The Principal-Agent Matching Market*

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Abstract

We propose a model based on competitive markets in order to analyze an economy with several principals and agents. We model the principal-agent economy as a two-sided matching game and characterize the set of stable outcomes of this principal-agent matching market. A simple mechanism to implement the set of stable outcomes is proposed. Finally, we put forward examples of principal-agent economies where the results fit into.

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1 Introduction

A large set of literature contributing to the theory of incentives analyzes optimal contracts in principal-agent relationships when there exist asymmetries of information. When this asymmetry concerns an action, or a decision to be made by an agent, a *moral hazard* problem emerges. Several works analyze optimal contracts when only one principal and one agent interact, including the seminal works by Pauly(1968), Mirlees(1976), and Harris and Raviv(1978). The *principal-agent contracts* involve the provision of incentives and typically lead to inefficiency due to the informational asymmetry.

The main goal of this paper is to propose a useful framework to analyze the relationship between each principal-agent pair not as an isolated entity but as a part of an entire market where several principals and agents interact. In this framework, the utilities obtained by each principal and each agent are determined endogeneously in the market. This allows us to improve over the previous approach where the agents' utilities are exogeneously given and the principals assume all the bargaining power. We consider the simultaneous determination of the identity of the parties who meet (i.e., which agent is contracted by which principal) and the contracts they sign in an environment where each relationship is subject to moral hazard.

We model the principal-agent economy as a two-sided matching game.[1] An outcome of this economy is an endogenous matching and a set of contracts, one for each principal-agent pair under the matching. Roughly speaking, an outcome is said to be *stable* if there is no individual or no relevant pair objecting the existing outcome. The paper studies the set stable outcomes of this principal-agent matching market. In particular, we consider an economy with several identical principals and several agents differentiated only with respect to their initial wealth. A pair of individuals, one principal and one agent, can enter into a relationship by signing a contract. This contract specifies the contingent payments that are to be made by the agent. Also it sets the level of investment, which together with a non-verifiable effort made by the agent, determines the probability of having a high return from the project the agent operates on. The initial wealth of the agent may not cover the amount to be invested and hence, the wealth differences imply differences in liability.

We begin by providing a complete characterization of the set of stable outcomes of the principal-agent economy. The first simple property we prove is that all the principals earn the same profit in a stable outcome. In particular, if the principals constitute the long side of the market, their profits are zero. The second feature is that the contracts offered in a stable outcome are Pareto efficient, i.e., it is not possible to increase the utility level of the principal without making the agent strictly worse-off. More interestingly, in a stable outcome, the matching itself is efficient, in the sense that it is the one that maximizes productive efficiency. For example, if the agents are in the long side of the market, only the wealthier ones, i.e., the more attractive ones (and all the principals) are matched. Third, the productive efficiency of a contract signed in a stable outcome increases with the wealth of a matched agent. That is, the richer the agent, closer is his contract to the first-best. The additional surplus generated due to this increase

[1] See also Serfes(2001). We will comment on this work later on in this section.

in efficiency accrues to the agents. Finally, the contracts signed in a stable outcome of this economy are more efficient than principal-agent contracts, i.e., the contracts signed when the principals assume all the bargaining power.

The previous characteristics of the set of stable outcomes have very relevant policy implications when applied to particular environments. For example, consider an economy where landowners (principals) contract with tenants (agents) who are subject to limited liability. Suppose that the government would like to improve the situations of the tenants by endowing the agents with some additional money. Our analysis suggests that the government will be interested in creating wealth asymmetries among tenants since otherwise, the landowners would appropriate all the incremental surplus intended to the tenants.

We establish a close relationship between the concept of stability and that of a competitive equilibrium. We define a competitive equilibrium as a situation where, given a set of contracts, one for each agent, the principals make their best choices. In our model, the set of stable outcomes coincides with the set of competitive equilibria in which only Pareto efficient contracts are signed.

From the point of view of matching theory, one can see our model as a generalization of the *assignment game* with several buyers and sellers described by Shapley and Shubik(1972).[2] In the current model, a relationship is established through a contingent contract, rather than a price. The first distinguishing feature is that the surplus of each principal-agent pair, in our model, is determined endogenously. Next, the utility cannot be transferred between a principal and an agent on a one-to-one basis. In other words, unlike the assignment game, our model is a non-transferable utility game. Notice that, in the assignment game the set of stable outcomes is equivalent to the set of competitive equilibria. But a similar notion of competitive equilibrium defined in our paper does not automatically guarantee the (Pareto) efficiency of the contracts signed. The difference is that while in the model of Shapley and Shubik(1972) any price induces an Pareto efficient sharing of the surplus between the buyer and the seller, in our framework a principal may take an inefficient contract if it is the only contract available.

We further consolidate stability as a reasonable solution concept for this principal-agent matching market by proposing a simple mechanism in which each of the agents proposes a contract and each principal chooses an agent. We show that the equilibrium outcomes of this mechanism coincide with the set of stable outcomes of the matching market.

Serfes(2001) analyzes an economy where the agents have different attitudes towards risk and the principals own assets which are subject to different exogeneous variability. He characterizes the stable outcome where the principals have all the bargaining power. In his model, a principal-agent pair cannot block an outcome with any contract, rather it is the principal who proposes

[2]The literature on matching models distinguishes two types of situations. In the first type, first analyzed by Gale and Shapley(1962), forming the matching does not involve any exchange between the parties, or equivalently, the amount exchanged is exogeneously fixed. In the second type, called assignment games, proposed by Shapley and Shubik(1972) the parties involved in matching endogenously decide the amount of money to exchange. Roth and Sotomayor(1990) provide an excellent review of the literature of matching models without and with money. In the present paper we extend the previous models by considering situations where the parties involved in a matching are linked by a contract (and not only by an exchange of money).

a contract once a blocking pair is formed. The predictions of the model by Serfes(2001) are different from those of the standard risk model where an isolated principal-agent pair is studied. In particular, there can be a positive, negative, or non-monotonic relationship between risk and incentives. A few other papers study agency problems with several principals and agents. In a tenancy relation Shetty(1988) proposes a model where a set of principals compete for a continuum of agents in the presence of limited liability. Mookherjee and Ray(2001) analyze the optimal short term contrats in an infinitely repeated interaction among principals and agents who are randomly matched at each period. Finally, the work of Barros and Macho-Stadler(1998) looks into a situation where several principals compete for an agent.

The paper is organized as follows. In Section 2 we lay out the basic model. We describe the main results in Section 3. In particular, we characterize the set of stable outcomes and establish the equivalence between the set of stable outcomes and the set of competitive equilibria where only Pareto efficient contracts are signed. In the following section we discuss the characteristics of contracts that are signed in a stable outcome. In Section 5 we propose a sequential mechanism that implements the set of stable outcomes. In Section 6 we put forward some examples of principal-agent economies where the findings fit into. In Section 7 we conclude the paper and indicate some avenues for future research.

2 The Model

2.1 Principals and Agents

We consider an economy with a (finite) set of risk neutral *principals*, $\mathcal{P} = \{P_1, P_2, ..., P_n\}$ and a (finite) set of risk neutral *agents*, $\mathcal{A} = \{A^1, A^2, ..., A^m\}$. A principal might be a landowner, a lender or an employer. An agent is a tenant, a borrower or a worker. Principals are of identical characteristics. Agents differ with respect to their initial wealth. An agent A^j has an initial wealth w^j, which is known to the principals. Without any loss of generality, we order the wealth level as $w^1 \geq w^2 \geq .... \geq w^m \geq 0$. The principals and the agents are matched in pairs and a contract is signed by each pair. We allow for the possibility that a principal or an agent can seek for an alternative partner and can sign a different contract. Hence, the matching is endogenous rather than being exogenous.

2.2 Projects

When all pairs are formed,[3] each agent operates on a project, chooses effort level e from the set $\{0, 1\}$, and makes an investment K, which is financed entirely by the principal he is assigned to. An agent incurs a disutility of e when he chooses the effort level e. The effort exerted is not contractible but the level of investment is.[4] Effort and investment influence the return of each project which is uncertain. Given an effort level e and investment K, let $\pi_e(K)$ be the

[3]There is a possibility that some principals and some agents remain unmatched.

[4]All our findings remain unaltered even if an agent with positive wealth finances part of the investment.

probability of the event of success (denoted by S) and $1-\pi_e(K)$, the probability of failure (denoted by F). Each project generates a return $y>0$ in case of success. In case of failure, the return is 0. We assume (a) $\pi_1(K)>\pi_0(K)$, for all $K>0$, (b) $0\leq\pi_e(K)\leq 1$, for all $K>0$ and $\pi_0(0)=0$ and (c) $\pi'_e(K)>0>\pi''_e(K)$ for all $K>0$ and $\lim_{K\to\infty}\pi'_e(K)=0$. Part (c) guarantees that the solution in K is interior. We denote by $\mathcal{M}\equiv\{\mathcal{P},\,\mathcal{A},\,w,\,\pi\}$ the market, where $w\equiv(w^1,\,...,\,w^m)$ denotes the vector of initial wealth of the agents in $\mathcal{A}$ and π represents the technology.

2.3 Contracts and Payoffs

A principal-agent pair $(P_i,\,A^j)$ signs a contract, c, which is a three dimensional vector $(\theta_S,\,\theta_F,\,K)$. We take the convention that the agent keeps the output. Then the first component of the contract, θ_S is the transfer to the principal in the event of *success* and the second component, the transfer in case of *failure*. The third component of c is the level of investment. Given a contract $c=(\theta_S,\,\theta_F,\,K)$ signed by a pair $(P_i,\,A^j)$, let e_c be defined as the effort that maximizes the agent's utility:[5]

$$e_c = \underset{e}{argmax}\;\{\pi_e(K)(y-\theta_S)-(1-\pi_e(K))\theta_F - e\}. \tag{IC}$$

For a contract c, the effort chosen by the agent will be e_c given that the effort is not contractible. This is the *incentive compatibility* constraint. Moreover, let R be the per unit opportunity cost of financing a project. Then the expected utilities of the principal P_i and the agent A^j when they sign the contract c will be:

$$\begin{aligned} u_{P_i}(A^j,\,c) &= \pi_{e_c}(K)\theta_S+(1-\pi_{e_c}(K))\theta_F - RK \\ u_{A^j}(P_i,\,c) &= \pi_{e_c}(K)(y-\theta_S)-(1-\pi_{e_c}(K))\theta_F - e_c. \end{aligned}$$

Notice that we have defined the expected utility of A^j *net* of the wealth w^j. The gross expected utility of A^j would be $u_{A^j}(P_i,\;c)+w^j$. For future notational convenience, we denote by $c^{null}=(0,\;0,\;0)$, the *null contract*. Under c^{null}, $u_{P_i}(A^j,\;c^{null})=u_{A^j}(P_i,\;c^{null})=0$. We assume that for an agent, signing a contract c^{null} is equivalent to the situation where he is not contracted by any principal, i.e., his *reservation utility* equals 0. Agent's liability is limited to his current wealth. This imposes restrictions on the set of contracts. *Limited liability* implies

$$\theta_S \leq y + w^j\;, \tag{LS}$$

$$\theta_F \leq w^j\;. \tag{LF}$$

The assumption of risk neutrality together with limited liability makes the incentive compatibility constraint costly and hence, it gives rise to *moral hazard* in agent's effort choice. A sensible contract for a principal-agent pair must satisfy the incentive compatibility and limited liability constraints. Furthermore, neither an agent nor a principal would accept a contract with negative expected utility. That is, a contract for a pair $(P_i,\,A^j)$ has to be acceptable to each member of the pair. We say that a contract c is *acceptable* for $(P_i,\,A^j)$ if $u_{P_i}(A^j,\,c)\geq 0$ and $u_{A^j}(P_i,\,c)\geq 0$. We club all these natural restrictions into the following definition.[6]

[5]Conventionally $e_c=1$ if both 1 and 0 maximizes the following expression.

[6]Notice that the limited liability constraints are agent specific.

Definition 1 *A contract is* **feasible for an agent** A^j *if it satisfies the restrictions of limited liability and acceptability.*

Denote by $\mathcal{X}^j$ the set of contracts feasible for agent A^j. From now on we will concentrate only on feasible contracts.

2.4 Matching

Principals and agents are matched in pairs and when a pair is formed, a contract is signed. The following three definitions describe a matching and a relevant outcome of this principal-agent economy.

Definition 2 *A (one-to-one)* **matching** *for* $\mathcal{M}$ *is a mapping* $\mu : \mathcal{P} \cup \mathcal{A} \to \mathcal{P} \cup \mathcal{A}$ *such that (i)* $\mu(P_i) \in \mathcal{A} \cup \{P_i\}$ *for all* $P_i \in \mathcal{P}$*, (ii)* $\mu(A^j) \in \mathcal{P} \cup \{A^j\}$ *for all* $A^j \in \mathcal{A}$ *and (iii)* $\mu(A^j) = P_i$ *if and only if* $\mu(P_i) = A^j$ *for all* $(P_i, A^j) \in \mathcal{P} \times \mathcal{A}$.

The definition implies that a matching for a market $\mathcal{M}$ is a mapping which specifies that either each individual of one side of the market is assigned to another individual of the other side or, the individual remains alone. We say that the pair (P_i, A^j) is *matched* under μ if $\mu(P_i) = A^j$ (or, equivalently, $\mu(A^j) = P_i$).

Definition 3 *A menu of contracts* $\mathcal{C}$ **compatible with a matching** μ *for* $\mathcal{M}$ *is a vector of contracts,* $\mathcal{C} = (c_1, ..., c_n, c^1, ..., c^m)$ *such that (a)* $c_i = c^j$ *if* $\mu(P_i) = A^j$ *and* c^j *is feasible for* (P_i, A^j)*, (b)* $c_i = c^{null}$ *if* $\mu(P_i) = P_i$ *and (c)* $c^j = c^{null}$ *if* $\mu(A^j) = A^j$.

Definition 4 *An* **outcome** $(\mu, \mathcal{C})$ *for the market* $\mathcal{M}$ *is a matching* μ *and a menu of contracts* $\mathcal{C}$ *compatible with* μ.

The outcomes of the market we describe here are endogenous. This endogeneity has two aspects. First, the contracts signed by the principals and the agents are endogenous. In the principal-agent theory, considerable attention has been paid in order to analyze the contracts that prevail in a given (isolated) principal-agent relationship. The second aspect is that the matching itself should be endogenous. We will approach this perspective in the same vein as the matching theory. We require that a *reasonable* outcome should be immune to the possibility of being *blocked* by any principal-agent pair (as well as by any single individual). Consider an outcome $(\mu, \mathcal{C})$. If there is a principal-agent pair which can sign a feasible contract such that both the principal and the agent are strictly better-off under the new arrangement compared to their situation in the outcome $(\mu, \mathcal{C})$, then such an outcome is not reasonable. This idea corresponds the notion of stability.

Definition 5 *An outcome* $(\mu, \mathcal{C})$ *for the market* $\mathcal{M}$ *is* **stable** *if there does not exist any pair* (P_i, A^j) *and any contract* $c' \in \mathcal{X}^j$ *such that* $u_{P_i}(A^j, c') > u_{P_i}(\mu(P_i), c_i)$ *and* $u_{A^j}(P_i, c') > u_{A^j}(\mu(A^j), c^j)$.

The above definition makes sure that there does not exist any principal-agent pair that can *block* the current outcome, signing a feasible contract c' between them.

3 The Set of Stable Outcomes

3.1 Results

In this section we characterize the set of stable outcomes of the market $\mathcal{M}$. We start by stating two important properties of a stable outcome. First, all the contracts in a stable outcome constitute a part of the set of contracts that are Pareto efficient. By Pareto efficiency we mean that there is no possibility of improving the utility of any individual without making the other individual worse-off. The following lemma states the Pareto efficiency property.

LEMMA 1 *All the contracts in a stable outcome are Pareto efficient.*

PROOF Suppose $(\mu, \mathcal{C})$ is stable, but the contract $c \in \mathcal{C}$ signed by P_i and A^j, where $\mu(A^j) = P_i$, is not Pareto efficient. Then there exists a contract c', feasible for (P_i, A^j) such that (i) $u_{P_i}(A^j, c') > u_{P_i}(A^j, c)$ and (ii) $u_{A^j}(P_i, c') > u_{A^j}(P_i, c)$. In that case (P_i, A^j) will block $(\mu, \mathcal{C})$ with c'. This contradicts the fact that $(\mu, \mathcal{C})$ is initially stable. ■

It is interesting to notice that the Pareto efficiency of a contract between a principal and an agent in any stable outcome is guaranteed by the possibility that the same pair can block the initial outcome with a different contract. Another property of stable outcomes is that no principal can gain more than any of her counterpart does. The profits of all the principals are equal. Lemma 2 proves this assertion.

LEMMA 2 *In any stable outcome $(\mu, \mathcal{C})$, $u_{P_i}(\mu(P_i), c_i) = u_{P_k}(\mu(P_k), c_k)$ for any $P_i, P_k \in \mathcal{P}$.*

PROOF Suppose $(\mu, \mathcal{C})$ is a stable outcome and $u_{P_i}(\mu(P_i), c_i) > u_{P_k}(\mu(P_k), c_k)$. We show that there exists a contract $c' \in \mathcal{C}$ such that $(P_k, \mu(P_i))$ blocks the outcome with c'. First, note that $\mu(P_i) \in \mathcal{A}$, otherwise $u_{P_i}(\mu(P_i), c_i) = 0$. Suppose $c_i = (\theta_S, \theta_F, K)$ and consider $c' = (\theta_S - \varepsilon, \theta_F - \varepsilon, K)$ with $\varepsilon > 0$.[7] It is easy to check that $e_{c_i} = e_{c'}$. Hence, for ε small enough, $u_{P_k}(\mu(P_i), c') = u_{P_i}(\mu(P_i), c_i) - \varepsilon > u_{P_k}(\mu(P_k), c_k)$ and $u_{\mu(P_i)}(P_k, c') = u_{\mu(P_i)}(P_i, c_i) + \varepsilon > u_{\mu(P_i)}(P_i, c_i)$. Therefore, $(P_k, \mu(P_i))$ blocks $(\mu, \mathcal{C})$ with c' and hence the lemma. ■

The above lemma states the intuitive property that, when the principals are identical, they must obtain the same profits in a stable outcome. This property is no longer valid if we consider some heterogeneity among the principals. One possible way to introduce heterogeneity is considering principals endowed with some attribute that makes some of them more attractive to the agents than other principals. Also, it can be the case that agents are more productive when

[7] In some proofs we will use the notation $c - \varepsilon$ to refer to the contract $(\theta_S - \varepsilon, \theta_F - \varepsilon, K)$, when $c = (\theta_S, \theta_F, K)$.

matched with some particular principals. In those situations, the principals with more valuable attributes will earn higher profits.

Lemma 1 implies that the contracts in a stable outcome must be Pareto optimal. Hence, a contract signed by a matched pair (P_i, A^j) must maximize the expected utility of one party taking into account that the other gets at least a certain utility level. The Pareto efficient contracts may require the agent to make any of the two possible efforts. In order to deal with interesting situations, we will assume, from now on, that the output y in case of success is high enough so that it is always optimal first, to establish a relationship and second, to set a contract that induces the agent to exert high effort. This implies that, for the Pareto efficient contracts, one can substitute the incentive compatibility constraint (IC) by the following:

$$(\pi_1(K) - \pi_0(K))(y - \theta_S + \theta_F) \geq 1. \tag{IC'}$$

We will denote by $\mathcal{Z}^j \subset \mathcal{X}^j$ the set of feasible contracts that also satisfy the incentive compatibility constraint (IC′). One particular class of Pareto efficient contracts are the *principal-agent* contracts, where the principal assumes all the bargaining power. The principal-agent contract for the pair (P_i, A^j), denoted c^{j*}, solves the following programme:

$$\max_{c \in \mathcal{Z}^j} u_{P_i}(A^j, c). \tag{P1}$$

Given the limited liability constraints, the moral hazard problem is typically costly for the principal, i.e., she earns lower profits than that in the *first best* situation, i.e., where she does not face any moral hazard problem. This happens if agent's wealth is below the level which makes the limited liability contraints no longer binding. Denote by w^0 this threshold level of initial wealth. Next, we show that if the principal has all the bargaining power, she strictly prefers hiring an agent with higher wealth if the *first best* has not been reached already.

PROPOSITION **1** *If* $w^j > w^k$ *and* $w^j < w^0$, *then* $u_{P_i}(A^j, c^{j*}) > u_{P_i}(A^k, c^{k*})$.

PROOF See Appendix B. ■

The utility possibility frontier is the set of utilities generated by the contracts that solve a programme similar to (P1) where the reservation utility of the agent can take value not only equal to zero as in (P1), but any number. The same set of Pareto efficient contracts results in if one maximizes agent's utility subject to a participation constraint of the principal (PCP). We will denote by $c^j(\widehat{u})$ the optimal contract that solves the following programme (as before we take agent's utility net of his wealth w^j):

$$\begin{cases} \max_{c \in \mathcal{Z}^j} u_{A^j}(P_i, c) \\ s.t. \;\; (PCP) \;\; u_{P_i}(A^j, c) \geq \widehat{u}. \end{cases} \tag{P2}$$

Notice that the contract that solves (P2) is acceptable for A^j only if $\widehat{u}$ is not too high. More precisely, $u_{A^j}(P_i, c^j(\widehat{u})) \geq 0$ if and only if $\widehat{u} \leq u_{P_i}(A^j, c^{j*})$. In the following theorems we characterize completely the set of stable outcomes. The properties that a stable outcome is Pareto efficient and that all principals earn equal profits provide a partial characterization. These help us complete the description of the set of stable outcomes. We distinguish among different cases. In Theorem 1, we consider the situation where there are more agents than principals ($m > n$) in the economy. In Theorem 2, we analyze the situations where there are same number of principals and agents and there are more principals than agents. Notice that the two lemmas stated above hold irrespective of the cardinalities of the set of principals and the set of agents.

THEOREM **1** *If $m > n$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold*
(a) $\mu(P_i) \in \mathcal{A}$ *for all* $P_i \in \mathcal{P}$, $\mu(A^j) \in \mathcal{P}$ *if* $w^j > w^{n+1}$ *and* $\mu(A^j) = A^j$ *if* $w^j < w^n$,
(b) $u_{P_i}(\mu(P_i), c_i) = \widehat{u} \in [u_{P_i}(A^{n+1}, c^{(n+1)*}), u_{P_i}(A^n, c^{n*})]$ *for all* $P_i \in \mathcal{P}$, *and*
(c) $c^j = c^j(\widehat{u})$ *if* $\mu(A^j) \in \mathcal{P}$ *and* $c^j = c^{null}$ *if* $\mu(A^j) = A^j$.

PROOF We first prove that **(a)-(c)** are necessary conditions for any stable outcome.

(a) Suppose first, that in a stable outcome $(\mu, \mathcal{C})$ any principal P_i is not matched. Then $u_{P_i}(\mu(P_i), c_i) = 0$. Now consider an agent A^j who is initially unmatched under μ. Then the contract $c^{j*} - \varepsilon \in \mathcal{Z}^j$ yields strictly higher payoffs to both P_i and A^j. Hence, (P_i, A^j) with $c^{j*} - \varepsilon$ blocks $(\mu, \mathcal{C})$. Second we show that A^j is matched if $w^j > w^{n+1}$. Suppose, on the contrary, that A^j is unmatched under μ and hence, $u_{A^j}(A^j, c^j) = 0$. Because of the previous proof, under μ there are n agents matched. Suppose, A^k is a matched agent such that $w^k \leq w^{n+1}$. Following Proposition 1, $u_{\mu(A^k)}(A^j, c^{j*}) > u_{\mu(A^k)}(A^k, c^{k*})$. Given that $u_{A^k}(\mu(A^k), c^k) \geq 0$ (since, the contract is feasible), $u_{\mu(A^k)}(A^k, c^k) \leq u_{\mu(A^k)}(A^k, c^{k*}) < u_{\mu(A^k)}(A^j, c^{j*})$. Take $c' = c^{j*} - \varepsilon$, with ε small enough. It is easy to see that $(\mu(A^k), A^j)$ with the contract c' will block the outcome which is a contradiction. For the last part of (a), suppose on the contrary that A^j is matched under μ and $w^j < w^n$. Since n agents are matched, take A^k such that this agent is not matched in a stable outcome and $w^k > w^n$. Applying the same argument as before, it is easy to show that $(\mu(A^j), A^k)$ with the contract $c^{k*} - \varepsilon$ will block the current outcome.

(b) We know that in all the stable outcomes the profits of the principals must be equal. Denote by $\widehat{u}$ the common profit of the principals. First we will show that in a stable outcome $(\mu, \mathcal{C})$, $\widehat{u} \geq u_{P_i}(A^{n+1}, c^{(n+1)*})$. Suppose on the contrary, $\widehat{u} < u_{P_i}(A^{n+1}, c^{(n+1)*})$. From part (a) of the theorem we know that any agent with less wealth than w^n cannot be matched in a stable outcome. Suppose this is A^{n+1} and consider any principal P_i. Then there is a contract $c' = c^{(n+1)*} - \varepsilon$, with ε small enough, such that (1) $u_{P_i}(A^{n+1}, c') = u_{P_i}(A^{n+1}, c^{(n+1)*}) - \varepsilon > \widehat{u}$ and (2) $u_{A^{n+1}}(P_i, c') \geq \varepsilon > 0 = u_{A^{n+1}}(\mu(A^{n+1}), c^{n+1})$. Hence, (P_i, A^{n+1}) blocks the outcome. Second, from Proposition 1 we know that $u_{P_i}(A^j, c^{j*}) > u_{P_i}(A^k, c^{k*})$ if and only if $w^j > w^k$. In a stable outcome $(\mu, \mathcal{C})$, an agent with wealth greater than w^{n+1}, say A^n is matched with some principal, say P_i. Then $u_{P_i}(A^n, c_i) = \widehat{u} > u_{P_i}(A^n, c^{n*})$ implies that $u_{A^n}(P_i, c_i) < u_{A^n}(P_i, c^{n*})$. This is not possible in a stable outcome.

(c) Let $(\mu, \mathcal{C})$ be a stable outcome. By Lemma 1, any contract $c \in \mathcal{C}$ is Pareto efficient and c^j is such a contract. So, given the stability of $(\mu, \mathcal{C})$, $c^j = c^j(\widehat{u})$ if $\mu(A^j) \in \mathcal{P}$.

We now prove that any outcome $(\mu, \mathcal{C})$ satisfying **(a)**-**(c)** is indeed stable. Suppose $\mu(A^j) \in \mathcal{P}$ and consider any principal P_i who, because of part (a), is matched. Clearly, (P_i, A^j) cannot block the outcome with any contract. Indeed, there does not exist a contract such that P_i gets more than $\widehat{u}$ and A^j gets more than $u_{A^j}(\mu(A^j), c^j)$ since $c^j(\widehat{u})$ is Pareto efficient by (c). Now suppose $\mu(A^j) = A^j$ and choose any arbitrary P_i (we can do so, since all principals have the same profit). By (a), we know that $w^j \leq w^{n+1}$. Then the maximum utility P_i can get by contracting A^j such that $u_{A^j}(..) \geq 0$ is $u_{P_i}(A^j, c^{j*}) \leq u_{P_i}(A^{n+1}, c^{(n+1)*})$. Given that $\widehat{u} \geq u_{P_i}(A^{n+1}, c^{(n+1)*})$ (because of (d)), there is no room for the pair (P_i, A^j) to block $(\mu, \mathcal{C})$. ■

In the following theorem, we restate Theorem 1 in cases where there are same number of principals and agents and where there are more principals than agents.

THEOREM 2 ***(i)*** *If $m = n$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold*
(a) *All principals and agents are matched,*
(b) $u_{P_i}(\mu(P_i), c_i) = \widehat{u} \in [0, u_{P_i}(A^n, c^{n*})]$ *for all $P_i \in \mathcal{P}$, and*
(c) $c^j = c^j(\widehat{u})$ *for any A^j.*

(ii) *If $m < n$, then an outcome $(\mu, \mathcal{C})$ is stable for the market $\mathcal{M}$ if and only if the following three conditions hold*
(a) *Only m principals and all the agents are matched,*
(b) $u_{P_i}(\mu(P_i), c_i) = 0$ *for all $P_i \in \mathcal{A}$, and*
(c) $c^j = c^j(0)$ *for any A^j.*

PROOF   Similar to the proof of Theorem 1. ■

The above theorems characterize the stable outcomes for this principal-agent economy. First important thing to note is the Pareto efficiency property of the contracts in the stable outcome. Efficiency in this market has in fact two aspects. The contracts signed are efficient for the parties involved. This was a property already established in Lemma 1. On the other hand, part (a) in both theorems makes sure that the matching itself is efficient too. This is the case because, in a stable outcome, all the individuals in the short side of the market are matched and, when there are more agents than principals, only the *best* (wealthier) agents are the ones who get contracted.

The second important property is that the profits of the principals are equal. In a stable outcome there emerges competition among the principals for the wealthier agents. In particular, when there are more principals than agents (Theorem 2(ii)), the profit of each principal is driven down to zero.

Third, in a stable outcome, all the agents whose wealth level is above the wealth of the poorest agent contracted obtain a strictly higher utility than that under a principal-agent contract. In fact, there are stable outcomes where the same is true even for the poorest agent contracted. To understand the reason for this property, notice that had the agents been symmetric, i.e., they had equal initial wealth, and they are large in number, the principals would assume all the bargaining power. In this case, the stable outcome would involve a principal-agent contract for each agent hired. The asymmetry among the agents does not let the principals appropriate all the incremental surplus generated in a principal-agent relationship, even when there are more agents than principals. Rather, the competition among principals makes the incremental surplus accrue to the agents. This competition is even more acute when there are a large number of principals compared with agents. In this case, it follows from Theorem 2 that the entire surplus generated in a relation accrues to the agent.

Finally, as is usual in the classical matching models, the set of stable outcomes in our economy has a nice structure. First, if $(\mu, \mathcal{C})$ is a stable outcome and μ' is an efficient matching, then $(\mu', \mathcal{C})$ is also stable. Second, if one stable outcome $(\mu, \mathcal{C})$ is better for an agent than another stable outcome $(\mu', \mathcal{C}')$, then $(\mu, \mathcal{C})$ is better than $(\mu', \mathcal{C}')$ for all the agents hired and worse for all the principals matched. In particular, there exists a stable outcome which is the best from the principals' point of view out of all the stable allocations and similarly for the agents. In this economy, the previous two extreme points in the set of stable allocations correspond to the outcomes in which the utilities of the principals are $\widehat{u} = u_{P_i}(A^n, c^{n*})$ and $\widehat{u} = u_{P_i}(A^{n+1}, c^{(n+1)*})$.[8] The first point is the principals' optimal stable outcome (we refer to this as *P-optimum*), while the second is the agents' optimal stable outcome (call this *A-optimum*).

3.2 Competitive Equilibrium

In matching models, the concepts of stability and of competitive equilibria are often similar. Also in our model there is a close relationship between these two paradigms. We show that the concept of stability reflects the idea of competitive equilibrium. In order to state properly this equivalence, we provide a definition of competitive equilibrium in our framework. Given a vector of *feasible* contracts $\bar{c} = (\bar{c}^1, ..., \bar{c}^m) \in \prod_{j=i}^{m} \mathcal{X}^j$ let $\mathcal{D}_i(\bar{c})$ denote the demand set of principal P_i which is given by

$$\mathcal{D}_i(\bar{c}) = \{A^j \in \mathcal{A} \cup \{P_i\} \mid u_{P_i}(A^j, \bar{c}^j) \geq u_{P_i}(A^k, \bar{c}^k) \text{ for all } A^k \in \mathcal{A}\}.$$

We use the following definition of a competitive equilibrium.

DEFINITION **6** *A tuple $(\mu, \bar{c})$ for the market $\mathcal{M}$ is a* **competitive equilibrium** *if* **(i)** $\mu(P_i) \in \mathcal{D}_i(\bar{c})$ *for all* $P_i \in \mathcal{P}$ *and* **(ii)** $\bar{c}^j = c^{j*}$ *if* $\mu(A^j) \notin \mathcal{P}$.

In our definition of a competitive equilibrium for the market $\mathcal{M}$, the vector of contracts $\bar{c}$ is playing the same role as the vector of prices does in a buyer-seller economy. Given the contracts (the prices), each principal should be able to choose (buy) one out of the set of her most preferred agents (objects). Moreover, the contract (price) offered by an unmatched agent (unsold object) must be the contract that gives all the bargaining power to the principal (all the surplus to the

[8] Remember that all the stable allocations provide the same utilities to the principals if $w^n = w^{n+1}$. In this case, the set of stable outcomes is a singleton.

buyer). That is, condition (ii) in the above definition corresponds to the zero-price condition for unsold objects.

It is important to notice that the previous definition does not guarantee that the contracts at equilibrium are Pareto efficient.

It is easy to go from the tuple $(\mu, \bar{c})$ to an outcome $(\mu, \mathcal{C})$ and viceversa in the following way. Given $(\mu, \bar{c})$, define $\mathcal{C}$ as the set of contracts actually signed in μ. Similarly, given $(\mu, \mathcal{C})$, take $\bar{c}^j = c^j$ if $\mu(A^j) \in \mathcal{P}$, and $\bar{c}^j = c^{j*}$, otherwise.

In the following theorem, we assert that the previous concept of stability and the concept of competitive equilibrium are similar for the principal-agent market $\mathcal{M}$.

THEOREM **3** **(a)** *If an outcome $(\mu, \mathcal{C})$ is stable, then the corresponding tuple $(\mu, \bar{c})$ is a competitive equilibrium.* **(b)** *If a tuple $(\mu, \bar{c})$ is a competitive equilibrium and the contracts are Pareto efficient, then the corresponding outcome $(\mu, \mathcal{C})$ is stable*

PROOF Consider a stable outcome $(\mu, \mathcal{C})$. First, the principals are indifferent among the signed contracts. Second, they obtain at least the same profits in the outcome $(\mu, \mathcal{C})$ than by signing a contract $\bar{c}^j = c^{j*}$ with any possible unmatched agent A^j. Hence, under $(\mu, \bar{c})$ each principal is choosing from her demand set.

Next consider a tuple $(\mu, \bar{c})$ which is a competitive equilibrium and the contracts are Pareto efficient. First, note that the corresponding outcome $(\mu, \mathcal{C})$ is individually rational. Also, all the principals obtain the same profits as they are identical and are choosing from their demand sets. Given that the contracts are Pareto efficient, it is not possible to find a blocking pair because there is no room for increasing the benefits of a principal without decreasing the utility of an agent. Hence, the outcome is stable. ■

In our framework, transactions occur via contracts. The major difference between this economy and a market where transactions go through prices (as in the assignment game analyzed by Shapley and Shubik(1972)) is that the total surplus produced in a particular relation does depend on the way in which the surplus is shared between the principal and the agent and on the design of the contract. The size of the surplus that accrues to the agent influences the extent to which the limited liability constraints are binding and hence the total surplus. On the other hand, unlike a market where the total surplus for a pair is *fixed* (and the price only determines the share of the surplus that goes to the seller), the contract signed by a principal-agent pair may not be Pareto efficient. Hence, a competitive mechanism in which contracts are endogenously determined by the principal-agent pair may lead to inefficient contracts. The previous theorem states that the set of stable outcomes is the same as the set of competitive equilibria in which the contracts are Pareto efficient. In a competitive equilibrium, the matching between principals and agents is efficient.

4 Characteristics of the Contracts in a Stable Outcome

In this section, we provide the characteristics of the contracts signed in a stable outcome. We have already shown that any such contract solves the maximization programme (P2). Now we turn on to analyze the characteristics of the solution to this programme. We will develop the analysis under the following assumption. In the appendix we comment on the qualitative changes if the opposite assumption holds.[9]

Assumption 1 $\pi_1(K)\pi_0'(K) - \pi_1'(K)\pi_0(K) > 0$ *for all* $K > 0$.

Assumption 1 implies that the derivative of $\frac{\pi_0}{\pi_1}$ with respect to K is positive. That is, the higher the level of investment, the lower is the difference between π_0 and π_1, and hence, the influence of making a high effort. The *first-best* level of investment, K^0 is given by the following equation:

$$\pi_1'(K^0)y = R. \tag{1}$$

In the first-best contract, K^0 is the level of investment that would be chosen if there was no moral hazard problem, or equivalently, if the limited liability constraints were not binding. In order to analyze the programme (P2), one can identify two disjoint ranges of values of w^j where the optimal solutions are different. First, for a very high level of agent's wealth both the incentive compatibility constraint and limited liability constraint (in the event of failure) are not binding.[10] This is equivalent to saying that there is no moral hazard problem. The threshold level of initial wealth, $w(\widehat{u})$, beyond which the optimal investment reaches its first-best level K^0 is:

$$w(\widehat{u}) \equiv -\pi_1(K^0)y + RK^0 + \widehat{u} + \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)}.$$

For low levels of initial wealth, $w^j \leq w(\widehat{u})$, both the incentive constraint and the limited liability constraint bind. In this region the moral hazard problem becomes important and hence, the optimal contract is lower than its first-best level. The optimal investment $\widehat{K}(w^j;\, \widehat{u})$ is implicitly defined by the following equation:

$$-\pi_1(K)y + RK + \widehat{u} + \frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} = w^j.$$

Given Assumption 1, the optimal investment increases with agents' wealth. The optimal investment is summarized in the following equation:

$$K = \begin{cases} \widehat{K}(w^j;\, \widehat{u}) & \text{if } w^j < w(\widehat{u}) \\ K^0 & \text{if } w^j \geq w(\widehat{u}). \end{cases}$$

We also describe in brief the characteristics of the state contingent transfers. Notice that, for $w^j \geq w(\widehat{u})$, any combination of $(\theta_S,\, \theta_F)$ that satisfies the constraints can be candidate for the

[9] The appendix provides a more complete analysis of the solution to (P2).

[10] One can easily check that the limited liability constraint in the event of success is automatically satisfied for the problem and that θ_S can be calculated from the (PCP).

optimum. One possible optimum corresponds to $\theta_F = w^j$. In case where the constraints (IC$'$) and (LF) are binding (for $w^j \leq w(\widehat{u})$), $\theta_F = w^j$ is also an optimum. Using the participation constraint of the principal, one can then easily calculate the optimal transfer in case of success which is given by the following.

$$\theta_S = \begin{cases} \frac{\widehat{u}+R\widehat{K}(w^j;\widehat{u})-(1-\pi_1(\widehat{K}(w^j;\widehat{u})))w^j}{\pi_1(\widehat{K}(w^j;\widehat{u}))} & \text{if } w^j < w(\widehat{u}) \\ \frac{\widehat{u}+RK^0-(1-\pi_1(K^0))w^j}{\pi_1(K^0)} & \text{if } w^j \geq w(\widehat{u}) \end{cases}$$

Once we know the characteristics of the solutions to program (P2), we use theorems 1 and 2 to provide a description of the contracts in a stable outcome. Consider first a situation with many agents where the wealth of most of them is zero, i.e., $m > n$ and $w^n = w^{n+1} = 0$. In this economy, the contracts signed in all the stable outcomes are the same. The contract signed by the hired agents with zero wealth will be the corresponding principal-agent contract, while the contract signed by the richer agents will correspond to the solution of program (P2), for $\widehat{u} = u_{P_i}(A^n, c^{n*})$. Figure 1 depicts the level of investments in the stable outcome.[11] For comparison, the diagram also includes the level of investments $K(w^j)$ that would be made if all the agents would sign a principal-agent contract. In this figure, $\overline{K}$ is the minimum level that would be invested by the agents with very low level of wealth (say, less than $\overline{w}$). The investment level is closer to the first-best level K^0 as the wealth of an agent is higher. That is, the productive efficiency of the relationship increases with the agent's wealth. The investment level coincides with the first-best level if the agent, say agent A^1, is rich enough, i.e., $w^1 \geq w(u_{P_i}(A^n, c^{n*}))$. It is worth noting also that these investments are always higher than those under principal-agent contracts, unless the agent's wealth is very large, $w \geq w^0$.

[Insert Figure 1 about here]

For the same economy, Figure 2 depicts agents' net and gross utility levels (the common principals' utility is $u_{P_i}(A^n, c^{n*})$). Agents' net utility increases with the wealth level (unless the level of wealth is already above $w(u_{P_i}(A^n, c^{n*}))$). The utility of wealthier agents is not only higher because of the initial wealth levels. They also profit from the increase in the surplus due to the more efficient (i.e., closer to the first-best) contracts.

[Insert Figure 2 about here]

For completeness, Figure 3 depicts the set of investment levels in the stable outcomes when $m > n$, $w^n > w^{n+1}$, and w^{n+1} is large. The line corresponding to the level of investments in a particular stable outcome, say $K^s(w^j)$ is quite similar to that in Figure 1 (although it starts from a level higher than $\overline{K}$). This line will be placed at a higher (or a lower) position depending if we are in a stable outcome closer to (or farther from) the *A-optimum*. In particular, the lowest line (that starts from $K(w^n)$) corresponds to the investment levels in the *P-optimum*.

[Insert Figure 3 about here]

[11] For sake of tangibility, all the figures are drawn for $\pi_1(K) = \frac{K}{1+K}$ and $\pi_0(K) = \frac{K}{2+K}$. Our results, although, hold good for a very general class of probability functions satisfying our assumptions.

The graphical representation of an economy with more principals than agents is very similar to that in figures 1 and 2. The levels of investment and of net and gross utilities are as in figures 1 and 2, with the only difference that they all start at a higher level than $\overline{K}$ and $\overline{w}$.

5 Implementing the Set of Stable Outcomes

In this section we further argue about stability as a reasonable solution concept for the market we analyze. We show that the set of stable outcomes that we have characterized in theorems 1 and 2 are also the equilibrium outcomes of a very simple and natural non-cooperative interaction between the principals and the agents. We propose a simple mechanism that implements the set of stable outcomes.[12] The mechanism, called Γ^A, is a two-stage game where in the first stage each agent proposes a contract. In the second stage of the game, each principal contracts an agent. Formally, at the first stage of the mechanism, agents send their messages simultaneously. The message of each agent is an element of the set of feasible contracts. A message $s^j \in \mathcal{X}^j$ of agent A^j should be understood as the contract he demands. At the second stage, knowing the messages of the agents, each principal P_i sends a message $s_i \in \mathcal{A} \cup \{P_i\}$. A message of a principal should be understood as the agent she wants to hire or she wants to stay unmatched. The outcome function $g(.)$ associates to each vector of messages, $s = (s_1, ..., s_n, s^1, ..., s^m)$ a matching, μ^s, and a menu of contracts, $\mathcal{C}(s)$, such that $\mu^s(A^j)$ is chosen with equal probability on the set $\mathcal{P}^j = \{P_i \in \mathcal{P} \mid s_i = A^j\}$ if $\mathcal{P}^j \neq \varnothing$ and $\mu^s(A^j) = A^j$, otherwise. Moreover,

$$c^j(s) = \begin{cases} s^j & \text{if } \mu^s(A^j) \in \mathcal{P} \\ c^{null} & \text{, otherwise.} \end{cases}$$

The natural solution concept used here is *Subgame Perfect Equilibrium.* We will analyze the Subgame Perfect Equilibria in pure strategies (SPE).

THEOREM 4 *The set of SPE outcomes of the mechanism Γ^A coincides with the set of stable outcomes for the market $\mathcal{M}$.*

PROOF See Appendix E. ■

From the point of view of implementation, the above theorem shows that one can propose a very simple mechanism which makes it possible to implement the set of stable outcomes of this principal-agent economy.

[12]Some recent papers that propose simple mechanisms to implement the set of stable outcomes in matching models are Alcalde, Pérez-Castrillo and Romero-Medina(1998), Alcalde and Romero-Medina(2000) and Pérez-Castrillo and Sotomayor(2001).

6 Applications

In this section, we propose two examples of principal-agent economies where our model fits into. In the first example, we consider optimal tenancy and credit contracts in agrarian communities. This economy consists of a few landowners and a large number of tenants. In the second example, we consider a typical banking sector where a large number of banks finance the projects that a set of entrepreneurs undertake.

6.1 A Landowner-Tenant Economy

In a seminal work, Shetty(1988) shows that wealth differences among tenants play a key role in determining the credit contracts when there exists a possibility of default on the rental commitments. Difference in initial wealth implies difference in liability of the tenants.[13] Hence, in the case where there is significant moral hazard problem due to limited liability, wealthier tenants are always preferred for a more efficient contractual structure, since possibility of default is less with wealthier tenants. Our results can be used to analyze similar situations when a set of landowners interacts with a set of tenants through tenancy relations. One feature is to note that the kind of contracts we use can often be observed in the less developed economies. It is very common that the same person acts as landowner-cum-moneylender in the villages by leasing land and lending money to the same person (here, the tenant). The contracts described for the market $\mathcal{M}$ also capture these components. The state contingent transfers, (θ_S, θ_F) are the payments made to the landowners and K is the amount borrowed from the landowners that is invested eventually in land. In this economy, the tenants cannot seek loans from the formal credit sector due to lack of sufficient collateral, while the landowners can. Consequently, the landowners become the only sources of credit to the hapless borrowers.

With these interpretations, our results imply:

(i) In a stable outcome, all the contracts signed between landowners and tenants are Pareto efficient and all the landowners and only the wealthier tenants are matched. All the landowners earn the same profit and the contracts maximize the expected utility of the tenants for the common profit level of the landowners. Wealthier the tenant is, the more efficient the contract he signs (closer to first-best). The above findings also conform to the findings of Shetty(1988).

(ii) The investments made in a stable outcome are, in fact, closer to first-best than those would be implemented if the tenants would sign principal-agent contracts. As landowners compete for the wealthier tenants, they are compelled to offer these tenants *better* contracts in order to attract them. Since the tenants obtain higher utility, the limited liability constraint is less stringent and hence the investment level approaches the first best. This phenomenon is described in figures 1 and 2. This comparison is relevant because the principal-agent contracts are the contracts that would have been offered, for example, if the landowners would collude.

The property highlighted in (ii) has important implications with respect to distributive

[13] The role of limited liability in tenancy contracts are also analyzed extensively by Basu(1992) and Sengupta(1997).

(in)equality and efficiency. It suggests that for a very low level of aggregate wealth, more inequality in the distribution of tenant's wealth, higher the total investment and more efficient the relationship. This is a typical feature of investment in wealth constrained societies.[14] From a normative point of view, the property also suggests that if the public authority has some money to distribute which could serve as collateral in tenancy relations, it may need to induce inequality among the tenants in order to increase both the efficiency of the contracts and the utility of (some of) the tenants. Suppose all the tenants had no initial wealth. If the public authority distributes to every tenant a small amount (less than $\overline{w}$), then in the stable outcome, all the tenants will sign principal-agent contracts investing a level $\overline{K}$. Hence, the efficiency of the relationship will remain the same as that prior to the distribution. Moreover, the gross utility of all the tenants hired will be the same as before. That is, the landowners will appropriate the additional amount distributed. On the other hand, if the public authority distributes the money among a few tenants (a number smaller than the number of landowners), then, first, the contracts signed by these tenants will be more efficient than before and, second, their gross utility will increase by more than the additional money they receive.

6.2 A Banker-Entrepreneur Economy

Liquidity constraints play a crucial role in the relationships between bankers and entrepreneurs. Bester and Hellwig(1987) and De Meza and Webb(1992), among others, show that these constraints can lead to credit rationing and to inefficient allocations of resources. Our analysis can also add some elements to this line of research. Let us interpret K as the observable amount of resources devoted to the project and e as a non-verifiable costly decision (or effort) to be taken by the entrepreneur (agent). Depending on personal or institutional circumstances, the collateral (the wealth level) that the entrepreneurs can risk in the project differs. Hence, the limited liability constraint of each entrepreneur differs according to his collateral.

With the above interpretation, our results imply the following. First, in a stable outcome, the investments made in the projects are inefficient with respect to the first-best level. This result is due to the influence of the level of investment in the manager's incentives. Since the bank cannot contract on the effort to be made by the entrepreneur, it influences the manager's incentives. Under assumption 1, there will be under-investment in the projects. If the opposite assumption holds, then there will be over investment in each project.

Second, the degree of inefficiency of investment decreases with the level of collateral posted by the entrepreneurs. In fact, in a stable outcome, this inefficiency disappears with a relatively smaller level of wealth compared to a situation where the banks assume all the bargaining power.

Finally, note that the previous conclusions do not depend on the fact that the banks make zero profits. In our model, banks make zero profits when they are large in number.

[14] See Perroti(1996).

7 Concluding Remarks

In this paper we model a principal-agent economy as a two-sided matching market and characterize completely the set of stable outcomes of this economy. As we have mentioned earlier, our model can be seen as a generalization of the assignment game described by Shapley and Shubik(1972). Our findings can easily be applied to various examples of principal-agent economies. We have already mentioned two of them in the previous section. The main task of this paper lies in suggesting a general (competitive) equilibrium model of a principal-agent economy. Using the restriction of limited liability should be taken as a very simple way to tackle incentive problems. This suggests that the qualitative findings will remain unaltered even under the hypothesis of risk-averse individuals. This paper also consolidates stability as a reasonable solution concept. In this regard, we show that our results are not only the outcome of a cooperative game, or a competitive equilibrium model, but can be reached through very simple non-cooperative interactions between the principals and the agents.

Our paper leaves several avenues open for further research. First, we have assumed that the principals are identical. Although some of the conclusions of our analyses can immediately be extended to apply to economies with heterogenous principals, the characteristics of the contracts signed in the stable outcomes can be quite different from those identified in the current work. On the one hand, the results that the contracts signed in a stable outcome are Pareto efficient and the matching itself is efficient (in the sense that it maximizes the total surplus) hold also in a framework with heterogenous principals. On the other, there is no unique way to model the differences among the principals and the contracts will be different depending on the type of heterogeneity one would like to introduce. Second, ours is a one-to-one matching model. If we consider the situation where several independent agents are matched with each principal, then the conclusions will remain unchanged. But these will be different in a more interesting situation where the action of an agent is dependent on that of others. This kind bears similarity with the multi-task agency problem. A natural way to analyze this would be to make use of a many-to-one matching model.

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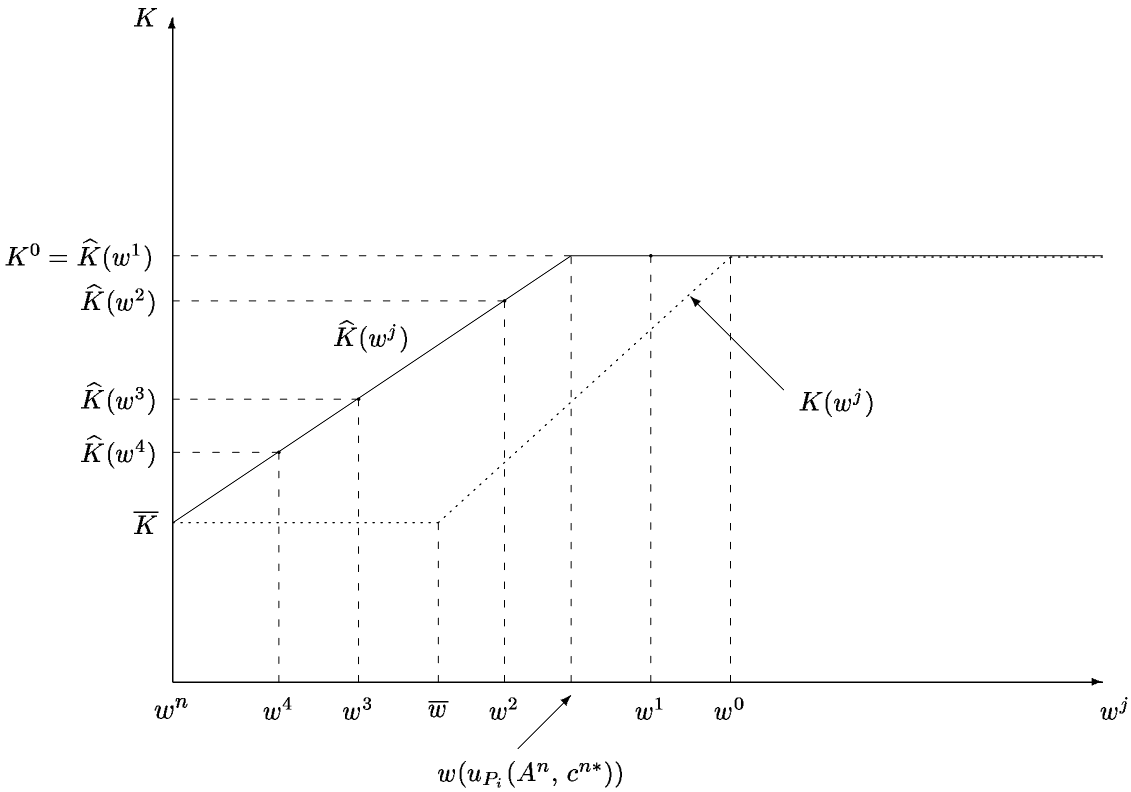

Figure 1: THE ENDOGENOUS INVESTMENT LEVELS WHEN $w^n = w^{n+1} = 0$

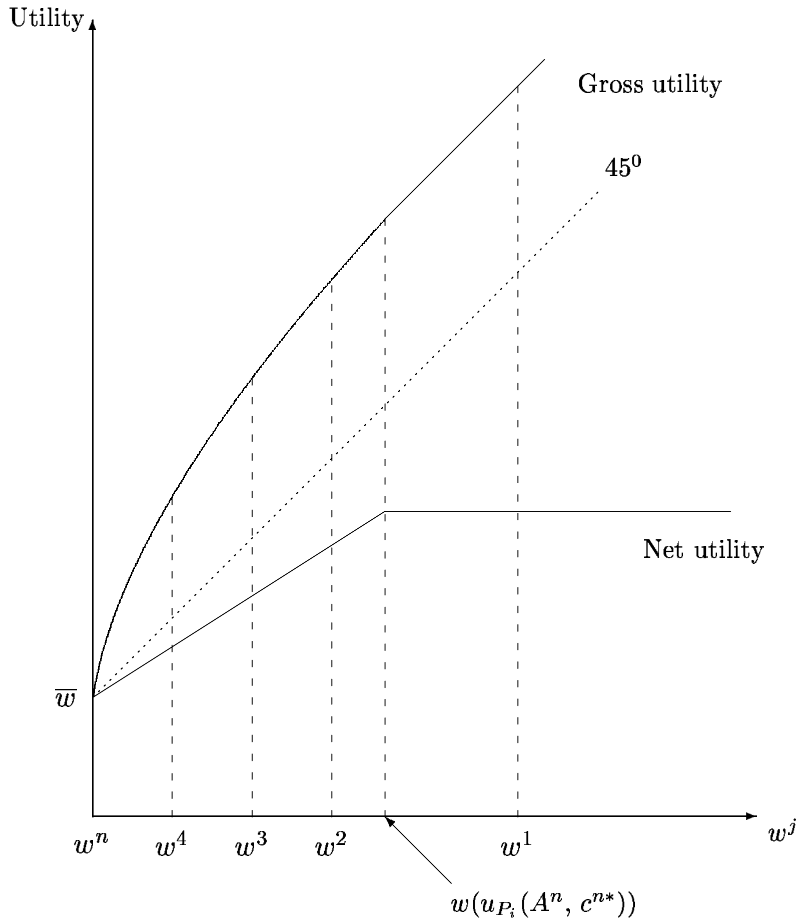

Figure 2: GROSS AND NET UTILITIES OF AN AGENT

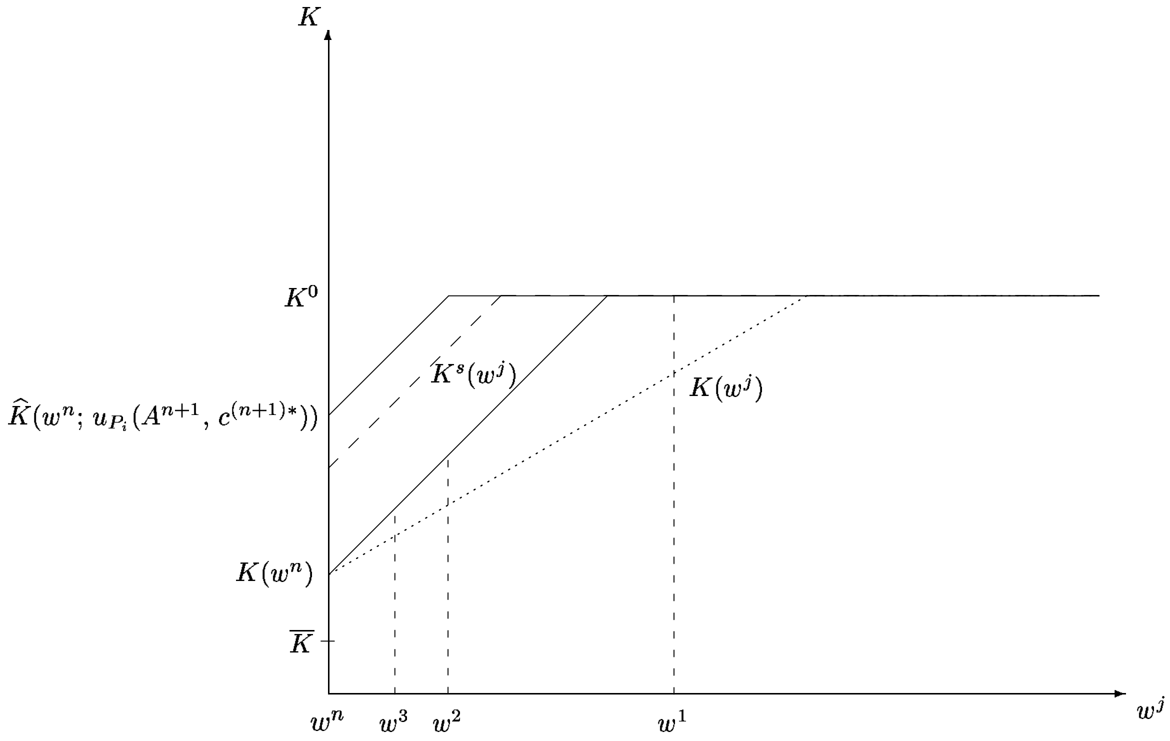

Figure 3: OPTIMAL INVESTMENT WHEN $w^n > w^{n+1}$

A The Principal-Agent Contracts

We solve for the optimal principal-agent contract for a pair (P_i, A^j):

$$\begin{cases} \underset{\{\theta_S, \theta_F, K\}}{\text{maximize}} & u_{P_i} = \pi_1(K)\theta_S + (1-\pi_1(K))\theta_F - RK \\ \text{subject to} & (PC) \quad \pi_1(K)(y - \theta_S + \theta_F) - \theta_F \geq 1 \\ & (IC') \quad [\pi_1(K) - \pi_0(K)](y - \theta_S + \theta_F) \geq 1 \\ & (LS) \quad \theta_S \leq y + w^j \\ & (LF) \quad \theta_F \leq w^j. \end{cases} \tag{P1}$$

At the optimum, (IC′) binds, so we write the constraint with equality.[15] Using this, one can replace θ_S in the objective function and the other three constraints. Moreover, if (PC) and (LF) are satisfied, (LS) also holds. Hence, the above programme reduces to the following:

$$\begin{cases} \underset{\{\theta_F, K\}}{\text{maximize}} & \pi_1(K)y - \frac{\pi_1(K)}{\pi_1(K)-\pi_0(K)} + \theta_F - RK \\ \text{subject to} & (PC') \quad \frac{\pi_1(K)}{\pi_1(K)-\pi_0(K)} - \theta_F - 1 \geq 0 \\ & (LF) \quad w^j - \theta_F \geq 0. \end{cases} \tag{P1$'$}$$

We denote μ_1 and μ_2 the Lagrangean multipliers of (P1′). Then, the Kuhn-Tucker (first-order) conditions are given by:[16]

$$y\pi_1' - R + (1-\mu_1)\frac{\pi_1'\pi_0 - \pi_1\pi_0'}{(\pi_1 - \pi_0)^2} = 0 \tag{2}$$

$$1 - \mu_1 - \mu_2 = 0 \tag{3}$$

$$\mu_1\left(\frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} - \theta_F - 1\right) = 0 \tag{4}$$

$$\mu_2\left(w^j - \theta_F\right) = 0 \tag{5}$$

$$\frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} - \theta_F - 1 \geq 0 \tag{6}$$

$$w^j - \theta_F \geq 0 \tag{7}$$

$$\mu_1, \mu_2 \geq 0 \tag{8}$$

Now we study different regions where the Kuhn-Tucker conditions can be satisfied. For simplicity, we develop the analysis when $\pi_1'\pi_0 - \pi_1\pi_0' < 0$.

Case 1: $\mu_1 = \mu_2 = 0$ (Both the constraints are non binding)

From (3), we can see that this case is not possible.

[15] To be more precise, (IC′) does not bind if w^j is very high, that is in the region where the limited liability constraints do not play any role and the first best contract is signed. This corresponds to Case 2 in the analysis that follows.

[16] The hypotheses on $\pi_1(K)$ and y make sure the optimal K must be interior and it satisfies the first-order conditions. The corner solution for θ_F is explicitly taken into account.

CASE 2: $\mu_1 > 0$, $\mu_2 = 0$ ((LF) is non-binding and (PC′) is binding)
From (3), $\mu_1 = 1$. Then from (2), we have $y\pi_1' - R = 0$, i.e., $K = K^0$, where K^0 is the first-best level of investment. Using (PC′) and (LF), one has

$$w^j \geq \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} - 1 \equiv w^0.$$

Hence, if $w^j \geq w^0$ a candidate for optimal solution exists involving $K = K^0$. In particular, an optimal payment vector is $(\theta_S = y + w^j - \frac{1+w^j}{\pi_1(K^0)}, \theta_F = w^j)$.

CASE 3: $\mu_1 = 0$, $\mu_2 > 0$ ((LF) is binding and (PC′) is non-binding)
From (3), $\mu_2 = 1$. Then (2) implicitly defines the level of optimum investment $\overline{K}$,

$$y\pi_1'(\overline{K}) - R + \frac{\pi_1'(\overline{K})\pi_0(\overline{K}) - \pi_1(\overline{K})\pi_0'(\overline{K})}{(\pi_1(\overline{K}) - \pi_0(\overline{K}))^2} = 0.$$

From (LF), we also have $\theta_F = w^j$. Moreover, θ_S is determined by (IC′) as $\theta_S = y + w^j - \frac{1}{\pi_1(\overline{K}) - \pi_0(\overline{K})}$. And from the non-binding (PC′) we have

$$w^j \leq \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} - 1 \equiv \overline{w}.$$

That is, the previous contract can only be a candidate if $w^j \leq \overline{w}$.

CASE 4: $\mu_1 > 0$, $\mu_2 > 0$ (Both the constraints are binding)
From (LF), $\theta_F = w^j$. Then (PC′) defines the optimal K as an implicit function of w^j. Denote this by $K(w^j)$, which must satisfy the following condition

$$\frac{\pi_1(K(w^j))}{\pi_1(K(w^j)) - \pi_0(K(w^j))} = w^j + 1. \tag{9}$$

Finally, θ_S is determined by (IC′). Previously found θ_F, θ_S and $K(w^j)$ are indeed the candidates for optimum if the Lagrange multiplier, μ_1, implicitly defined by (2) lies in the interval [0, 1] (so that constraints (3) and (8) are satisfied). Given that $\pi_1'\pi_0 - \pi_1\pi_0' < 0$, $\mu_1 < 1$ if and only if

$$y\pi_1'(K(w^j)) - R > 0 \Rightarrow K(w^j) < K^0.$$

Again using $\pi_1'\pi_0 - \pi_1\pi_0' < 0$, $K(w^j) < K^0$ is optimal if

$$\frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} < w^j + 1 \Rightarrow w^j < w^0.$$

Similarly, $\mu_1 > 0$ if and only if

$$y\pi_1'(K(w^j)) - R + \frac{\pi_1'(K(w^j))\pi_0(K(w^j)) - \pi_1(K(w^j))\pi_0'(K(w^j))}{(\pi_1(K(w^j)) - \pi_0(K(w^j)))^2} < 0.$$

The above inequality implies $K(w^j) > \overline{K} \Rightarrow \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} < 1 + w^j \Rightarrow w^j > \overline{w}$. Hence, the optimal contract corresponds to the solution found in Case 3 when $w^j < \overline{w}$, is the candidate found in Case 4 when $\overline{w} < w^j < w^0$, and it is the first-best contract of Case 2 when $w^0 \leq w^j$.

B Proof of Proposition 1

We are to show that if $w^j > w^k$ in the region $w^j < w^0$, then $u_{P_i}(A^j,\, c^{j*}) > u_{P_i}(A^k,\, c^{k*})$. From the previous section one can write the value function $v(w^j) = u_{P_i}(A^j,\, c^{j*})$. Using the Envelope theorem, we get $v'(w^j) = \mu_2 > 0$ and hence the proposition.

C Contracts in a Stable Outcome

Let us rewrite (P2):

$$\begin{cases} \underset{\{\theta_S,\theta_F,K\}}{\text{maximize}} & u_{A^j} = \pi_1(K)(y-\theta_S) - (1-\pi_1(K))\theta_F - 1 \\ \text{subject to} & (PCP) \quad \pi_1(K)\theta_S + (1-\pi_1(K))\theta_F - RK \geq \widehat{u} \\ & (IC') \quad [\pi_1(K)-\pi_0(K)](y-\theta_S+\theta_F) \geq 1 \\ & (LS) \quad \theta_S \leq y + w^j \\ & (LF) \quad \theta_F \leq w^j. \end{cases} \tag{P2}$$

As we have pointed out in the paper, this programme is individually rational for the agent only if $\widehat{u} \leq u_{P_i}(A^j,\, c^{j*})$. Denote by $w^{min}(\widehat{u})$ the level of wealth such that $\widehat{u}$ is the utility of a principal that hires an agent with this wealth under a principal-agent contract. Programme $(P2)$ is only well defined for $w^j \geq w^{min}(\widehat{u})$. At the optimum, (PCP) binds. Hence, one can substitute for θ_S in the objective function and the rest of the constraints. Also, if both (IC$'$) and (LF) hold, then (LS) becomes redundant. Then one has the above programme reduced as the following:

$$\begin{cases} \underset{\{\theta_F,K\}}{\text{maximize}} & \pi_1(K)y - \widehat{u} - RK - 1 \\ \text{subject to} & (IC'') \quad \pi_1(K)y - \frac{\pi_1(K)}{\pi_1(K)-\pi_0(K)} + \theta_F - RK - \widehat{u} \geq 0 \\ & (LF) \quad \theta_F \leq w^j. \end{cases} \tag{P2$'$}$$

Let ν_1 and ν_2 be the Lagrange multipliers for (IC$''$) and (LF), respectively. The Kuhn-Tucker (first-order) conditions are

$$y\pi_1' - R + \nu_1\left(y\pi_1'(K) - R + \frac{\pi_1'(K)\pi_0(K) - \pi_1(K)\pi_0'(K)}{(\pi_1(K)-\pi_0(K))^2}\right) = 0 \tag{10}$$

$$\nu_1\left(\frac{\pi_1(K)-\pi_0(K)}{\pi_1(K)}\right) - \nu_2 = 0 \tag{11}$$

$$\nu_1\left((\pi_1(K)-\pi_0(K))\left(y - \frac{\widehat{u}-\theta_F+RK}{\pi_1(K)}\right) - 1\right) = 0 \tag{12}$$

$$\nu_2(w^j - \theta_F) = 0 \tag{13}$$

$$\left((\pi_1(K)-\pi_0(K))\left(y - \frac{\widehat{u}-\theta_F+RK}{\pi_1(K)}\right) - 1\right) \geq 0 \tag{14}$$

$$w^j - \theta_F \geq 0 \tag{15}$$

$$\nu_1,\, \nu_2 \geq 0 \tag{16}$$

Now we study different regions for the Kuhn-Tucker conditions to be satisfied.

CASE 1: $\nu_1 = 0$, $\nu_2 > 0$ ((LF) is binding and (IC$''$), non-binding)
Using (11), one can see that this case is not possible.

CASE 2: $\nu_1 > 0$, $\nu_2 = 0$ ((LF) is non-binding and (IC$''$), binding)
From (11), it is clear that this case is not possible either.

CASE 3: $\nu_1 = \nu_2 = 0$ (Both the constraints are non-binding)
From (10), $K = K^0$, the first best level of investment.The payment made to the principal in case of failure, θ_F is calculated from (PCP). For example, $\theta_F = w^j$ and $\theta_S = \frac{\widehat{u}+RK^0-(1-\pi_1(K^0))w^j}{\pi_1(K^0)}$ are optimal. From (IC$''$) and (LF), the above is only possible if

$$w^j \geq -\pi_1(K^0)y + RK^0 + \widehat{u} + \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} \equiv w(\widehat{u}).$$

CASE 4: $\nu_1 > 0$, $\nu_2 > 0$ (Both the constraints are binding)
In this case, $\theta_F = w^j$ and optimal investment is a function of w^j, $\widehat{K}(w^j;\widehat{u})$, that is implicitly defined by the condition

$$-\pi_1(K)y + RK + \widehat{u} + \frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} = w^j. \tag{17}$$

Notice that, from (10), for $K \leq K^0$, $y\pi_1'(K) - R + \frac{\pi_1'(K)\pi_0(K)-\pi_1(K)\pi_0'(K)}{(\pi_1(K)-\pi_0(K))^2} \geq 0$. From the above expression, this immediately implies that $\widehat{K}(.)$ is increasing in w^j. The previous values of θ_F, θ_S and K are optimal solutions to the above programme if the multipliers ν_1 and ν_2 defined in equations (10) and (11) satisfy (16), i.e., they are non-negative. Notice that (10) implies $\nu_2 > 0$ if and only if $\nu_1 > 0$. To check when $\nu_1 > 0$, notice that if $w^j > w(\widehat{u})$, then it is necessary that

$$\begin{aligned} w^j &= -\pi_1(\widehat{K}(w^j;\widehat{u}))y + R\widehat{K}(w^j;\widehat{u}) + \widehat{u} + \frac{\pi_1(\widehat{K}(w^j;\widehat{u}))}{\pi_1(\widehat{K}(w^j;\widehat{u})) - \pi_0(\widehat{K}(w^j;\widehat{u}))} \\ &> -\pi_1(K^0)y + RK^0 + \widehat{u} + \frac{\pi_1(K^0)}{\pi_1(K^0) - \pi_0(K^0)} \equiv w(\widehat{u}). \end{aligned}$$

Now we can characterize the optimal contract as follows.

$$K = \begin{cases} \widehat{K}(w^j;\widehat{u}) \text{ if } w^j \leq w(\widehat{u}) \\ K^0 \text{ if } w^j \geq w(\widehat{u}) \end{cases}$$

$$\theta_S = \begin{cases} \frac{\widehat{u}+R\widehat{K}(w^j;\widehat{u})-(1-\pi_1(\widehat{K}(w^j;\widehat{u})))w^j}{\pi_1(\widehat{K}(w^j;\widehat{u}))} \text{ if } w^j \leq w(\widehat{u}) \\ \frac{\widehat{u}+RK^0-(1-\pi_1(K^0))w^j}{\pi_1(K^0)} \text{ if } w^j \geq w(\widehat{u}) \end{cases}$$

and $\theta_F = w^j$
Here we also want to prove that for any level of $w^j \geq w^{min}(\widehat{u})$, $\widehat{K}(w^j) \geq K(w^j)$. First of all we know that, $\widehat{K}(w^j) > \overline{K}$. Comparing (9) and (17), it is clear that proving $\widehat{K}(w^j) \geq K(w^j)$ is

equivalent to showing that $\pi_1(\widehat{K})y - R\widehat{K} - \widehat{u} \geq 1$. Suppose that $w^{min}(\widehat{u}) \leq \overline{w}$. Then $\widehat{u}$ is given by

$$\widehat{u} = \pi_1(\overline{K})y - \frac{\pi_1(\overline{K})}{\pi_1(\overline{K}) - \pi_0(\overline{K})} + w^{min}(\widehat{u}) - R\overline{K}.$$

Using the above together with (6), it is easy to see that $\pi_1(\widehat{K})y - R\widehat{K} - \widehat{u} > 1$. This also proves that $w(\widehat{u}) \leq w^0$. We now do the same considering $w^{min}(\widehat{u}) > \overline{w}$. Notice that, in this case $\widehat{u} = \pi_1(K(w^{min}(\widehat{u})))y - RK(w^{min}(\widehat{u}))$. Also, $[\pi_1(\widehat{K})y - R\widehat{K}] - [\pi_1(K(w^{min}(\widehat{u})))y - RK(w^{min}(\widehat{u}))] > 0$, since investment is increasing in wealth. These previous two facts imply the above assertion that $\widehat{K}(w^j) \geq K(w^j)$ for all $w^j \geq w^{min}(\widehat{u})$.

D The Case when $\pi_1(K)\pi'_0(K) < \pi'_1(K)\pi_0(K)$

In the paper we have analyzed our model under the assumption that $\pi_1\pi'_0 > \pi'_1\pi_0$. We also asserted that, all the qualitative results of our model would hold good under the assumption that $\pi_1\pi'_0 < \pi'_1\pi_0$. Under this assumption, the findings in Appendix A imply $\overline{K} > K(w^j) > K^0$ and $K(w^j)$ is decreasing for $w^j \in (\overline{w},\ w^0)$. The reason behind this is the following. When $\pi_1(K)$ is increasing relative to $\pi_0(K)$, for a high level of initial investment, giving incentives is much easier. Because of this, for low level of wealth, the principal gives over incentives to the agent by lending more money (equivalently, the optimal investment is higher). Similarly, under this assumption, the findings of Appendix C imply that $\widehat{K}(w^j;\, \widehat{u}) > K^0$ for $w^j > w(\widehat{u})$.

E Proof of Theorem 4

First we prove that each SPE outcome is stable. We do that through several claims. **(a)** At any SPE, the contracts accepted are (among) the ones yielding the highest utility to the principals. Otherwise, a principal accepting a contract that yields lower utility would have incentives to switch to a better contract that has not already been taken. **(b)** At any SPE, all the contracts that are accepted provide the same utility to all the principals. Otherwise, on the contrary, consider one of the (at most $n-1$) contracts that gives the maximum utility to the principals. If one of the agents slightly decreases the payments offered to the principal, his contract will still be accepted at any Nash equilibrium (NE) of the second-stage game for the new set of offers. **(c)** At any SPE, precisely n contracts are accepted. To see this, suppose on the contrary that at most $n-1$ contracts are accepted. Then there is a (unmatched) principal with zero utility. This is not possible since **(b)** holds. **(d)** The contracts that are finally accepted are those offered by the wealthiest agents. Suppose $w^k > w^j$ and the contract offered by A^j is accepted, but not the one by A^k. Then A^k can offer a slightly better (for the principals) contract than s^j. Given **(a)**-**(c)**, this new contract will be accepted at any NE of the second-stage game. This is a contradiction. **(e)** Finally, any SPE outcome is stable. It only remains to prove that the common utility level of the principals at an SPE, denoted by $\widehat{u}$, lies in $[u_{P_i}(A^{n+1},\, c^{(n+1)*}),\, u_{P_i}(A^n,\, c^{n*})]$. First, $\widehat{u} \leq u_{P_i}(A^n,\, c^{n*})$, because otherwise, some agents would be better-off by not offering any contract. Secondly, $\widehat{u} \geq u_{P_i}(A^{n+1},\, c^{(n+1)*})$ for agent A^{n+1} not to have incentives to propose a contract that would have been accepted. We now prove that any stable outcome can be supported by an SPE strategy. Let $(\mu, \mathcal{C})$ be a stable allocation where each principal gets utility $\widehat{u}$. Consider the following strategies of each agent A^j for all j and of each principal P_i for all i:

$$\widehat{s}^j = \begin{cases} c_{\mu(A^j)} & \text{if } \mu(A^j) \in \mathcal{P} \\ \widehat{c} \text{ s.t. } u_{P_i}(A^j, \widehat{c}) = \widehat{u} \text{ for any } P_i \in \mathcal{P}, & \text{otherwise.} \end{cases}$$

And $\widehat{s}_i = \mu(P_i)$ if $\widehat{s}$ is played in the first stage. Otherwise, principals select any agent compatible with an NE in pure strategies given any other message s sent in the first period. These strategies constitute an SPE yielding the stable outcome $(\mu, \mathcal{C})$. To see this, notice that given any message $s^j \neq \widehat{s}^j$, principals play their NE strategies. Given that $\widehat{s}$ is played in the first stage, by deviating any principal P_i she cannot gain more than $\widehat{u}$. This is true because any contract offered in the first stage yields the same utility $\widehat{u}$ to any principal. Now consider deviations by the agents. Given that $\widehat{u} \geq u_{P_i}(A^{n+1}, c^{(n+1)*})$, by stability, there does not exist any contract that would be offered by an unmatched agent that guarantees him a positive utility while yielding at least $\widehat{u}$ to a principal. Hence, unmatched agents do not have incentives to deviate. Also, given the efficiency of the contracts in a stable allocation, there does not exist a different contract that a matched agent could offer at which he could have strictly improved while still guaranteeing at least $\widehat{u}$ to the principals. If there is a plethora of contracts that yields utility $\widehat{u}$ to the principals, it is easy to check that there is no NE of the game at which a contract providing utility lower than $\widehat{u}$ is accepted by a principal. Hence, the matched agents do not also have any incentive to deviate from $\widehat{s}$.