Consumption Externalities, Habit Formation, and Equilibrium Efficiency*

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Abstract

We analyze the welfare properties of the competitive equilibrium in a capital accumulation model where individual preferences are subjected to both habit formation and consumption spillovers. We also discuss how consumption externalities and habits interact to generate an inefficient dynamic equilibrium. Finally, we characterize optimal tax policies aimed to restore efficient decentralized paths.

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1. Introduction

This paper analyzes the welfare properties of the competitive equilibrium in a capital accumulation model where individual preferences are subjected both to a process of habit formation and to spillovers from the other agents' consumption. Intertemporal general equilibrium models have traditionally assumed that individual tastes are exogenous in the sense that, in each period, agents derive utility exclusively from the absolute level of their own current consumption. In our model tastes are however endogenous because the utility derived from a given level of present consumption depends on a reference level, which can be viewed as a (time-varying) standard of living. In fact, a growing number of papers in macroeconomics have introduced endogenous preferences in order to account for some economic facts that cannot be reconciled with the more traditional theories based on exogenous preferences.[1]

The introduction of habit formation means that the individuals of our model derive utility from the comparison of the current level of own consumption with that in the previous period. Therefore, when individuals choose their current consumption, they are simultaneously setting a standard of living that will be used to evaluate the utility accruing from the level of future consumption. We assume that past consumption imposes a minimum level for future consumption and, hence, we use the "subtractive" functional form to introduce habits. This is in contrast to the "multiplicative" specification considered by other authors like, for instance, Abel (1990), Carroll et al. (1997, 2000), and Carroll (2000). The reason for our choice is that the subtractive formulation allows us to maintain the usual concavity property of the utility function and does not make the analysis substantially more cumbersome than under the traditional specification of preferences. Moreover, some empirical studies have argued in favor of the subtractive specification to reconcile theory with consumption data.[2]

We assume that the benchmark level of consumption is also driven by either "egoistic" or "altruistic" motivations, since individuals care about the level of consumption of their neighbors. More precisely, our individuals' utility depends both on the lagged and on the current levels of average consumption in the economy. These consumption spillovers may either reduce or increase the felicity that each individual obtains from his own (habit adjusted) consumption. Our model thus encompasses both the "catching up with the Joneses" and the "keeping up with the Joneses" features introduced by Abel (1990) and Galí (1994), respectively. In the former case the lagged value of others' consumption makes more valuable the marginal increase of current own consumption, while in the later case the current value of others' consumption increases that private marginal utility. In order to be consistent with

[1]Examples of this literature are the papers by Abel (1990, 1999) and Galí (1994), who look at the implications for the equity premium puzzle; Lettau and Uhlig (2000), who analyze some stylized business cycle facts; Ljungqvist and Uhlig (2000), who examine the effects of fiscal policy; and Carroll et al. (1997 and 2000) and Shieh et al. (2000), who study how the patterns of growth are modified when preferences are time dependent.

[2]See, for instance, Ferson and Constantinidies (1991) and Heaton (1995).

the functional form used to introduce habits, we also use the subtractive specification for consumption spillovers. Therefore, the argument in individuals' utility function turns out to be an additive combination of the current level of own consumption, the lagged value of own consumption, the current level of average consumption, and the lagged value of average consumption.

The rest of the model has the typical features of a deterministic economy with infinitely lived agents. The production side is modeled with a standard neoclassical production function with constant returns to scale that uses raw labor and capital as inputs. The equilibrium exhibits transitional dynamics driven by the decreasing returns to capital and the time dependence of preferences. At a given period the standard of living derived from past consumption is fixed and, thus, the consumption does not adjust instantaneously to the benchmark level. Therefore, when habits are introduced, the assumption of decreasing returns to capital is not necessary to generate transitional dynamics since the preferences alone can give raise to non-instantaneous adjustment paths.

Consumption externalities constitute a potential source of equilibrium inefficiency since individuals do not take them into account when they choose their individual consumption paths. In particular, the consumption externalities considered in our model affect the future standard of living and, thus, they have consequences for the individuals' willingness to substitute consumption across periods. Since a benevolent social planner would internalize these consumption spillovers, the marginal rate of substitution between consumption at different periods of an individual behaving competitively differs from that of the social planner. As a consequence, the competitive path of consumption is not efficient. It should be noted that the key element for the existence of inefficiency is the interaction of consumption externalities with time dependent preferences. Thus, the externality accruing from the current average consumption does not generate inefficiency whenever individuals' preferences are neither subject to a process of habit formation nor to spillover effects from the others' past consumption. Under time independent preferences, contemporaneous externalities do not have intertemporal effects on the marginal utility of own consumption and, thus, the functional form of the marginal rate of substitution of a competitive economy is identical to that of the social planner.

The existence of inefficient competitive paths calls for implementing a fiscal policy aimed to replicate the socially planned solution. We characterize the optimal rates of both a capital income tax and a consumption tax. We show that, if individuals' willingness to shift present consumption to the future is suboptimally low along the socially planned solution, equilibrium efficiency will be achieved by means of either subsidizing capital income or introducing a tax on consumption with a tax rate that falls over time. These tax policies decrease the relative price of future consumption and encourage individuals to shift consumption from the present to the future. Moreover, the optimal rates of the capital income tax and of the consumption tax converge to zero and to a constant value, respectively, since no inefficiencies appear at a steady state.

The outline of the paper is as follows. Section 2 presents the benchmark model where preferences are endogenously determined. In Section 3 we derive the equilibrium path of the competitive economy, while in Section 4 we characterize the solution of the socially planned economy. Section 5 discusses the efficiency properties of the competitive equilibrium. We characterize the optimal taxation policy in Section 6. Section 7 concludes the paper.

2. The Model

The economy is composed of a continuum of identical individuals facing an infinite lifetime in discrete time. The population grows at a constant, exogenous growth rate $n > -1$. Individual preferences differ from the traditional specification where individuals derive utility only from their absolute level of consumption in each period. We assume thus that individuals derive utility from the comparison between current own consumption and a reference level. This reference level is determined by the lagged value of individuals' own consumption and by current and lagged average consumption in the economy. We thus posit the following instantaneous utility function:

$$u\left(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1}\right) = \frac{\left(c_t - \gamma c_{t-1} - \alpha \overline{c}_{t-1} - \theta \overline{c}_t\right)^{1-\sigma}}{1-\sigma}, \tag{2.1}$$

where c_t is the consumption at period t of the individual under consideration and $\overline{c}_t$ is the average consumption of the economy at period t. We assume that the preference parameters satisfy $\sigma > 0$, $\gamma \in (0,1)$, $\alpha \in (-1,1)$ and $\theta \in (-1,1)$. The parameter σ becomes the inverse of the elasticity of intertemporal substitution when $\gamma = \alpha = \theta = 0$. The particular case with $\gamma > 0$ and $\alpha = \theta = 0$ corresponds to the habit formation model where the amount of own consumption in the previous period becomes a standard of living that is used to evaluate the utility accruing from current consumption. The parameter γ thus measures how important is the reference set by past own consumption. As Constantinidies (1990) and Campbell and Cochrane (1999) among others, we use a subtractive specification for modelling habit formation instead of the multiplicative one suggested by other authors like Abel (1990, 1999) and Carroll et al. (1997, 2000). The later formulation would force us to consider only the case where the parameter σ takes a value larger than one in order to obtain interior solutions for the competitive consumption path.[3] Our specification of habit formation avoids this problem at the cost of having a different one, namely, that the habit adjusted consumption could be negative and, hence, the utility function would not be well defined in that case (see Carroll, 2000). In our deterministic framework, this problem is easily solved by imposing the appropriate parametric conditions that ensure a positive value of the habit adjusted consumption.

If $\alpha \neq 0$, individuals care about the lagged value of average consumption in the economy. In particular, when we assume $\alpha > 0$ and $\gamma = \theta = 0$ in (2.1), the model reduces to the simple formulation of "catching up with the Joneses" introduced by Abel (1990).[4] If $\theta \neq 0$, individuals' utility is affected by an externality accruing from the current average consumption in the economy, as in Galí (1994). The case with $\theta > 0$ and $\alpha = \gamma = 0$ corresponds to the "keeping up with the Joneses" model

[3] Alonso-Carrera et al. (2001) deal with this issue.

[4] When preferences display a "catching up with the Joneses" feature, it is also said that they are subjected to a process of "external habit formation".

where the others' consumption increases the marginal utility of own consumption. In the case with $\theta < 0$ and $\alpha = \gamma = 0$ average consumption displays negative externalities since the others' consumption makes less valuable an additional unit of own consumption.

Note that, in order to be consistent with the specification used for the process of habit formation, we posit the subtractive formulation to introduce consumption spillovers. As we have pointed out above, we will have to impose conditions that ensure a positive value of the argument of the utility function (2.1) along the equilibrium path of consumption. By simply imposing that $\alpha + \gamma + \theta < 1$, we will guarantee that the utility function will be well defined around a stationary consumption path. From now on, we assume that the previous inequality holds.

Each individual inelastically supplies one unit of labor in each period. At each date a single good is produced according to a constant returns to scale technology that uses labor and capital as inputs. Gross output per capita y_t is thus a function of capital per capita k_t,

$$y_t = f(k_t),$$

where the per capita production function f satisfies the standard neoclassical properties, $f'(k_t) > 0$ and $f''(k_t) < 0$, and the usual Inada conditions for $k_t > 0$. The single good of the economy can be either consumed or added to the capital stock.

The government in this economy sets flat rate taxes on capital income and on consumption. We allow both tax rates to be time-varying. The fiscal revenues are returned to individuals by means of a lump-sum subsidy. Hence, the government faces the following budget constraint:

$$\tau_t^k r_t k_t + \tau_t^c c_t = S_t, \tag{2.2}$$

where τ_t^k and τ_t^c are respectively the income tax rate and the consumption tax rate at time t, r_t is the rental rate of capital, and S_t is a lump-sum transfer per capita. The budget constraint of an individual is thus

$$(1 + \tau_t^c)c_t = w_t + \left[1 + \left(1 - \tau_t^k\right) r_t - \delta\right] k_t + S_t - (1 + n)\, k_{t+1}, \tag{2.3}$$

where w_t is the rental rate of labor and $\delta \in (0, 1)$ is the depreciation rate of the capital stock.

In the following two sections we will analyze the competitive equilibrium and the socially planned solution of this economy.

3. The Competitive Equilibrium

In the competitive economy, factor prices are equated to marginal productivities so that

$$w_t = f(k_t) - f'(k_t)\, k_t, \tag{3.1}$$

and

$$r_t = f'(k_t). \tag{3.2}$$

Each individual chooses a sequence of consumption $\{c_t\}_{t=0}^{\infty}$ aimed to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1}),$$

subject to the budget constraint (2.3), for a given sequence $\{\overline{c}_t\}_{t=-1}^{\infty}$ of average consumption and for the given initial conditions on capital k_0 and on consumption c_{-1}. The parameter $\beta \in (0,1)$ is the subjective discount factor. The Lagrangian corresponding to this problem is the following:

$$\begin{aligned} L(c,k,\lambda) \quad = \quad & \sum_{t=0}^{\infty} \Big\{ \beta^t u(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1}) \\ & + \lambda_t \Big[w_t + \Big(1 + \Big(1 - \tau_t^k\Big) r_t - \delta\Big) k_t + S_t - (1+\tau_t^c) c_t - (1+n) k_{t+1} \Big] \Big\}, \end{aligned}$$

where $c = \{c_t\}_{t=-1}^{\infty}$, $k = \{k_t\}_{t=0}^{\infty}$ and $\lambda = \{\lambda_t\}_{t=0}^{\infty}$ is the infinite sequence of positive Lagrange multipliers. Let us define $u(t) \equiv u(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1})$, $u_1(t) = \frac{\partial u(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1})}{\partial c_t}$ and $u_2(t) = \frac{\partial u(c_t, c_{t-1}, \overline{c}_t, \overline{c}_{t-1})}{\partial c_{t-1}}$. The first order conditions of the previous problem are thus

$$\frac{\partial L}{\partial c_t} = \beta^t u_1(t) + \beta^{t+1} u_2(t+1) - \lambda_t (1+\tau_t^c) = 0, \tag{3.3}$$

and

$$\frac{\partial L}{\partial k_{t+1}} = -(1+n)\lambda_t + \lambda_{t+1}\Big(1 + \Big(1-\tau_{t+1}^k\Big) f'(k_{t+1}) - \delta\Big) = 0, \tag{3.4}$$

for all t.

The competitive equilibrium is defined by the positive paths of c_t, k_t and λ_t satisfying conditions (3.3) and (3.4), in addition to the budget constraint (2.3), the budget constraint of the government (2.2), the profit-maximizing conditions (3.1) and (3.2), the transversality conditions

$$\lim_{t\to\infty} \lambda_t k_{t+1} = 0, \tag{3.5}$$

$$\lim_{t\to\infty} \beta^t u_1(t) c_t = 0, \tag{3.6}$$

and the initial conditions on k_0 and c_{-1}.

Combining the equations (3.3) and (3.4), and plugging (3.2) in the resulting equation, we get

$$\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{u_1(t+1) + \beta u_2(t+2)}{u_1(t) + \beta u_2(t+1)}\right) = \frac{1+n}{\beta\left[1 + (1-\tau_{t+1}^k) f'(k_{t+1}) - \delta\right]}, \tag{3.7}$$

which is the typical Euler condition equating the marginal rate of substitution (MRS, henceforth) of consumption between periods t and $t+1$ with the corresponding marginal rate of transformation (MRT, henceforth). Note that the previous equation differs from the Euler equation appearing in standard models of capital accumulation because here consumers take into account the effect that current consumption has in setting the reference for next period consumption. According to (2.1), an using the fact that in a symmetric equilibrium $c_t = \overline{c}_t$ for all t, we observe that in equilibrium,

$$u_1(t) = \frac{u(t)}{(1-\theta)c_t - (\gamma+\alpha)c_{t-1}}, \tag{3.8}$$

and

$$u_2(t) = -\gamma u_1(t). \tag{3.9}$$

Substituting (3.8) and (3.9) into (3.7), we obtain

$$\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{1-\gamma\beta\frac{u_1(t+2)}{u_1(t+1)}}{1-\gamma\beta\frac{u_1(t+1)}{u_1(t)}}\right)\left(\frac{u_1(t+1)}{u_1(t)}\right) = \frac{1+n}{\beta\left[1+\left(1-\tau_{t+1}^k\right)f'(k_{t+1})-\delta\right]}. \tag{3.10}$$

Let us define the gross rate of growth of the marginal utility $u_1(t)$,

$$\phi_t = \frac{u_1(t+1)}{u_1(t)}, \tag{3.11}$$

and we can rewrite (3.10) as

$$\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{1-\gamma\beta\phi_{t+1}}{1-\gamma\beta\phi_t}\right)\phi_t = \frac{1+n}{\beta\left[1+\left(1-\tau_{t+1}^k\right)f'(k_{t+1})-\delta\right]}. \tag{3.12}$$

Next, we use the definition of ϕ_t to obtain

$$\phi_t = \frac{u_1(t+1)}{u_1(t)} = \frac{\left(c_{t+1}(1-\theta)-(\gamma+\alpha)c_t\right)^{-\sigma}}{\left(c_t(1-\theta)-(\gamma+\alpha)c_{t-1}\right)^{-\sigma}}. \tag{3.13}$$

Let us define the gross rate of growth of consumption,

$$x_t = \frac{c_t}{c_{t-1}}, \tag{3.14}$$

so that equation (3.13) becomes

$$(x_t)^{-\sigma}\left(\frac{x_{t+1}-\frac{\gamma+\alpha}{1-\theta}}{x_t-\frac{\gamma+\alpha}{1-\theta}}\right)^{-\sigma} = \phi_t,$$

which can also be written as

$$x_{t+1} = \left(\frac{\gamma+\alpha}{1-\theta}\right) + \phi_t^{-\frac{1}{\sigma}}\left(1-\left(\frac{1}{x_t}\right)\left(\frac{\gamma+\alpha}{1-\theta}\right)\right). \tag{3.15}$$

Combining the government and the individual budget constraints (2.2) and (2.3), and substituting (3.1) and (3.2) in the resulting equation, we obtain the resource constraint

$$k_{t+1} = \frac{f(k_t)+(1-\delta)k_t - c_t}{1+n}. \tag{3.16}$$

The system of first order difference equations (3.12), (3.14), (3.15) and (3.16), together with the transversality conditions (3.5) and (3.6) and the initial conditions on k_0 and c_{-1}, fully characterize the equilibrium path of the variables x_t, ϕ_t, c_t and k_t.

Let us assume now that the government follows a stationary fiscal policy, that is, $\tau_t^c = \tau^c$ and $\tau_t^k = \tau^k$ for all t. At a steady state of the system of dynamic equations (3.12), (3.14), (3.15) and (3.16) the variables x_t, ϕ_t, c_t and k_t are all constant. Making $k_t = k$, $c_{t-1} = c$, $x_t = x$ and $\phi_t = \phi$ for all t in the system of equations (3.12), (3.14), (3.15) and (3.16), we get the following stationary values of the variables of the model:

$$x = \phi = 1, \tag{3.17}$$

$$f'(k) = \frac{1 + n - \beta(1-\delta)}{\beta\left(1-\tau^k\right)}, \tag{3.18}$$

and

$$c = f(k) - (n+\delta)\,k. \tag{3.19}$$

Lemma 1. *Let $\beta\left(1-\tau^k\right) \in (0,1)$. An interior steady state exists if and only if the following condition holds:*

$$1 + n > \beta(1-\delta). \tag{3.20}$$

Proof. Since $f'(k) > 0$ and $\beta\left(1-\tau^k\right) > 0$, equation (3.18) implies that condition (3.20) should hold. Moreover, concavity implies that $\frac{f(k)}{k} > f'(k)$, whereas (3.18) implies that $f'(k) > n + \delta$ whenever $\beta\left(1-\tau^k\right) \in (0,1)$. Thus, $\frac{f(k)}{k} > n + \delta$, which means that $c > 0$ as follows from (3.19). ■

We will next show that the previous steady state is indeed well defined. Note first that the argument in the instantaneous utility (2.1) is always strictly positive in the steady state since $x = 1$ and $\alpha + \gamma + \theta < 1$. Therefore, the objective function of each individual is well defined around a competitive steady state.

Using (3.8), the transversality condition (3.6) becomes

$$\lim_{t\to\infty} \beta^t u(t) \left(\frac{x_t}{(1-\theta)\,x_t - (\gamma+\alpha)}\right) = 0.$$

Since x_t and $u(t)$ are constant at the steady state, the previous condition holds because β belongs to the open interval $(0,1)$ and $\alpha + \gamma + \theta < 1$.

According to (3.9) and (3.11), the first order condition (3.3) at the steady state becomes

$$\lambda_t = \frac{\beta^t u_1(t) - \gamma\beta^{t+1} u_1(t+1)}{1+\tau^c} = \beta^t u_1(t) \left(\frac{1-\gamma\beta\phi}{1+\tau^c}\right). \tag{3.21}$$

Since $\phi = 1$, we observe that $\lambda_t > 0$ if and only if $\gamma\beta < 1$. The previous inequality always holds since both γ and β belong to the open interval $(0,1)$. Therefore, since $\lambda_t > 0$, the discounted sum of utilities is increasing in the amount of current consumption c_t (see (3.21)). Finally, substituting (3.21) into the transversality condition (3.5), we can also conclude that this transversality condition is also satisfied at the steady state.

Concerning the stability properties of the steady state we just have to notice that the dynamic system formed by the difference equations (3.12), (3.14), (3.15) and (3.16) displays saddle path stability whenever $\gamma = \alpha = \theta = 0$, since in this case the model reduces to the standard neoclassical model of capital accumulation. Thus, the steady state of the model considered in this paper is saddle-path stable for values of the vector $(\gamma, \alpha, \theta) \in \mathbb{R}^3$ lying in a sufficiently small neighborhood of the vector $(0, 0, 0)$. From now on, we will assume that the steady state of our model is saddle-path stable so that, for given initial conditions on k_0 and c_{-1}, the equilibrium path converges to the steady state characterized above.

Finally, we can characterize the long run effects of changes in the stationary tax rates τ^c and τ^k. These effects immediately follow from applying a comparative static analysis over the expressions of c and k in (3.18) and (3.19). We observe that k is decreasing in τ^k. Moreover, changes in the tax rate τ^c have no long run effects. The results coincide with those obtained in the standard neoclassical growth model.[5] In fact, the steady state derived in this model with endogenous tastes coincides with the steady state of the standard neoclassical model. This occurs because the steady state is independent of the preference parameters α, γ and θ.

4. The Efficient Solution

In this section we turn our attention to the solution that a time consistent social planner would implement. This solution is also called the efficient solution. The planner internalizes the consumption spillovers and, thus, he perceives the following utility function:

$$\hat{u}(c_t, c_{t-1}) \equiv u(c_t, c_{t-1}, c_t, c_{t-1}) = \frac{\left((1-\theta)\, c_t - (\alpha + \gamma)\, c_{t-1}\right)^{1-\sigma}}{1-\sigma}. \tag{4.1}$$

Moreover, the aggregate resource constraint per capita faced by the social planner is just equation (3.16). Therefore, the Lagrangian for the social planner's problem is given by

$$L\left(c, k, \hat{\lambda}\right) = \sum_{t=0}^{\infty} \left\{ \beta^t \hat{u}\left(c_t, c_{t-1}\right) + \hat{\lambda}_t \left[f(k_t) + (1-\delta)\, k_t - c_t - (1+n)\, k_{t+1} \right] \right\},$$

where $\hat{\lambda} = \left\{\hat{\lambda}_t\right\}_{t=0}^{\infty}$ is the infinite sequence of positive Lagrange multipliers. Define $\hat{u}(t) \equiv \hat{u}(c_t, c_{t-1})$, $\hat{u}_1(t) = \frac{\partial \hat{u}(c_t, c_{t-1})}{\partial c_t}$ and $\hat{u}_2(t) = \frac{\partial \hat{u}(c_t, c_{t-1})}{\partial c_{t-1}}$. The first order conditions for the social planner's problem are thus

$$\frac{\partial \hat{L}}{\partial c_t} = \beta^t \hat{u}_1(t) + \beta^{t+1} \hat{u}_2(t+1) - \hat{\lambda}_t = 0, \tag{4.2}$$

and

[5] See, for instance, Hall (1971) and Brock and Turnovsky (1981), among many others.

$$\frac{\partial \hat{L}}{\partial k_{t+1}} = \left(1 + f'(k_{t+1}) - \delta\right)\hat{\lambda}_{t+1} - (1+n)\,\hat{\lambda}_t = 0, \tag{4.3}$$

for all t. The socially planned equilibrium is defined by the positive paths of $\widehat{c}_t$, $\widehat{k}_t$ and $\hat{\lambda}_t$ satisfying conditions (4.2) and (4.3) in addition to the resource constraint (3.16), the transversality conditions

$$\lim_{t\to\infty} \hat{\lambda}_t k_{t+1} = 0, \tag{4.4}$$

$$\lim_{t\to\infty} \beta^t \hat{u}_1(t) c_t = 0, \tag{4.5}$$

and the initial conditions on k_0 and c_{-1}.

Combining equations (4.2) and (4.3), we get the typical Euler condition

$$\frac{\hat{u}_1\,(t+1) + \beta\hat{u}_2\,(t+2)}{\hat{u}_1\,(t) + \beta\hat{u}_2\,(t+1)} = \frac{1+n}{\beta\left[1 + f'(\hat{k}_{t+1}) - \delta\right]}. \tag{4.6}$$

According to (4.1), we observe that

$$\hat{u}_1\,(t) = \hat{u}\,(t)\left(\frac{1-\theta}{(1-\theta)\,\hat{c}_t - (\gamma+\alpha)\,\hat{c}_{t-1}}\right), \tag{4.7}$$

and

$$\hat{u}_2\,(t) = -\hat{u}_1\,(t)\left(\frac{\gamma+\alpha}{1-\theta}\right). \tag{4.8}$$

Plugging (4.8) in (4.6), we obtain

$$\left(\frac{1 - \left(\frac{\beta\,(\gamma+\alpha)}{(1-\theta)}\right)\left(\frac{\hat{u}_1(t+2)}{\hat{u}_1(t+1)}\right)}{1 - \left(\frac{\beta\,(\gamma+\alpha)}{(1-\theta)}\right)\left(\frac{\hat{u}_1(t+1)}{\hat{u}_1(t)}\right)}\right)\left(\frac{\hat{u}_1\,(t+1)}{\hat{u}_1\,(t)}\right) = \frac{1+n}{\beta\left[1 + f'(\hat{k}_{t+1}) - \delta\right]}. \tag{4.9}$$

Let us define $\hat{\phi}_t = \frac{\hat{u}_1(t+1)}{\hat{u}_1(t)}$. Note that the definition of $\hat{\phi}_t$ is the exact counterpart of that of ϕ_t given in (3.11) for the competitive economy. We can rewrite (4.9) as

$$\left(\frac{\varepsilon - \hat{\phi}_{t+1}}{\varepsilon - \hat{\phi}_t}\right)\hat{\phi}_t = \frac{1+n}{\beta\left[1 + f'(\hat{k}_{t+1}) - \delta\right]}, \tag{4.10}$$

where $\varepsilon = \dfrac{1-\theta}{\beta\,(\gamma+\alpha)}$.

Using the definition of $\hat{\phi}_t$ and the functional form of the utility function, we obtain

$$\hat{\phi}_t = \frac{\hat{u}_1\,(t+1)}{\hat{u}_1\,(t)} = \frac{\left(\hat{c}_{t+1}\,(1-\theta) - (\gamma+\alpha)\,\hat{c}_t\right)^{-\sigma}}{\left(\hat{c}_t\,(1-\theta) - (\gamma+\alpha)\,\hat{c}_{t-1}\right)^{-\sigma}}. \tag{4.11}$$

Let us define now the gross rate of consumption growth of the social planner's solution

$$\hat{x}_t = \frac{\hat{c}_t}{\hat{c}_{t-1}}. \tag{4.12}$$

Then, equation (4.11) becomes

$$(\hat{x}_t)^{-\sigma} \left(\frac{\hat{x}_{t+1} - \frac{\gamma+\alpha}{1-\theta}}{\hat{x}_t - \frac{\gamma+\alpha}{1-\theta}} \right)^{-\sigma} = \hat{\phi}_t,$$

which can also be written as

$$\hat{x}_{t+1} = \left(\frac{\gamma+\alpha}{1-\theta} \right) + \hat{\phi}_t^{-\frac{1}{\sigma}} \left(1 - \left(\frac{1}{\hat{x}_t} \right) \left(\frac{\gamma+\alpha}{1-\theta} \right) \right). \tag{4.13}$$

Finally, from the resource constraint (3.16), we obtain

$$\hat{k}_{t+1} = \frac{f\left(\hat{k}_t\right) + (1-\delta)\hat{k}_t - \hat{c}_t}{1+n}. \tag{4.14}$$

The system of first order difference equations (4.10), (4.12), (4.13) and (4.14), together with the transversality conditions (4.4) and (4.5) and the initial conditions k_0 and c_{-1}, fully characterize the dynamics of the variables $\hat{\phi}_t$, $\hat{x}_t$, $\hat{c}_t$ and $\hat{k}_t$. We can easily check that the difference equations (4.12), (4.13) and (4.14) characterizing the socially planned solution are the exact counterparts of equations (3.14), (3.15) and (3.16) characterizing the competitive equilibrium. Therefore, the competitive and the socially planned solutions only differ in the equations characterizing the evolution of ϕ_t and $\hat{\phi}_t$ (see (3.12) and (4.10)).

In order to find the steady state of the previous dynamic system, we evaluate equations (4.10), (4.12), (4.13) and (4.14) at $\hat{\phi}_t = \hat{\phi}$, $\hat{x}_t = \hat{x}$, $\hat{c}_t = \hat{c}$ and $\hat{k}_t = \hat{k}$ for all t. We thus obtain the following stationary values for the variables of the model:

$$\hat{x} = \hat{\phi} = 1, \tag{4.15}$$

$$f'(\hat{k}) = \frac{1+n-\beta(1-\delta)}{\beta}, \tag{4.16}$$

and

$$\hat{c} = f(\hat{k}) - (\delta+n)\,\hat{k}. \tag{4.17}$$

We see from looking at (3.17), (3.18) and (3.19), that $\hat{\phi} = \phi$, $\hat{x} = x$, $\hat{c} = c$ and $\hat{k} = k$ whenever $\tau_t^k = 0$ and $\tau_t^c = \tau_{t+1}^c$ for all t. Therefore, when the tax rate on capital income is zero and the tax rate on consumption is constant, the steady state of the competitive solution coincides with that of the efficient solution. As we pointed out when discussing the competitive equilibrium, the steady state of the socially planned solution is saddle-path stable for values of the vector (γ, α, θ) sufficiently close to the vector $(0,0,0)$.

Note that condition (3.20), which was imposed to ensure an interior steady state for the competitive economy, is also required to obtain a socially planned solution displaying an interior steady state. Moreover, the transversality conditions (4.4) and (4.5) hold at the steady state. Using (4.7), the transversality condition (4.5) becomes

$$\lim_{t\to\infty} (1-\theta)\,\beta^t \hat{u}(t) \left(\frac{\hat{x}_t}{(1-\theta)\,\hat{x}_t - (\gamma+\alpha)} \right) = 0.$$

Since $\hat{x}_t$ and $\hat{u}(t)$ are constant at the steady state, the previous condition is satisfied because $\beta \in (0,1)$ and $\alpha+\gamma+\theta < 1$.

According to (4.8) and the definition of $\hat{\phi}_t$, the first order condition (4.2) at the steady state becomes

$$\hat{\lambda}_t = \beta^t \hat{u}_1(t) - \beta^{t+1} \hat{u}_1(t+1) \left(\frac{\gamma+\alpha}{1-\theta} \right) = \beta^t \hat{u}_1(t) \left(1 - \frac{(\gamma+\alpha)\,\hat{\phi}}{1-\theta} \right). \tag{4.18}$$

Since $\phi = 1$ and $\alpha+\gamma+\theta < 1$, we observe that $\lambda_t > 0$. Therefore, since $\hat{\lambda}_t > 0$ the discounted sum of utilities faced by the social planner is increasing in the amount of current consumption $\hat{c}_t$ (see (4.18)). Finally, plugging (4.18) in the transversality condition (4.4), we immediately see that this transversality condition is also met at the steady state.

5. Equilibrium Efficiency

The competitive equilibrium may be inefficient in our model because individuals do not internalize the spillovers arising form the other agents' consumption. In fact, there are two potential sources of inefficiency: the externality accruing from the lagged value of average consumption and that generated by the current level of average consumption. When individuals choose the present level of consumption, they are affecting present and future marginal utilities in a way that is not completely internalized. In this section, we study the conditions under which a non-efficient path arises in equilibrium.

Comparing the equations characterizing the competitive equilibrium with the ones characterizing the social planner's solution, we observe that the only difference is in the equations that relate the growth rate of marginal utility with the stock of capital (see (3.12) and (4.10)). More precisely, the difference between the two solutions lies in the Euler equations. On the one hand, the Euler condition (3.7) for the competitive economy without taxes becomes

$$\frac{u_1(t+1) + \beta u_2(t+2)}{u_1(t) + \beta u_2(t+1)} = \frac{1+n}{\beta\left[1 + f'(k_{t+1}) - \delta\right]}. \tag{5.1}$$

On the other hand, the Euler equation for the socially planned economy is given by the equation (4.6). Since the right hand sides of (5.1) and (4.6) are identical, the competitive path of consumption $\{c_t\}_{t=0}^{\infty}$ would be efficient if and only if the functional forms of the MRS's of the two economies are identical,

$$\frac{\hat{u}_1(t+1) + \beta \hat{u}_2(t+2)}{\hat{u}_1(t) + \beta \hat{u}_2(t+1)} = \frac{u_1(t+1) + \beta u_2(t+2)}{u_1(t) + \beta u_2(t+1)}.$$

Therefore, the competitive equilibrium is efficient if and only if

$$\hat{u}_1(t) + \beta \hat{u}_2(t+1) = \psi [u_1(t) + \beta u_2(t+1)], \tag{5.2}$$

for all t and for some constant ψ along the competitive equilibrium path of consumption. Recalling that $u_2(t) = -\gamma u_1(t)$ and $\hat{u}_2(t) = -\hat{u}_1(t) \left(\frac{\gamma+\alpha}{1-\theta}\right)$, condition (5.2) reduces to

$$\hat{u}_1(t) - \beta \hat{u}_1(t+1) \left(\frac{\gamma+\alpha}{1-\theta}\right) = \psi [u_1(t) - \gamma \beta u_1(t+1)]. \tag{5.3}$$

Using the definition $\phi_t = \frac{u_1(t+1)}{u_1(t)}$, and dividing by $u_1(t)$, the efficiency condition (5.3) becomes

$$\frac{\hat{u}_1(t)}{u_1(t)} - \beta \phi_t \left(\frac{\gamma+\alpha}{1-\theta}\right) \left(\frac{\hat{u}_1(t+1)}{u_1(t+1)}\right) = \psi [1 - \gamma \beta \phi_t]. \tag{5.4}$$

Given that externalities enter in the utility function in a subtractive way, $u_1(t)$ and $\hat{u}_1(t)$ are linearly dependent for all t. In particular, according to (3.8) and (4.7), it holds that $\hat{u}_1(t) = (1-\theta) u_1(t)$ along the competitive equilibrium path of consumption. Therefore, the efficiency condition (5.4) simplifies to

$$\beta [\alpha + \gamma (1-\psi)] \phi_t = 1 - \theta - \psi. \tag{5.5}$$

Let us emphasize two important conclusions from the efficiency condition (5.5). First, the growth rate of the marginal utility is not constant off the steady state of the competitive economy. Therefore, the efficiency condition (5.5) does not hold along the transition to the steady state and, thus, there is role for imposing taxes aimed to restore efficiency. Second, the externality accruing from the present average consumption does not generate inefficiencies unless individuals' preferences be time dependent. The following proposition states precisely the result:

Proposition 1. *Let $\tau^c = \tau^k = 0$. Then,*

(a) The competitive equilibrium is efficient at the steady state.

(b) The competitive equilibrium is efficient off the steady state if and only if at least one of the following sets of conditions holds: (i) $\alpha = \theta = 0$ or, (ii) $\alpha = \gamma = 0$.

Proof. (a). Since the variable ϕ_t is constant at the steady state, the statement in part (a) follows directly from condition (5.5).

(b) First, when $\alpha = \theta = 0$, there are no externalities and, hence, there is not any source of inefficiency. Second, when $\alpha = \gamma = 0$, the efficiency condition (5.3) simplifies to $\hat{u}_1(t) = \psi u_1(t)$, which is satisfied as follows from (3.8) and (4.7). ■

Under saddle-path stability, the statement of part (a) of the previous proposition implies that the competitive equilibrium converges to a stationary path that is efficient. In fact, this property was already obtained in the previous section, since we saw there that the stationary competitive solution with no taxes is identical to the stationary efficient solution. Part (b) of Proposition 1 tells us that inefficiency of the competitive equilibrium requires some consumption externality combined with time dependent preferences. The externality accruing from the contemporaneous value of average consumption is thus not sufficient to generate inefficiency. If preferences are time independent ($\alpha = \gamma = 0$), the previous contemporaneous externality will appear as a scale factor affecting symmetrically the marginal utility of own consumption in all periods and, hence, the MRS of the competitive economy will be equal to the efficient MRS. In contrast, if the contemporaneous externality coexists with a process of habit formation ($\theta \neq 0$ and $\gamma \neq 0$), then the externality also affects the standard of living of next period. Therefore, since the social planner internalizes the spillovers from present consumption, he can affect at some extend how important is the past standard of living for current utility. The social planner can minimize the effect of habits on future consumption while maximizing simultaneously the utility from current consumption. In other words, in the socially planned solution habits turn out to be less important than in the competitive equilibrium and, thus, the MRS of the competitive equilibrium differs from that of the socially planned economy.

If an externality arises from the lagged value of average consumption ($\alpha \neq 0$), current consumption affects the future standard of living in a way that individuals do not fully internalize. In this case, habits in the competitive equilibrium are again more important than in the efficient solution. Therefore, the MRS of the competitive economy would also differ from that of the socially planned economy.

6. Optimal Taxation

We have just shown that the competitive equilibrium can be inefficient along the transition to the steady state. Inefficiency comes from the discrepancy between the Euler equation of the competitive economy and that of the socially planned economy (see (3.7) and (4.6)). In particular, what is different in these equations are the functional forms of the MRS's of both economies. The government can thus design a fiscal policy that restores the efficiency of the competitive equilibrium by driving the competitive MRS to its efficient value. We will see next that the consumption tax and the capital income tax are alternative instruments that allow the decentralized economy to reach an efficient equilibrium path.

Let MRS^d and MRS^p be the MRS corresponding to the competitive economy and to the socially planned economy, respectively, that is,

$$MRS^d\left(c_{t+2}, c_{t+1}, c_t, c_{t-1}\right) = \frac{u_1\left(c_{t+1}, c_t, c_{t+1}, c_t\right) + \beta u_2\left(c_{t+2}, c_{t+1}, c_{t+2}, c_{t+1}\right)}{u_1\left(c_t, c_{t-1}, c_t, c_{t-1}\right) + \beta u_2\left(c_{t+1}, c_t, c_{t+1}, c_t\right)}$$

and

$$MRS^p\left(c_{t+2}, c_{t+1}, c_t, c_{t-1}\right) = \frac{\hat{u}_1\left(c_{t+1}, c_t\right) + \beta \hat{u}_2\left(c_{t+2}, c_{t+1}\right)}{\hat{u}_1\left(c_t, c_{t-1}\right) + \beta \hat{u}_2\left(c_{t+1}, c_t\right)}.$$

Therefore, the Euler equation (3.7) of the competitive economy becomes

$$\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) MRS^d\left(c_{t+2}, c_{t+1}, c_t, c_{t-1}\right) = \frac{1+n}{\beta\left[1+\left(1-\tau_{t+1}^k\right) f'\left(k_{t+1}\right)-\delta\right]}, \qquad (6.1)$$

while the Euler equation (4.6) of the socially planned economy becomes in turn

$$MRS^p\left(\hat{c}_{t+2}, \hat{c}_{t+1}, \hat{c}_t, \hat{c}_{t-1}\right) = \frac{1+n}{\beta\left[1+f'\left(\hat{k}_{t+1}\right)-\delta\right]}. \qquad (6.2)$$

From inspection of the right hand sides of equations (6.1) and (6.2), we see that the functional forms of the MRT's of the two economies are identical under zero tax rates. Evaluating equation (6.1) along the efficient path, and dividing the resulting equation by (6.2), we obtain the following optimal taxation condition:

$$\frac{MRS^d\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right)}{MRS^p\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right)}=\left(\frac{1+\hat{\tau}^c_{t+1}}{1+\hat{\tau}^c_t}\right)\left(\frac{1+f'(\hat{k}_{t+1})-\delta}{1+\left(1-\hat{\tau}^k_{t+1}\right)f'(\hat{k}_{t+1})-\delta}\right), \tag{6.3}$$

where $\hat{\tau}^c_t$ and $\hat{\tau}^k_t$ denote the optimal rates of the consumption tax and of the capital income tax at period t, respectively. The next proposition characterizes the optimal tax rates:

Proposition 2. *(a) If* $MRS^d\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right) > (<) MRS^p\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right)$ *and* $\hat{\tau}^c_{t+1}=\hat{\tau}^c_t$*, then* $\hat{\tau}^k_t > (<) 0$.

(b) If $MRS^d\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right) > (<) MRS^p\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right)$ *and* $\hat{\tau}^k_t = 0$*, then* $\hat{\tau}^c_{t+1} > (<) \hat{\tau}^c_t$.

(c) If $MRS^d\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right) = MRS^p\left(\hat{c}_{t+2},\hat{c}_{t+1},\hat{c}_t,\hat{c}_{t-1}\right)$*, then to set* $\hat{\tau}^k_t=0$ *and* $\hat{\tau}^c_{t+1}=\hat{\tau}^c_t$ *for all* t *constitutes an optimal tax policy.*

Proof. The proposition follows directly from condition (6.3). ■

If the competitive MRS evaluated along the efficient path turns out to be larger than the efficient MRS evaluated along the same path, then the individuals' willingness to shift present consumption to the future would be too high. In this case, the efficient path can be reached by the decentralized economy through taxes that raise the price of future consumption in terms of present consumption in order to prevent consumption from being postponed. This can be achieved by means of either a positive tax rate on capital income or a sequence of tax rates on consumption that increase over time. Note that an increasing sequence of tax rates on consumption directly increases the after-tax relative price of future consumption. Moreover, a tax on capital income increases the cost of shifting resources to future periods and, thus, increases also the relative price of future consumption. If the MRS of the competitive economy along an efficient path is smaller than that of the socially planned economy, then the individuals' willingness to shift present consumption to the future will be suboptimally low. In this case, condition (6.3) establishes that a welfare-maximizing government must either subsidize capital or impose a tax on consumption with a rate falling over time. This optimal fiscal policy makes future consumption cheaper in terms of present consumption and, hence, it optimally drives individuals' willingness to shift consumption to the future up.[6]

We have proved in the previous section that the steady state value of the MRS corresponding to the competitive economy without taxes coincides with that of the MRS of the socially planned economy. Therefore, Proposition 2 also establishes that zero tax rates on capital income coupled with constant tax rates on consumption constitute an optimal policy at the steady state.

[6] Note that these results on optimal taxation are similar to those obtained by Fisher and Hof (2000), who disregard habit formation and consider externalities accruing only from the contemporaneous average consumption. These authors obtain a characterization of the optimal taxes in terms of the intertemporal elasticity of substitution of consumption.

7. Conclusion

In this paper we have analyzed the welfare properties of the competitive equilibrium of an economy with capital accumulation where we have assumed that individuals' preferences vary over time due to a process of habit formation and to the presence of consumption spillovers. Individuals will not derive utility from their absolute level of consumption at a given period but from the change of consumption with respect to a reference level. This reference consumption is determined by an additive combination of the past own consumption, the lagged value of average consumption and the current average consumption.

This departure from the more traditional formulations of preferences has consequences for the dynamic behavior of consumption and capital. In particular, consumption externalities could be an obvious source of inefficiency. More precisely, these externalities affect the welfare properties of the competitive equilibrium only if preferences are time dependent. Hence, contemporaneous consumption spillovers do not generate any kind of sub-optimality whenever individuals' utility is neither subjected to a process of habit formation nor to consumption spillovers from the lagged value of average consumption. This occurs because, in this case, the functional form of the marginal rate of substitution between consumption at different periods is identical to the efficient marginal rate of substitution. Consumption spillovers only break down the previous identity between the two marginal rates of substitution through their effect over the future standard of living. Obviously, this discrepancy calls for some public intervention aimed to restore efficiency. If consumption spillovers affect habits in a way that the individuals' willingness to shift consumption to the future along an efficient path is below (above) the efficient one, the government maximizes welfare by means of either subsidizing (taxing) capital or taxing consumption with tax rates that fall (increase) over time. Furthermore, the optimal rates of the capital income tax and of the consumption tax tend to zero and to a constant value, respectively.

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