Bidding for the surplus: Realizing efficient outcomes in general economic environments*

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Abstract

In this paper, we consider two classes of economic environments. In the first type, agents are faced with the task of providing local public goods that will benefit some or all of them. In the second type, economic activity takes place via formation of links. Agents need to both form a network and to decide how to share the output generated. For both scenarios, we suggest a bidding mechanism whereby agents bid for the right to decide upon the organization of economic activity. The subgame perfect equilibria of this game generate efficient outcomes.

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1 Introduction

A substantial amount of economic activity takes place in settings that are very different from the perfect competition model. A natural concern for such environments is the attainment of efficient outcomes despite the difficulties stemming from strategic behavior and the market power participating agents have. In this paper we consider two such environments and resolve the problem of achieving efficiency by constructing appropriate bidding mechanisms whose equilibria generate efficient outcomes. The mechanisms proposed for these environments are based on the proposal developed in Pérez-Castrillo and Wettstein [19].

The first environment is a local public goods economy where agents produce the goods and share the production costs. Standard competitive behavior typically leads to inefficient outcomes. We suggest a simple sequential mechanism to decide upon the amounts produced and consumed and show that its equilibria yield efficient outcomes.

In the second environment the main economic activity consists of the formation of links among agents.[1] The total output produced is a function of the final network formed. The network formed is efficient if the amount of output it produced exceeds or equals the amount of output produced by any other network. We again suggest a simple sequential mechanism with the property that its equilibrium outcomes generate an efficient network.

The mechanisms suggested have as their first stage a bidding game. The winner of that bidding obtains the right to organize the economic activity; however he is not a dictator. He proposes an organization of the activity (either the amount of public goods for a coalition, or the network structure) and a vector of transfers. If the agents accept his proposal, it is carried out. In case the offer is refused the winner is removed from the game and is left on his own. The remaining agents play the same game again starting from the bidding stage. The subgame perfect equilibria of both mechanisms generate efficient outcomes for their respective environments. The payoffs received by the agents coincide with their Shapley values in appropriately defined cooperative games.

Several previous papers were concerned with achieving efficiency in environments similar to ours. Bagnoli and Lipman [3], Jackson and Moulin [10], Bag and Winter [2], and Mutuswami and Winter [15] for the public good environment and Jackson and Wolinsky [11], Dutta and Mutuswami [9], Currarini and Morelli [7] and Mutuswami and Winter [15] for the network formation framework. Sequential mechanisms that were constructed by Moore and Repullo [13] and Maniquet [12] to realize general social choice functions would work for our environments as well. Due to their large scope

[1] This type of structure has been used in different contexts, as the analysis of the internal organization of firms or cost allocation schemes. See, for instance, the doctoral dissertation of van den Nouweland [18].

of coverage they are however more complex than the other mechanisms mentioned. We compare our proposal with the previous mechanisms and discuss its advantages in the corresponding sections.

2 A local public goods economy

We consider environments with a set $N = \{1, \ldots, n\}$ of agents that consume m *local public goods* and one private good. The preferences of the agents are quasi-linear in the private good. Agent i's preferences are given by $U_i(y, S, x_i) = u_i(y, S) - x_i$, where $S \subseteq N$ denotes the coalition to which the agent belongs, $y \in \Re^m_+$ the level of public goods produced by the members of S, and $x_i \in \Re$ agent i's contribution towards the production of the public goods. The local nature of the public good implies that an agent only enjoys the public good produced by a coalition if he belongs to it. That is, each coalition can bar agents not belonging to the coalition from consuming the public goods produced by it. Also, the utility of an agent depends on both the level of public goods and the identity of the partners in the coalition.

The technology is given by a cost function $c(y, S)$, that describes the cost (in terms of the private good) of producing y by the members of S. Hence it could be that different coalitions have different costs for producing identical amounts of public goods. Differences may stem from size or from the availability of different technologies.

An *efficient partition and production plan* for this economy is a feasible allocation that maximizes the total payoff members in N could obtain, by possibly splitting into coalitions that would produce public goods for their members. Therefore, an efficient partition and production plan for N solves:

$$\max \left\{ \sum_{S_j \in \pi} \left(\sum_{i \in S_j} u_i(y_j, S_j) - c(y_j, S_j) \right) | \pi \text{ is a partition of } N \text{ and } y_j \in \Re^m_+ \text{ for all } j \right\}$$

We can model the previous environment as a cooperative game with transferable utility. Denote by $w(T)$ the value associated with any coalition $T \subseteq N$:

$$w(T) = \max \left\{ \sum_{i \in T} u_i(y, T) - c(y, T) | y \in \Re^m_+ \right\}$$

The function $w(T)$ thus measures the maximum total surplus that the members of the coalition T can obtain by producing on their own some vector of public goods. The resulting cooperative game (N, w) need not be super-additive. We construct the super-additive cover of this cooperative game and denote its characteristic function by $W(T)$ for $T \subseteq N$:

$$W(T) = \max \left\{ \sum_{S_j \in \pi} w(S_j) | \pi \text{ is a partition of } T \right\}$$

Note that $W(T)$ is the maximal total payoff that members of T can generate, possibly splitting into several subcoalitions. The *Shapley value* of every agent in the cooperative game with the characteristic function $W(S)$ is denoted by $\phi_i(N, W)$, that is:

$$\phi_i(N, W) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [W(S \cup \{i\}) - W(S)]$$

The Shapley value allocation, hence, generates an efficient partition and produces the corresponding efficient levels of public goods. Moreover, it shares the production costs in an equitable manner.

For the local public goods economy, we suggest a mechanism whose equilibrium outcome would generate an efficient partition and production plan for N. We view this as an implementation problem in an environment with complete information. The mechanism constructed will in effect implement the Shapley value $\phi(N, W)$.

The *bidding mechanism* proceeds as follows: In stage 1 the agents bid for the right of being the proposer. Each agent bids by submitting an $(n-1)$-tuple of payments to be made to all other players conditional of his being chosen as the proposer. At the end of stage 1, based on the bids, one of the agents is chosen as the proposer. Before moving to stage 2, the proposer pays out the bids he made. In stage 2 the proposer offers a vector of payments (typically negative) to all other players, chooses a coalition he wants to form, and proposes a vector of public goods that will be produced and enjoyed by the members of that coalition. The offer is accepted if all the other players agree. In case of acceptance the coalition is formed, the proposer produces the public goods and the players outside the coalition proceed to play the same game again among themselves. In the case of rejection all the players other than the proposer play the same game again.

Formally, if there is only one player $\{i\}$, he chooses a vector y of public goods. (If only one player plays, there is no bidding stage.)

Given the rules of the mechanism for at most $n-1$ players, the mechanism for $N = \{1, \ldots, n\}$ proceeds as follows:

$t = 1$: Each player $i \in N$ makes bids $b^i_j \in \Re$ for every $j \neq i$. Hence, at this stage, a strategy for player i is a vector $(b^i_j)_{j \neq i} \in \Re^{n-1}$.

For each $i \in N$, define the *net bid* of player i by $B^i = \sum_{j \neq i} b^i_j - \sum_{j \neq i} b^j_i$. Let $\alpha = \text{argmax}_i(B^i)$ where an arbitrary tie-breaking rule is used in the case of a non-unique maximizer. Once he has been chosen, player α pays b^α_i to every player $i \neq \alpha$.

$t = 2$: Player α chooses a coalition S_α with $\alpha \in S_\alpha$, a production plan $y_\alpha \in \Re^m_+$ and makes an offer $x^\alpha_i \in \Re$ to every player $i \neq \alpha$.[2]

[2]For the players in S_α, $-x^\alpha_i$ is the payment made by player i to the player α who bears the whole cost of producing the public goods vector. For players outside S_α, x^α_i is the payment necessary (positive or negative) to induce them to stay outside of S_α.

$t = 3$: The players other than α, sequentially, either accept or reject the offer. If a player rejects it, then the offer is rejected. Otherwise, the offer is accepted.

If the offer is accepted, each player $i \neq \alpha$ receives y_i^α, player α forms the coalition S_α and produces y_α bearing the cost $c(y_\alpha, S_\alpha)$. After this, players in $N \backslash S_\alpha$ proceed to play the game again among themselves. Therefore, the final payoff to a player $i \in S_\alpha \backslash \{\alpha\}$ is $u_i(y_\alpha, S_\alpha) + x_i^\alpha + b_i^\alpha$, player α receives $u_\alpha(y_\alpha, S_\alpha) - c(y_\alpha, S_\alpha) - \sum_{i \neq \alpha} x_i^\alpha - \sum_{i \neq \alpha} b_i^\alpha$, and the final payment for a player $i \in N \backslash S_\alpha$ will be the sum of the bid b_i^α, the offer x_i^α, and the payoff he will obtain in the game played by $N \backslash S_\alpha$. On the other hand, if the offer is rejected, all players other than α proceed to play the same game where the set of players is $N \backslash \{\alpha\}$ and player α is on his own. The final payoff to α is what he can obtain by himself (that is, $\max_{y \in \Re_+^m} [u_\alpha(y, \alpha) - c(y, \alpha)]$) minus the bids already paid. The final payoff to any player $i \neq \alpha$ is the sum of the bid b_i^α and the payoff he obtains in the game played by $N \backslash \{\alpha\}$.

Theorem 1 *The subgame perfect equilibria of the bidding mechanism result in an efficient partition and production plan for the local public goods economy. Moreover, the mechanism implements the Shapley value of* (N, W).

Proof: We have described the underlying environment as a cooperative game and considered its super-additive cover. This description allows us to stablish a relationship between the implementation of (N, W) in the local public goods set up and the implementation of the Shapley value of the super-additive cover of a transferable utility game provided in Section 5 of Pérez-Castrillo and Wettstein [19] (PC&W hereafter). Therefore, we propose here a proof that is similar to that of Theorem 3 in PC&W.

The proof proceeds by induction on the number of players n. It is easy to see that the theorem holds for $k = 1$. We assume that it holds for all $k \leq n-1$ and then consider the following strategies for the case of n players:

At $t = 1$, each player i, $i \in N$, announces $b_j^i = \phi_j(N, W) - \phi_j(N \backslash \{i\}, W)$ for every $j \neq i$.

At $t = 2$, player i, if he is the proposer, chooses a coalition S_i such that $S_i \in \mathrm{argmax}_{S \subseteq N, S \ni i} \{w(S) + W(N \backslash S)\}$, a production plan y_i efficient for S_i, and offers $x_j^i = \phi_j(N \backslash \{i\}, W) - u_j(y_i, S_i)$ to every $j \in S_i \backslash \{i\}$ and $x_j^i = \phi_j(N \backslash \{i\}, W) - \phi_j(N \backslash S_i, W)$ to every $j \notin S_i$.

At $t = 3$, player i, if player $j \neq i$ is the proposer and $i \in S_j$, accepts any offer greater than or equal to $\phi_i(N \backslash \{j\}, W) - u_i(y_j, S_j)$ and rejects it otherwise. If player $j \neq i$ is the proposer and $i \notin S_j$, player i accepts any offer greater than or equal to $\phi_i(N \backslash \{j\}, W) - \phi_i(N \backslash S_j, W)$ and rejects it otherwise.

The induction argument ensures that player $i \notin S_\alpha$ will obtain $\phi_i(N \backslash S_\alpha, W)$ if the game is played among the players in $N \backslash S_\alpha$. Also, player $i \in S_\alpha$ obtains the utility derived from enjoying the public good, $u_i(y_\alpha, S_\alpha)$ plus the payment x_i^α. Then, it is easy to check that the total utility (taking into account the bid) obtained by any player different from the proposer is $\phi_i(N, W)$ when all the players follow the previous strategies. Also, these strategies lead to an efficient partition and to an optimal production plan of public goods for each coalition. Hence, the proposer also obtains his Shapley value.

We now prove that the previous strategies constitute an SPE. The induction argument makes it clear that the strategy at $t = 3$ is a best response for any player different from the proposer. At $t = 2$, given the strategies that the other players will follow at $t = 3$, player i's best decision (if he is the proposer) is to choose a subset S_i (such that $i \in S_i$), a production plan $y_i \in \Re_+^m$ and payments x_j^i to every $j \neq i$.[3] The coalition and production plan are chosen so as to maximize

$$\begin{aligned} u_i(y_i, S_i) - c(y_i, S_i) - \sum_{j \neq i} x_j^i \equiv & \Big[u_i(y_i, S_i) - c(y_i, S_i) - \sum_{j \neq i} \phi_j(N \backslash \{i\}, W) + \\ & + \sum_{j \notin S_i} \phi_j(N \backslash S_i, W) + \sum_{j \in S_i \backslash \{i\}} u_j(y_i, S_i) \Big] \\ \equiv & \Big[\sum_{j \in S_i} u_j(y_i, S_i) - c(y_i, S_i) + \\ & + W(N \backslash S_i) - W(N \backslash \{i\}) \Big] \end{aligned} \tag{1}$$

The payments offered to all other players are as follows:

$$x_j^i = \begin{cases} \phi_j(N \backslash \{i\}, W) - u_j(y_i, S_i) & \text{if } j \in S_i \backslash \{i\}, \\ \phi_j(N \backslash \{i\}, W) - \phi_j(N \backslash S_i, W) & \text{if } j \notin S_i. \end{cases}$$

The coalition S_i and the level of public good y_i that maximize (1) are the ones proposed in the candidate strategy.

Finally, consider the strategies at $t = 1$. Notice that the balanced contributions property (that is, $\phi_i(N, W) - \phi_i(N \backslash \{j\}, W) = \phi_j(N, W) - \phi_j(N \backslash \{i\}, W)$ for all $i, j \in N$. On this, see Myerson [17]) implies that $B^i = 0$ for all $i \in N$. Given this, if player i increases his net bid $B^i = \sum_{j \neq i} b_j^i$, he will be chosen as the proposer with certainty, but his payoff will decrease (it will be lower than $\phi_i(N, W)$). If he decreases B^i, another player will be chosen as the proposer, and player i's payoff would still equal his Shapley value. Finally, any change in his bids that leaves B^i constant will influence the identity of the proposer but will not alter player i's payoff.

[3] Notice that the proposer always has the possibility of proposing an allocation that will not be accepted by the other players. This possibility is equivalent to proposing $S_i = \{i\}$ and then $x_j^i = \phi_j(N \backslash \{i\}, W) - \phi_j(N \backslash \{i\}, W) = 0$ to every $j \neq i$ and so it is implicitly taken into account in what follows. Notice also that the bids made by the proposer are a sunk cost at this stage and so do not enter the analysis at this point.

To show that any SPE yields the Shapley value, denote the "effective offer" to player $i \neq \alpha$ in stage 2 when player α is the proposer by z_i^α:

$$z_i^\alpha = \begin{cases} x_i^\alpha + u_i(y_\alpha, S_\alpha) & \text{if } i \in S_\alpha \backslash \{\alpha\}\ , \\ x_i^\alpha + \phi_i(N \backslash S_\alpha, W) & \text{if } i \notin S_\alpha. \end{cases}$$

By the induction argument, the effective offer is the total utility (without taking into account the bid already received) that a player will obtain (at equilibrium) if the offer is accepted. We proceed by a series of claims. We state the claims without proof, since they are similar to those in Theorems 1 and 3 of PC&W.

Claim (a). In any SPE, any player $j \neq \alpha$ accepts the offer at $t = 3$ if $z_i^\alpha > \phi_i(N \backslash S_\alpha, W)$ for every $i \neq \alpha$. If $z_i^\alpha < \phi_i(N \backslash S_\alpha, W)$ for some $i \neq \alpha$, then the offer is rejected.

Claim (b). In any SPE of the game that starts at $t = 2$, α will choose a coalition S_α that is part of an efficient partition. Player α will announce offers such that $z_i^\alpha = \phi_i(N \backslash \alpha, W)$ for all $i \neq \alpha$. Finally, at $t = 3$, every player $i \neq \alpha$ accepts any offer such that $z_i^\alpha \geq \phi_i(N \backslash \alpha, W)$.[4] The final payoffs to players α and i are $W(N) - W(N \backslash \{\alpha\}) - \sum_{j \neq \alpha} b_j^\alpha$ and $\phi_i(N \backslash \{\alpha\}, W) + b_i^\alpha$ respectively.

Claim (c). In any SPE, $B^i = 0$ for all $i \in N$.

Claim (d). In any SPE, each player's payoff is the same regardless of who is chosen as the proposer.

Claim (e). In any SPE, the final payment received by player i is $\phi_i(N, W)$.

These claims complete the proof of the theorem. ■

Bagnoli and Lipman [3], Jackson and Moulin [10], Bag and Winter [2], and Mutuswami and Winter [15] also proposed mechanisms that realize efficient outcomes in environments with (local) public goods. We now discuss the advantages and distinguishing features of the bidding mechanism.

The first distinguishing feature of our mechanism is that it does not need a planner. The players do not submit messages to a planner who then implements the final outcome. In the bidding mechanism, the messages and offers from a player are made directly to the other players. The agents can play the mechanism by themselves. The only role that a third party might play is to act as a court in case some player does not fulfill his commitments. This is an easy task here because the actions by the players are just contracts (offers) which are easy to verify.

A second advantage of the proposed mechanism is that in contrast to the mechanisms suggested in the previous contributions, it can handle environments with local public goods. It allows the participating agents the

[4]To be rigorous, if $\{\alpha\}$ is part of any efficient partition, then there exist other equilibria in addition to the previous ones. Any strategy profile such that at $t = 2$, α makes offers such that $z_j^\alpha \leq \phi_j(N \backslash \alpha, W)$ to a particular player $j \neq \alpha$ and at $t = 3$, player j rejects any effective offer less than or equal to $\phi_j(N \backslash \alpha, W)$ also constitutes an SPE.

possibility of splitting up into smaller coalitions. The outcome generated by the mechanism specifies not only the production/consumption plan the individuals will follow but also the coalition structure that will prevail. Since the outcome is efficient, coalitions other than the grand coalition can form in equilibrium.

Third, the equilibria of bidding mechanism give rise to efficient outcomes even when the utility of a player depends on the identity of the partners he is with[5] and the costs of production the public good different for different coalitions. The bidding mechanism thus obtains efficiency in a larger class of environments than previous contributions.

The fourth feature is that equilibrium payoffs received by the agents in our mechanism coincide with the Shapley value of the super-additive cover of the corresponding cooperative game. It is important to point out that, in this value, the total contributions made by the agents belonging to a coalition do not necessarily match with the cost of the public good produced by this coalition. In our mechanism therefore, there can be cross subsidies across coalitions. Due to the possibility of cross subsidies, the value of a player is a measure of his strategic possibilities not only inside the coalition to which he ends up belonging, but also with agents outside this coalition.

In the two-stage mechanisms proposed by Bag and Winter [2] and Mutuswami and Winter [15], the equilibrium payoffs coincide with the Shapley values of the original cooperative game (not its super-additive cover) for some environments.[6] We think that in environments where forming the grand coalition is not efficient, it is the super-additive cover which is the relevant measure of social surplus and consequently, in looking for efficient and equitable outcomes, it is the Shapley value of the super-additive cover which is relevant. Moreover, in these papers, the Shapley value can be obtained as *actual* payoffs only by assuming that each agent prefers the game ending in Stage 1 (which gives the Shapley value as actual payoffs) to ending in Stage 2 (which gives the value as expected payoffs). No such assumption is required here: we always obtain the Shapley value as the actual payoffs.

Finally, it is worthwhile to remark that the equilibrium strategies in the bidding mechanism are basically unique, eliminating the problem of coordination that exists when strategies are not unique.

[5]See the papers by Banerjee, Konishi and Sonmez [5] and Bogomolnaia and Jackson [6] for an analysis of "pure hedonic coalitions." In these papers, the players have (ordinal) preferences over the coalitions in which they are members, and the objective is to find conditions on preferences under which there exist a "stable partition" of players.

[6]Of course, the two solution concepts coincide if the game is super-additive. We observe that in the environment analyzed by Bag and Winter [2], the corresponding cooperative game is *convex* which is, of course, stronger than super-additivity.

3 Forming networks

Let $N = \{1, \ldots, n\}$ be the set of agents. For any $S \subseteq N$, let g^S denote the set of all subsets of S of size 2. A graph or network, denoted generically by g is some subset of g^N. If $g \subset g^S$ where $S \subsetneq N$, then we shall say that g is a graph restricted to S. A graph, therefore, is a structure of *bilateral relations* among agents. Clearly, agents i and j have a bilateral relation only if $\{i, j\} \in g$. We shall refer to the subset $\{i, j\}$ of g as the *link* between i and j and denote it as (ij).[7] We let G_S denote the set of all graphs involving links only between members of S: $g \in G_S$ and $(ij) \in g$ implies that $\{i, j\} \subset S$.

We need the following graph-theoretic terminology for what follows. Fix a graph g. Players i and j are said to be *connected in* g if there exists a sequence of agents $i = i_0, i_1, \ldots, i_K = j$ such that $(i_k i_{k+1}) \in g$ for all $k = 0, \ldots, K-1$. Let $N(g) \equiv \{i|$ There exists j such that $(ij) \in g\}$ denote the set of agents who have at least one bilateral relation. The graph $h \subset g$ is said to be a *connected component* of g if all agents in $N(h)$ are connected to each other in h, and for all $i \in N(h), j \in N \backslash N(h), (ij) \notin g$. The set of all connected components of g is denoted $C(g)$.

A value function is a mapping $v : G_N \to \Re$. We can think of the value of a graph g as representing the total surplus produced by agents when they form a set of bilateral relationships represented by g. We will restrict attention to value functions satisfying *component additivity*, that is, $v(g) = \sum_{h \in C(g)} v(h)$. Component additivity can be interpreted as absence of externalities between different components. We let V denote the set of component additive value functions. Given $v \in V$, a graph g is *strongly efficient* if $v(g) \geq v(g')$ for all $g' \in G_N$. An allocation rule is a mapping $Y : V \times G \to \Re^n$ satisfying $\sum_{i \in N} Y_i(v, g) = v(g)$. An allocation rule simply specifies the division of total surplus for each possible graph. An allocation rule Y is *component balanced* if $\sum_{i \in N(h)} Y_i(v, g) = v(h)$ for every $h \in C(g)$.

An example of an allocation rule is the one proposed by Jackson and Wolinsky [11] which associates to each graph, the Shapley value of a transferable utility game associated with the graph. Formally, fix v. For any graph g and $S \subseteq N$, let $g|S \equiv \{(ij)|(ij) \in g \text{ and } \{i, j\} \subseteq S\}$ denote the restriction of g to S. Define the transferable utility game (N, w_g) by $w_g(S) = \sum_{h \in C(g|S)} v(h)$ for all $S \subset N$. The Jackson-Wolinsky allocation rule for any graph g is the Shapley value of (N, w_g).[8] Jackson and Wolinsky [11], extending an earlier result of Myerson [16], show that this is the unique allocation rule satisfying component balance and *equal bargaining power*.[9]

[7] Our emphasis on bilateral relationships means that the links in our framework are *non-directed*. We say more on the applicability of this mechanism to situations involving directed links later.

[8] This value is also referred to in the literature as the Myerson value.

[9] The allocation rule Y satisfies equal bargaining power if $Y_i(v, g) - Y_i(v, g - (ij)) =$

They also note that this allocation rule may arise naturally if the allocations result from bargaining between players. However, this bargaining is not modeled explicitly.

In the game (N, w_g), the worth of coalition S is the surplus generated by looking at the restriction of g to S: in other words, the graph $g|S$. This procedure, however, does not take into account the fact that the agents in S can form many other graphs besides $g|S$ and ideally one would like to take this into consideration.[10] However, the way to do this is not clear for an arbitrary graph g. Suppose however that we restrict attention to graphs which are strongly efficient. In this case, a natural possibility is to associate to each coalition S the *maximum surplus that can be derived by the members of S acting on their own*. One can now consider the transferable utility game (N, W) defined by $W(S) = \max\{v(g)|g \in G_S\}$ for all $S \subset N$ and the corresponding Shapley value. The game (N, W) can be easily seen to be super-additive.

It is easy to see that the restricted graph $g|S$ is not necessarily the graph that maximizes the surplus for S even if g itself is strongly efficient. The two approaches outlined above are thus bound to give different results. The following example illustrates this possibility.

Example 1 Consider the following value function taken from Jackson and Wolinsky [11]. Let $N = \{1, 2, 3\}$, and the component-additive value function given by $v((ij)) = v(g^N) = 1$ and, for $i \neq j, i \neq k, j \neq k$, $v((ij), (jk)) = 1+\epsilon$ where $0 < \epsilon < 1/6$. The strongly efficient networks are of the form $g^j = \{(ij), (jk)\}$. The Jackson-Wolinsky procedure applied to any g^j gives the allocation $(x_i, x_j, x_k) = ((1 + 2\epsilon)/6, (2 + \epsilon)/3, (1 + 2\epsilon)/6)$ while the Shapley value of (N, W) gives the uniform payoff vector $((1+\epsilon)/3, (1+\epsilon)/3, (1+\epsilon)/3)$. This example also illustrates that, in contrast to the Jackson-Wolinsky rule, the allocation rule proposed here need not be component balanced. We view this as a consequence of having to take into account the strategic possibilities open to an agent *outside* the component to which he belongs.

The *bidding mechanism* that we propose for the network environment can be considered as a model of network formation in which a bargaining process is modeled explicitly. In this mechanism, connected components are formed sequentially. Formally, the bidding mechanism for network formation operates as follows:

$Y_j(v, g) - Y_j(v, g - (ij))$ where $g - (ij)$ is the graph obtained by removing the link (ij) from g.

[10] Typically, the literature on social and economic networks assumes that agents have the right to decide which links they want to form. See for instance, the papers of Jackson and Wolinsky [11], Currarini and Morelli [7] or Dutta and Mutuswami [9]. In this context, it is thus necessary that the complete strategic possibilities open to an agent be considered.

If there is only one player i (say), he can only form the empty graph, $g = \emptyset$ and therefore, he obtains $v(\emptyset) = 0$.[11]

Given the rules for at most $n-1$ agents, the mechanism for $N = \{1, \ldots, n\}$ works as follows:

$t = 1$: Same as in the mechanism constructed in Section 2.

$t = 2$: Player α chooses a subset of players S_α (such that $\alpha \in S_\alpha$), a graph $g^*_\alpha \in G_{S_\alpha}$ (such that g^*_α is connected on S_α) and offers $x^\alpha_i \in \Re$ to every $i \neq \alpha$.

$t = 3$: The players other than α, sequentially, either accept or reject the offer. If an agent rejects it, then the offer is rejected. Otherwise, the offer is accepted.

If the offer is accepted, then the final payoff to player $i \in S_\alpha \backslash \{\alpha\}$ is $x^\alpha_i + b^\alpha_i$, player α receives $v(g^*_\alpha) - \sum_{i \neq \alpha} x^\alpha_i - \sum_{i \neq \alpha} b^\alpha_i$ and players in $N \backslash S_\alpha$ receive $x^\alpha_i + b^\alpha_i$ plus what they obtain in the game played by $N \backslash S_\alpha$. If the offer is rejected, the final payoff to α is $-\sum_{i \neq \alpha} b^\alpha_i$ and final payoff to any $i \neq \alpha$ is the sum of b^α_i and the payoff obtained in the game played by $N \backslash \{\alpha\}$.

Theorem 2 *At any subgame perfect equilibrium of the bidding mechanism, a strongly efficient graph is always proposed and formed. Moreover, the payoffs to the agents are uniquely given by the Shapley value of the game* (N, W).

Proof:. The proof is essentially identical to that of Theorem 1, and is thus, omitted here. ■

Remark 1 In principle, one can apply the bidding mechanism to situations involving directed links. However, we need an additional assumption: namely, that establishing links needs the permission of both concerned parties. This assumption may be valid in some circumstances. For instance, a telephonic connection (directed link) may be initiated by one party, but it requires the cooperation of both to carry forth a conversation. A similar consideration, we think, is valid with e-mail. Equally, there may be situations where links can be established unilaterally – in such cases, our mechanism is not valid because a player whose proposal is rejected may reenter by unilaterally establishing links. Bala and Goyal [4] and Dutta and Jackson [8] both consider network models involving directed links.

A recent literature on social and economic networks, stemming from the paper of Jackson and Wolinsky [11] has also focused on the problem of generating efficient networks. It tries to resolve the tension between efficiency

[11] Component additivity implies that the value of an isolated player (and therefore, the empty graph) is zero.

and stability of networks.[12] Jackson and Wolinsky [11] showed that if the allocation rule is required to satisfy both anonymity and component balancedness, then it is not possible to reconcile this tension. In a subsequent paper, Dutta and Mutuswami [9] showed that this tension can be resolved by using a *mechanism design* approach. Their formulation assumes the presence of a social planner who can decide the allocation rule to be followed but cannot compel the agents to form the links that she desires. Dutta and Mutuswami [9] showed that it is possible to construct an allocation rule which, if proposed by the planner, will ensure that agents (acting in their own self-interest) form an efficient network.[13]

In another paper, Currarini and Morelli [7] examine this problem when there is no social planner. They analyze two sequential-move games.[14] In both games, an agent's strategy consists of two components. The first component specifiying the set of agents with whom the agent wants to form links is common to both games. The second component is a payoff demand and can be either a single absolute payoff demand or a vector of demands, one for each proposed link. Thus, the payoff division in Currarini and Morelli [7] is endogenous, in contrast to the analysis is Jackson and Wolinsky [11] and Dutta and Mutuswami [9]. They show that all subgame perfect equilibria of their games give rise to efficient networks. However, the payoff division is highly asymmetric being sensitive to the order in which agents move.

Our proposal is in the same line of research as Currarini and Morelli [7]. The advantages of our mechanism with respect to theirs are, first, that the payoff division are equitable corresponding to the Shapley value of a game which takes into account the various network options available to the agents. Second, our result holds for *all* component additive value functions while their result holds only for anonymous value functions satisfying size monotonicity. Finally, our mechanism is simpler in that it does not require out-of-equilibrium free disposal.

[12] Stability of a network can be understood as meaning that there does not exist a deviation for some group of agents which makes all deviating members better off. Jackson and Wolinsky [11] restrict deviations to coalitions of size 2 while Dutta and Mutuswami [9] allow for deviations by all coalitions. The reader is referred to their papers for details.

[13] Mutuswami and Winter [14] analyze a model of network formation in an explicitly mechanism design setting. They present mechanisms which implement outcomes identical to those of the bidding mechanism. However, their setting is different from ours. Our setting is identical to that of Jackson and Wolinsky [11] and used among others by Currarini and Morelli [7] and Dutta and Mutuswami [9]. This setting, in contrast to that of Mutuswami and Winter [14] is not one of mechanism design because there is no informational asymmetry between the agents and the planner.

[14] Aumann and Myerson [1] also analyze a network formation model with sequential moves. The main difference between their model and that of Currarini and Morelli [7] is that in their work, the payoff division is exogenous, being given by the Myerson value.

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