

Competition between Conglomerate Firms in a Multimarket Oligopoly

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Abstract: The paper provides a static analysis of multimarket competition trying to extend classical models of oligopolistic competition including a multimarket effect in firms' decision problem. After a short definition of what are multimarket oligopolies, we define a multimarket effect as a relation between cross market variables that can be internalised by firms. In case of interrelated costs this will be seen as a sort of externality linked to joint production economies, while in case of independent costs and demands it is modelled as an expected rival cross market reaction. In both cases it modifies competitors' optimal behaviour.

1. Introduction*

In the last twenty years many works have pointed out that diversification can be an optimal choice for a firm either for exploiting economies of scope, or as a strategic pre-emptive weapon against potential rivals. Among others Schmalensee (1978), Scherer (1979) and Judd (1985) show that brand proliferation can effectively crowd out potential competitors. Kreps and Wilson (1982) and Milgrom and Roberts (1982) argue, from a different perspective, that multimarket firms may be able to deter entry in market X by developing a reputation for being aggressive in market Y. In a different vein operating in many markets can be a profit maximising strategy where there are excess resources to be employed (Cairns and Mahabir (1988)), if resources have a public good character (i.e. consumer goodwill and managerial skills) or, simply, if some joint production economies can be exploited.

In all these cases the existence of a multimarket spillover, seen as a kind of externality between two or more markets, can raise extra profits or losses and can modify firm's optimal behaviour with respect to a situation in which she operates only in one market (or in many unrelated markets).

In particular, a relevant change in the competitive game played by firms occurs whenever two of them face simultaneously each other into more than one market. If they recognise their mutual interdependence in several markets, they may decide to facilitate collusion moving toward cooperative outcomes (Bernheim and Whinston (1990)). The higher is the degree of market concentration, in a framework of multimarket competition, the higher will be the degree of collusion implemented by players (Scott (1982)).

It seems then reasonable to model competition in presence of a *multimarket effect*, trying to cover an existent hiatus in the economic literature. On one hand in fact economic analysis has used repeated game models in order to address the issue, first stressed by Edwards (1955), that multimarket contacts may affect firms' ability to sustain cooperative outcomes; on the other hand some contributions have tried to depict qualitatively the effect on economic behaviour of multimarket competition (Van Witteloostuijn (1993), Van Witteloostuijn and Van Wegber (1992)) using a strategic management approach where a multimarket reaction is seen as business strategy in a complex environment. As we will see, few contributions have tried to modify existent static models of competition taking formally into account a multimarket effect.

The paper is organised as follows: section 2 gives account of such models and states some preliminary definitions and distinctions. In section 3 a formal model of oligopolistic competition with

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a multimarket effect is presented extending the seminal contribution of Bulow, Geneakopulos and Klemperer (1985). We will try to define possible strategic links between strategic variables, referred to different markets, which can modify firm's optimal conduct in front of an higher rival's aggressivity. As we will see, the presence of a multimarket effect in case of interrelated costs can influence oligopolistic firms' behaviour by affecting its reaction curves. In section 4 we present a different way to see a multimarket effect, now not considered a sort of external effect linked to costs but an expectation about rival's behaviour on the parallel market. Applying this idea to classical quantity and price competition models it is possible to provide a very simple analysis of its effect on equilibrium. Section 5 summarises our central conclusions.

2. Oligopolistic Multimarket Competition: Some Preliminary Definitions

A multimarket effect can be seen as a sort of externality among markets that relates a firm conduct on market X with her possible actions and rival's replies on market Y. In principle it is possible to model this linkage indirectly, using interdependent demands for products, or directly supposing a functional relation between strategic variables.

Loomis (1997) analyses price competition between two firms which face each other in two markets. Demands for products are dependent and a price change in one market will modify the quantity sold in that market and consequently other market demands. Firms' optimal decisions will take into account this relation and equilibrium outcomes will be dependent on the degree of interdependence between demands and strategic complementarity of firms' variables.

In what follows we are not going to use interdependent demands but we will concentrate on the possible interconnections between each player variables in any oligopolistic market where they operate. Demand functions are then independent (i.e. there are no technical relation between goods) and each firm sells products on two markets in which it faces the same competitor.

The two markets are not vertically related and neither they share common regulatory schemes as in Phillips and Mason (1996) or in Cowan (1997). They are seen as parallel markets in which each firm has decided to enter in order to diversify her bundle of supplied goods. More precisely,

Definition 1: A parallel oligopoly is a set of markets formed by two, or more, separate markets where each firm operates using common intangible assets, know how, managerial skills etc...

This particular characterisation of market structure and the following analysis fit well for particular sectors or firm activities: local public utilities competing in border zones in fact produce a bundle of services (gas, water etc...) each of them related to a certain market with an independent demand as well as big international firms actually compete not only in core business market, but also

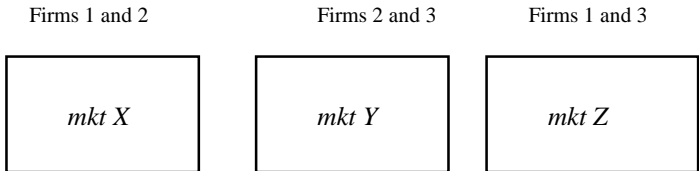
in new markets quite far away from their original activity (i.e. banks, telecommunication firms etc.). Independently by the examples that can be found, there is a general tendency observed in the world economy in the last years: the increasing number of mergers and acquisitions has created (or is creating) many big conglomerate firms with a complex organisational structure which operate in diversified markets with a quite high level of concentration. Some subsets of these markets can be seen as separated, strategically related and hence quite well described by our definition.

Finally we can say that

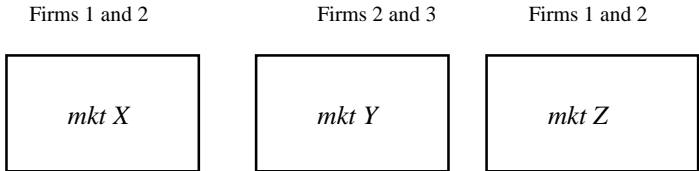
Definition 2: A parallel oligopoly is a *multimarket oligopoly* if:

- (i) a cross market relation between strategic variables is recognised by the competitors
- (ii) in all markets operate the same set of players

In the next section we'll refer to this idea of oligopoly; the following figure gives a quick intuition of the last definition.



a) The set $\{X, Y, Z\}$ is a parallel but not a multimarket oligopoly



b) $\{X, Y\}$ and $\{Y, Z\}$ are not multimarket oligopolies. $\{X, Z\}$ is a multimarket oligopoly

Figure 1

3. Interrelated Costs and Multimarket Effect as an Externality

Following Bulow et al. (1985), suppose a conglomerate firm A that operates into a monopolistic market (mkt. 1) and two oligopolies (mkts. 2 and 3) where she faces the same competitor (firm B). Each firm chooses strategic variables S^k with $k = A, B$ for each market in which it is active and both firms have rational expectations in the sense that perfectly anticipate the effect of their decisions on profits. Assume that a higher level chosen for strategic variables indicates a more aggressive play; the interpretation of this assumption is quite straightforward in the case of quantities and advertising, while in the case of prices an aggressive play must coincide with the inverse of prices charged.

Without loss of generality we can assume that $S_1^A = q_1^A$ because as a monopolist in market one, firm A can select a certain price level choosing one point on the demand function. A shock variable Z affects the profitability of market 1: a one unit increase (decrease) of Z shifts firm A's marginal revenue upward (downward) by one unit or, equivalently, his marginal cost downward (upward). Finally demands for each market are supposed to be independent.

In any oligopolistic market (2 and 3) each firm revenues depend on the value of his own strategic variables and rivals' ones on that market. Furthermore the conduct of one firm in a market is assumed to be influenced by rival's behaviour in the other one. In this case a line of action followed by a firm in one of the two oligopolies changes what a firm can do in the other in two ways: first modifying its set of possible actions given its costs and some other possible constraints (*indirect effect of first type*), secondly influencing rival's reactions (*indirect effect of second type*). These markets are then a multimarket oligopoly and they are linked by a multimarket effect. More precisely:

Definition 3: A Multimarket Effect is as a relation between a firm strategic variable in market i and its rival's one in market j , that is¹

$$S_i^k = S_i^k(S_j^{-k}) \text{ for } i = 1, 2 \quad i \neq j \text{ and } k = A, B$$

Each firm faces a two step decision problem: in a first stage of the market game it has to decide whether taking into account this existent effect or not and hence behaving as if it operates in two distinct markets; then decisions on strategic variables are taken. What it is going to decide in

¹ In fact we are using for tractability a one degree multimarket effect breaking the possible chain of relations among strategic variables at the first step.

period $t-1$ will influence its behaviour in period t , then a new decision, whether or not internalise a multimarket effect, must be taken².

Profit functions of firms A and B, assumed continuous and twice continuously differentiable, if they take into account (*internalise*) such a multimarket effect are then given by the following expressions:

$$\begin{aligned}\Pi^A &= R_1^A(S_1^A) + \sum_{\substack{i=2 \\ i \neq j}}^3 R_i^A[S_i^A(S_j^B); S_i^B(S_j^A)] - C^A(S_1^A; S_i^A(S_j^B); S_j^A(S_i^B)) + ZS_1^A \\ \Pi^B &= \sum_{\substack{i=2 \\ i \neq j}}^3 R_i^B[S_i^B(S_j^A); S_i^A(S_j^B)] - C^B(S_i^B(S_j^A); S_j^B(S_i^A));\end{aligned}\tag{1}$$

The revenue in each market is perceived and then assumed to be dependent only on that market decisions while production is managed jointly for all markets, given some internal division of common costs. In firm A's profit obviously there is a term (ZS_1^A) that expresses the effect of a change in Z on Π_A . Assuming differentiability then there are first order conditions (focs) that must hold at an interior Nash equilibrium, these are given by

$$\begin{aligned}\frac{\partial \pi^A}{\partial S_1^A} &= \frac{\partial R_1^A}{\partial S_1^A} - \frac{\partial C^A}{\partial S_1^A} + Z = 0 \\ \frac{\partial \pi^A}{\partial S_i^A} &= \frac{\partial R_i^A}{\partial S_i^A} + \frac{\partial R_i^A}{\partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} - \frac{\partial C^A}{\partial S_i^A} = 0 \\ \frac{\partial \pi^B}{\partial S_i^B} &= \frac{\partial R_i^B}{\partial S_i^B} + \frac{\partial R_i^B}{\partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} - \frac{\partial C^B}{\partial S_i^B} = 0\end{aligned}\tag{2}$$

for $i = 2, 3$ and $i \neq j$

In order to analyse the effects of a shock that makes market 1 marginally more profitable for firm A we can compute the total differential of the focs³. In doing this we will assume that each firm looks at each market separately taking into account the existence of a multimarket effect related to his choice of S_i and her marginal impact on total costs. There are two class of reasons for this assumption: first a higher level of tractability, second the effective managerial organisation of

² In fact in a static framework we have a *one shot market game*, Bernheim and Whinston (1990) proposes a repeated game analysis of multimarket competition.

³ A different argument but similarly based on second order effects on profits of different firm's behaviour is given by Fudenberg and Tirole (1984).

multimarket firms⁴. It actually coincides with assuming a multidivisional organisation of a big conglomerated firm where each division manages, in a fully decentralised way, her activities in one market. For any division in isolation only direct and indirect effects of possible changes of his own strategic variables related to his market are relevant and what is going on in related oligopolies is taken as given; more shortly

Assumption 1 (Separability): Each competitor separates markets in which it operates considering parallel market strategic variables as exogenously fixed, that is for $i=2,3$ $dS_{-i}^k = 0$ with $k=A,B$

In the case of the monopolistic market for firm A, we can imagine that a change in S_1^A modifies the focus of each oligopolistic market and that an higher or lower profitability in that market can change profit maximising choices in market 1. In other words the conduct of the division that manages market 1 can separately affect strategic choices of each multimarket oligopolist (or more precisely of each division that manages one of the two parallel oligopolies). With respect to any separate market and for both competitors the total differential of profit maximising conditions is given by

$$\begin{aligned}
& \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_1^A} dS_1^A + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_i^A} dS_i^A + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} dS_i^A + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_i^B} dS_i^B + \\
& + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} dS_i^B + \frac{\partial^2 \pi^A}{\partial S_1^A \partial Z} dZ = 0 \\
& \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_1^A} dS_1^A + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_i^A} dS_i^A + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} dS_i^A + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_i^B} dS_i^B + \\
& + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} dS_i^B + \frac{\partial^2 \pi^A}{\partial S_i^A \partial Z} dZ = 0 \tag{3} \\
& \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^A} dS_i^A + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} dS_i^A + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^B} dS_i^B + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} dS_i^B = 0 \\
& \text{for } i = 2,3 \text{ and } i \neq j
\end{aligned}$$

these can be written, as shown by Bulow et al. (1985), as

⁴ In this context we are assuming a divisional organisation; many contributions have studied a decentralised or centralised organization as a profit maximizing firm's choice. See among others Bárcena Ruiz and Espinosa (1996) and (1999).

$$\begin{pmatrix} \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_1^A}; \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_1^A} + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A}; \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_i^B} + \frac{\partial^2 \pi^A}{\partial S_1^A \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} \\ \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_1^A}; \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_i^A} + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A}; \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_i^B} + \frac{\partial^2 \pi^A}{\partial S_i^A \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} \\ 0 \quad ; \quad \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^A} + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A}; \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^B} + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B} \end{pmatrix} \begin{pmatrix} dS_1^A \\ dS_i^A \\ dS_i^B \end{pmatrix} = \begin{pmatrix} -dZ \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

We assume that the equilibrium is locally strictly stable which implies that the determinant of the matrix in (4) is negative and that a more aggressive play of one firm reduces rival's profit

(*i.e.* $\frac{\partial \Pi^k}{\partial S_i^{-k}} < 0$ for $i = 2, 3$ and $k = A, B$). For each division of any firm we can traditionally

have joint economies (diseconomies) of scope between her markets

(*i.e.* $\frac{\partial^2 \Pi^k}{\partial S_i^k \partial S_{-i}^k} = -\frac{\partial^2 C^k}{\partial S_i^k \partial S_{-i}^k} > (<) 0$). In the first case profit maximisation suggests that an

increase in both variables can be optimal (being more aggressive in one market augments the marginal profit from being more aggressive in the other market), conversely in the second one (being less aggressive in one market augments the marginal profit from being more aggressive in the other).

Solving (4) it is possible to determine that, exactly as in Bulow et al. (1985), $\frac{dS_1^A}{dZ} > 0$: a positive

shock in the monopolistic market implies a more aggressive play of firm A in that market in order to obtain higher profits.

More difficult is to evaluate the effect of a shock on the behaviour of each firm division into his oligopoly. In fact we have that

$$\begin{aligned} \text{sign}\left(\frac{dS_i^A}{dZ}\right) &= \text{sign}\left[\left(\frac{\partial^2 \pi^A}{\partial S_1^A \partial S_i^A}\right) \left(1 + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B}\right)\right] \\ \text{sign}\left(\frac{dS_i^B}{dZ}\right) &= \text{sign}\left[\left(\frac{\partial^2 \pi^A}{\partial S_1^A \partial S_i^A}\right) \left(\frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^A} + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A}\right)\right] \end{aligned} \quad (5)$$

where the first term in the right hand square brackets is the joint economies effect (positive or negative) while $\frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^A}$ represents the change in market i of firm B's marginal profitability due to a

more aggressive conduct in front of an aggressive rival in that market. If we have strategic

complementarity between S_i^B and S_i^A this term will be positive whereas with strategic substitutes it will be negative.

Nevertheless in order to determine above expressions sign we have to discuss the remaining terms related with the existence of a multimarket effect. Let me start by defining a basic relation between cross market strategic variables that specifies how a multimarket effect can be internalised:

Definition 5: A strategic variable S_i^k is a *cross substitute (complement)* to S_j^{-k} if $\frac{\partial S_j^{-k}}{\partial S_i^k} < 0$ (> 0)

Each division of the two firms perfectly knows the kind of strategic linkage between its conduct and rival's reaction of the parallel market, that is equivalent to suppose a perfectly rational expectation on existing cross market relations between strategic variables. After a shock Z it will react modifying S_i^k and this will affect equilibrium outcomes on both markets; in other words internalising a multimarket effect, indirect effects of a certain decision on the other market are taken into account on the basis of the expected cross markets reaction of the rival. For instance an aggressive play by a firm in one market can be followed by an aggressive reply of the rival in the parallel oligopoly that induces there a profit reduction for the former. In that case a firm may prefer not to be aggressive in order to induce a more accommodating conduct of the rival and an increase of her profits. The relation between firms' strategic variables and profit can be stated as follows:

Definition 6: Strategic variables S_i^k and S_{-i}^{-k} for $i = 2,3$ and $k = A, B$ are *cross strategic substitutes (complements)* if

$$\frac{\partial}{\partial S_{-i}^{-k}} \left(\frac{\partial \Pi_i^k}{\partial S_i^k} \right) < 0 \text{ (} > 0 \text{)}$$

Thus with cross strategic substitutes (resp. complements) firm k 's best reply in market i to an aggressive play of the rival in market $-i$ is to be less (resp. more) aggressive in order to maximise profit. As quite usual, each firm division will be concerned with the effect of his choices on marginal profitability of his market and, given rival's behaviour and the kind of existing multimarket effect, on what can happen in the parallel oligopoly.

Given that taking in account a certain multimarket relation instead of simply ignoring it during profit maximising decisions is a firm's rational choice, it is obvious that

Remark 1: A multimarket effect is internalised only if it is profitable

Thus profit maximisation purposes allow us to state that

Remark 2: For an internalised multimarket effect, cross substitutes (complements) are cross strategic (complements) substitutes

Again the intuition is immediate: if two variables are internalised as cross substitutes (complements) this must be a profit maximisation consistent decision and hence an increase in S_i^k in front of a rival more aggressive play must augment firm k ' profits⁵.

From this framework we can arrive to some predictions on possible firms reactions to a shock Z ; proposition 1 is referred to market 1 monopolist

Proposition 1: For any internalised multimarket effect by firm A her profit maximising reaction to a shock depends on whether or not each oligopolistic market exhibits joint economies

Proof: For any internalised cross market relation between strategic variables S_i^B and S_j^A we will

always have that $\left(1 + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^A} \frac{\partial S_j^A}{\partial S_i^B}\right) > 0$ and then from (5) we have that

$sign\left(\frac{dS_i^A}{dZ}\right) = sign\left[\left(\frac{\partial^2 \pi^A}{\partial S_i^A \partial S_i^A}\right)\right]$ where the right hand term is what we have called joint economies or

diseconomies.

This result is perfectly consistent with that obtained by Bulow et al. (1985) with only two markets. We know that, with a positive shock, in equilibrium firm A will sell more in market 1, this will lead A to be more or less aggressive in each parallel oligopoly whether or not there are joint economies related to that market.

Differently, for firm B a reaction to a shock will depend on several terms; in particular in front of a more aggressive play of A $\left(\frac{dS_i^A}{dZ} > 0\right)$ ⁶ we will have that

⁵ In fact if we drop the assumption that firms have rational expectations it is possible that a wrong decision will be taken. Nevertheless it seems to me reasonable to depict as perfectly rational big conglomerate firms.

⁶ The opposite case is symmetric so we omit it. For a description of the symmetric case see Lanzi (1999).

$$\text{sign}\left(\frac{dS_i^B}{dZ}\right) = \text{sign}\left[\left(\frac{\partial^2 \pi^B}{\partial S_i^B \partial S_i^A} + \frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A}\right)\right] \quad (6)$$

where the first term in the right hand brackets indicates if variables for market i are strategic complements or substitutes while the second one is our multimarket effect linked with a term related to joint economies or diseconomies.

Proposition 2: Firm B, that has internalised a certain multimarket effect, reacts aggressively in market i to a shock and to an aggressive play of the rival in that market if and only if (i) market i variables are strategic complements or (ii) market i variables are strategic substitutes but the multimarket effect on profit is more than compensative.

Proof:

Case (i): Suppose that market i variables are strategic complements then $\frac{\partial^2 \Pi^B}{\partial S_i^A \partial S_i^B} > 0$. For an

internalised relation between cross market variables we have that $\frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} > 0$. This term is the

second order effect on profit of a certain multimarket reaction: with joint economies (diseconomies) firm B will increase (reduce) S_j enhancing the marginal profitability of being more aggressive in

market i . Hence expression (6)'s right hand brackets have positive sign and $\frac{dS_i^B}{dZ} > 0$. The

multimarket effect will lead to a more aggressive reaction compared to the one in case of single market competition.

Case (ii): In opposition to the previous case suppose that $\frac{\partial^2 \Pi^B}{\partial S_i^A \partial S_i^B} < 0$, we will have $\frac{dS_i^B}{dZ} > 0$ also

when $\frac{\partial^2 \pi^B}{\partial S_i^B \partial S_j^B} \frac{\partial S_j^B}{\partial S_i^A} > 0 > \frac{\partial^2 \Pi^B}{\partial S_i^A \partial S_i^B}$. That is a marginal increase in profit induced by an internalised

multimarket effect that more than compensate a marginal reduction in profit due to a reverse conduct with respect to what suggested by best responses referred to market i . As above, with joint economies

(diseconomies) firm B will increase (reduce) S_j enhancing marginal profits obtained from being a little more aggressive in the other market. If this effect is not too high, this simply means a smaller

reduction of S_i^B but for a sufficiently high second order effect on profit the optimal conduct of the firm can be reversed.

The main point of last proposition is to show how a sufficiently strong multimarket effect can modify a firm's optimal reaction; if market i variables are strategic complements it simply increases how a player will be aggressive, but for strategic substitutes it can suggest an aggressive reply even if this would not be optimal under single market competition. This effect will be due to interrelated costs and it will be not ignored by firms if it is profit maximising consistent.

Proposition 2 can be immediately applied to more specific frameworks. If we consider linear demands and increasing marginal costs it is well known that prices can be seen as strategic complements and quantities as strategic substitutes within a certain market. Therefore in case of price competition the existence of a multimarket effect will not modify the sign of firm's reaction function slope only increasing his value. It is then obviously proved the following corollary

Corollary 1: With linear demands and interrelated costs an internalised multimarket effect under price competition will increase firms' reaction function slope

More interesting is the case of quantity competition: taking again linear demands of the form

$$p_l = a_l - b_l \left(\sum_k q_l^k \right) \text{ for } l = i, j \quad (7)$$

interrelated costs which exhibit joint economies or diseconomies⁷

$$TC^k = c \left(\sum_l q_l^k \right)^\alpha \text{ for } k = A, B \text{ and } \alpha \in R_+ \setminus \{1\} \quad (8)$$

and assuming for simplicity that a multimarket effect has the following form $\frac{\partial q_{-i}^{-k}}{\partial q_i^k} = m$ with m negative or positive; we can show that

Corollary 2: With linear demands and interrelated costs an internalised multimarket effect under quantity competition reverses firms' reaction function slope if produced quantity is larger than a threshold Q^ .*

⁷ It can be easily checked that this cost function exhibits joint economies (diseconomies) when $\alpha < (>)1$

Proof: Writing profits as in expression (1) with specific assumptions about the economy given by (7)

and (8), we have that $\frac{\partial^2 \Pi^k}{\partial q_i^k \partial q_i^{-k}} = -b_i - \alpha(\alpha - 1)Q_k^{\alpha-2}m$. In the case of joint economies

(diseconomies) $(\alpha - 1) < (>)0$, $m > (<)0$ and then in both cases $-\alpha(\alpha - 1)m = \vartheta > 0$. Firm k 's

reaction function will be then increasing when $Q^k > Q^* = \left(\frac{b_i}{\vartheta}\right)^{\frac{1}{\alpha-2}}$, otherwise it is traditionally downward sloping and a multimarket effect only reduces its slope.

Intuitively if a firm is sufficiently large to obtain, because of interrelated costs, a more than compensative benefit from being more aggressive in market i , given its internalised cross market reaction to a rival aggressive play, its best reply function can be upward sloping even under quantity competition.

In summary an internalised multimarket effect seems to increase firms' aggressivity in front of an aggressive play and their propensity to accommodate ahead accommodating rivals.

4. No Interrelated Costs and Multimarket Effect as an Expected Rival's Reaction

Until now we have seen a multimarket effect as a sort of externality that relates conglomerate firms actions in parallel oligopolies; its sign was determined in a profit maximisation consistent way through interrelated costs and its effect was to strengthen reciprocal aggressive or accommodating conducts. But not in all circumstances costs or demands are interrelated; as empirically noticed, *inter alia*, by Parker and Roller (1997) and Evans and Kessides (1994), even in those market where it does not seem reasonable to allow for such explicit interdependence some effects on firms' behaviour of operating in a multimarket framework emerge. As pointed out these cases can be explained using a repeated game set up trying to capture what we can call dynamic multimarket interdependence. As best as I know no attempts exist to deal with static multimarket interdependence in absence of cross market relations between costs or demands. In what follows we will try to modify traditional static models of oligopolistic competition taking into account a multimarket effect completely not related to costs or demands. As Scott (1991) has stressed, contacts across markets can change firm's conjectures about what is an optimal conduct within a market and

thereby change equilibrium outcomes⁸. We will refer analysis to a simple linear economy⁹ since our argument is not related, as in the previous section, to any second order effect on profits of a given conduct.

Definition 7: A *Linear Economy* is characterised by linear demands functions and linear total costs of the form¹⁰ $p_l = a_l - b_l \left(\sum_k q_l^k \right)$ for $l = i, j$ and $TC^k = c \left(\sum_{l=i}^j q_j^k \right)$ for $k = A, B$

We assume again that two symmetric conglomerate firms compete in a multimarket oligopolies in which they are incumbents. Each firm is sufficiently expert to have noticed a certain line in rival's conduct into market j in response of her behaviour in market i even without any explicit linkage between costs or demands. Let us call *multimarket incumbency (MMI)* a set Ω of possible conjectures, based on observed practices or reputation, on rival's possible reactions in the parallel market and an internalised single conjecture ω *multimarket expected rivalry (MMER)*. Suppose that there exist three kinds of conjecture: a rival can be *warmonger (W)* if it increases his aggressivity in market j in response of an aggressive play in market i , *scared (S)* if it plays less aggressively in the parallel market after an aggressive play on the other or *neutral (N)* if it does not consider multimarket contacts. Formally $\omega \in \Omega = \{W, S, N\}$; firms are not allowed to mix possible rival's reactions. Some further definitions depict our set up.

Definition 8: A *MMER* is *bilateral* if it is symmetrically internalised by firms

Definition 9: A *MMER* $\tilde{\omega}$ is *profitable* if $\Pi^k | \tilde{\omega} > \Pi^k | \omega$ for $\forall \omega \neq \tilde{\omega}$

Analogously to the last definition we will refer to a *MMER* as *welfare enhancing* if its associated social welfare is maximum.

Definition 10: A *Multimarket Effect (MME)* is a relation among cross market strategic variables taken into account when a certain *MMER* is internalised.

More precisely we will deal with a *positive multimarket effect* if firm i expects a warmonger rival, *negative multimarket effect* with a scared rival and *null multimarket effect* in the remaining case.

⁸ For a discussion on how multimarket contacts affect even indirectly conglomerate firms's conducts see applied works by Adams (1974), Scherer (1979), Mueller (1987). For an application in banking see Pita Barros (1999).

⁹ Conclusions do not change for decreasing and concave demands, while absence of interrelated costs suggest to consider linear cost functions.

¹⁰ Traditional conditions for existence of a Cournot equilibrium hold: $a_l, b_l > 0$ $a_l > c \forall l$.

Recalling Definition 5 it is immediate to notice that a positive MME coincides with cross complements case and a negative one with strategic substitutes.

Definition 11: An profile of equilibrium strategies $\{\bar{S}_l^k\}_{l=i,j;k=A,B}$ is *MMER Optimal* if

$$\bar{S}_l^k | \omega = \arg \max_{S_l^k} \Pi^k | \omega, \forall \omega \in \Omega, \forall k, \forall l.$$

In what follows we will refer to quantity competition focalising on pure strategy equilibria, since a randomisation between strategies seems here not realistic¹¹. An internalised MMER enters in firm's profit maximisation problem as a relation between cross markets variables as above. Firms still have a divisional organisation and each division that manage a market can separately have a different conjecture about rival's reaction in the parallel market

Definition 12: A MMER is *internally shared* if it is identically internalised by any division of each firm and it is *market shared* if it is identically internalised by all firms with respect to the same market.

In some sense it seems natural to require a more stringent condition on internally shared expectations than on market shared ones; in the first case it is reasonable to see them as imposed on each division by a common ownership, in the second as left to market specific considerations done by each division.

We will denote a MME for division $l = i, j$ of firm $k = A, B$ as $\frac{\partial S_{-l}^{-k}}{\partial S_l^k} = m_l^k \in R$. If a MMER is *internally shared* then $m_l^k = m_{-l}^k$, if it is *market shared* then $sign(m_l^k) = sign(m_{-l}^k)$, if it is *reciprocally shared* then $sign(m_l^k) = sign(m_{-l}^{-k})$ ¹². Finally when $m_l^k = m \forall l, \forall k$ a multimarket rivalry is commonly expected by firms in all markets. We will refer to this last case as a *public multimarket effect* (and respectively to a public MMER).

Proposition 3: In a linear economy under quantity competition a positive (resp. negative) public multimarket effect will decrease (resp. increase) firms' aggressivity. Moreover in equilibrium the

¹¹ For price competition in order to avoid the *Bertrand Paradox* two directions can be followed: or we assume that exists an internal solution in a normal price competition model with linear direct demands and related costs, but in this case results don't change (see Lanzi (1999)) or we have to complexify our framework allowing for product differentiation. See for example Lal and Matutes (1989).

¹² Trivially it is possible to show that internally or market shared MME of the same intensity, respectively, for both firms or both markets are always public hence reciprocal MME. Then considering a public multimarket effect coincides with a totally symmetric multimarket competition.

reduction (resp. increase) of quantity produced by each firm will increase with multimarket effect intensity

Proof: Consider expressions two and three in (2) for $S_i^k = q_i^k \forall l, \forall k$ and in a linear economy. These will be firms' reaction curves of the form $2b_l q_i^k - a_l + b_l q_i^{-k} + c + b_{-l} m_i^k = 0 \forall l, \forall k$. Hence a multimarket effect will translate upward or downward firms' reaction curves. If there is a public multimarket effect $m_i^k = m \forall k, \forall l$ hence it is easy to compute Cournot-Nash equilibria of these symmetric multimarket oligopolies: this will be a quantity level that is MMER optimal $\bar{q}_i^k | \omega = \frac{a_l - c}{3b_l} - \frac{b_{-l} m}{3b_l} \forall l, \forall k$. As we can notice the first term on last expression right hand side is exactly a one market Cournot-Nash equilibrium that coincides with a reciprocally null multimarket effect. Thus we can write $\bar{q}_i^k | \omega = \bar{q}_i^k | N - \frac{b_{-l} m}{3b_l} \forall l, \forall k$. In front of a warmonger rival each firm takes in account a positive multimarket effect, both firms' reaction curve will move downward in both markets and in equilibrium we will have that $\bar{q}_i^k | W - \bar{q}_i^k | N = -\frac{b_{-l} m}{3b_l} < 0$. Each oligopolist will reduce his quantity (*less aggressivity*) in each market in order not to induce aggressive reaction of the rival in the parallel one and this will give a new equilibrium in each market with lower quantities produced and a higher equilibrium price. In opposition, facing a scared rival each firm will augment his production (*more aggressivity*) in order to induce reduction of other firm's quantity in the parallel market; thus the multimarket effect will be negative, reaction curves translated upward and $\bar{q}_i^k | S > \bar{q}_i^k | N \forall l, \forall k$. In the new Cournot-Nash equilibrium aggregate quantities will be higher and equilibrium prices lower.

Furthermore it is true for both cases that $\frac{\partial [\bar{q}_i^k | \omega - \bar{q}_i^k | N]}{\partial m} < 0 \forall \omega \in \Omega - \{N\}$.

Corollary 3: In a linear economy under quantity competition a public MMER is profitable only if firms are reciprocally warmonger and it is welfare enhancing only if firms are reciprocally scared.

Proof: With a public multimarket effect, computing profits in the two new Nash equilibria we have that $\Pi^k | W > \Pi^k | N > \Pi^k | S$, hence $\omega = \{W\}$ is a profitable MMER that leads firms to a less competitive equilibrium with restricted quantities and higher prices. Symmetrically the unique Nash equilibrium with scared rivals will be more competitive and hence welfare enhancing.

The last corollary gives us a very useful intuition: under multimarket competition firms will arrive to a more collusive equilibrium if both have some reputation for aggressivity, otherwise multimarket contacts can lead to higher aggressivity and a more competitive outcome. Even in presence of constant costs a possible effect on equilibrium quantities and prices can raise from multimarket contacts in a static model. When firms internalise a public warfare rivalry they will be induced to collude towards a mutual low aggressivity equilibrium, raising prices and increasing profits. This creates, in a dynamic perspective, a correct incentive for each firm to build a reputation of aggressivity on all markets. Hence it can intuitively explain why in dynamic models the only effect of multimarket contacts is to induce collusive behaviour.

As noticed above, in case of a positive MME equilibrium prices will rise and their increase will be higher the stronger such an effect is; the same kind of correlation has been stressed by several empirical works¹³. Another accepted conclusion in the economic analysis of multimarket competition is confirmed by our static model: almost all contributions underline how a relevant relation exists between concentration in markets and firms ability to coordinate towards a collusive equilibrium. Our Cournot-type model seems to be consistent with this view for two kind of reasons: first because if we suppose perfectly competitive one of our two oligopolies any multimarket effect disappears and in the other a traditional Cournot equilibrium is reached; secondly because we use a Cournot equilibrium notion proved to be quasicompetitive.

What happen if some asymmetries are introduced on expected multimarket reactions ? Is it in this case always profitable for a firm to have a reputation of being aggressive ? If such a reputation can be built only in one market where is more convenient to do so ? The following two propositions try to answer, within our framework, to these related questions

Proposition 4: With internally shared MMERs, to be considered warmonger is profitable for a firm k if her rival is more concerned about multimarket competition, that is $|m_l^k| > |m_l^{-k}| \forall l$.

Proof: Suppose internally shared MMERs then it must be that $sign(m_l^k) \neq sign(m_l^{-k}) \forall l$ otherwise we will have a public MMER. Suppose $m_l^k > 0$, $m_l^{-k} < 0$ that means firm k expects a warmonger rival and firm $-k$ a scared one¹⁴. A new equilibrium is given by the following system, for $\forall l$

$$\begin{cases} 2b_l q_l^k - a_l + b_l q_l^{-k} + c + b_{-l} m_l^k = 0 \\ 2b_l q_l^{-k} - a_l + b_l q_l^k + c + b_{-l} m_l^{-k} = 0 \end{cases}$$

from which equilibrium quantities and prices can be computed

¹³ See among the other Mester (1987), Gelfand and Spiller (1987), Jans and Rosenbaum (1997).

¹⁴ Obviously the argument is symmetric.

$$\bar{q}_i^{-k}|S = \bar{q}_i^{-k}|N + \frac{b_{-i}(m_i^k - 2m_i^{-k})}{3b_i}; \quad \bar{q}_i^k|W = \bar{q}_i^k|N + \frac{b_{-i}(m_i^{-k} - 2m_i^k)}{3b_i}; \quad \bar{p}_i = \bar{p}_i|N + \frac{b_{-i}(m_i^{-k} + m_i^k)}{3}$$

then profits for a warmonger firm $-k$

$$\Pi^{-k} = \Pi^{-k}|N + \sum_i \frac{b_{-i}b_i(a_i - c)(2m_i^k - m_i^{-k}) + b_{-i}^2(m_i^{-k} + m_i^k)}{9b_i}$$

Thus if $|m_i^k| > |m_i^{-k}|$, or firm k 's has internalised a more intense MME, to have a reputation of being warmonger induces higher profits (lower for the other firm).

Proposition 4¹⁵ gives some insights into profitability of a multimarket reputational effect: in case of complete symmetry expected warmonger rivals can achieve higher profits, while in a externally asymmetric case a warmonger competitor facing a scared rival can increase his production, while his rival reduces it. This will end in a global increase in profits for the former firm and a reduction for other, with respect to an ordinary Cournot case, only if a scared rival quantity reaction with an internalised multimarket effect is stronger than quantity adjustment of a warmonger competitor. If this is not the case it is more convenient to have a reverse reputation. As a device for obtaining a leadership, a multimarket reputational effect of being aggressive must be sufficiently intense. Intuitively the more (less) afraid a rival is of competing with an aggressive firm, the more (less) convenient is to appear warmonger (in some cases it is profitable not to appear warmonger at all).

Proposition 5: When market shared MMERs have opposite signs, it is profitable for each firm k to have a warmonger reputation in the market where there is a more intense multimarket effect i.e. $|m_i^k| > |m_{-i}^k|$.

Proof: For a market shared MMER we have a MME of the following form $m_i^k = m_i^{-k}$; $m_{-i}^k = m_{-i}^{-k}$. If these have opposite sign, without loss of generality we can suppose $m_i^k > 0$, $m_{-i}^k < 0 \forall k$. Using Proposition 3 we can immediately say that $\Pi_i^k|W > \Pi_i^k|N \forall k$ and $\Pi_{-i}^k|S < \Pi_{-i}^k|N \forall k$. Then $\Pi_i^k|W - \Pi_{-i}^k|S > 0 \forall k$ only if $|m_i^k| > |m_{-i}^k| \forall k$ that is a more intense MME on market with $\omega = \{W\}$.

¹⁵ Some more cases can raise if we allow for not shared MMER or MMEs with more controversial signs.

Under the possibility for a firm to build a strong reputation in only one market it will be convenient to do so in the more reactive one, where firms' reaction to a certain MMER is stronger, otherwise it is preferable to behave as if the two markets are ordinary Cournot oligopolies.

The next proposition deals with a particular type of firms' behaviour called *follow the leader strategy* (Encarnation (1987)) where each firm assume a role of leader in one market and of follower in the other¹⁶. A "follow the leader" equilibrium will raise only in a particular case:

Proposition 6: Under opposite reciprocally shared MMERs firms reach a follow the leader equilibrium. Furthermore if $|m_{-l}^F| \geq |m_l^L| \quad \forall l$ this equilibrium is profitable for each leader.

Proof: Suppose opposite reciprocally shared MMERs then, without loss of generality, we can take the following MMEs $sign(m_i^k) = sign(m_{-i}^{-k}) > 0$, $sign(m_{-i}^k) = sign(m_i^{-k}) < 0$. For any value of MME we know from Proposition 3 that $q_i^k |W < q_i^k |N$, $q_i^{-k} |S > q_i^{-k} |N$ and $q_{-i}^{-k} |W < q_{-i}^{-k} |N$, $q_{-i}^k |S > q_{-i}^k |N$. Hence in terms of market shares, we have that firm $-k$ assume a role of a quantity leader in market i and firm k in market $-i$. If the follower reduces his quantity of an amount larger than (or equal to) leader's increase, i.e. $|m_{-l}^F| \geq |m_l^L| \quad \forall l$, then in both markets the new equilibrium will be less (equally) competitive, not welfare enhancing and leader's profits will be higher.

Proposition 6 suggests that a "follow the leader" equilibrium coincides only with a particular MMER and that it is optimal for each firm to assume the role of a leader in one market only if the follower is sufficiently submissive. For particular values of MMEs it could be also possible that for a firm profits reduction in the market where she is a follower was larger than profits increase in the other market where she plays as a leader. In this case nobody will assume an explicit leadership.

5. Conclusions

¹⁶ Bernheim and Whinston (1990) call this possibility, as suggested by Edwards (1955), development of *spheres of influence*.

In the preceding sections we have analysed some consequences of a multimarket effect on competition between conglomerate oligopolists. The attempt was to answer to an existent hiatus, only partially covered in the case of interdependent demands, in economic analysis of multimarket competition from a static point of view.

First we have tried to modify a model of oligopolistic competition for multimarket oligopolies where some interrelated costs suggest to firms to internalise a multimarket effect view as a sort of externality linked to joint economies. Using this approach a primary conclusion emerges: multimarket competition can augment firms' aggressivity in front of warmonger rivals and their tendency to accommodate ahead of not aggressive ones. This can also lead to some cases where firms reaction functions are upward sloping even in case of strategic substitutes.

Secondly we have assumed no interrelated costs and a multimarket effect simply related to a firms' belief about rival reactions in the parallel market. Even in this case some high aggressivity equilibria can raise in the case of expected scared rivals. A multimarket version of Cournot's traditional quantity competition model allows us to deal with different kind of expected multimarket reactions and with different firms' reputational status.

Some empirical regularities can be at least partially explained with both static approaches and some conclusions of dynamic models can be seen as perfectly consistent with insights of the static analysis. Finally, from a social welfare perspective multimarket competition is desirable only when it leads to more competitive equilibria and it reduces firms' willingness to collude. Therefore regulatory policies have to take in account what kind of competition is going on between firms in order to not intervene improperly in markets.

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