A Foundation Model for Marxian Theories of the Breakdown of Capitalism

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Abstract: Marx and the writers that followed him have produced a number of theories of the breakdown of capitalism. The majority of these theories were based on the historical tendencies: the rise in the composition of capital and the share of capital and the fall in the rate of profit. However these theories were never modeled with main stream rigour. This paper presents a constant wage model, with capital, labour and land as factors of production, which reproduces the historical tendencies and so can be used as a foundation for the various theories. The use of Chaplygin's theorem in the proof of the main result also gives the paper a technical interest.

JEL Classification: B24,E11,O41.

I. Introduction.

One of the major themes of Marxian economics is that capitalism will breakdown for reasons connected with conflicts between workers and capitalists. Mainstream economics, with its paradigm of methodological individualism, is ill suited for considering this essentially class based critique of capitalism. Marx and the writers who followed him have produced a number of theories of how these conflicts will arise but have failed to underpin them with a model of mainstream rigor. The aim of this paper is to provide such a model. This project has clear relevance for third world capitalism and the analysis of revolutions, but it is at first hard to see what insight it might afford into the workings of currently healthy first world capitalism. The answer is that, in a way that mainstream analysis can not do, it will allow one to judge the extent that first world capitalism is also in peril of a Marxian demise.

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The proposed model has one sector, a neo-classical production function and three factors of production, labour, capital and land. The supply of labour is infinitely elastic at a fixed subsistence wage, capital accumulates because of capitalist saving and the supply of land, which may be thought of as resources in general, is fixed. The result is that, under a simple assumption on the production function, as capital accumulates, the ratio of capital to the payment to labour , called the composition of capital, rises, the rate of profit falls and the capitalist share of income[2] rises. That is, what Duménil and Lévy (1999) have dubbed the " historical tendencies", are generated.[3]

The paper is organised as follows. Section II explains why the model is suitable for its role. First it argues that a subsistence wage and the importance of agriculture are pervasive elements in Marx's thought. Then it briefly describes a number of breakdown theories and notes how they all depend on one or more of the historical tendencies. Section III sets out the model and presents a referee's proof using a variation of Chaplygin's method. Section IV, which concludes comments on the problem of constructing a model which generates the historical tendencies and on the application of the model to revolutions and third and first world capitalism. Finally there is a brief appendix that shows the correctness of a claim, first made by Meek (1967), that in a model with both technical progress and land, the profit rate rises before it starts to fall.

II. The Suitability of the Model.

It is important to argue that the model corresponds to Marx's thought. This is because the intention is to use the model as a foundation for Marxian theories of the breakdown. If the foundation is non-Marxian there is much less justification for the whole procedure.

1. The Subsistence Wage.

The model is constructed by adding land to one of the many versions of the neo-Marxian model set out by Marglin (1984). Thus in a strict sense only the inclusion of

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[2] This is essentially the rate of surplus value.

[3] Their list is slightly longer than the one given here. They have, in addition, that labour productivity and the share of profits devoted to saving grows and that the rate at which capital accumulates falls. In the model presented below the first of these additions is equivalent to a rising capital share. I think it is the concept rather than the details that are important and thus have expropriated their name.

land need be justified. However since much of current research has focussed on rising wage models a few words are in order.[4]

Marx is ambivalent about whether the real wage will rise of not and sections of his writing can be cited to support either position. However when the focus is on the historical tendencies and the breakdown, I think that the subsistence wage assumption is the most suitable. I give four reasons: First, the argument that capitalism will breakdown because of the impoverished condition of the workers is much easier to sustain in a model with a constant subsistence wage.[5] Second, a reading of Volume I of Capital, the last of Marx's major works, can leave no doubt that Marx thought the then current situation of British workers was close to subsistence. Third in his major attempt to link the falling rate of profit to the rising composition of capital, Marx went to great lengths to keep the real wage constant.[6] Forth, in the Solow model, when the elasticity of substitution is greater than one and the model is out of steady state with a growing capital to labour ratio, one has all the historical tendencies but with a rising real wage; in the standard Ricardian model with no physical capital or technical progress one has a constant wage but the share of capital falls to zero. The challenge is, in large part, to find a model that reproduces the historical tendencies in the context of a constant wage.

2. Land.

It is the fixed quantity of land in the model that is responsible for the falling rate of profit and the rest of the historical tendencies. This poses the question of whether this mechanism at all reflects Marx's thinking. The majority of writers think it does not[7], while a minority claim it does. My view is that it does, but only to some extent.[8]

Marx wanted to locate the falling rate of profit in the nature of capitalist development, that is in the accumulation of capital, but he never managed to construct a completely convincing argument. At the same time he provided, in numerous places, accounts of the falling rate of profit based on growing resource scarcity. Thus the model which adds land does embody a mechanism which can be found in Marx's writing, but it is not the one upon which Marx placed his main emphasis.

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[4] See Duménil and Lévy (1993,1995,2000), Skott (1992), Michl (1994, 1999) and Skillman (1997).
[5] See Foley (1986) for a cogent argument that a revolution can occur with a rising wage.
[6] The argument of Marx which is referred to here is not well known and only appeared in English recently (Marx and Engels (1991)). Petith (2001b) provides a detailed analysis.
[7] A possible reason why models like the present one have not already appeared.
[8] Petith (2001b) reviews the opinions of the majority and the minority and gives evidence for the opinions expressed in the next paragraph.

3. Breakdown Theories.[9]

The theories can be divided into those that have capitalism ending as a result of an evermore violent series of business cycles (called crises) and those that have it ending because of trends. In the business cycle group the first argues that the rise in the composition of capital will cause a continual shift of demand from consumption to investment goods, that supply will not adequately react and that the resulting over supply of consumption goods will lead to increasingly violent business cycles. An additional part of this theory is that the falling rate of profit will cause increasingly risky investment ventures to be chosen which will add to the amplitude of the cycles. The second theory works through the rising capital share. Since capitalists are thought to invest rather than consume this continually increases the proportion of aggregate demand that is devoted to investment. Since investment demand is more volatile, this increases the instability of the economy and leads to more violent fluctuations.

In the trend group the first is that capitalists will put pressure on workers as they try to avoid the fall of their rate of profit and that this will increase social conflict. The second is that the rise in the composition of capital will somehow make large holdings of capital more efficient and thus centralise ownership. This, combined with the rising share of capital, will cause an increasingly unequal distribution of income with ever fewer increasingly wealthy capitalists on the one hand and growing mass of impoverished workers on the other. The third is that the composition of capital will rise in a way to ensure that there is a sufficiently large number of unemployed workers, called the reserve army, to keep the wages from rising about subsistence level. The last is that the rate of profit will fall to such an extent that capitalism itself will not be viable.

These descriptions show that all of the breakdown theories depend on one or more of the historical tendencies.

III. The Model, Result and Proof.

1. The Model and the Result.

The model and the result are set out in this section. The model is an augmented and specialised version of Marglin's (1984, chap. 9) one sector neo-Marxian model with continuous substitution. It is augmented by adding technical progress and land and it is

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[9] The information in this section is taken from Part III of Petith (2001a) which describes the historical development of these theories. Part III in turn is taken from Clarke (1994) for Marx, and Howard and King (1989,1992) for Marxist writers.

specialised by using a particular production function and the simplest version of the savings function.

The specific production function that produces output Y is CES in a Cobb-Douglas capital/labour aggregate and land, where K, L, and M are capital, labour and land.

(1) $$Y=[\alpha(K^{\beta}L^{1-\beta}e^{\gamma t})^{-\rho}+(1-\alpha)(Me^{\delta t})^{-\rho}]^{-1/\rho}$$

The aggregate experiences factor augmenting technical progress at rate $\gamma > 0$ while land experiences it at rate $\delta \geq 0$. The elasticity of substitution between the aggregate and land is $\sigma = \frac{1}{1+\rho}$, $-1 \leq \rho \leq \infty$, where $0 \leq \alpha, \beta \leq 1$ are parameters and t is time. The constant real wage w and the rental on capital r are equal to their respective marginal products,

(2) $$w = \partial Y/\partial L$$

(3) $$r = \partial Y/\partial K.$$

The rate of return on investment in land, which is its marginal product plus the capital gain $\dot{P}$ divided by the price, is equal to the return on capital,[10]

(4) $$r = \left(\frac{\partial Y}{\partial M} + \dot{P}\right)/P,$$

where P is the price of land in terms of the good. Capitalists are assumed to own the land as well as capital. Their rate of profit R is defined as

(5) $$R=\frac{Y+\dot{P}M-wL}{K+PM}.$$

It is easily shown that $r = R$ and from this point on r will be call the rate of profit. Savings are provided only by capitalists who save all their income. Their savings are equal to the accumulation of wealth,

(6) $$\dot{P}M+\dot{K}=Y+\dot{P}M-wL.$$

The assumption that all profits are saved removes $\dot{P}M$ from the accumulation equation and immensely simplifies the model. This seems justified since the effect of changing land prices on capitalist saving seems to play no role in Marx's writing. It is assumed that factor augmenting technical progress is slower for land than for the aggregate,

(7) $$\delta < \gamma.$$

Marx (TSV vol. III, 1972, pp. 300-1) specifically assumed that this was the case and, moreover, one must have this if the fixed stock of land is to lead, in some cases, to a

[10] $\dot{x}$ is the derivative of x with respect to time.

falling rate of profit. Finally the definitions of the two other Marxian concepts are as follows. The composition of capital is $K/wL = k$ and a measure of the share of capital e is

(8) $$e = \frac{Y}{wL} - 1.$$

This concludes the presentation of the model.

The model yields a single differential equation in K in the following manner. (2) determines L as a function of K and t, $L(K,t)$. Thus output also depends on these two variables, $Y(K,L,t) = \underline{Y}(K,L(K,t),t)$. Substituting these into (6) gives the non-autonomous differential equation

(9) $$\dot{K} = f(K,t) \qquad K(0) = K_0 > 0$$

where K_0, the initial capital stock, is assumed to be positive. The initial-value problem (9) has a solution $K(t)$. Taking account of the dependence of L on K and t, this solution implies the time paths of the key variables, $k(t)$, $r(t)$ and $e(t)$.

The characteristics of these time paths are given by the following theorem.

<u>Theorem</u> : *For the model of equation (1)-(8), there exists a t^*, such that, for $t > t^*$:*

a) If $\rho > 0$ ($\sigma < 1$) then $k \to$ and $\dot{k} > 0$, $r \to 0$ and $\dot{r} < 0$ and $e \to$ and $\dot{e} > 0$.

b) If $\rho = 0$, ($\sigma = 1$) then $k \to cst > 0$, $r \to cst > 0$ and $e = cst > 0$.

c) If $\rho < 0$ ($\sigma > 1$) then $k \to 0$ and $\dot{k} < 0$, $r \to$ and $\dot{r} > 0$ and $e \to cst > 0$ and $\dot{e} < 0$.

2. The Proof (Referee A).

The proof of the theorem uses the following theorem due to Chaplygin.[11]

<u>Theorem</u> (Chaplygin). *For an equation of the form $\dot{y} = f(y,t)$, $y(t_0) = y_0$, if the differential inequalities*

$$\dot{u}(t) - f(u(t),t) < 0$$
$$\dot{v}(t) - f(v(t),t) > 0$$

hold with $t > t_0$ and $u(t_0) = v(t_0) = y_0$, then

$$u(t) < y(t) < v(t)$$

holds for all $t > t_0$.

The exposition of the proof is divided into four parts.

I. Equation (1) can be rewritten:

[11] See Mikhlin and Smolitskiy (1967) pp.9-12 or Zwillinger (1989) pp. 388-91.

(10) $$Y = e^{\delta t} f(x)$$

with $$f(x) = \frac{c_1 c_2}{\left(c_2 + x^{\rho}\right)^{1/\rho}}, \quad x = K^{\beta} L^{1-\beta} e^{(\gamma-\delta)t}, \quad c_1 = \frac{M}{(1-\alpha)^{1/\rho}}, \quad \text{and} \quad c_2 = \alpha\left(c_1\right)^{\rho}.$$

A few properties of the function *f(x)* will be useful later:

(11) $$f'(x) = \frac{c_1 x}{\left(c_2 + x^{\rho}\right)^{1+1/\rho}}, \quad f''(x) = -\frac{c_1 c_2 (1+\rho) x^{\rho-1}}{\left(c_2 + x^{\rho}\right)^{2+1/\rho}}$$

$$f(x) - (1-\beta) x f'(x) = \frac{c_1 x\left(\beta c_2 + x^{\rho}\right)}{\left(c_2 + x^{\rho}\right)^{1+1/\rho}}$$

$$g(x) = \frac{f'(x) - \frac{1-\beta}{\beta} x f''(x)}{\left(f(x)\right)^{1/\beta}} = \frac{1}{\left(c_1 c_2\right)^{(1-\beta)/\beta}} \frac{c_2 + \mu x^{\rho}}{\left(c_2 + x^{\rho}\right)^{1+(1-\mu)/\rho}} \text{ with } \mu = \frac{1+\rho-\beta\rho}{\beta}.$$

Equations (2) and (3) can be rewritten as:

$$w = e^{\delta t} f'(x)(1-\beta)\frac{x}{L}$$

$$r = e^{\delta t} f'(x)\beta\frac{x}{K}.$$

One can now express the main variables as functions of *x*.

$$L = \frac{1-\beta}{w} e^{\delta t} x f'(x)$$

(12) $$\frac{Kr}{\beta} = \frac{Lw}{1-\beta} \text{ or k} = \frac{\beta}{1\text{-}\beta}\frac{1}{r}.$$

(13) $$e = \frac{1}{1-\beta}\frac{c_2 + x^{\rho}}{c_2} - 1$$

$$\Pi = Y - wL = e^{\delta t}\left(f(x) - (1-\beta) x f'(x)\right) = c_1 e^{\delta t}\frac{x\left(\beta c_2 + x^{\rho}\right)}{\left(c_2 + x^{\rho}\right)^{1+1/\rho}}$$

(14) $$K = \left(\frac{x e^{-(\gamma-\delta)t}}{L^{1-\beta}}\right)^{\frac{1}{\beta}} = \left(\frac{w}{1-\beta}\right)^{\frac{1-\beta}{\beta}} e^{(\delta-\phi)t}\frac{x}{\left(f'(x)\right)^{(1-\beta)/\beta}}, \text{ with } \phi = \frac{\gamma}{\beta}.$$

(15) $$r = \frac{Lw}{K}\frac{\beta}{1-\beta} = \beta\left(\frac{1-\beta}{w}\right)^{(1-\beta)/\beta} e^{\phi t}\left(f'(x)\right)^{1/\beta}$$

II. We now determine the differential equation accounting for the accumulation of capital. Differentiating equation (14) one obtains:

$$\dot{K} = \left(\frac{w}{1-\beta}\right)^{\frac{1-\beta}{\beta}} e^{(\delta-\phi)t}\left((\delta-\phi)\frac{x}{\left(f'(x)\right)^{(1-\beta)/\beta}} + \dot{x} g(x)\right).$$

From the equation $\dot{K} = \Pi$ we can derive the differential equation satisfied by x :

(16) $$\dot{x} = e^{\phi t} F(x) + G(x) \quad \text{with initial condition} \quad x(t_0) = x_0$$

with

$$F(x) = \frac{a\mu x\left(x^{\rho} + \beta c_2\right)}{\left(c_2 + x^{\rho}\right)^{\mu/\rho}\left(c_2 + \mu x^{\rho}\right)}, \quad a = \frac{\left(c_1 c_2^{\,1-\beta}\right)}{\mu}\left(\frac{1-\beta}{w}\right)^{(1-\beta)/\beta}$$

$$G(x) = (\phi - \delta)\frac{x\left(c_2 + x^{\rho}\right)}{c_2 + \mu x^{\rho}}.$$

We define $\tilde{F}(x)$ and $\tilde{G}(x)$ by:

(17) $$F(x) = \frac{a}{x^{\mu - 1}}\tilde{F}(x) \quad \text{and} \quad G(x) = \frac{\phi - \delta}{\mu} x\tilde{G}(x).$$

These two functions satisfy:

$$\lim_{x \to \infty} \tilde{F}(x) = \lim_{x \to \infty} \tilde{G}(x) = 1.$$

III. As a preliminary to the study of the differential equation (16), we first consider a more simple equation in which the asymptotical behaviour of $\tilde{F}(x)$ and $\tilde{G}(x)$ have been substituted for *F(x)* and *G(x)*, i.e. 1 has been substituted for $\tilde{F}(x)$ and $\tilde{G}(x)$:

(18) $$\dot{x} = \frac{ae^{\phi t}}{x^{\mu-1}} + \frac{\phi - \delta}{\mu} x \quad \text{with the initial condition } x(T) = X.$$

In order to determine the solutions of equation (18), we substitute the function y(t) for x(t), with $y(t) = a\mu \frac{e^{\phi t}}{\left(x(t)\right)^{\mu}}$. The function y(t) satisfies the following differential equation:

$$\dot{y} = y(\delta - y)$$

which can be easily integrated:

$$\bar{y}(t) = \frac{\delta}{1 - Ce^{-\delta t}}.$$

Thus, the solutions of equation (18) can be determined:

$$\bar{x}(t; a, \delta, T, X) = \left(\frac{a\mu}{\delta} e^{\phi t}\left(1 - C(a, \delta, T, X)e^{-\delta t}\right)\right)^{1/\mu}$$

in which the constant *C(a,*δ*, T, X)* is determined by the initial condition $\bar{x}(T; a, \delta, T, X) = X$. We also define the function $\bar{\bar{x}}(t)$ whose asymptotical behaviour is identical to that of $\bar{x}$ when t tends to infinity:

(19) $$\bar{\bar{x}}(t) = \left(\frac{a\mu}{\delta} e^{\phi t}\right)^{1/\mu}.$$

The following differential equation will be used in what follows:

(20) $$\dot{x} = \lambda\left(\frac{ae^{\phi t}}{x^{\mu-1}} + \frac{\phi-\delta}{\mu}x\right) \text{ with the initial condition } x(T) = X.$$

This equation can be rewritten as:

$$\dot{x} = \frac{a_\lambda e^{\phi t}}{x^{\mu-1}} + \frac{\phi-\delta_\lambda}{\mu}x \quad \text{with } a_\lambda = \lambda a \text{ and } \delta_\lambda = \lambda\delta + (1-\lambda)\phi$$

which allows for the determination of the solutions of (20): $\bar{x}(t;a_\lambda,\delta_\lambda,T,X)$.

IV. Contrary to equations (18) or (20), it is not possible to obtain explicit solutions for equation (16). From the solutions of equation (20), one can, however, derive bounds for the solutions of equation (16), using the method of Chaplygin.

If *x(t)* is the solution of (16), and if $\tilde{F}(x)$ and $\tilde{G}(x)$ are bounded upward and downward by constant terms, for *t > T*, bounds can be derived for *x(t)*, for *t > T*, from the solutions of equation (20). More specifically, we will prove the following lemma:

<u>Lemma.</u> If x(t) is the solution of equation (16), if T > t_0 , and if $\tilde{F}(x)$ and $\tilde{G}(x)$ satisfy:

(21) $$m < \tilde{F}(x) < M \ \text{ and } \ m < \tilde{G}(x) < M \ \text{ if } \ x > x(T)$$

then

(22) $$\bar{x}_{m,T}(t) < x(t) < \bar{x}_{M,T}(t) \ \text{ if } t > T$$

with $\bar{x}_{m,T}(t) = \bar{x}(t;a_m,\delta_m,T,x(T))$ *and* $\bar{x}_{M,T}(t) = \bar{x}(t;a_M,\delta_M,T,x(T))$.

Proof:

Since *x(t)* satisfies equation (16) with $x(t_0)=x_0 > 0$, $\dot{x}(t)$ is always positive and *x(t)* is increasing and positive. The same is true of $\bar{x}_{m,T}$ and $\bar{x}_{M,T}$ which satisfy equation (20) with the initial condition $\bar{x}_{m,T}(T) = \bar{x}_{M,T}(T) = x(T) > 0$, which are also increasing functions. It follows that $\bar{x}_{m,T}(t)$ and $\bar{x}_{M,T}(t)$ are strictly larger than *x(T)* for *t > T*. Consequently, using equations (20), (21) and (17), one has

$$\dot{\bar{x}}_{m,T} = m\left(\frac{ae^{\phi t}}{(\bar{x}_{m,T})^{\mu-1}} + \frac{\phi-\delta}{\mu}\bar{x}_{m,T}\right) < e^{\phi t}F(\bar{x}_{m,T}) + G(\bar{x}_{m,T})$$

as well as:

$$\dot{\bar{x}}_{M,T} > e^{\phi t}F(\bar{x}_{M,T}) + G(\bar{x}_{M,T}).$$

Chaplygins's conditions are satisfied, and this brings the proof of equation (22) to completion.

First application of the lemma.

Over the entire interval $t \geq t_0$, the functions $\tilde{F}(x)$ and $\tilde{G}(x)$ are always defined and bounded when their arguments tend to infinity, both upward and downward by m_0 and M_0 respectively. Using the lemma, a bound for *x(t)* can be obtained for all $t \geq t_0$. Thus, one obtains the following corollary:

Corollary 1: If x(t) is the solution of (16), then,

$$\bar{x}_{m_0,t_0}(t) < x(t) < \bar{x}_{M_0,t_0}(t) \;\; if \;\; t > t_0.$$

Therefore, *x(t)* is bounded both upward and downward by functions asymptotically proportional to $\bar{\bar{x}}$ (equation (19)).

Second application of the lemma.

It is possible to improve this result and prove that the asymptotic behaviour of *x(t)* is $\bar{\bar{x}}(t)$.

Corollary 2. If x(t) is the solution of (16), then:

$$\lim_{x \to \infty} \frac{x(t)}{\bar{\bar{x}}(t)} = 1.$$

Proof:

We want to prove that, for any ε (which can be assumed to be smaller than 1), one can find T_ε such that

(23) $$(1-\varepsilon)\bar{\bar{x}}(t) < x(t) < (1+\varepsilon)\bar{\bar{x}}(t) \qquad t > T_\varepsilon .$$

We define:

(24) $$\nu = 3 + 4\frac{\varphi - \delta}{\delta}.$$

Since $\tilde{F}(x)$ and $\tilde{G}(x)$ tend to 1 when *x* tends to infinity, X_ε exists such that:

$$1 - \frac{\varepsilon}{\nu} < \tilde{F}(x) < 1 + \frac{\varepsilon}{\nu} \quad \text{and} \quad 1 - \frac{\varepsilon}{\nu} < \tilde{G}(x) < 1 + \frac{\varepsilon}{\nu} \qquad x > X_\varepsilon .$$

We define T_ε^1 by $x(T_\varepsilon^1) = X_\varepsilon$, and apply the lemma for $T = T_\varepsilon^1$ and for :

(25) $$m = 1 - \frac{\varepsilon}{\nu} \quad \text{and} \quad M = 1 + \frac{\varepsilon}{\nu}.$$

We must now prove that a value of T_ε can be determined such that:

(26) $$(1-\varepsilon)\bar{\bar{x}}(t) < \bar{x}_{m,T}(t) \quad \text{and} \quad \bar{x}_{M,T}(t) < (1+\varepsilon)\bar{\bar{x}}(t) \qquad t > T_\varepsilon .$$

1. With ν defined by equation (24), $\varphi - \delta$ larger than 0 (since $\varphi = \gamma/\beta > \gamma > \delta$), m and M defined by equation (25) and ε smaller than 1, one can prove that:

$$\left(1 - \frac{\varepsilon}{3}\right)\frac{a}{\delta} < \frac{a_m}{\delta_m} \quad \text{and} \quad \frac{a_M}{\delta_M} < \left(1 + \frac{\varepsilon}{3}\right)\frac{a}{\delta}.$$

2. If $C_{m,T}$ and $C_{M,T}$ are the two constants in the expressions of $\bar{x}_{m,T}(t)$ and $\bar{x}_{M,T}(t)$:

$$C_{m,T} = C(a_m, \delta_m T, x(T)) \text{ and } C_{M,T} = C(a_M, \delta_M T, x(T))$$

and defining T_ε^2 and T_ε^3 by:

$$\left|C_{m,T_\varepsilon^1}\right| e^{-\delta_m T_\varepsilon^2} = \frac{\varepsilon}{3} \quad \text{and} \quad \left|C_{M,T_\varepsilon^1}\right| e^{-\delta_M T_\varepsilon^3} = \frac{\varepsilon}{3}$$

then one has:

$$1-\frac{\varepsilon}{3} < 1 - C_{m,T_\varepsilon^1} e^{-\delta_m t} \quad \text{and} \quad 1 - C_{M,T_\varepsilon^1} e^{-\delta_M t} < 1 + \frac{\varepsilon}{3} \qquad t > max\left(T_\varepsilon^2, T_\varepsilon^3\right).$$

3. Since μ is larger than 1, or $1/\mu$ smaller than 1, and still assuming that ε is smaller than 1, one has:

$$1-\varepsilon < (1-\varepsilon)^{1/\mu} < \left(\left(1-\frac{\varepsilon}{3}\right)^2\right)^{1/\mu} \quad \text{and} \quad \left(\left(1+\frac{\varepsilon}{3}\right)^2\right)^{1/\mu} < (1+\varepsilon)^{1/\mu} < 1+\varepsilon.$$

Choosing $T_\varepsilon = max\left(T_\varepsilon^1, T_\varepsilon^2, T_\varepsilon^3\right)$, the inequalities (26) are satisfied, and this brings the proof of corollary 2 to completion.

<u>Proof of the Theorem:</u>

If $\rho = 0$, one can get an explicit solution for the differential equation in K. The reported results then follow from this in a straight forward way.

If $\rho \neq 0$, then consider the asymptotic behaviour. From corollary 2, equations (15) and (19) and $f'(x) \approx c_1 c_2 x^{-(1+\rho)}$

$$r \approx cste^{-\frac{\phi}{\mu}\rho t}.$$

If $\rho > 0$ then r falls and approaches 0, by (12) k rises and approaches infinity and by corollary 2, and equations (13) and (19), e rises and approaches ∞. If $\rho < 0$ the opposite movements occur with e approaching a positive constant. This completes the proof.

It is worth noting how different both the problem and the method of solution are from those of conventional growth theory.

IV Conclusion.

The model of this paper is important for two reasons: first it provides a hitherto missing account of the historical tendencies in the context of a constant wage and second it opens the door to an ambitious Marxian research program.

The intrinsic problem in modelling the historical tendencies in the context of a Marxian model is the generation of the dynamics. The neo-classical growth model does this by having the exogenously determined rate of growth of labour be different from the endogenously determined rate of growth of capital when the model is out of steady state. The natural Marxian assumptions of a fixed real wage and an infinitely elastic supply of labour that characterise the formal Marxian models of the 1970s and 1980s[12] meant that this mechanism could not operate. The only equilibrium of these models was a steady state in which the labour supply increased in tandem with the capital stock and the organic composition of capital, the rate of exploitation and the rate of profit all remained constant. Duménil and Lévy (2000) broke this impasse in an eminently Marxian way by inserting their model of endogenous technical change. Specifically, in the face of symmetric technical possibilities, the relatively higher labour share generates technical change that raises the organic composition of capital and, in steady state, the wage in a way that the share remains constant so that the rate of profit falls. They thus reproduce two out of the three central relations but at the cost of a rising wage. The model of this paper returns to a version of the neo-classical mechanism: the infinitely elastic labour supply still adjusts to keep the real wage constant but now it is the difference between an exogenously determined rate of growth of land in efficiency units and the endogenously determined rate of growth of capital that supplies the dynamics. This mechanism generates the complete central relations in the context of a constant wage.

There are three ways in which the model may be used: to formalise the breakdown theories, to model past revolutions and current revolutionary situations and finally, to gain insights into the presently successful capitalism of the industrialised world.

First the model could be used as a foundation for the formalisation of each of the breakdown theories since all of them depend on one or more of the historical tendencies. This may seem an excessive production of counterfactual models when one considers only the industrialised world. However the project has considerable relevance since the instability and conflictive class relations that these theories describe reflect the actual conditions of many third world capitalist countries.[13]

The second way in which the model can be used is for the analysis of revolutions. The model is suitable for this both because of the subsistence wage assumption and because land plays an important role, but one extension must be made. Population pressure on the land played a role in Marx's writing on revolutions, Theda Skocpol´s

[12] See Morishima (1973), Steadman (1977), Roemer (1981) and Marglin(1984).

seminal work (1979) emphasised the importance of this for the French (p.56), Russian (p.132) and Chinese (p.47) revolutions, and this theme has continued in the descriptions of recent revolutionary situations in Vietnam (Popkin, 1988), Latin America (Wickham-Crowley, 1992), southern Mexico and the former Zaire (Renner,1997). However a common theme of all these later writers (not to mention Lenin and Mao) is the difference between the roles played by the peasants and the urban workers. All these events had deep economic causes and much is known about what happened but they have not been formally analysed. If the model of the present paper were extended so as to treat peasants as a separate group, it could provide the basis for a formal description of these events.

The third way the can be used is to analyse the capitalism of the industrialised world. Three problems immediately arise: this capitalism is currently enjoying good health, the subsistence wage assumption is far from reality and the historical tendencies are absent: the rate of profit and the composition of capital have fluctuated with little long term trend while the share of capital has fallen.[14] These problems can be surmounted by generalising the model so that slight changes in either parameter values or some of its mechanisms enable it to reproduce either the condition of current capitalism or the Marxian scenario. Some examples follow.

In Petith (2000) I added a Marxian population theory to a simplified version of the model of this paper with the result that, depending on the speed of technical progress, it either exhibited the characteristics of first world capitalism or a falling wage and rate of profit. However one of the drawbacks caused by the Cobb-Douglas production function was that technical progress was not factor specific.[15]

The model of this paper highlights the importance of relative speeds of technical progress and Foley (2000) give precise insights into the relation between these and the historical tendencies. Anyone familiar with the Duménil-Lévy endogenous technical change mechanism might think that its insertion in the present model would make the crucial difference in rates disappear. But the mechanism itself is delicate and its functioning depends on, among other things, the correct pricing of factors. Foley has

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[13] To my knowledge the existence of the historical tendencies in the third world has not been investigated.
[14] For example this is the situation in the United States according to the following studies: Duménil and Lévy (1995) for the rate of profit with data 1869-1989; Duménil, Glick and Rangel (1987) for the share of capital with data 1899-1978; and Wolf (1999) for the composition of capital with data 1947-1996. It is possible to find studies like Moseley (1991) that provide support for the historical tendencies.
[15] This also meant that the share of capital was constant so that all the historical tendencies were not present. In this respect the present model is an advance.

interpreted land as environmental services and has shown that, when they are priced, normal capitalism appears but when they are un-priced a path results with a falling wage and characteristics remarkably like the historical tendencies.

Finally the theme of human capital fits well into this way of thinking. One of the breakdown theories emphasises the centralisation of wealth and its no-occurrence is, it would seem, one of the reasons for the current non-conflictive state of first world capitalism. One of the obvious reasons is that roughly half of the stock of capital is human and more difficult to centralise. The mechanisms that lead to its accumulation are currently a popular research topic and Galor and Moav (2000) in the wonderfully titled "Das Human Kapital" have actually argued that capitalist self interest has been responsible for the decentralised spread of human capital. But once again these mechanisms are delicate and it is not hard to imagine a configuration of the underlying technical possibilities that would lead to all human capital being concentrated in a small intellectual elite.

With respect to the first world, the broad research project suggested by the model of this paper is the construction of models that are capable of producing either current conditions or those of the breakdown scenarios. It would be good to know, even on the level of theory, how fragile the current good health of capitalism[16] is and the Marxist perspective is well suited to the task.

Appendix on Meek's Theorem.

Theorem (Meek). *For the model of equations (1)-(8), if* $\rho > 0$ *(*$\sigma < 1$*) then* $\lim_{K(0)\to 0} \dot{r}(0) > 0.$

Proof: Differentiating (15) at $t=0$ gives

$$\dot{r}(0) = \phi\left(f'\right)^{1/\phi} + \frac{1}{\beta}\left(f'\right)^{\frac{1}{\beta}-1} f''\dot{x}.$$

From (14) $\lim_{K\to 0} x = 0$. Then from (11) and (16): $\lim_{K\to 0} f' > 0$, $\lim_{K\to 0}\left(f'\right)^{\frac{1}{\beta}-1} f''\,\dot{x} = 0.$

References.

[16] As an example of fragility, Duménil and Lévy (1995) calculate that the US trend rate of profit fell from about 40% in 1870 to close to 20% in 1920 and then rebounded due to a wave of technical change. One wonders what would have become of American capitalism in the absence of the wave.

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