coalitions, we assume that production is a short-term decision. Also, at each period, two randomly chosen coalitions can merge in the existing partition. A merger means forming a cartel in which the partners decide on production jointly. The decision on the merger is made by taking the long-term profits into account.

As Salant, Switzer, and Reynolds (1983) point out, two firms (or coalitions) will not be interested in merging if they only consider the present period profits and if there are already at least three firms (coalitions) in the industry. Their result extends easily to our model: If the firms' discount rate is low enough, they will not merge at any period in the unique subgame perfect equilibrium of the game. Hence, the outcome is that all of the firms remain singletons.

The situation when firms are forward-looking is more interesting. In such a case, the firms may want to merge even if they lose profits in the short run. In fact, we show that when firms are patient enough, and there are enough firms in the industry, the final outcome of any subgame perfect equilibria is "the grand coalition". The firms form coalitions sequentially, growing gradually, so that finally they all end up together. We characterize the sequences of mergers that the firms will undertake in equilibrium. In those sequences, firms will accept some of the mergers and will reject others.

The fact that, in a linear Cournot model, "the grand coalition" can result as the equilibrium of a game of coalition formation, is in contrast with other results on mergers presented in the literature. Interestingly, it is the fact that *only* a pair of coalitions can merge at each period (so bigger coalitions cannot be formed immediately) which allows them to reach "the grand coalition". Even if this restriction to bilateral agreements is ex-ante damaging to the formation of coalitions, since it reduces the choice set of the firms, it ex-post results in an impulse for the process of merging. The reason for this is that the bilateral nature of the mergers induces the coalitions to be formed gradually, in such a way that the incentives for all of the players to "free ride" are always outweighed by the future benefits of the mergers.

Bloch (1996) and Ray and Vohra (1999), also analyze an infinite-horizon sequential game. In their model, payoffs are only realized after the coalitions have been formed. In the coalition formation game previous to the production, the first agent, according to a rule of order, makes an offer to other agents to join him in a coalition. If all members accept the offer, the partnership is formed and the partners in the coalition leave the game. The first agent in the set of remaining players then makes a partnership proposal, and the game continues following the same rule until all of the players have left the game. If someone rejects, he will then have to make the next proposal. This model applies to general games. For the linear Cournot game, Bloch (1996) proves that, when players are ex-ante symmetric and the discount rate is high enough, the coalition structures that result from the stationary symmetric perfect equilibria in pure strategies contain a coalition whose size is about 80% of the market, while the other firms remain isolated. Hence, "the grand coalition" is not formed, since some individual firms are able to appropriate the positive externalities generated by the large coalition, while the firms in the large coalition are better-off inside than outside the coalition.

There are three main differences between the game proposed by Bloch (1996), and by Ray and Vohra (1999) and our proposal. First, in their games, a player may make an offer to any set of partners. Secondly, if the offer is accepted, the coalition leaves the game. And thirdly, production takes place only after the coalitions have been formed. This third difference, however, is not relevant. In the last section of the paper, we propose a variation of our game in which firms only produce after the coalition-formation game has been played. We verify that our results hold in this environment as well. We also show that our analysis can be easily adapted to cope with situations in which the identity of the firms or coalitions that can merge at any given period is not random but follows a deterministic protocol. On the other hand, the fact that a player may propose a merger to any set of players, and that any coalition that is formed leaves the game and will not be approached again by any proposer, are key features of the model by Bloch, and the one by Ray and Vohra. We show that when players who form coalitions do not leave the game, and the coalition formed at a given period may have any size, "the grand coalition" is just one outcome of the game, but many other outcomes may arise in equilibrium.[1]

[1] Note that this is the only interesting alternative since if players meet two by two and leave the game when a merge occurs, only very particular outcomes may arise in equilibrium.

Note, also, that the characteristics of our game allow us to analyze all of the subgame perfect equilibria, without restricting our attention to stationary strategies. All the results remain true if we concentrate on the stationary subgame perfect equilibria of our coalition formation game.

In the literature on mergers, several authors have addressed the question of the coalition structures that would prevail in Cournot games with homogenous goods and linear demand by analyzing the stability of the coalition structures.[2] This literature suggests that there would eventually be one large coalition and a few players as singletons. Our game never has these intermediate results: If there is a small number of players, or if the discount rate is low, all of the players remain as singletons, while "the grand coalition" is the only final outcome when both the set of players and the discount rate are large enough. In fact, "all singletons" and "the grand coalition" are the only two possible subgame perfect equilibrium outcomes of our game.

Some authors have considered the sequential formation of mergers by studying how merger decisions are inter-connected over time. Pesendorfen (2000) considers a model of merger formation in the line of Kamien and Zang (1990), where certain firms acquire others by submitting bids and asking prices. Pesendorfen (2000) allows for entry and shows that some mergers may be more profitable if future mergers are expected. In his model, "the grand coalition" cannot be formed in a single period, not because agreements by a fixed number of firms are allowed, but because all the firms are not present in the market from the beginning of the game. He concludes that even if frequent mergers are not profitable when the number of firms in the industry is small, they can become profitable as the number of firms increases. Gowrisankaran (1999) considers a dynamic model in which mergers arise endogenously. He focuses on the dynamic links between the process

[2] In simultaneous games, we can refer to four stability concepts (Aumann (1967) and Hart and Kurtz (1983)). A coalition structure is α-stable if no group of firms can guarantee an improvement, independently of what the others do. A partition is β-stable if no group of firms has, for any possible reaction of the external players, a strategy that can improve its situation. A coalition structure is γ-stable (respectively, δ-stable) if no set of players has incentives to deviate when the players of their original coalitions split up (respectively, they still form a coalition). In the linear Cournot game, α-stable, β-stable, and γ-stable outcomes always have the form $\{s, 1, ..., 1\}$ with s being higher or equal to 80% of the market. The set of δ-stable outcomes, on the other hand, is empty.

of endogenous mergers and the decisions on production, entry, exit and investment. The complexity of the game forces him to solve it numerically. In spite of this, the model provides interesting insights into the dynamic interaction between the different firm's decisions. Gowrisankaran and Holmes (2000) analyze the steady states of an endogenous merger game, in which a dominant firm takes merger decisions regarding a competitive fringe. They assess the importance of the different elasticities (both of supply and demand), as well as the alternative discount factor, on the evolution of the market. They show that monopoly and perfect competition always belong to the set of steady states in the game.[3]

Our work is not only in line with the literature that analyzes the formation or stability of coalition structures in Cournot games, but also with Gul (1989), author analyzes a transferable utility economy in which random bilateral meetings occur. At each meeting, one of the agents makes a proposal to the other which he can either accept or reject. If the proposal is accepted, the resources of both agents are in the hands of the proposer from this moment on, otherwise, both players stay in the game. Gul (1989) shows that, under some conditions, all the players will eventually end up together and the expected payoff of each player in an efficient sequential perfect equilibrium is his Shapley value.[4]

In the following section we present the coalition-formation game. In Section 3, we analyze the outcomes of the game when firms are myopic, while in Section 4 we do the analysis when firms are forward looking. In Section 5, we show how our results can be extended to several variations of our game.

[3]Nilssen and Sorgard (1998) focus on sequential merger decisions by two disjoint groups of firms that are exogenously fixed. They study how a merger may encourage or discourage other mergers in the future.

[4]Seidmann and Winter (1998) also analyze gradual coalition-formation in games without externalities, although the agreements are not bilateral. Other papers that study coalition-formation in characteristic function games are: Chatterjee *et al.* (1993), Hart and Mas-Colell (1996), and Okada (1996). They all use stationary subgame perfect equilibrum as solution concept given that the set of subgame perfect equilibria in multilateral bargaining problems, as the ones they study, is typically very large.

2 The Coalition-Formation Game

We study the sequential formation of coalitions between firms competing à la Cournot in a framework in which only *bilateral agreements* are allowed. We assume that, at each moment in time only two of the existing coalitions can decide to merge.

At the beginning of the game, there are n identical firms, with $n \geq 2$. We denote the set of firms by $N = \{1, ..., n\}$. Firms can form coalitions following a certain protocol that will be described later. Hence, at any point in time, these n firms form a partition of N, i.e., they constitute a *coalition structure*.

Let $\prod$ denote the set of coalition structures over N. Denote $\pi \in \prod$ an element of this set, that is, $\pi = \{S_1, ..., S_r\}$, with $S_a \subset N$ for all $a = 1, ..., r$, $\cup_{a=1}^{r} S_a = N$, and $S_a \cap S_b = \emptyset$ for all $S_a, S_b \in \pi$, with $S_a \neq S_b$. We denote by s_a the size of coalition S_a. Among the set of partitions, a particular coalition structure is the one in which all the agents are alone, i.e., all the coalitions are singletons. We denote such a partition by π^n and "the grand coalition" by $\pi^1 \equiv N$, i.e., the coalition structure with just one element. We denote by $(\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$, the coalition structure that results when we replace two elements of π, namely S_a and S_b, by their union. Therefore, if π is formed by r coalitions, $(\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ consists of $(r-1)$ coalitions.

Firms make decisions at any time $t = 0, 1, 2, ...$ At time t, the present profits of a firm depend on the whole coalition structure that is formed at that time. We assume, for the sake of simplicity, that firms face a linear demand function and bear equal constant average costs. That is, the inverse demand function at time t is:

$$P(\sum_{j=1}^{n} q_j) = \alpha - \beta \sum_{j=1}^{n} q_j.$$

The production costs of firm i are given by:

$$C_i(q_i) = c q_i.$$

When firms merge, they form a cartel. That is, merging allows the firms to co-ordinate their quantity decisions. We calculate the firms' profits at any point in time, given a cartel structure (i.e., a coalition structure) $\pi = \{S_1, ..., S_r\}$. We assume that production is a short-term decision, being taken by short-term managers.[5] Given that there are r cartels in this structure and that marginal costs are equal for all firms in a cartel, cartel S_a chooses the total level of production q_a of its firms by solving the following maximization program:

$$\max_{q_a} \left\{ \left(\alpha - \beta \sum_{b=1}^{r} q_b \right) q_a - c q_a \right\}. \tag{1}$$

From this program we find that the equilibrium quantities are equal for all of the cartels and that they are equal to: $q^r = \frac{\alpha - c}{\beta(r+1)}$. Hence, the Cournot profits per-cartel V^r in a coalition structure with r cartels are:

$$V^r = \frac{(\alpha - c)^2}{\beta(r+1)^2}.$$

We normalize $\frac{(\alpha-c)^2}{\beta} = 1$, so:

$$V^r = \frac{1}{(r+1)^2}.$$

It can be easily verified that the efficient outcome, from the industry's point of view, is arrived at when all the firms merge, and "the grand coalition" is formed.

We assume that the sharing of profits among the firms that form the cartel is exogenously fixed and egalitarian. Therefore, the individual profits $V_i(\pi)$ of any firm i belonging to the cartel $S_a \in \pi$, with a size s_a, when there are r cartels in the coalition structure π, are:

$$V_i(\pi) = \frac{1}{(r+1)^2 s_a}. \tag{2}$$

Firms value future payoffs with a homogeneous discount factor $\delta \in [0, 1)$. Therefore, if π_t is the coalition structure existing at time t, for $t \geq t^\circ$, the discounted payoff of firm

[5] It is well known that, in an infinite game like ours, there are strategies by which firms may reach implicit collusion in production if the discount rate is high enough (notice, however, that the set of equilibrium outcomes is usually very large). Our objective in this paper is the analysis of the incentives for coalition formation, so we will abstract from the possibility of collusion by assuming that production is a short-term decision. An equivalent assumption is that firms use Markov, or stationary, strategies when they decide their production level. In Section 5, we analyze a simpler game in which this assumption is not necessary because production takes place only once. In this game, all our results still hold.

i at time t° is $\sum_{t=t^\circ}^{\infty} \delta^{(t-t^\circ)} V_i(\pi_t)$.[6] We shall also discuss, later on, the particular case of the players being perfectly patient ($\delta = 1$) and evaluate future profits with the "time-average criterion". That is, firm i maximizes:

$$\lim_{T\to\infty} \inf \frac{\sum_{t=0}^{T} V_i(\pi_t)}{T}.$$

We study the outcome of a *process of sequential coalition formation.* This infinite-horizon process is undertaken according to the following protocol. At each period t, there is first the decision to merge (stages $t.1$ and $t.2$) and secondly, (stage $t.3$), there is the decision on production. We have already described the result of the production stage, summarized by the profit function $V_i(\pi_t)$. More precisely:

At $t = 0$:

0.1 Two different firms i and j are randomly selected. All the firms have the same probability of being selected.

0.2 Firms i and j sequentially decide whether to merge or not. The merger occurs if both players agree.

The coalition structure at time $t = 0$ is then either $\pi_0 = (\pi^n \backslash \{\{i\}, \{j\}\}) \cup \{i, j\}$ if firms i and j have merged or $\pi_0 = \pi^n$ if they have not.

0.3 Each firm $k \in N$ obtains, at $t = 0$, profits $V_k(\pi_0)$.

Let us now consider any time $t \geq 1$. The coalition structure existing at $t-1$ was π_{t-1}. If $\pi_{t-1} = N$, then $\pi_t = N$. Otherwise:

$t.1$ Two coalitions S_a and S_b in π_{t-1} are randomly selected. All of the coalitions in π_{t-1} have the same probability of being selected.

$t.2$ Firms in coalitions S_a and S_b sequentially decide whether to merge. The merger is carried-out if all of the firms in coalitions S_a and S_b agree to it.[7]

[6] When $\delta = 0$, the discounted payoff of player i at time t^o is $V_i(\pi_{t^o})$.

[7] The firms are the players of our game. When they decide on the merger, the members in S_a and S_b do not face a co-ordination problem because they chose sequentially. Therefore, if it is optimal for all of them, they will sequentially choose to merge. If the merger is not profitable for the firms in one of the coalitions the merger will not happen, because one of the firms will not accept it.

The coalition structure at time t is either $\pi_t = (\pi_{t-1} \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if coalitions S_a and S_b have merged or $\pi_t = \pi_{t-1}$ if they have not.

t.3 Each player $k \in N$ obtains profits $V_k(\pi_t)$ at time t.

The solution concept that we consider is *Subgame Perfect Equilibrium* and we concentrate on *pure strategies*. We denote the set of Subgame Perfect Equilibria in pure strategies by SPE.

We must point out that the proposed process for the formation of coalitions is irreversible, in the sense that the players cannot dissolve a merger once it has been formed. Allowing for mergers to split up enlarges the set of possible SPE considerably.

Given the irreversibility of the coalition-formation process, the game will arrive at a situation in which the existing coalition structure at that specific period will remain forever, with probability one. We will refer to such a coalition structure as a *final coalition structure* or a *final outcome*. If there are SPE strategies that lead to a particular final outcome, then we say that it is an *SPE final outcome*.

3 Myopic Firms

The objective of this paper is to look at the SPE final outcomes of the game of sequential formation of coalitions. The easiest analysis is done in the simple benchmark where players have a completely *myopic behavior*. This is equivalent to assuming that $\delta = 0$, the case in which we have the static version of our game.

If the players are myopic, the firms in two coalitions S_a and S_b in partition π will decide to merge (if they are chosen by the protocol) at any period, if and only if:[8]

$$V_i(\pi) < V_i\left((\pi \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}\right) \text{ for all } i \in S_a \cup S_b.$$

Let us suppose that the coalition structure π is formed by $r \geq 2$ coalitions. Then, the firms of S_a and S_b will want to merge and move to a structure with $r-1$ coalitions if:

$$\max\left\{\frac{1}{(r+1)^2 s_a}, \frac{1}{(r+1)^2 s_b}\right\} < \frac{1}{r^2(s_a + s_b)}.$$

[8]For convention, we make the implicit assumption that a player will only be willing to join a coalition if he makes a strictly positive gain by doing so.

Let us assume, without loss of generality, that $s_a \leq s_b$. The condition then becomes:

$$\frac{1}{(r+1)^2 s_a} < \frac{1}{r^2(s_a+s_b)},$$

or equivalently:

$$s_a > \frac{r^2}{2r+1} s_b.$$

Note that the previous equation implies $s_a > s_b$ as long as $r \geq 3$, which would be in contradiction with our hypothesis that $s_a \leq s_b$. Therefore, two coalitions of firms will never be interested in merging if they only care about present profits and if there are at least three existing coalitions in the industry. This is a well-known result in static games that goes back to Salant *et al.* (1983). In addition to this, and for the case $r = 2$, the previous inequality shows that two coalitions will merge to monopoly if and only if their sizes are not very different. More precisely, the required condition is that $s_a > (4/5)s_b$, for the case $s_a \leq s_b$.

The previous observation implies that *if there are at least three firms* in the market, *the only myopic final outcome of the game of coalition formation is "all singletons"*. That is, when $\delta = 0$ no merger will occur.

For low enough discount rates, a firm is not interested in compensating short-term loses with long-term gains. Therefore, the myopic final outcome will also be the SPE final outcome when the discount parameter δ is low enough. We state this result formally in the following proposition:

Proposition 1 *If $n \geq 3$ and the discount rate δ is low enough, then the only SPE final outcome of the process of sequential coalition-formation in the linear Cournot setting is that all firms remain singletons.*

Proof. Immediate, after the discussion for the case $\delta = 0$. ■

4 Forward-Looking Firms

When firms are forward-looking, they may be interested in merging even if they lose profits in the short run, if by doing so they anticipate higher profits in the future. A (non-profitable) merger by two firms or two coalitions may further other mergers. Hence, although the initial merging firms (or coalitions) lose profits because of the first merger, they may improve their situation later on if other mergers are carried-out.

The following proposition restricts the set of potential SPE final outcomes of the sequential game for any discount rate. It shows that, in equilibrium, firms will surely not start merging to end up in a coalition structure with more than one coalition.

Proposition 2 *The SPE final outcome of the game of coalition formation in a Cournot competition model is either a monopoly or "all singletons".*

Proof. We do the proof by contradiction. Let us suppose that the final outcome is a coalition structure π formed by r coalitions, with $2 \leq r \leq n-1$. Denote by S_a and S_b the last two coalitions that merged, say at period t^0, with $s_a \leq s_b$. In Section 3, we saw that, for a firm $i \in S_a$, $V_i(\pi) > V_i((\pi \backslash \{S_a \cup S_b\}) \cup \{S_a, S_b\})$. In addition, firms in S_a would even get strictly higher profits if, at any period after t^0, other mergers not involving S_a take place. Therefore, for firms in S_a, the strategy of merging with S_b at t^0 (leading to the final outcome $\pi \neq \pi^1$) is strictly dominated by the strategy of not accepting any merger from t^0 on. Therefore, the firms in S_a have a profitable deviation. Hence, no SPE strategy profile can lead to a final outcome with r coalitions, for $2 \leq r \leq n-1$. ■

Proposition 2 shows that the process of coalition-formation in a linear Cournot model will only begin if it leads to full integration (monopoly). Otherwise, all of the firms will remain singletons. The reason for this result is that no pair of coalitions wants to be the last to merge (unless the merger leads to a monopoly). In equilibrium, therefore, a merger can only happen if the firms involved anticipate that it will be followed by another, and yet another, until "the grand coalition" is formed.

We are now interested in finding out when the SPE final outcome of the game of coalition-formation is a monopoly. We know that a necessary condition for a monopoly to emerge is that the discount rate should be high enough, since no merger takes place in equilibrium when the discount rate δ is low enough, as was shown in Proposition 1.

Given Proposition 2, we also know that two coalitions will never merge if there is not a sequence of unions leading up to full integration. Another necessary condition for the mergers to arise therefore, is that for every value of r, for $2 \leq r \leq n$, there must exist a coalition structure with r coalitions, such that at least two of them obtain smaller profits in this sort of structure than they would in a monopoly.

The profits of the members of a coalition of size s in a coalition structure with r cartels are strictly smaller than their profits in a monopoly if:

$$\frac{1}{(r+1)^2 s} < \frac{1}{4n}, \text{ i.e., } s > \frac{4n}{(r+1)^2}.$$

Given that s is a natural number, the condition can be re-written as:[9]

$$s \geq \underline{s}^r \equiv int\left\{\frac{4n}{(r+1)^2}\right\} + 1.$$

Hence, in a partition with r coalitions, a necessary condition for two coalitions to merge is that the size of each one be at least $\underline{s}^r$. This necessary condition has to be verified for every $r \geq 2$.

To formally state the conditions under which a monopoly might be the SPE outcome, let us denote by $\mathcal{M} \equiv \mathcal{M}^n$ the set of sequences of coalition structures $M = \{\pi^r\}_{r=1}^n$ such that π^n is "all singletons" and, for all $r = 2, ..., n$, $\pi^{r-1} = (\pi^r \backslash \{S_a^r, S_b^r\}) \cup \{S_a^r \cup S_b^r\}$, for some S_a^r and S_b^r in π^r satisfying $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$.

Similarly, for any $r^\circ = 1, ..., n$, we denote by $\mathcal{M}^{r^\circ}$ the set of sequences of coalition structures $M^{r^\circ} = \{\pi^r\}_{r=1}^{r^\circ}$ such that π^{r° is any partition of N with r° coalitions and, for all $r = 2, ..., r^\circ$, $\pi^{r-1} = (\pi^r \backslash \{S_a^r, S_b^r\}) \cup \{S_a^r \cup S_b^r\}$, for some S_a^r and S_b^r in π^r satisfying $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$.

According to the previous definition, $\mathcal{M}^1 = \{N\}$. Also, if the sequence $\{\pi^r\}_{r=1}^{r^\circ} \in M^{r^\circ}$, then $\{\pi^r\}_{r=1}^{r'} \in M^{r'}$, for any $r^\circ = 1, ..., n$ and $r' \leq r^\circ$.

Proposition 3 *For any n, there exists a $\bar{\delta} < 1$, such that $\forall \delta \geq \bar{\delta}$, the SPE strategy profiles of the process of sequential coalition-formation satisfy the following properties. Consider a subgame in which the existing partition π^r contains r coalitions:*

(a) If coalitions S_a and S_b are chosen by the mechanism, the merger will not be accepted if $\min\{s_a, s_b\} < \underline{s}^r$ or if $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ does not belong to any sequence of coalitions in $\mathcal{M}^{r-1}$.

[9] We use $int\{m\}$ to denote the integer of $m \in \mathbb{R}$.

(b) If π^r belongs to some sequence of coalitions in $\mathcal{M}^r$, there are two coalitions S_a and S_b in π^r, such that the firms in S_a and S_b accept the merger if they are selected by the mechanism.

(c) The final outcome will be a monopoly if and only if π^r belongs to some sequence of coalitions in $\mathcal{M}^r$. Otherwise, the final outcome will be π^r.

Proof. We prove the proposition by induction over r.
($r = 2$) Take any subgame where only two coalitions S_a and S_b are left, i.e., $\pi^2 = \{S_a, S_b\}$. In such a case, the merger of the two coalitions is N, hence it is always in $\mathcal{M}^1$.

(2.a) If $\min\{s_a, s_b\} < \underline{s}^r$, any firm in the smallest coalition prefers to stay as a duopoly rather than become part of a monopoly. Therefore, every SPE involves rejection of the merger.

(2.b) If π^2 belongs to some sequence of coalitions in $\mathcal{M}^2$, then $\min\{s_a, s_b\} \geq \underline{s}^2$. All the firms in S_a and S_b obtain higher profits by merging. As a consequence, accepting this merger is the only SPE strategy in this subgame.

(2.c) Immediate, after (2.a) and (2.b).

We now assume, by the induction hypothesis, that properties (a), (b), and (c) hold for any $r' < r$ and we prove that they are also satisfied for r, where $r = 3, ..., n$.
(r) Let π^r be the existing partition.

($r.a$) Suppose that the coalitions S_a and S_b in π^r have been chosen by the mechanism and the firms in these coalitions must decide whether to merge or not. If the partition $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ does not belong to any $M^{r-1} \in \mathcal{M}^{r-1}$ then, according to the induction hypothesis (c), the final outcome will be $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$. On the other hand, if the firms in one of the coalitions choose never to merge (not necessarily the optimal strategy, but one possibility), they obtain, from this moment on, at least the benefits that they have under the structure π^r. Given that $r > 2$, $V_i(\pi^r) > V_i((\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\})$ for either every firm in S_a or every firm in S_b. Therefore, merging is not the optimal strategy for any of the firms, either in S_a or in S_b.
Similarly, let us suppose that $\min\{s_a, s_b\} < \underline{s}^r$. According to the induction hypothesis (c), the final outcome will be either a monopoly or $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ if S_a and S_b merge. In both cases, the firms of the smallest coalition will obtain lower profits than in

π^r. Hence, here also, merging is a strategy that is strictly dominated (for the firms in the smallest coalition) by the strategy of never merging from this moment on.

($r.b$) We now prove that if π^r belongs to some $M^r \in \mathcal{M}^r$ then the strategies of the members of (at least) two coalitions S_a and S_b in π^r will be to accept the merger if they are selected by the mechanism. We do the proof by contradiction. If no merger happens, the final outcome is π^r. Take a pair of coalitions S_a and S_b in π^r, such that $\min\{s_a, s_b\} \geq \underline{s}^r$ and $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ belongs to some $M^{r-1} \in \mathcal{M}^{r-1}$ (the existence of such a pair of coalitions is guaranteed by the definition of $\mathcal{M}^r$). The members of S_a and S_b obtain higher profits in a monopoly than staying in π^r, since $\min\{s_a, s_b\} \geq \underline{s}^r$. Also, a monopoly is the final outcome if $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ is reached, given that it belongs to some $M^{r-1} \in \mathcal{M}^{r-1}$ and the induction hypothesis (c) holds. Therefore, if δ is close enough to 1, the members of S_a and S_b strictly prefer to arrive at a monopoly after some periods than to stay in S_a and S_b forever. They will therefore, have incentives to change their strategy and accept the merger.

($r.c$) is a direct consequence of ($r.a$), ($r.b$), and the induction hypothesis (c). ■

Proposition 3 gives a lot of information about SPE when the discount factor δ is high. It provides the two main characteristics of the SPE outcome. First, in a SPE strategy profile, the members of two randomly chosen coalitions will only decide to merge if the resulting coalition structure belongs to some sequence $M^r \in \mathcal{M}^r$. Secondly, when it is possible to keep the chain of coalitions in a sequence in $\mathcal{M}^r$, then at least one pair of coalitions will decide to merge. The two properties together imply that, if we start from a partition in some $\mathcal{M}^{r\circ}$, the firms will form coalitions and end up all together.

Hence, form Proposition 3 we can conclude that if the "all singletons" coalition π^n belongs to some sequence $M \in \mathcal{M}$, then a monopoly is the final outcome. Moreover, a monopoly can only be reached through sequences in $\mathcal{M}$.

The next natural question is whether the set $\mathcal{M}$ that we have identified exists or not. To see that it is sometimes empty, it is sufficient to verify that it is empty for $n = 3$ or $n = 4$. The following lemma provides a sufficient condition for $\mathcal{M}$ to contain some sequence of coalition structures.

Lemma 1 *If n is large enough, then $\mathcal{M}$ is non-empty.*

Proof. The proof proceeds as follows: First, we construct a sequence $M = \{\pi^r\}_{r=1}^n$ by starting from "the grand coalition", $\pi^1 = N$, and splitting up one coalition each time. Secondly, we prove that the sequence M belongs to the set $\mathcal{M}$.

a) We denote by S_a^r and S_b^r the two coalitions that are split from π^{r-1}, i.e., $\pi^r = (\pi^{r-1} \backslash \{S_a^r \cup S_b^r\}) \cup \{S_a^r, S_b^r\}$ (the interpretation is that S_a^r and S_b^r are the "candidates" for a merger if the coalition structure π^r emerges).

We divide $S^1 = N$ into S_a^2 and S_b^2 with $s_a^2 = n - \underline{s}^2$ and $s_b^2 = \underline{s}^2$. From this point on, we divide the selected coalition into two, of equal sizes, or at least as equal as possible. For $r = 3$, S_a^3 and S_b^3 are obtained by dividing S_a^2 in such a way that $s_a^3 = s_b^3 = \frac{n-s_a^2}{2}$ if s_a^2 is even and $s_a^3 = s_b^3 + 1 = \frac{n-s_a^2+1}{2}$ if s_a^2 is odd. For $r \geq 4$, we split the largest coalition in π^{r-1}, which corresponds to the largest coalition of those with the smallest index in π^{r-1}. Formally, we divide $S_b^{\frac{r}{2}}$ if r is even, and $S_a^{\frac{r+1}{2}}$ if r is odd (see Figure 1).

[Insert Figure 1.]

b) We relegate the proof that the sequence M, previously constructed, belongs to $\mathcal{M}$ when n is large enough to the Appendix. ■

We denote the set of natural numbers for which the set $\mathcal{M}$ is non-empty by $\mathcal{N}$. According to Lemma 1, for a sequence to exist in $\mathcal{M}$, the number of initial players is crucial. In fact it can be shown that the set $\mathcal{N}$ contains all the numbers higher than or equal to 37.[10] Let us explain why starting with a large number of firms facilitates arriving at a monopoly. Two coalitions must not be very different in size to be willing to merge, but this is a requirement to be fulfilled throughout the entire sequence of mergers. If, at any stage all of the coalitions are too similar, when two of them merge they create a great coalition compared to the others, and the small ones may stop the process by free-riding on the big one. With many players, there is a way of having coalitions whose sizes are balanced enough at every stage.

To highlight the previous argument, consider the case of three firms. In order to reach "the grand coalition", a firm of size 2 has to merge with the a firm of size 1. This process will, however, not be completed because the duopoly is very asymmetric. The firm alone receives higher profits in the duopoly than the third it would obtain from the monopoly

[10] One can also check that $\mathcal{N}$ also includes 15, 22, 23, 26, 29 to 31, and 33 to 35.

profits. Consider now, the case $n = 39$. For the same reason as stated before, a sequence of mergers that leads to a duopoly with a firm of size 26 and another of size 13 will never arrive at "the grand coalition". However, a path yielding a duopoly with firms of sizes 21 and 18 will eventually end up as a monopoly. In the previous step (a triopoly), two firms of sizes 10 and 11, for instance, are not too small and so they prefer to reach "the grand coalition" than to stay in the triopoly.

The next proposition states the main result of this paper by combining Proposition 3 and Lemma 1. It shows that if $n \in \mathcal{N}$ and δ is large enough, then the firms will enter into a sequential process of forming coalitions that will end up in the creation of a monopoly.

Proposition 4 *If $n \in \mathcal{N}$, there exists a $\bar{\delta} < 1$, such that $\forall \delta \geq \bar{\delta}$, the final outcome of any SPE of the process of sequential coalition-formation is "the grand coalition".*

Proof. Immediate, after Proposition 3 and Lemma 1 ■

In our coalition-formation game, only the extreme coalition structures, "all singletons" or "the grand coalition", can be equilibrium outcomes. Proposition 4 shows that, when the number of initial players is high enough and these players are patient enough, the efficient outcome is the only equilibrium outcome. That is, under these two conditions, the possibility of establishing bilateral agreements sequentially makes the firms merge in such a way that they end up being a monopoly. This result is in contradiction with previous results of merger games. Indeed, as we have discussed in the Introduction, "the grand coalition" is often not an equilibrium (or stable) outcome and, when it is, it is not the only one.

It is difficult to give a complete characterization of the set of SPE strategies. The reason for this is that the members of two coalitions may have incentives to wait to merge (even if they keep the coalition structure on a "good path") in order to obtain short-term profits when they know that some other coalitions will eventually start merging and lead to a monopoly. The next proposition specifies some SPE strategies for the coalition-formation game in the particular case of the players being perfectly patient ($\delta = 1$).

Proposition 5 *If firms evaluate future profits according to the "time-average criterion", then the following strategy profile is a SPE profile in the game of coalition formation:*

At any period at which the members of the coalitions S_a and S_b in partition π^r have to chose whether to merge or not, they will merge if and only if $\min\{s_a, s_b\} < \underline{s}^r$ *and the resulting partition* $(\pi^r \backslash \{S_a, S_b\}) \cup \{S_a \cup S_b\}$ *belongs to some sequence of coalitions in* $\mathcal{M}^{r-1}$.

Proof. Straightforward, after the proof of Proposition 3. ■

Note that the above-stated equilibrium strategy profile is symmetric. Moreover, it is stationary. As we have pointed out in the Introduction, our results remain valid if we only allow for stationary SPE strategies.

5 Comments and Extensions

In this paper, we have shown that when the initial number of firms is large enough and they are forward-looking, a sequential process of bilateral agreements will lead to the creation of a monopoly ("the grand coalition"). In this section, we discuss the main ingredients of our model by proposing several processes of gradual agreements. We introduce modifications that affect the timing of the coalition formation and the production stages, the protocol that chooses the candidates for mergers, the exogenous sharing rule, and the bilateral nature of the agreements.

5.1 Timing of the Production Stage

Let us consider the case where production takes place and profits are realized only after the whole process of coalition formation has ended. This is the framework that most models in the literature have considered.[11] The difference between this game and the one described in Section 2 is that, in the latter, production takes place at every period, while in the former, it is only undertaken once the coalition-formation stage has finished. In fact, this variant makes the analysis simpler.

To adapt our model to this framework, let us assume the same protocol for coalition-formation as before. Staying in the process of coalition-formation is costly, since players discount future payoffs. Firms, hence, can decide to end the coalition formation stage

[11] See Bloch (1996), Ray and Vohra (1999), and Montero (1999).

and move to the production stage if they wish. We consider the following natural rule to allow the firms to quit the coalition-formation stage: *"at the beginning of each round, firms are asked sequentially whether to move to the production stage, or to continue with the coalition-formation process. The formation of coalitions continues only if all the firms agree to it."*

In this set-up, all of our results still hold. In fact, the coalition formation process is fostered in this framework, since the absence of intermediate payoffs reduces the possibility of free-riding. Moreover, the equilibrium players' strategies are easier to characterize. For example, in the result that would parallel Proposition 3, firms will all decide to merge if and only if such decision minimizes the expected losses from discounting. Notice that, in this case, all the firms share the same objective function when they decide whether to merge or not. Finally, the players' strategies when the existing coalition structure is π^r, specify that they will decide to move to the production stage if and only if π^r does not belong to any sequence of coalitions in $\mathcal{M}^r$.

5.2 Protocol

Another feature of our coalition-formation game which is not crucial for obtaining the results, is the random choice of the coalitions that are chosen in any given period. If a deterministic protocol selects the identity of the two coalitions that can merge, the results still hold, provided that the protocol is exhaustive in the set of possible couples for each coalition structure (i.e., all the possible pairs of coalitions in any coalition structure are called by the protocol at some moment).

The analysis is equally robust in scenarios where the protocol selects (either randomly or deterministically) one of the coalitions, which then has the possibility to offer a merger to any other coalition. Finally, we can also consider situations in which a particular player is in charge of naming, at each period, the two coalitions that may merge. In this case, at the SPE of the game with a high level of patience, "the grand coalition" will be attained through the sequence of mergers that is most favorable to such a player. The same happens if, for example, the coalitions that decide whether to merge or not at time t are chosen by some player belonging to one of the coalitions selected at time $t-1$.

5.3 Endogenous Sharing Rule

We have chosen to study the outcomes of a coalition formation procedure when the payoffs of the players, at any moment, depend exclusively on the coalition structure prevailing at that moment. Indeed, we have assumed an exogenous equal-sharing rule that is independent of the history. We could also study the outcomes of a similar procedure allowing for endogenous sharing rules that would depend on the bargaining power of the coalitions at the moment when they have to decide whether to merge or not. Although it may seem at first sight that allowing for endogenous sharing rules should help the formation of coalitions, since it allows for compensating players in any way, this possibility makes forming coalitions more difficult. The reason for this is that merging at an early stage lowers the bargaining power of the players in the continuation of the game. Hence, although the final mergers are easier to implement, the players have no incentive to start the process.

The SPE outcome of the linear Cournot game with endogenous sharing rule is that all the players remain as singletons. To illustrate this result, consider a variation of our coalition formation game in which, of the two coalitions that have to decide whether to merge or not, one of them is chosen randomly and must make a proposal to the other concerning the sharing of the surplus. In expected terms, the possible surplus will be shared equally between the two coalitions (not necessarily equally among the firms, since the coalitions can be of different sizes). Imagine a situation where all the players have been merging continually until the structure in the market is of three coalitions. The sum of the payoffs of the firms in each of the coalitions is $1/16$. If a duopoly is formed, the two coalitions will have incentives to merge, each eventually obtaining an expected payoff of $1/8$, since they share the benefits of the monopoly, i.e., $1/4$. But this implies that no two coalitions in a triopoly will have any incentive to merge, forming first a duopoly in which their join profits decrease, to end up obtaining the same profit. Note that this argument is independent of the size of the existing coalitions, which is not true in the game proposed in our paper where the two smallest coalitions in a triopoly may have incentives to merge.

5.4 Multilateral Agreements

The bilateral nature of the agreements is a key feature of our analysis. The results obtained in this paper do not extend if the players have the possibility of forming coalitions of any size in a single round.

Consider the following variant of our coalition formation game. At each period t, one of the existing coalitions S_a in π_t is randomly chosen. The firms in S_a choose any set of potential partners. That is, they select any subset Z of π_t such that $S_a \in Z$. All of the firms in Z sequentially decide whether to accept the offer or not. The merger is formed if all the firms agree on it, otherwise the same coalition structure remains until period $t+1$. Other than this modification, the coalition formation and production game is undertaken under the rules described in Section 2. Recall that this is the game proposed in Bloch (1996), with the difference that production takes place at each and every period (as discussed previously, this difference is innocuous) and that once a coalition has been formed it does not leave the game, so it can be part of another future merger. This last difference is shown to be crucial.

In this framework, the set of SPE outcomes (as well as the set of stationary SPE outcomes) is quite large. In particular, monopoly can be sustained as a SPE outcome. Indeed, suppose that, when π_t is "all singletons", each player's strategy prescribes accepting an offer if and only if $Z = \pi^1$ and proposing $Z = \pi^1$ if he has been chosen by the protocol. Also, players follow any SPE strategy profile in those subgames where at least one merger has already taken place. One can prove, by using the one-stage deviation principle,[12] that the previous profile is an SPE whose outcome is monopoly, independently of the discount rate. However, many other coalition structures are also SPE outcomes for every discount rate. For instance, take $S \subset N$ such that $s \geq s^*$, where s^* is the minimum size of any profitable coalition, defined by Salant, Switzer, and Reynolds (1983) that represents about 80% of the market. Consider strategies where, when π_t is "all singletons", the players in S accept any offer involving at least s players, and propose $Z = S$ if they are selected by the protocol, while players outside S never accept nor offer any merger, and complete the profile by taking any SPE profile for the remaining subgames. This profile

[12] For a formal statement of this principle see, for example, Fudenberg and Tirole (1991).

also constitutes an SPE that yields a coalition structure with a coalition of s players and $(n-s)$ singletons. For the particular case when $s = s^*$, this is precisely the outcome of the game proposed by Bloch (1996).

Many equilibrium profiles that induce more than one merger can also be devised. In particular, any duopoly in which the size of the smallest coalition is less than $(4/9)n$ (so that the members of this coalition are better off in duopoly than in monopoly) can be sustained by SPE strategies if δ is high enough. The SPE profile will be such that the small coalition is formed first, and then the big coalition forms. Clearly, similar arguments allow more complex coalition structures.

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6 Appendix

Proof of Lemma 1.b) We prove that the sequence M constructed in part a) of the proof of Lemma 1 belongs to $\mathcal{M}$ when n is large enough. We do the proof by induction over r. For each r, we provide conditions over n under which the two "candidate" coalitions S_a^r and S_b^r satisfy $\min\{s_a^r, s_b^r\} \geq \underline{s}^r$. Note that, since the minimum size of a coalition is 1, when $\underline{s}^r = 1$, the previous condition imposes no restriction on the size of the coalitions. This is the case if

$$int\left\{\frac{4n}{(r+1)^2}\right\} = 0, \text{ i.e., } r > r_{\max}(n) \equiv \sqrt{4n} - 1.$$

Therefore, we concentrate on $r \in [2, r_{\max}(n)]$.
$(r = 2)$ $\min\{s_a^2, s_b^2\} \geq \underline{s}^2$ holds if and only if $s^2 \geq \underline{s}^2$, that is $s^1 = n \geq 2\underline{s}^2$, i.e.,

$$n \geq 2\left(int\left\{\frac{4n}{9}\right\} + 1\right).$$

A sufficient condition for the above inequality to hold is $n \geq 18$.
$(r = 3)$ The condition $\min\{s_a^3, s_b^3\} \geq \underline{s}^3$ is always satisfied if:

$$\frac{s_a^2 - 1}{2} \geq \underline{s}^3, \text{ i.e., } n - \left(int\left\{\frac{4n}{9}\right\} + 1\right) - 1 \geq 2\left(int\left\{\frac{n}{4}\right\} + 1\right).$$

It can be shown that the above inequality holds as soon as n is large enough.

For any $r \geq 4$, the sizes of the coalitions S_a^r and S_b^r (which are as equal as possible) sum up to the size of the largest coalition in π^{r-1}. Since the size of such a coalition is at least $\frac{n}{r-1}$, we have that $\min\{s_a^r, s_b^r\} \geq \frac{\frac{n}{r-1} - 1}{2}$. Therefore,

$$\min\{s_a^r, s_b^r\} \geq \underline{s}^r \text{ if } \frac{n}{r-1} - 1 \geq 2\left(int\left\{\frac{4n}{(r+1)^2}\right\} + 1\right).$$

This inequality holds if:

$$f(r) \equiv \frac{3\,(r+1)^2\,(r-1)}{(r-3)^2} \leq n.$$

Since the function $f(r)$ is first decreasing (from $r = 4$ on) and then increasing, the previous inequality holds for all relevant r if it is satisfied at the extreme values $r = 4$ and $r = r_{\max}(n)$. It can be shown that this happens as long as n is large enough.

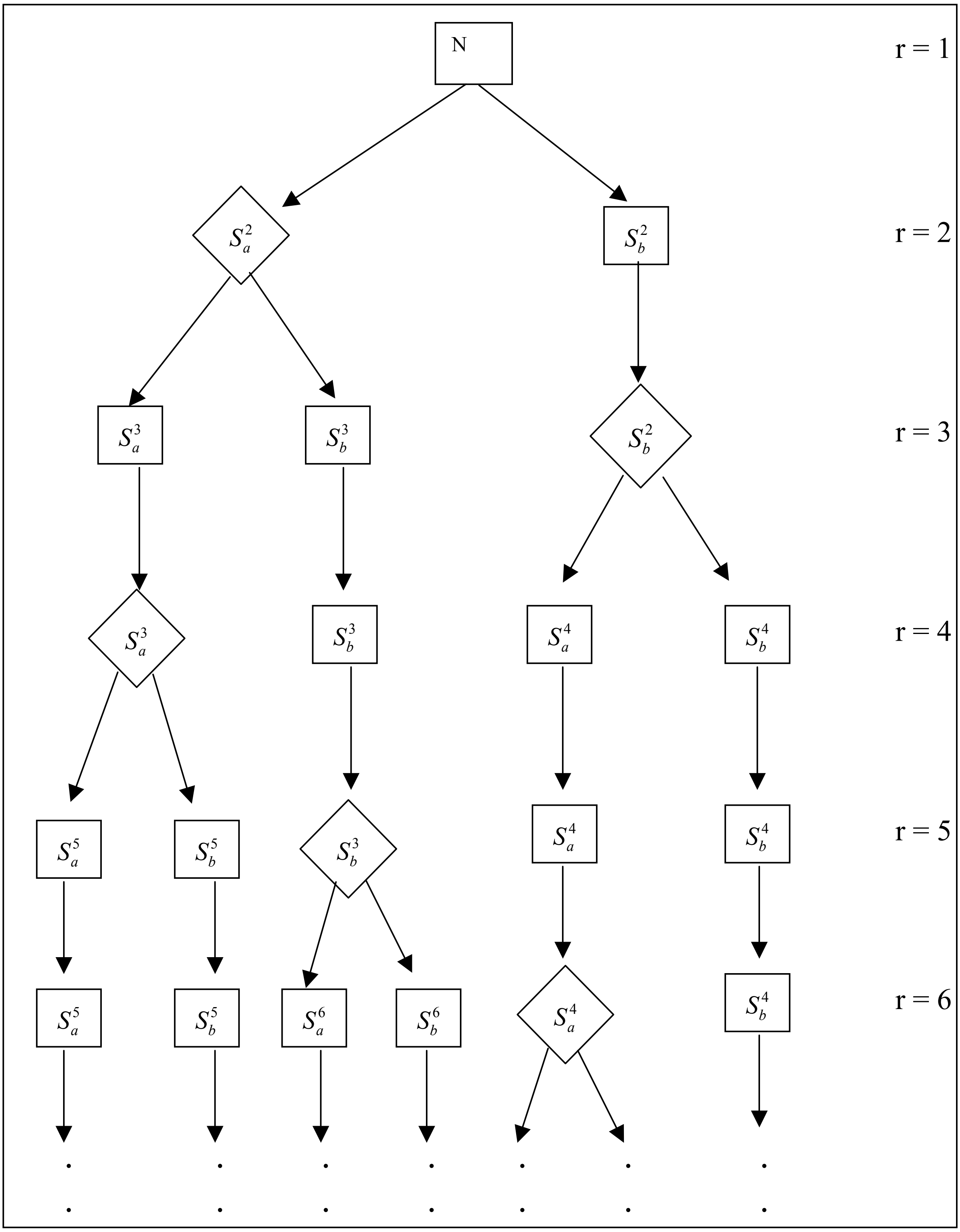

Figure 1: Outline of the Sequence of Moves