

# Settlement in Tax Evasion Prosecution<sup>1</sup>

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## **Abstract**

It is often argued that even if optimal ex-post, settlement dilutes deterrence ex-ante. We analyze the interest for the tax authority of committing, ex-ante, to a settlement strategy. We show that to commit to the use of settlements is ex-ante optimal when the tax authority receives signals that provide statistical information about the taxpayers' true tax liability. The more informative the signal, the larger the additional expected revenue raised by the tax authority when using settlement as a policy tool.

# 1 Introduction

Many tax systems rely on voluntary compliance. Taxpayers are required to report their income honestly and to pay taxes according to their reported income. This system is actively enforced by the tax administration, which can impose substantial penalties for non-compliance when auditing allows to identify evasion.<sup>1</sup> However, it is often the case in practice that tax-evaders and the tax authority avoid further audits (and part of the sanctions) by a procedure in which the taxpayers agree to pay a lesser amount than the total charge associated to their evasion.<sup>2</sup> This practice is called *settlement*.<sup>3</sup>

The first intuition on the effects of this practice is that settlements are ex-post efficient (once the income reports have been made) since they save costs. The cost-saving is associated to exempting the taxpayer to be audited, but it has also been related in more general frameworks to the fact that going to court is costly to the parties, and the trial outcome is random.<sup>4</sup> While the reasons why settlement can be optimal ex-post have been often discussed, its ex-ante effect has been much less analyzed. One reason for this forsake is that settlements, in order to be accepted, must propose a reduction in the fine. The anticipation of such a reduction in the penalty can only have negative effects on the deterrence target. This argument has been extensively used to say that settlement will lead to a reduction in the administration revenue. In words of Polinsky and Shavell (2000): “settlement dilute deterrence: for if insurers desire to settle, it must be because the expected disutility of sanctions is lowered for them”. Franzoni (1999) explicitly analyzes a model of tax enforcement where settlement is viewed as a

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<sup>1</sup>See Andreoni, Erard, and Feinstein (1998) for a review on the literature of tax compliance.

<sup>2</sup>For example, Andreoni (1991) reports that between 1919 and 1952 the IRS has maintained an explicit policy of granting criminal immunity to tax-evaders who voluntarily paid their due taxes.

<sup>3</sup>It is also called plea bargaining. Settlement, or plea bargaining, differ from an amnesty in that it is offered to one individual who has already been selected for investigation. A depart from this view is taken in Chu (1990), who considers a plea bargaining offered from the beginning of the game, and it is not contingent to any audit result or to any information. In fact, his concept of plea bargaining is equivalent to a change in the tax function.

<sup>4</sup>See Grossman and Katz (1985) and Reinganum (1988) for an analysis that considers the welfare effects of plea bargaining when the government is concerned about the costs of erroneous convictions and acquittals, and there are limited resources to punish criminals.

renegotiation of the initial contract.<sup>5</sup> He shows that “settlements prove to be a poor enforcement tool” in his model.

However, it is so often a practice, that it is worthwhile to look for the rationale for the tax authority to use settlement offers. In our opinion, there is one fact that the argument given above misses out. Settlements often occur, not only when the individual has been selected for investigation, but also *when the tax authority has obtained some* (maybe partial, maybe soft) *information about the taxpayer’s true income*. In other words, settlement occurs when the authority has some preliminary, incriminatory evidence that the taxpayer may have evaded. We will argue that this fact has important consequences on the ex-ante optimality of settlement.

In a very stylized principal-agent model, we consider the *ex-ante* optimality of using settlement as part of the enforcement strategy. We assume that an exogenous random mechanism provides the tax authority with information correlated with the taxpayer’s true income.<sup>6</sup> This information is ‘soft’, and cannot be used to prosecute the individual as a tax-evader. However, this informative signal can be useful to threaten the tax-evader with a tough audit strategy at the settlement stage. Taxpayers have not control on the realization of the signal.<sup>7</sup>

The exogenous random variable is a simplification of the idea that the authority has access to lots of information on taxpayers that can be valuable to guess who may be evading taxes and to select individuals for audit. Often, for example, the information in the income report is introduced in a computer prior to any selection for audit. From the analysis of these data, or the search of inconsistencies, the tax administration has

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<sup>5</sup>The possibility of renegotiation cannot increase the ex-ante payoff to the principal except if it occurs after some new information becomes available to the parties. Papers like Reinganum (1993) and Miceli (1996) consider plea bargaining deterrence effects on a different framework. Miceli (1996) takes also the ex-ante perspective in the analysis of the effects of plea bargaining on crime deterrence when prosecutors are independent agents concerned by different objectives.

<sup>6</sup>Macho-Stadler and Pérez-Castrillo (2001) is a first step in analyzing the use of statistical information about taxpayers’ income. To avoid confusion let us stress that ours is not a signalling model: the signal received on a taxpayer is an exogenous random variable.

<sup>7</sup>In a trial context, there is some papers considering the information transmission prior to a settlement, such as Shavell (1989). This author considers voluntary sharing of information by a plaintiff with a defendant prior to settlement negotiations (then the defendant makes a take-or-leave-it offer).

sometimes a signal that a taxpayer may be underreporting. In fact, tax administrations tend to adopt a statistical approach to the problem of how to use the information at hand, as manifested in the DIF score method used in the USA.

In our model the random signal can take two values: “up” and “down”, and the realization may be correlated to the true tax liability of the taxpayer. If it is, we say that the signal is informative.

The tax administration designs a policy for each income level. This policy includes the initial payment (the report), the audit probability, the settlement probability and the settlement offer, as a function of the realization of the signal. We show that the optimal policy concentrates first in auditing the taxpayers whose realized signal is “up” in order to more effectively dissuade the high-income taxpayers from evading. In fact, the tax administration only audits taxpayers with signal “up” when the penalty rate is high or the budget is low.

We also show that, when the signal is informative, the use of settlements is optimal. When the signal is not informative (or, it is not available), settlement is never optimal, as it was argued in the previous literature.

To understand the benefits of offering settlement agreements, consider first the situations where the tax authority concentrates the auditing in taxpayers with signal “up”. That is, the tax authority does not expend resources in auditing the taxpayers whose realized signal is “down”. To settle (that in this case just occurs when “up” is observed) has the same effect as a random device that selects more often high than low-income taxpayers. The tax authority has very much interest in using such a device in the contract addressed to low-income taxpayers in order to dissuade high-income taxpayers to sign this contract. Hence, the optimal policy consists of threatening to audit with a high probability each time that the signal is “up”, but to offer a settlement deal with some frequency. This way, the effective audits can be kept in the numbers allowed by the budget. Higher budgets will translate in less frequent settlement because high-income taxpayers are even more dissuaded by a real audit than they are by a settlement.

Consider now the situation where the tax authority does not concentrate on taxpayers with signal “up” but assigns some audit probability to taxpayers whose realized signal is “down”. In this case, settlement offers are rather made to the taxpayers with

signal “down”. The rationale is that this strategy allows to concentrate more resources on auditing the taxpayers with a higher expected evasion (the ones that generate signal “up” more often). The settlement offer is contingent to the observed signal. Here, taxpayers with a low signal are offered a better deal. This is consistent with the intuition (and casual observation) that prosecutors usually do not allow all suspects to plead guilty to the same amount.

Finally, we also prove that using settlements may allow the tax administration to achieve full compliance at no cost. This happens when the signal is informative enough or the difference between high and low tax liabilities is small.

The paper is organized as follows. In Section 2 we present a simple model of tax evasion where, after the taxpayers have made their tax report, the tax authority receives a signal of the true income of each individual. In Section 3, we analyze the optimal enforcement policy. In Section 4, we discuss the value of using an informative signal, and we distinguish between the benefits due to the use of signals in the auditing policy and the benefits associated to settlement. In Section 5 we conclude. Finally, the Appendix contains all the proofs.

## 2 The Model

There exists a population of taxpayers characterized by their true tax liability  $i$ . The true tax liability  $i$  can take two different values,  $i \in \{L, H\}$ , with  $0 < L < H$ . There are  $h_i$  taxpayers with tax liability equal to  $i$ . The tax authority knows the distribution of tax liability, but it does not know the true tax liability of a particular taxpayer.

Taxpayers fill a tax return and pay the income tax liability corresponding to the return. We will denote a taxpayer’s reported tax liability by  $r_i$ . If a taxpayer’s true income is discovered, he has to pay the evaded tax liability (if any),  $(i - r_i)$ . On top of the tax due, an evader must pay a penalty. We assume that the marginal penalty rate is constant, i.e., penalties equal  $(f - 1)$  times the level of tax evasion for  $f > 1$ . There are no bonuses for over-reporting.

Taxpayers are risk neutral. They choose how much income to report in order to maximize their expected net income. Under risk neutrality this is equivalent to saying

that taxpayers minimize their expected payment.

In addition to the tax return, the tax authority receives a signal on the taxpayer's true tax liability. This signal is the realization of a random variable. The signal is non-verifiable and, for now, we assume that it is also free. The taxpayer does not know the realization of the signal at the time at which he makes his report, he only knows that the tax authority will receive and use the signal. The signal can take two values,  $s \in \{d, u\}$ , where  $d$  stands for “down” and  $u$  stands for “up”. The probability of the realization of the signal depends upon the true income of the taxpayer in the following way:

$$Prob(s = u|i) = \alpha_i \text{ for } i \in \{L, H\}.$$

We assume that  $0 < \alpha_L \leq \alpha_H \leq 1$ , i.e., the realization of the signal is non-negatively correlated to the true income. Taxpayers cannot manipulate the realization of the signal. When  $\alpha_L = \alpha_H$  the signal is not informative, that is, the tax administration does not receive any relevant information about the taxpayers' tax liability. This is the situation considered in the tax evasion literature. When  $\alpha_L < \alpha_H$  the tax authority receives a signal correlated to the true income. A realization “up” is more probable when the tax liability is high than when it is low. The signal is statistically informative.

The tax authority maximizes the revenue collected through taxes and penalties. The main tool of the enforcement policy is the auditing of taxpayers. We suppose that the tax audit is so effective that, when applied, it finds out the true tax liability of the taxpayer in a verifiable way.<sup>8</sup> The tax authority is subject to a given audit budget  $B$ . We normalize to 1 the cost of auditing one taxpayer; that is,  $B$  is the number of audits that the budget allows to make. The probability that the tax authority will audit a particular taxpayer can be a function of both: the report and the realization of the

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<sup>8</sup>This is the audit technique that is typically considered in the literature on tax auditing, see, for example, Scotchmer (1987) and Sánchez and Sobel (1993). See also Macho-Stadler and Pérez-Castrillo (1997) for a departure from this assumption. We characterize the optimal auditing strategy under the assumption that the tax authority has access to this perfect auditing technology. In our model, this is not a crucial assumption. We will discuss in the conclusion a possible interpretation of our result in a different environment.

signal.

As part of its enforcement policy, the tax authority, also as a function of the tax return and the realization of the signal, can offer a settlement to the taxpayer. If the taxpayer is offered a settlement and he accepts the offer, he pays the corresponding amount and the game ends. If he refuses, or there is no settlement, the tax authority can send officials to audit the taxpayer.

Finally, we assume that the tax authority can commit to its enforcement strategy. In other words, we take the principal-agent approach for this adverse selection problem. The revelation principle implies that the best the tax administration can possibly do is to offer a menu of contracts, each addressed to a group of taxpayers with the same tax liability. Each agent is asked to announce his true income,  $L$  or  $H$ , by choosing one of the two contracts. Given the set of enforcement policy instruments that we have already introduced, the menu of contracts is  $\{C_L, C_H\}$ , where  $C_i \equiv (r_i, p_{is}, \gamma_{is}, b_{is})_{s=u,d}$  for  $i = L, H$ . The report  $r_i$  needs not be equal to the true tax liability for each type. We denote by  $p_{is}$  the probability that the tax administration will audit a taxpayer with tax liability  $i$  when the signal is  $s$ , by  $\gamma_{is}$  the probability that this taxpayer will be offered a settlement, and by  $b_{is}$  the settlement offer. We assume all the contract terms to be non-negative. This implies, in particular, that no reward is given to truthful revelation, and that no settlement can give back money to the taxpayer.

We can interpret the menu of contracts as follows. There are two possible reports,  $\{r_L, r_H\}$ . If a taxpayer chooses a report  $r_i$ , then he will be subject to the enforcement policy  $(p_{is}, \gamma_{is}, b_{is})_{s=d,u}$ . That is, the auditing pressure suffered by a taxpayer, the probability of a settlement, and the deal he is offered if settlement takes place do depend upon the report he makes ( $r_i$ ) and the realization of the signal  $s$ .

Let us denote by  $EP_i(C_j)$  the expected payment of a taxpayer with tax liability  $i$  when making the report  $r_j$  (i.e., when choosing contract  $C_j$ ). Formally:

$$EP_i(C_j) = r_j + \alpha_i \left[ \gamma_{ju} \min\{b_{ju}, p_{ju}f(i - r_j)\} + (1 - \gamma_{ju})p_{ju}f(i - r_j) \right] + \\ (1 - \alpha_i) \left[ \gamma_{jd} \min\{b_{jd}, p_{jd}f(i - r_j)\} + (1 - \gamma_{jd})p_{jd}f(i - r_j) \right].$$



That is, first of all the taxpayer pays  $r_j$ . With probability  $\alpha_i$ , the tax authority receives a signal “up” about the taxpayer, in which case it will offer a settlement of  $b_{ju}$  with probability  $\gamma_{ju}$ . When a settlement is offered, the taxpayer only accepts it if it implies a lower expected payment. With probability  $(1 - \gamma_{ju})$ , or if the taxpayer is offered a settlement but does not accept the deal, he is subject to an audit with probability  $p_{ju}$ , in which case his true income is discovered. Hence, the taxpayer will accept the settlement if  $b_{ju} \leq p_{ju}f(i - r_j)$ . Similarly, with probability  $(1 - \alpha_i)$  the tax authority receives the signal “down” and then it applies the policy  $(p_{jd}, \gamma_{jd}, b_{jd})$ .<sup>9</sup>

Let us consider now the constraints that the tax authority must observe. For a taxpayer to reveal his type, the expected payment when announcing his true type must not be superior to the expected payment when announcing other type, these are the incentive compatibility constraints. In addition, the contract addressed to a taxpayer cannot imply an expected payment that is superior to his true tax liability. Formally, the constraints for taxpayers with tax liability  $L$  are:

$$EP_L(C_L) \leq EP_L(C_H), \quad (1)$$

$$EP_L(C_L) \leq L. \quad (2)$$

The constraints for taxpayers with tax liability  $H$  are:

$$EP_H(C_H) \leq EP_H(C_L), \quad (3)$$

$$EP_H(C_H) \leq H. \quad (4)$$

Note that constraints (2) and (4) and the fact that all the contract terms are non-negative imply  $r_L \leq L$  and  $r_H \leq H$ . In addition to the previous constraints, the tax authority faces a budget constraint. If we denote the expected cost of contract  $C_i$  by  $ECost(C_i)$ , and consider that the constraints (1) to (4) are fulfilled, the budget constraint (BC) can be written as:

$$ECost(C_L) + ECost(C_H) \leq B \quad (5)$$

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<sup>9</sup>Formally, the previous expression is only correct if  $r_j \leq i$ . For  $r_j > i$ ,  $EP_i(C_j) = r_j + \alpha_i p_{ju}(i - r_j) + (1 - \alpha_i)p_{jd}(i - r_j)$ , which is never lower than  $i$ . This is why, the fact that there are no bonuses for over-reporting implies that no taxpayer will chose a contract  $C_j$  with  $r_j > i$ .

with:

$$ECost(C_i) = h_i[\alpha_i q_{iu} + (1 - \alpha_i)q_{id}],$$

where  $q_{is} = (1 - \gamma_{is})p_{is}$  if  $b_{is} \leq p_{is}f(i - r_i)$ , while  $q_{is} = p_{is}$  if  $b_{is} > p_{is}f(i - r_i)$ . That is,  $q_{is}$  is the probability that an audit actually takes place in the contract addressed to a taxpayer with tax liability  $i$  if the signal is  $s$ . For convenience, we have assumed that a taxpayer accepts a settlement offer when it involves the same expected payment than being subject to the auditing pressure he would otherwise suffer.

### 3 Optimal Enforcement Policy

The tax authority maximizes revenues, that is, it maximizes  $\{h_L EP_L(C_L) + h_H EP_H(C_H)\}$  subject to constraints (1) to (4), and to the BC (5). Typically, low-income taxpayers do not have incentives to pick up the contract addressed to high-income taxpayers. This is also true in our model. Hence, we will characterize the optimal contract without taking into account the constraint (1) and we will check that this constraint is actually satisfied by the contract we will propose.

In what follows, we identify some characteristics of an optimal menu of contracts. We proceed by eliminating some contractual possibilities that are dominated by others, in the sense that the second possibilities are always at least as good as the first ones. Therefore, we could be eliminating some policies that are in fact equivalent to the optimal policy that we will characterize.

First, we show that, given that the contract designed for high-income taxpayers is not appealing for low-income agents, the tax administration is not interested in expending resources to audit high-income reports. That is, the easiest and cheapest contract addressed to taxpayers with the highest tax liability only involves to pay a fixed amount  $r_H$ .

**Lemma 1** *The tax authority can restrict attention to optimal enforcement policies where the only term in contract  $C_H$  that is different from zero is  $r_H$ .*

The tax authority's revenue on taxpayers with income  $H$  is  $r_H$ . Therefore,  $r_H$  will

be set at the maximum level compatible with constraints (3) and (4). Formally:  $r_H = \text{Min} \{H, EP_H(C_L)\}$ .

Second, we show that we can concentrate on those contracts where the settlement  $b_{Ls}$  offered to a taxpayer with tax liability  $L$  is the maximum amount that makes him accept the deal.

**Lemma 2** *The tax authority can restrict attention to optimal enforcement policies with  $b_{Ls} = p_{Ls}f(L - r_L)$ , for  $s = d, u$ .*

The intuition for this result is the following. On the one hand, offering a settlement to low-income taxpayers that is never accepted is equivalent to not making any offer at all, that is, to stating a zero probability of settlement. Moreover, the possibility of settlement for the low-type can only give incentives for high-income taxpayers to sign the low-type contract. Therefore, it is possible to propose another policy without settlement (and, in particular,  $b_{Ls} = p_{Ls}f(L - r_L)$ ) that is at least as good as the previous one. On the other hand, if the settlement offer is lower than the maximum acceptable, then the tax authority may set an equivalent policy (in terms of payments) involving a lower audit probability, a higher settlement offer (and a lower probability of settlement) in which the settlement offer is equal to the maximum acceptable.

Lemma 2 allows us to write the settlement offer as a function of the other contract terms. Also, it allows simplifying the expression of the budget constraint, since it makes sure that a settlement is always accepted. Finally, notice that if a taxpayer with tax liability  $H$  would sign the contract  $C_L$ , then he would also always accept the settlement.

Lemma 2 has implications on another parameters of the contract. Given Lemma 2 and  $r_L \leq L$ ,  $EP_L(C_L) = r_L + (\alpha_L p_{Lu} + (1 - \alpha_L) p_{Ld}) f(L - r_L)$ . Therefore, if  $r_L < L$ ,  $EP_L(C_L) \leq L$  is equivalent to:

$$f(\alpha_L p_{Lu} + (1 - \alpha_L) p_{Ld}) \leq 1. \quad (6)$$

Condition (6) imposes an upper-bound on the auditing pressure (in terms of probability of auditing) that can be exerted on the low reports when  $r_L < L$ . It does not impose, however, any constraint concerning the probability of settlement. Next lemma gives us some information about the optimal use of settlement offers.

**Lemma 3** *The tax authority can restrict attention to enforcement policies where  $\gamma_{Ld} < 1$  implies  $\gamma_{Lu} = 0$ .*

To understand the intuition for this result, suppose for a moment that  $p_{Ld} = p_{Lu}$ , which means that the settlement amount would be the same independently of the realization of the signal. The cost of the enforcement policy can be kept constant by substituting plea bargaining in one state of the nature by the other in the appropriate proportion,  $\alpha_L/(1 - \alpha_L)$ . Remember also that the revenue on the taxpayers with income  $L$  is the same whatever the decision on the probability of settlement. Hence, the key issue to understand whether it is better to settle when the realization of the signal is "up" or when it is "down" is to see what happens concerning taxpayers with income  $H$  when they sign the contract  $C_L$ . These taxpayers pay more when there is not settlement ( $p_{Ls}(H - r_L)$ ) than where there is ( $p_{Ls}(L - r_L)$ ). Since for them the signal "up" is more often observed (as compared to low-income taxpayers) than the signal "down", the tax authority prefers to settle when "down" rather than when "up". In this way, it can ask for a higher  $r_H$ . Note that this argument is based on the informative content of the signal: if  $\alpha_H = \alpha_L$  this argument leads to an equivalent policy, but not to a superior one. Note also that a similar argument works when  $p_{Ld}$  and  $p_{Lu}$  are not same, the only difference is that the appropriate proportion for substituting settlement in one state by the other is now  $\alpha_L p_{Lu}/(1 - \alpha_L) p_{Ld}$ .

The fact that the signal conveys statistical information about the true income not only biases the interest of the tax administration concerning the probability of settlement. It also affects its decision about the probability of auditing, as next lemma shows.

**Lemma 4** *The tax authority can restrict attention to optimal enforcement policies with  $p_{Ld} > 0$  only if  $p_{Lu} = 1$ .*

Lemma 4 states an intuitive result: given that the tax administration uses the auditing to dissuade and catch evasion and that the signal is correlated with the income, it will first go after the taxpayers whose signal is "up".<sup>10</sup>

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<sup>10</sup>This result was also true in the framework where settlement is not allowed, analyzed in Macho-Stadler and Pérez-Castrillo (2001).

The last lemma before presenting an optimal enforcement policy states another intuitive result: the expected payment of low-income taxpayers is equal to  $L$ .

**Lemma 5** *In the optimal enforcement policy  $EP_L(C_L) = L$ .*

Lemma 5 implies that the participation constraint of taxpayer  $L$ , constraint (2), is always binding. This characteristic is common to adverse selection problems but remark that here the problem is not an standard one, since agents do not have the same reservation utility.

Now we can tackle the analysis of the optimal enforcement policy. As it is usual, full compliance can be achieved if the budget is high enough. We denote  $B^{\max}$  the level of budget that allows implementing full compliance in our model. Proposition 1 states an optimal enforcement policy for the relevant values of the budget, that is, for  $B \leq B^{\max}$ . Corollary 3 will identify  $B^{\max}$  in some particularly interesting cases.

**Proposition 1** *For  $B \leq B^{\max}$ , the following policy addressed to low-income taxpayers is optimal:*

- (a) If  $\alpha_L \geq 1/f$ , then  $r_L = 0$ ,  $p_{Lu} = \frac{1}{f\alpha_L}$ ,  $p_{Ld} = 0$ ,  $\gamma_{Lu} = 1 - \frac{fB}{h_L}$ ,  $\gamma_{Ld}$  any.
- (b) If  $\alpha_L < 1/f$ , then  $r_L = 0$ ,  $p_{Lu} = 1$ ,  $p_{Ld} = \frac{1-f\alpha_L}{f(1-\alpha_L)}$ . In addition,
  - (b.1) if  $B \leq h_L\alpha_L$ , then  $\gamma_{Lu} = 1 - \frac{B}{h_L\alpha_L}$  and  $\gamma_{Ld} = 1$ .
  - (b.2) if  $B > h_L\alpha_L$ , then  $\gamma_{Lu} = 0$  and  $\gamma_{Ld} = 1 - \frac{f(B-h_L\alpha_L)}{1-f\alpha_L}$ .

Before commenting on Proposition 1, remember that an optimal enforcement policy addressed to high-income taxpayers is  $r_H = \text{Min}\{H, EP_H(C_L)\}$ ,  $p_{Hu} = p_{Hd} = \gamma_{Hu} = \gamma_{Hd} = 0$ . Also notice that equation (1), that we have not taken into account in the analysis, is trivially satisfied, since it is equivalent to  $r_H \geq L$ , which is always true.

In order to see the interest of committing to a settlement strategy ex-ante, and to discuss its characteristics as a function of the quality of the signal, let us identify the optimal policy without settlement. Without settlement, the optimal policy is described in part B) of the proof of Proposition 1, and takes the form:

For  $B \leq h_L\alpha_L$ , then  $r_L = L$ ,  $p_{Lu} = \frac{B}{h_L\alpha_L}$ ,  $p_{Ld} = 0$ .

For  $B > h_L\alpha_L$ , then  $r_L = L$ ,  $p_{Lu} = 1$ ,  $p_{Ld} = \frac{B-h_L\alpha_L}{1-\alpha_L}$ .

It can be easily checked that for  $\alpha_L = \alpha_H$  the policy depicted in the Proposition is equivalent to the policy without settlement. If  $\alpha_L < \alpha_H$ , this is not longer true. We state this result in the following corollary.

**Corollary 1** *An optimal policy must include the possibility of settlement whenever  $\alpha_H > \alpha_L$ . Including the possibility of settlement in the auditing policy is not profitable when  $\alpha_H = \alpha_L$ .*

Let us now explain the reasons behind the optimality of the use of settlements and the main intuitions and results of Proposition 1. Remember that the contract terms discussed in Proposition 1 refer to the contract offered to low-income taxpayers. High-income taxpayers pay a fixed amount equal to their expected payment when accepting the low-income contract. So, the optimal contract for low-income taxpayers is very much determined by the incentives provided to taxpayers with income  $H$ , i.e., by the expected payment that it induces from high-income taxpayers.

Consider first the case where the penalty rate  $f$  is high enough (as compared to  $1/\alpha_L$ ) so that the tax authority only needs to audit when the realization of signal is “up”. Given the budget  $B$ , the tax authority has resources to audit each low-income taxpayer whose realized signal is “up” with probability  $q_{Lu} = B/h_L\alpha_L$ . It can choose not to settle at all and audit with probability  $p_{Lu} = q_{Lu}$ . However, this policy is not optimal. Instead, the tax authority can offer an acceptable settlement with some probability  $\gamma_{Lu} > 0$ , increasing the probability of (real or threaten) audit accordingly,  $p_{Lu} = q_{Lu}/(1 - \gamma_{Lu})$ . Since  $\alpha_H > \alpha_L$ , the settlement would be offered more often to high-income taxpayers if they chose the contract  $C_L$  than to low-income taxpayers. In addition, the settlement offer increases with the announced probability of audit  $p_{Lu}$  and it decreases with the fixed amount  $r_L$ . Therefore, the optimal policy involves  $r_L = 0$  and  $p_{Lu}$  as high as possible. The maximum  $p_{Lu}$  is imposed by the low-income taxpayers’ participation constraint. Indeed, for these taxpayers not to pay more than  $L$ , the audit probability cannot exceed  $p_{Lu} = 1/f\alpha_L$  when the penalty is high. The budget only determines the probability of settlement. Finally, the effective audits increase with the budget, which means that the probability of settlement decreases with  $B$ . Indeed, high-income taxpayers are more afraid of a real audit than they are of a settlement.

Take now the case of a low penalty rate. The last arguments also apply here. However, now even if  $p_{Lu} = 1$ , low-income taxpayers pay less than  $L$  so the tax authority also announces audits when the signal is “down”. The effective audits are then  $\alpha_L (1 - \gamma_{Lu}) + (1 - \alpha_L) (1 - \gamma_{Ld}) p_{Ld}$  if  $p_{Lu} = 1$ . Again settlement allows to control the number of effective audits. Since the tax authority prefers to effectively audit when the signal is “up” than when the signal is “down”, settlement is rather offered when “down” is observed (this eliminates the need to audit when “down”). The probability of settlement is here again determined by the budget. When the budget is low, a settlement is offered any time that “down” is observed, and the probability of settlement when “up” decreases with  $B$ . When the budget is high, a signal “up” never receives a settlement offer, and the probability of settlement when “down” decreases with  $B$ .<sup>11</sup>

The reasons provided in the previous paragraphs also explain most of the intuitions behind the results stated in Corollary 2.

**Corollary 2** *The additional revenues due to the settlement practice are increasing in  $L$  and  $\alpha_H$ , non-decreasing in  $f$ , and decreasing in  $\alpha_L$  (unless full compliance is already achieved with settlements). They are also increasing with the population of high-income taxpayers,  $h_H$ , but independent from the budget  $B$ .*

For an economy, the amount of extra revenue associated to the settlement strategy only depends on the informational content of the signal, the penalty rate, the low-income tax liability, and the size of the high-income taxpayers’ population. It does not depend on the budget  $B$ . That is, using settlements implies a fixed gain in revenue terms. As explained before, using settlements is similar to using a random device that is weighted against high-income taxpayers. The additional revenues raised through this device do not depend on the budget allocated to really audit taxpayers.

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<sup>11</sup>Since auditing (and settlement) when “down” is observed is aimed at inducing low-income taxpayers to pay their tax compliance, a fair question is why the tax authority does not simply raise the report  $r_L$ . This raise would allow to obtain the same income from low-income taxpayers. However, it would be suboptimal since raising  $r_L$  makes  $C_L$  more attractive for high income taxpayers. In other words, it reduces their evasion when taking  $C_L$  and, consequently, this change would reduce the threat on, and the expected revenue from, high income taxpayers.

It is worthwhile to notice that, if the signal is informative enough, using settlements allows the tax administration to achieve full compliance at no cost. When the realization of the signal is very correlated with the true tax liability, a settlement is much more often suffered by high than by low-income taxpayers. Threatening with a frequent settlement is so dissuasive that no real auditing is necessary. We state this result in Corollary 3, whose proof is immediate after Proposition 1.

**Corollary 3** *Full compliance is achieved with a budget of zero, that is  $B^{\max} = 0$ , whenever:*

- (a)  $\frac{H}{L} \leq \frac{\alpha_H}{\alpha_L}$  if  $\alpha_L \geq 1/f$ ,
- (b)  $\frac{H}{L} \leq 1 + \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} f$  if  $\alpha_L < 1/f$ .

Finally, given the parameters that characterize the optimal probabilities of settlement and announced audit, Lemma 2 directly tells us about the optimal settlement offers. The most interesting aspect of these offers is that they are contingent on the realization of the signal. When the tax authority offers a settlement under both realizations of the signal (which happens if  $\alpha_L < 1/f$  and  $B \leq h_L \alpha_L$ ), it offers a better deal to a taxpayer whose realization is "down".

**Corollary 4** *The settlement offer that a taxpayer receives is increasing with the realization of the signal. Formally,  $b_{Lu} > b_{Ld}$ .*

## 4 The value of the signal

In this section, we discuss the benefits of using a signal. We have seen above the advantages of committing to a settlement strategy versus not using settlement in a world where the tax authority receives a useful signal  $(\alpha_L, \alpha_H)$ , with  $\alpha_L < \alpha_H$ . In order to determine the value of using signals, let us note that the optimal auditing policy without signals consists in offering a contract for low-income taxpayers equal to  $(r_L = L, p_L = \frac{B}{h_L})$  i.e., it requires to pay the true tax liability and announces the highest possible auditing probability given the budget. We compare these revenues with the revenues with signals (using settlement).



**Proposition 2** *When  $B \leq B^{\max}$ , the additional profits due to the existence of the signal are the following:*

(i) *When  $\alpha_L \geq 1/f$ , then  $\Delta R = h_H \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} \left[ L + \frac{f}{h_L} (H - L)B \right]$ .*

(ii) *When  $\alpha_L < 1/f$  and*

(iia)  *$B \leq h_L \alpha_L$ , then  $\Delta R = h_H (\alpha_H - \alpha_L) \left[ \frac{(f-1)}{(1-\alpha_L)} L + \frac{f}{\alpha_L h_L} (H - L)B \right]$ .*

(iib)  *$B > h_L \alpha_L$ , then  $\Delta R = h_H \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} \left[ (fH - L) - \frac{f}{h_L} (H - L)B \right]$ .*

To understand the benefits associated to the signal it is useful to identify two different advantages. The first one is associated to the possibility of conditioning the audit probability to the realization of the signal. The second advantage relates to the possibility of using the signal to build up a settlement strategy. From Corollary 2, we know that settlement implies a fixed (respective to the budget) gain with respect to not committing to settle. It is easy to check that in the expression  $\Delta R$  the term that is independent of  $B$  corresponds to the gains due to the possibility of settlement while the term in  $B$  reflects the benefits from the use of the signal in the optimal auditing probability. For example, in the case  $\alpha_L \geq 1/f$ , the term  $h_H \frac{\alpha_H - \alpha_L}{1 - \alpha_L} L$  corresponds to the additional revenue raised by using the settlement, while  $h_H \frac{\alpha_H - \alpha_L}{1 - \alpha_L} \frac{f}{h_L} (H - L)B$  corresponds to the extra revenue associated to the use of the signal in the auditing strategy.

Note that since low income taxpayers fully comply and high income taxpayers pay less than their tax due, the effective system is less progressive than the nominal one. However, high income taxpayers compliance is increasing in  $(\alpha_H - \alpha_L)$  (see Proposition 1 and Corollary 2). Hence, signals, both through the increase in the efficiency of the audit policy and through settlement, allow to counter the regressive bias. Note that as it was stated by Scotchmer (1992) the regressive bias is a common phenomena in tax evasion (independently of the tax authority's commitment capacity). In a model with more than two incomes, signals may not only decrease the regressive bias but may also lead to effective taxes that are more progressive than the nominal ones for some intervals of incomes (see, Macho-Stadler and Pérez-Castrillo, 2001).

We have developed our analysis under the assumption that the signal is free. If obtaining the signal for every taxpayer implies a fixed cost, the tax administration will buy the signal  $(\alpha_L, \alpha_H)$  if the fixed cost is lower than its expected revenue. Consider

the decision of a tax administration that has to decide whether to buy a signal (say, it can buy a big computer to run the DIF program) out of its budget. The results suggest that the highest the budget allocated to the tax authority, the highest the incentives to buy such a signal. This conclusion holds unless the penalty rate is low ( $\alpha_L < 1/f$ ) and the budget is high ( $B > h_L \alpha_L$ ). Of course, if the cost of the signal depends on its quality, then the quality of the signal chosen by the tax authority will be the one that equalizes the marginal expected revenue and the marginal cost.

## 5 Conclusion

The aim of this paper is to highlight the interest for the tax authority of committing, ex-ante, to a settlement strategy. Our claim is that the answer depends on the quality of the information that the tax authority has on taxpayers true income. This information takes, in our model, a very simple form; we model it as an exogenous random variable, correlated with the true income. The more informative is the signal, the more raises the expected revenue of the tax authority when using settlement as a policy tool.

We identify the benefits associated to the existence of a signal correlated with the taxpayers' income. In fact, we identify two different advantages. The first one is associated to the possibility to condition the audit probability to the realization of the signal. The second advantage relates to the possibility of using the signal to build up a settlement strategy. While the first advantage is a function of the tax authority budget, the advantage associated to settlement is independent of the budget.

A common criticism to the principal-agent approach in tax enforcement frameworks is the legitimacy of the assumption asserting that the tax authority commits ex-ante to the audit policy. This criticism can obviously be extended to the commitment ex-ante on the settlement policy. However, the full commitment case can be understood as the best an enforcement authority can do, since any other policy (with no commitment on one or more elements of the audit policy) will give a lower expected revenue.

In our model, settlement is used once the signal is observed but before the auditing policy is implemented. In particular, the tax authority offers the following type of deal to the taxpayer: “either you pay this fixed amount or you will be audited with

probability  $p$  (with a technique that allows me to identify your true income with no error)". Following the usual criticism to ex-ante optimal auditing, threatening with a probability may not seem credible. However, the model can be reinterpreted as follows. The tax authority chooses, for each possible report, the audit technology. If the tax authority chooses technology " $p$ " then it will identify the true income with probability  $p$  (or, equivalently, it will under-cover a fraction  $p$  of the taxpayer's true income). The cost of the technology  $p$  is  $p$  (since in the model we normalize the cost of auditing to one). Then, the settlement is offered in the following way: "either you pay this fixed amount or you will be audited for sure through technology  $p$ ".

## 6 Appendix

**Proof. of Lemma 1.-** Given that we do not take into account constraint (1), the contract  $C_H$  only appears in BC (5) and in the definition of the expected payment  $EP_H(C_H)$ . It is clear that, if  $EP_H(C_H) > r_H$ , we can substitute  $C_H$  by another contract  $C'_H$  in which  $r'_H = EP_H(C_H)$ ,  $p'_{Hu} = p'_{Hd} = 0$ ,  $\gamma'_{Hu} = \gamma'_{Hd} = 0$ . This new contract implies the same expected payment for the taxpayer and  $ECost(C'_H) \leq ECost(C_H)$ . ■

**Proof. of Lemma 2.-** If  $b_{Ls} > p_{Ls}f(L - r_L)$ , the settlement is not accepted by a low-income taxpayer, i.e., the taxpayer is audited with probability  $p_{Ls}$  when the signal  $s$  is observed. An equivalent policy for this taxpayer consists in keeping the same  $p_{Ls}$  and setting  $\gamma'_{Ls} = 0$ . Both, the payment  $EP_L(C'_L)$  made by taxpayer  $L$  and the cost of his contract  $ECost(C'_L)$ , are the same as before. Moreover,  $EP_H(C'_L)$  is either higher or equal than  $EP_H(C_L)$  (since the possibility of accepting the settlement when cheating disappears). Therefore, constraint (3) for taxpayer  $H$  either remains unchanged or it is easier to satisfy. Hence, changing  $C_L$  by  $C'_L$  cannot be in detriment of the administration.

If  $b_{Ls} < p_{Ls}f(L - r_L)$ , then it is also the case that  $b_{Ls} < p_{Ls}f(H - r_L)$ , so taxpayers  $L$  and  $H$  would both accept the settlement if they sign  $C_L$ . We propose an alternative contract  $C'_L$  in which  $b'_{Ls} = p'_{Ls}f(L - r_L)$  and that is equivalent to  $C_L$  in the sense that  $EP_L(C'_L) = EP_L(C_L)$ ,  $EP_H(C'_L) = EP_H(C_L)$ , and  $ECost(C'_L) = ECost(C_L)$ . The new parameters  $b'_{Ls}$ ,  $p'_{Ls}$ , and  $\gamma'_{Ls}$  are such that  $(1 - \gamma'_{Ls})p'_{Ls} = (1 - \gamma_{Ls})p_{Ls}$ ,  $\gamma'_{Ls}b'_{Ls} = \gamma_{Ls}b_{Ls}$ , and  $b'_{Ls} = p'_{Ls}f(L - r_L)$ . It is easy to check that such parameters are feasible and

they are:  $b'_{Ls} = \gamma_{Ls}b_{Ls} + (1 - \gamma_{Ls})p_{Ls}f(L - r_L) > b_{Ls}$ ,  $\gamma'_{Ls} = \gamma_{Ls}b_{Ls}/b'_{Ls} < \gamma_{Ls}$ , and  $p'_{Ls} = b'_{Ls}/f(L - r_L) < p_{Ls}$ . Therefore, without loss of efficiency, we can restrict attention to policies where  $b_{Ls} = p_{Ls}f(L - r_L)$ . ■

**Proof. of Lemma 3.-** Suppose  $\gamma_{Ld} < 1$  and  $\gamma_{Lu} > 0$ . Remark first that if  $p_{Ld} = 0$ , then setting  $\gamma_{Ld} = 1$  is without consequences, and Lemma 3 holds. Now assume  $p_{Ld} > 0$ . Consider a marginal change in the probability of both settlements. Take  $d\gamma_{Lu} = -\delta$  with  $\delta > 0$ , and choose  $d\gamma_{Ld}$  such that  $dECost(C_L) = 0$ , i.e.,

$$-\alpha_L p_{Lu} d\gamma_{Lu} - (1 - \alpha_L) p_{Ld} d\gamma_{Ld} = 0 \iff d\gamma_{Ld} = \frac{\alpha_L p_{Lu}}{(1 - \alpha_L) p_{Ld}} \delta.$$

Given that the probabilities  $p_{Lu}$  and  $p_{Ld}$  do not change,  $EP_L(C_L)$  is the same after the marginal change in  $\gamma_{Ld}$  and  $\gamma_{Lu}$ . As to the expected payment  $EP_H(C_L)$ , note that Lemma 2 implies:

$$EP_H(C_L) = r_L + \alpha_H [\gamma_{Lu}(L - r_L) + (1 - \gamma_{Lu})(H - r_L)] p_{Lu} f + \\ (1 - \alpha_H) [\gamma_{Ld}(L - r_L) + (1 - \gamma_{Ld})(H - r_L)] p_{Ld} f.$$

Therefore:  $dEP_H(C_L) = -\alpha_H p_{Lu} f(L - H)\delta + (1 - \alpha_H) p_{Ld} f(L - H) \frac{\alpha_L p_{Lu}}{(1 - \alpha_L) p_{Ld}} \delta$ .

Hence, if  $p_{Lu} > 0$ , then  $dEP_H(C_L) \geq 0$  if and only if  $\frac{\alpha_H}{(1 - \alpha_H)} \geq \frac{\alpha_L}{(1 - \alpha_L)}$ , which is always true (if  $p_{Lu} = 0$ , then  $dEP_H(C_L) = 0$ ). Therefore, the proposed change is (weakly) improving for the tax authority. ■

**Proof. of Lemma 4.-** (a) Assume that  $p_{Ld} > 0$  and  $0 < p_{Lu} < 1$ . Consider a marginal change in the policy involving  $dp_{Ld} = -\delta$ , with  $\delta > 0$ . Also, change  $p_{Lu}$  such that  $EP_L(C_L)$  is unchanged, i.e.,  $dp_{Lu} = \frac{(1 - \alpha_L)}{\alpha_L} \delta$ . Finally, choose  $\gamma_{Lu}$  to keep  $ECost(C_L)$  constant, i.e.,  $d\gamma_{Lu} = \frac{(1 - \alpha_L)[\gamma_{Ld} - \gamma_{Lu}]}{\alpha_L p_{Lu}} \delta$ . We know by Lemma 3 that  $d\gamma_{Lu} \geq 0$  since  $\gamma_{Ld} \geq \gamma_{Lu}$ . Now, by computing  $dEP_H(C_L)$  we can check that  $dEP_H(C_L) \geq 0$ , which is equivalent to  $\frac{\alpha_H}{(1 - \alpha_H)} \geq \frac{\alpha_L}{(1 - \alpha_L)}$ , which is always true.

(b) Assume now that  $p_{Ld} > 0$  and  $p_{Lu} = 0$ . We can take  $\gamma_{Lu} = 0$ , since  $b_{Lu} = 0$ . We consider the same marginal changes on  $p_{Ld}$  and  $p_{Lu}$  as in (a), but we increase  $\gamma_{Ld}$  to keep constant the cost:  $d\gamma_{Ld} = \frac{(1 - \gamma_{Ld})}{p_{Ld}} \delta$ . It is then easy to check that  $dEP_H(C_L) \geq 0$  if and only if  $\alpha_H \geq \alpha_L$ .

In both cases, (a) and (b), we can proceed with marginal changes until either  $p_{Ld} = 0$  or  $p_{Lu} = 1$ . And those changes cannot be harmful for the tax administration. ■

**Proof. of Lemma 5.-** Note first that, when  $r_L < L$ , Lemma 4 implies that  $p_{Ld} < 1$  because otherwise  $p_{Lu} = 1$  and  $f(\alpha_L p_{Lu} + (1 - \alpha_L)p_{Ld}) = f > 1$  which contradict (6). We do the proof of Lemma 5 by contradiction. Suppose that  $EP_L(C_L) < L$ , i.e.,  $r_L < L$  and  $f(\alpha_L p_{Lu} + (1 - \alpha_L)p_{Ld}) < 1$ . We can propose a change in the policy:  $dp_{Ld} = \delta$ ;  $d\gamma_{Ld} = \frac{(1-\gamma_{Ld})}{p_{Ld}}\delta$  which keeps the cost of  $C_L$  constant (i.e.,  $(1 - \gamma_{Ld})p_{Ld}$  does not change), but  $d(\gamma_{Ld}p_{Ld}) = \delta > 0$ . This change increases expected revenues from low-income taxpayers, since  $EP_L(C_L)$  increases because  $dp_{Ld} > 0$ . Moreover,  $EP_H(C_L)$  also increases because  $d(\gamma_{Ld}p_{Ld}) > 0$ . Therefore,  $r_H$  can also increase accordingly (unless high-income taxpayers were already paying  $H$ , in which case  $r_H$  would be the same as before), which is desirable by the tax authority. ■

**Proof. of Proposition 1.-** We proceed in several steps. First, we compute the optimal policy for different regions, in particular we distinguish the regions where  $r_L < L$  and where  $r_L = L$ . In these regions, we compute the expected revenue from taxpayers with true tax liability  $H$ . Note that for computing the optimal policy we can concentrate in this revenue since low income taxpayers pay in expectation always  $L$ . Then we compare the expected revenues in order to identify the optimal enforcement policy as a function of the parameters.

**A)** Consider first the region where  $r_L < L$ . By Lemma 5, we know that  $f(\alpha_L p_{Lu} + (1 - \alpha_L)p_{Ld}) = 1$ . This implies, in particular, that  $\frac{\partial EP_H(C_L)}{\partial r_L} \leq 0$ , since:

$$\frac{\partial EP_H(C_L)}{\partial r_L} \leq 0 \iff f(\alpha_H p_{Lu} + (1 - \alpha_H)p_{Ld}) \geq 1.$$

The previous inequality always holds given that  $\alpha_H \geq \alpha_L$  and  $p_{Lu} \geq p_{Ld}$ . Moreover,  $EP_L(C_L) = r_L$  independently of  $r_L$ . Hence, it is optimal to set  $r_L = 0$ . To characterize the other parameters, two cases are possible:

**Ai)** If  $\alpha_L \geq 1/f$ ,  $p_{Lu} = \frac{1}{f\alpha_L}$  because, by Lemma 4,  $p_{Ld} = 0$  unless  $p_{Lu} = 1$ . Also, since  $p_{Ld} = 0$ ,  $\gamma_{Ld}$  can be any. Finally, (BC) determines  $\gamma_{Lu}$ :  $\gamma_{Lu} = 1 - \frac{fB}{h_L}$  if  $B \leq \frac{h_L}{f}$ , and  $\gamma_{Lu} = 1$  otherwise.

The expected revenue of this policy from a high-income taxpayer is:

$$EP_H^{Ai}(C_H) = \text{Min} \left\{ H, EP_H^{Ai}(C_L) \right\} = \text{Min} \left\{ H, \frac{\alpha_H}{\alpha_L} \left[ L + f \frac{B}{h_L} (H - L) \right] \right\}.$$

Note that in case Ai, if  $H \leq \frac{\alpha_H}{\alpha_L} L$ , full compliance is achieved at no cost. If  $H > \frac{\alpha_H}{\alpha_L} L$ , then the minimum budget (that in this region allows) to ensure full compliance is  $\bar{B}$  defined by:

$$\frac{\alpha_H}{\alpha_L} \left[ L + f \frac{\bar{B}}{h_L} (H - L) \right] = H \iff \bar{B} = \frac{(\alpha_L H - \alpha_H L) h_L}{\alpha_H (H - L) f}.$$

Notice that  $\bar{B} \leq \frac{h_L}{f}$ . Since the region where  $B > \bar{B}$  does not make sense,  $\gamma_{Lu} = 1 - \frac{fB}{h_L}$ . Also,  $\bar{B} \leq h_L \alpha_L$  when  $\alpha_L \geq 1/f$ .

**Aii)** If  $\alpha_L < 1/f$ ,  $p_{Lu} = 1$  and  $p_{Ld} = \frac{1-f\alpha_L}{f(1-\alpha_L)}$ . (BC) determines the probability of settlement:

$$\begin{aligned} h_L [\alpha_L (1 - \gamma_{Lu}) + (1 - \alpha_L) (1 - \gamma_{Ld}) p_{Ld}] &= B \\ \iff h_L \left[ \alpha_L (\gamma_{Ld} - \gamma_{Lu}) + \frac{(1 - \gamma_{Ld})}{f} \right] &= B. \end{aligned}$$

Hence, to determine the optimal probabilities for settlement, again two case are possible.

**Aiia)** If  $B \leq h_L \alpha_L$ , then  $\gamma_{Ld} = 1$  and  $\gamma_{Lu} = 1 - \frac{B}{h_L \alpha_L}$ . After easy calculations, we can write the expected revenue from a high-income taxpayer as:

$$EP_H^{Aiiia}(C_L) = \text{Min} \left\{ H, \frac{\alpha_H}{\alpha_L} f \frac{B}{h_L} (H - L) + L \left[ 1 + \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} (f - 1) \right] \right\}.$$

If  $H \leq L \left[ 1 + \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} (f - 1) \right]$ , then full compliance is achieved at no cost. In the other case, the minimum budget that allows to ensure full compliance is implicitly defined by:

$$\frac{\alpha_H}{\alpha_L} f \frac{B}{h_L} (H - L) + L \left[ 1 + \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} (f - 1) \right] = H.$$

This level of budget may be higher or lower than  $B = h_L \alpha_L$ .

**Aiib)** If  $B > h_L \alpha_L$ , then  $\gamma_{Ld} = \frac{h_L - fB}{h_L(1-f\alpha_L)}$  and  $\gamma_{Lu} = 0$ . The expected revenue is:

$$EP_H^{Aiiib}(C_L) = \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} f H + \frac{(1 - \alpha_H)}{(1 - \alpha_L)} \left[ L + f \frac{B}{h_L} (H - L) \right].$$

**B)** Consider now the region where  $r_L = L$ . The expected payment for  $H$  when choosing  $C_L$  is:  $EP_H(C_L) = L + [\alpha_H (1 - \gamma_{Lu}) p_{Lu} + (1 - \alpha_H) (1 - \gamma_{Ld}) p_{Ld}]$ . The cost can be written as:

$$ECost(C_L) = h_L [\alpha_L (1 - \gamma_{Lu}) p_{Lu} + (1 - \alpha_L)(1 - \gamma_{Ld}) p_{Ld}].$$

We know that if  $\gamma_{Ld} < 1$  and  $p_{Ld} > 0$  then  $\gamma_{Lu} = 0$  and  $p_{Lu} = 1$ . Hence, two cases are possible.

**Ba)** If  $B \leq h_L \alpha_L$ , then  $(1 - \gamma_{Lu}) p_{Lu} = \frac{B}{h_L \alpha_L}$ . For example, the optimal policy may not involve the use of settlement agreements:  $p_{Lu} = \frac{B}{h_L \alpha_L}$  and  $\gamma_{Lu} = 0$ . Here, the expected revenue from taxpayers with true tax liability  $H$  is:

$$Min \{H, EP_H^{Ba}(C_L)\} = Min \left\{ H, L + \frac{\alpha_H}{\alpha_L} f \frac{B}{h_L} (H - L) \right\}.$$

**Bb)** If  $B > h_L \alpha_L$ ,  $p_{Lu} = 1$  and  $\gamma_{Lu} = 0$ , and  $(1 - \gamma_{Ld}) p_{Ld} = \frac{B - h_L \alpha_L}{h_L (1 - \alpha_L)}$ . For example, an optimal policy may not use plea bargaining:  $p_{Ld} = \frac{B - h_L \alpha_L}{h_L (1 - \alpha_L)}$ , and  $\gamma_{Ld} = 0$ . The expected revenue from high-income taxpayers is:

$$Min \{H, EP_H^{Bb}(C_L)\} = Min \left\{ H, L + \left[ \alpha_H + \frac{(1 - \alpha_H)}{(1 - \alpha_L)} \left( \frac{B - h_L \alpha_L}{h_L} \right) \right] f (H - L) \right\}.$$

*COMPARISON:* Now we compare the expected revenues.

(i) Let us consider first the case where  $\alpha_L \geq 1/f$ . We compare possibilities Ai) and B). As we have seen, possibility Bb) does not make sense here since Ai) is always superior because a budget  $h_L \alpha_L$  allows to obtain full compliance in Ai).

$$EP_H^{Ai}(C_L) \geq EP_H^{Ba}(C_L) \iff \frac{\alpha_H}{\alpha_L} \geq 1.$$

Hence, the optimal policies characterized in regions Ai) and Ba) are equivalent if  $\alpha_L = \alpha_H$ . The optimal policy in region Ai) is strictly better than that in Bi) if  $\alpha_L < \alpha_H$ .

(ii) When  $\alpha_L < 1/f$ , it is also easy to check that both  $EP_H^{Aii}(C_L) \geq EP_H^{Ba}(C_L)$  and  $EP_H^{Aib}(C_L) \geq EP_H^{Ba}(C_L)$  if and only if  $(\alpha_H - \alpha_L) f \geq (\alpha_H - \alpha_L)$ . Hence, the optimal policies in regions Aii) and Bb) are equivalent if  $\alpha_L = \alpha_H$ . The policy in Aii) is strictly better than that in Bb) if  $\alpha_L < \alpha_H$ . ■

**Proof. of Corollary 1.-** Let us define by  $E\Delta$  the difference between the expected payment of a taxpayer  $H$  when using settlement or not in the optimal enforcement policy. It is immediate to check that:

$$E\Delta_a = \left(\frac{\alpha_H}{\alpha_L} - 1\right) L \text{ if } \alpha_L \geq 1/f$$

$$E\Delta_b = (f - 1) \frac{(\alpha_H - \alpha_L)}{1 - \alpha_L} L \text{ if } \alpha_L < 1/f. \blacksquare$$

**Proof. of Corollary 4.-** From Lemma 2 we know that  $b_{Ls} = p_{Ls}f(L - r_L)$  for  $s = d, u$ . From Proposition 1 we have that for  $\alpha_L \geq 1/f$ ,  $b_{Lu} = \frac{1}{f\alpha_L}i > 0 = b_{Ld}$  and for  $\alpha_L < 1/f$ ,  $b_{Lu} = fi > \frac{1-f\alpha_L}{1-\alpha_L}i = b_{Ld}$ , which is always true for  $f > 1$ . ■

**Proof. of Proposition 2.-** The proof is the simple difference between the expression of the revenues in the different cases. The revenues obtained by the tax authority when it does not use signals are:

$$R^0 = h_L L + h_H \text{Min} \left\{ H, L + \frac{1}{h_L} f(H - L)B \right\}.$$

The revenues with the revenues with signals (using settlement) are:

When  $\alpha_L \geq 1/f$  :

$$R = h_L L + h_H \text{Min} \left\{ H, \frac{\alpha_H}{\alpha_L} L + \frac{1}{h_L} \frac{\alpha_H}{\alpha_L} f(H - L)B \right\}.$$

When  $\alpha_L < 1/f$  and  $B \leq h_L \alpha_L$  :

$$R = h_L L + h_H \text{Min} \left\{ H, \left( \frac{1 - \alpha_H}{1 - \alpha_L} + f \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} \right) L + \frac{1}{h_L} \frac{\alpha_H}{\alpha_L} f(H - L)B \right\}.$$

When  $\alpha_L < 1/f$  and

$$B > h_L \alpha_L : R = h_L L + h_H \text{Min} \left\{ H, \left( \frac{(1 - \alpha_H)}{(1 - \alpha_L)} L + f \frac{(\alpha_H - \alpha_L)}{(1 - \alpha_L)} H \right) + \frac{1}{h_L} \frac{(1 - \alpha_H)}{(1 - \alpha_L)} f(H - L)B \right\}.$$

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