Unemployment and Wage Formation in a Growth Model with Public Capital*

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Abstract

We study the relation between public capital, employment and growth under different assumptions concerning wage formation. We show that public capital increases economic growth, and that, if there is wage inertia, employment positively depends on both economic growth and public capital.

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1. Introduction

In the economic growth literature some papers analyze the relationship between growth and unemployment. Pissarides (1990), using a matching model of the labor market, finds a positive effect of growth on employment via the capitalization effect[1]. Aghion and Howitt (1994) adds to this positive effect a negative one due to the creative destruction effect. In this paper, using a monopoly union model, we find that there is also a positive effect of growth on employment if there is wage inertia, that is, when the wage set in one period depends on previous wages. This occurs because, due to the wage inertia, the increases in productivity, which are associated to economic growth, do not fully translate into wage increases that prevent employment growth.[2]

In this model sustained growth is due to the introduction of public capital. Increases in public capital enhance growth which positively affects employment when there is wage inertia. More precisely, when the wage depends on both past wages and the unemployment benefit, the employment rate converges to a long-run steady state value which increases with public capital. In contrast, when the wage only depends on past wages, the employment rate grows at a constant rate until full-employment is achieved. In this case, increasing public capital enhances employment growth during the transition to full-employment. Finally, when the wage only depends on the unemployment benefit and, thus, there is no wage inertia, the employment rate is constant and does not depend on public capital.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and shows how the properties of the equilibrium depend on the wage formation process. Also in this section, we compare the effects on growth and employment of increasing public capital when different modes of government financing are considered. Section 4 concludes.

2. The Economy

Firms produce the only good of the economy using the following production function introduced by Barro (1990):

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} g_t^{1-\alpha}, \ \alpha \in (0,1),$$

where Y_t is aggregate output, K_t is the aggregate stock of capital, L_t is the labor force, and g_t measures the services derived from the stock of public capital in the

[1] "an increase in growth raises the rate at which the returns for creating a plant (or a firm) will grow and hence increases the capitalized value of those returns, thereby encouraging more entry by new plants and therefore more job creation"(Aghion and Howitt (1998) pp. 127).

[2] Bean and Pissarides (1993), Eriksson (1997) and Daveri and Tabellini (2000) present models that analyze the influence of different exogenous variables on growth and unemployment. Nevertheless, none of these models includes wage inertia.

economy. Profit maximization implies that factor prices are equated to marginal productivities so that the interest rate is

$$r_t = \frac{\partial Y_t}{\partial K_t},$$

and the labor income is

$$w_t = \frac{\partial Y_t}{\partial L_t}. \tag{2.1}$$

Equation (2.1) characterizes the labor demand.

Following many others, we assume that the unions' preferences are characterized by the following Stone-Geary utility function:

$$V(w_t) = ((1-\tau_w)(1-\tau)w_t - w_t^r)^\gamma L_t, \; \gamma \in (0,1).$$

Thus, unions' utility depends on both employment and the difference between the wage net of taxes and a reservation wage, w_t^r (see de la Croix et al. 1996). The parameter γ is a measure of the concavity of the utility function with respect to the difference between the wage and the reservation wage, τ_w is a tax on the labor income that employed workers pay to finance the unemployment benefit and τ is an income tax. Unions choose a wage that maximizes the utility function taking into account that the labor demand depends on the wage (union's monopoly model). The solution of the program is

$$w_t = \frac{w_t^r}{(1-\tau_w)(1-\tau)(1-\gamma\alpha)},$$

where $-\frac{1}{\alpha}$ is the elasticity of the labor demand which is constant.

Following de la Croix et al. (1996), we assume that the reservation wage is a weighted average of the unemployment benefit and of the wage in the previous period[3]

$$w_t^r = \phi d_t + (1-\phi)(1-\tau_w)(1-\tau)w_{t-1},$$

where d_t is the unemployment benefit net of taxes and $\phi \in [0,1]$ provides a measure of the intensity of past wages in the wage formation process. It follows that the wage equation is

$$w_t = \frac{\phi d_t + (1-\phi)(1-\tau_w)(1-\tau)w_{t-1}}{(1-\tau_w)(1-\tau)(1-\gamma\alpha)}. \tag{2.2}$$

The previous wage equation shows the existence of wage inertia provided $\phi < 1$. Wage inertia could also be derived in an efficient wage model where workers' disutility depends on the comparison between current and past wages (see Collard et al. 2000 and de la Croix et al. 2000). Therefore, the assumption that drives the results is not

[3] We could also interpret w_t^r as an aspiration wage. In this case the weights associated to the previous wage and the unemployment benefit would be positive constants that eventually could be larger than one. As noted by Blanchard and Katz (1997), the reservation wage is not observable and as they say "models based on fairness suggest that the reservation wage may depend on factors such as the level and the rate of growth of wages in the past, if workers have come to consider that wage increase as fair. Perhaps a better word than reservation wage in that context is aspiration wage" (see pp. 54).

the wage setting under unionism but that agents' utility depends on the comparison between present and past wages.

On the consumers' side, we consider a standard overlapping generations model (OLG, henceforth). We assume that each consumer lives for two periods. In the first period, consumers inelastically supply one unit of labor, consume, and save. In the second period, they consume the income generated by the savings accumulated during the first period. Moreover, we assume that in each period t, there are N_t consumers in their first period of life, and that population grows at a constant growth rate, $n \geq -1$. For simplicity, we assume that consumers' utility function is homothetic so that the savings function is a constant fraction of income, i.e., $s_t = sI_t$ where $s \in (0,1)$, and $I_t = (1-\tau_w)(1-\tau)w_t$ when the consumer is employed and $I_t = d_t$ when the consumer is unemployed. Because each agent inelastically supplies one unit of labor in the first period, the aggregate labor supply is equal to N_t, and aggregate savings are equal to

$$S_t = s\left(L_t(1-\tau_w)(1-\tau)w_t + (N_t - L_t)d_t\right),$$

where $N_t - L_t$ are the unemployed workers that receive the unemployment benefit.

The government collects taxes in order to finance both the unemployment benefit and a public input. More precisely, the unemployment benefit, as we said, is financed by means of taxes on the labor income payed by workers[4], i.e.

$$(N_t - L_t)d_t = \tau_w w_t L_t, \tag{2.3}$$

and government revenues, R_t, are equal to

$$R_t = \tau_k r_t K_t + (1-\tau_w)\tau w_t L_t,$$

where τ_k is the tax on the interest rate. We also assume that the government devotes a fraction v of the production to the public input and that the services derived from the public input are congested by the number of workers in the economy.[5] This implies that the services derived from the public input are

$$g_t = \frac{vY_t}{L_t}. \tag{2.4}$$

Furthermore, we assume that the government budget constraint is balanced in each period, i.e.

$$vY_t = R_t. \tag{2.5}$$

3. The Equilibrium

In this section, we derive the equations that characterize the equilibrium of this economy. To this end, we first derive the equilibrium production function and the equilibrium government budget constraint.

[4] De la Croix et al. (1996) assumes that the unemployment benefit is financed by means of taxes on the labor income payed by both workers and firms. For simplicity, we assume that only workers pay taxes to finance the unemployment benefit.

[5] The introduction of a congestion effect avoids scale effects which are not empirically supported.

Substituting (2.4) into the production function and solving for g_t, we obtain

$$g_t = (vA)^{\frac{1}{\alpha}} \left(\frac{K_t}{L_t}\right).$$

Plugging the previous expression into the production function, we derive the production function in equilibrium

$$Y_t = BK_t, \tag{3.1}$$

where $B = A(vA)^{\frac{1-\alpha}{\alpha}}$ measures total factor productivity. Next, combining (2.1) and (2.5), we derive the equilibrium government budget constraint

$$v = \alpha\tau_k + (1-\alpha)\,\tau\,(1-\tau_w)\,.$$

In equilibrium, the savings accumulated by the consumers are the next period stock of capital, i.e., $K_{t+1} = S_t$. Using the aggregate savings function and (2.3), we get

$$K_{t+1} = s\left((1-\tau_w)(1-\tau)+\tau_w\right) w_t L_t.$$

Combining (2.1) with (3.1), we obtain the growth rate of capital

$$\frac{K_{t+1}}{K_t} = s\,(1-\alpha)\,B\,(1-\tau\,(1-\tau_w)) = G, \tag{3.2}$$

which coincides with the rate of growth of output as follows from (3.1).

Using (2.1), we derive the labor demand

$$w_t = (1-\alpha)\,B\left(\frac{K_t}{L_t}\right).$$

Using the previous equation and (3.2), we obtain

$$\frac{w_t L_t}{w_{t-1}L_{t-1}} = G. \tag{3.3}$$

Equation (3.3), derived from the labor demand, implies that the aggregate labor income grows at a constant growth rate which coincides with the economic growth rate. The reason is that the aggregate labor income is a constant fraction of production and, thus, it grows with production. The increase in the aggregate labor income may imply either larger wages or larger employment. We will show that if there is no wage inertia then an increase in economic growth fully translates into wage growth and there is no increase in the employment rate. Therefore, only when there is wage inertia, economic growth causes employment growth.

Next, we combine the wage equation, (2.2), with (2.3) to derive the growth rate of wages

$$\frac{w_t}{w_{t-1}} = \frac{\frac{\lambda_2}{1+n}}{1-\lambda_1\left(\frac{L_t}{N_t-L_t}\right)}, \tag{3.4}$$

where

$$\lambda_1 = \frac{\phi\tau_w}{(1-\tau_w)(1-\tau)(1-\gamma\alpha)} \text{ and } \lambda_2 = \frac{(1+n)(1-\phi)}{1-\gamma\alpha}.$$

Equation (3.4) shows that the growth rate of wages negatively depends on the unemployment rate. According to Blanchard and Katz (1999) this wage equation is empirically supported by US data.

Let us define the employment rate by $l_t = \frac{L_t}{N_t}$. Combining (3.3) and (3.4), we obtain the dynamic equation that characterizes the equilibrium rate of employment

$$G = \frac{\lambda_2 \left(\frac{l_t}{l_{t-1}}\right)}{1-\lambda_1 \left(\frac{l_t}{1-l_t}\right)}. \tag{3.5}$$

An equilibrium of this economy is a set of sequences $\{l_t, K_t\}_{t=0}^{\infty}$ such that jointly satisfy (3.2), (3.5), an initial condition on the stock of capital, K_0, and an initial condition on the labor income, w_{-1}.[6] Moreover, a balanced growth path equilibrium (BGP, henceforth) is an equilibrium path where capital grows at a constant growth rate and the employment rate remains constant. The following proposition characterizes the BGP:

Proposition 3.1. *There exists a unique BGP equilibrium. Along this path,* $\frac{K_t}{K_{t-1}} = G$ *and*

$$l = 1 - \frac{\lambda_1}{1+\lambda_1-\frac{\lambda_2}{G}}.$$

Proof. The proof follows from (3.2) and imposing $l_t = l$ for all t in (3.5).

In order to guarantee for a well defined BGP, that is $l \in [0,1]$, we must assume that the parameters satisfy the following relation: $G > \lambda_2$. Next, in the proposition below, stability of the BGP is discussed.

Proposition 3.2. *Assume that* $\phi \in (0,1)$. *Then, the BGP equilibrium is globally stable. Thus, the dynamic equilibrium converges to the BGP from any initial condition.*

Proof. Using (3.5), it can be shown that if $l_t = l$ then $\frac{\partial l_t}{\partial l_{t-1}} \in (0,1)$. This means that the BGP is locally stable and because there is a unique BGP it is also globally stable.

While the growth rate of both capital and output is constant along the equilibrium path as follows from (3.2), the employment rate changes along the transition to the BGP if $\phi \in (0,1)$. Moreover, because $\lambda_2 > 0$, when $\phi \in (0,1)$ and thus there is inertia in the wage formation process, the employment rate positively depends on the economic growth rate and on public capital as follows from (3.2). This result points out the importance of the wage formation process in driving the dynamics of employment. Actually, when $\phi = 1$ and thus the reservation wage does not depend on past wages, $l_t = l = \frac{1}{1+\lambda_1}$ for all t. In this case, the employment rate does not

[6] Unions set the initial wage using the value of the wage in the previous period, i.e. w_{-1}.

exhibit transition and the growth rate does not affect the rate of unemployment which implies that an increase in public capital causes more growth but does not affect the rate of unemployment. In contrast, when $\phi = 0$ and thus the reservation wage coincides with the past wage, (3.5) simplifies into the following equation: $\frac{l_t}{l_{t-1}} = \frac{G}{\lambda_2}$. Because of the assumptions made, this gross growth rate is larger than one implying that the employment rate monotonically grows until full employment is achieved. Moreover, economic growth increases the growth rate of employment during the transition. Therefore, the equilibrium path crucially depends on the value of the parameter ϕ. Interestingly, the empirical literature finds that the value of ϕ differs substantially between countries and depending on the use of micro and macro data (see, for example, Blanchard and Katz (1997), (1999)).

We proceed to derive the long run effects on employment and on the growth rate of increasing public capital when $\phi \in (0,1)$. Furthermore, we compare the effects of increasing public capital under different modes of government financing and we discuss the effects of increasing the unemployment benefit. The results are given in the following proposition:

Proposition 3.3. *Assume that* $\phi \in (0,1)$ *and let* $v^2 = (1-\alpha)(1-\alpha(1-\tau_k))$, $v^1 \in (0, v^2)$, *and* $v^3 > v^2$. *Then,*

a) $\frac{\partial G}{\partial \tau_k} > 0$, *and if* $v > (<)\, v^2$ *then* $\frac{\partial G}{\partial \tau} < (>)\, 0$ *and* $\frac{\partial G}{\partial \tau_w} > (<)\, 0$.

b) $\frac{\partial l}{\partial_{\tau_k}} > 0$, *if* $v < (>)\, v^1$ *then* $\frac{\partial l}{\partial \tau} > (<)\, 0$, *and if* $v < (>)\, v^3$ *then* $\frac{\partial l}{\partial \tau_w} < (>)\, 0$.

Proof. Part a follows from the definition of the long run growth rate in (3.2). Part b follows from the definition of the long run employment rate in Proposition 3.1.

Increasing public capital enhances the marginal product of labor and, hence, the wage increases. The increase in wages accelerates savings which explains the positive effect of public capital on economic growth. Furthermore, because of the assumptions made on the utility function, a tax on the interest rate does not reduce savings. This explains the result in Part a of Proposition 3.3 that shows that increasing public capital always results in a larger growth rate when it is financed by means of taxes on the interest rate. In contrast, an increase in the tax on the labor income drives two opposite forces that affect growth. On the one hand, it increases public capital which accelerates growth. On the other hand, an increase in the tax rate on the labor income reduces the income of the people who save which deters growth as savings are reduced. This explains the ambiguity on the growth effects of increasing this tax. Finally, increasing the tax that is used to finance the unemployment benefit has exactly the opposite effects. It increases the income of the people who accumulate private capital and reduces public capital. Again, this explains that the growth effects of increasing this tax rate are ambiguous and depend on the value of the fraction of production devoted to public capital.

Increasing public capital may enhance the employment rate when it increases economic growth. When the increase in public capital is financed by means of taxes on the interest rate, the employment rate always increases with this tax as growth unambiguously increases. In contrast, when the increase in public capital is financed by means of taxes on the labor income, the employment rate decreases with this tax when the fraction of production devoted to public capital, v, is large. This negative effect may occur because taxes on the labor income enhance the wage payed by firms and, hence, reduce the labor demand. Finally, an increase in the tax used to finance the unemployment benefit results in a reduction in the employment rate unless the fraction of production devoted to public capital is very large. This negative effect occurs because increasing the unemployment benefit makes the wage larger and, hence, reduces the employment rate.

4. Conclusions

In this economy increasing public capital enhances growth and may increase the employment rate. While the first result does not depend on the wage formation process, the second result crucially depends on this process. Moreover, we have pointed out that the properties of the equilibrium both in the long run and during the transition crucially depend on the wage formation process.

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