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The Production of Goods in Excess of Demand : a Generalization of Self Protection

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First version : september 1996
This version : february 1999
441.99

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Keywords : reserve goods, self protection, insurance.
JEL Classification : D81, D21

Abstract

We consider a risk-averse firm producing a limited number of goods which can be defective. The firm must determine its level of production before knowing which goods will be defective. This is the case for example for a producer of telecommunication satellites. The problem under scrutiny can be interpreted as a generalization of self protection for more than two states of nature. In our model, the firm determines jointly its level of production and its demand for insurance. It is shown that, under reasonable assumptions, the two strategies are complements.

Resumen

Consideramos una empresa aviersa al riesgo que produce un número linitado de bienes que pueden ser defectuosos. La empresa debe determinar su nivel de producción antes de saber que bienes saldrán defectuosos. Un ejemplo es el caso de un fabricante de satélites de telecomunicaciones. El problema que es estudia se puede interpretar como une generalización de auto protección cuando hay más de dos estados de la naturaleza. En nuestro modelo, la empresa determina simultáneamente su nivel de producción y su demanda de seguro. Se demuestra que, bajo supuestos razonables, las dos esrategias son complementarias.

Introduction

Consider a firm facing a known demand for a randomly defective product who must decide on the quantity of goods to produce before knowing how many goods will be defective. The firm can decide to engage in a reserve goods strategy to modify the distribution of the number of non defective products. The more the firm produces, the greater is the probability of getting a given number of successful products. This problem is standard in the operation management literature: it is the single-period inventory problem under production uncertainty.

We examine the inventory choice for a risk averse firm so that we can study the interactions between an inventory decision and an insurance decision footnote . Several studies have examined how demand uncertainty affects production or pricing decisions of a risk averse firm footnote . In all these studies, the decision variable for the firm cannot affect the distribution of the random variable. We consider a different problem : the decision variable for the firm directly affects the distribution of the random variable.

An excess production strategy can be interpreted as a generalization of self protection for more than two states of nature. Ehrlich and Becker (1972) define self protection as an action or expenditure that reduces the probability of a loss. Although this definition has a general character, self protection is usually studied in models with two states of nature. Dionne and Eeckhoudt (1985) and Bryis and Schlesinger (1991) have analysed the impact of an increase in risk aversion on the optimal level of self protection. Boyer and Dionne (1983, 1989) and Chang and Ehrlich (1985) have studied the interaction between self insurance and self protection with and without an insurance market. All these studies have considered models with two states of nature and cannot offer predictions about the form of the optimal insurance contract under self protection. Winter (1991) however proposes to study self protection in a general model in a moral hazard context. He presents a different generalization of the concept : self protection refers to an increase in the probability of zero-loss, with no change in the conditional distribution of the loss.

It is shown that the optimal insurance contract involves a total insurance of losses with a deductible. The indemnity is independent of the number of reserve goods. On the contrary, the premium is lower when the number of reserve goods is high because producing more reserve goods reduces the expected loss in revenue. The analysis of the interactions between the two strategies show that these strategies are complements rather than substitutes. Under reasonnable assumptions, we show that the quantity of reserve goods produced will be higher if the firm has insurance. Similarly, the amount of insurance purchased will be higher if the firm produces reserve goods. Finally, an increase in the production costs of one of the two strategies causes the insurance demand and the reserve goods demand to adjust in the same direction.

An interesting application of this model is the space industry footnote . Space firms, generally facing contracts for a serial production of satellites, are characterized by a high degree of variability in their revenue. This riskiness arises from possible technical failure during the launch and the in orbit operation. Production delays being important, it is necessary for firms to determine the number of satellites to produce before the launching phase and therefore, before knowing which satellites will be defective. We can observe that the behavior of space industry firms fits the predictions of the model: they jointly insure the risk of financial loss on the insurance market and produce one or several reserve satellites.

In section 2, we introduce the notation and the model of reserve goods demand used throughout the analysis. The optimality condition guaranteeing a positive level of reserve goods is derived and we propose comparative statics results. The joint use of insurance and reserve goods is investigated in section 3. The optimal insurance contract and the optimal choice of reserve goods are determined. In section 4, we investigate the comparative statics properties of the model. Section 5 concludes.

The model

Consider a firm with initial wealth w who produces goods at a unit cost c and resells them at a price p. The firm faces a known demand and can not sell more than a quantity D of goods. Production requires delays. We can consider a one period game: the goods are produced during the period and sold at the end of the period.

The production process may yield defective goods. The quality of a good, i.e. defective or successful, is known by the firm at the end of the period. The firm must therefore determine its level of production before knowing which goods will be defective. The firm will at least produce a quantity D of goods but it can also produce a quantity n of additional goods. These additional goods are called reserve goods.

The firm is then endowed with the following profit function.

$$\Pi(x,n) = \begin{cases} \Pi_-(x,n) = w + px - c(D+n) & \text{if } x \leq D \\ \Pi_+(x,n) = w + pD - c(D+n) & \text{if } x > D \end{cases}$$

Let denote $F(x,n)$ the cumulative distribution function of x when the production of the firm is $D+n$. The firm obtains $\Pi_-(x,n)$ with probability $F_x(x,n)$ and obtains the maximum profit $\Pi_+(x,n) \equiv \Pi(D,n)$ with probability $1 - F(D,n)$.

By increasing its level of production, the firm modifies the distribution of the number of successful goods in the sense of First-Order Stochastic Dominance footnote (FSD). The probability of getting a given quantity of successful goods x is higher when the firm uses the reserve goods strategy. The firm can then decide to produce an important quantity of reserve goods and increase the probability to get an important number of successful goods or prefer not to engage in a costly production strategy and face a less favorable successful goods' distribution. Typically this strategy is a self-protection activity : by incurring an additional expenditure, the firm reduces the probability of loss on its payoff $p(D-x)$.

We assume that the firm is risk averse. The utility function $U(.)$ is assumed to be monotonic increasing, three times continuously differentiable and concave $(U' > 0, U'' \leq 0)$. The objective of the firm is as follows :

$$\max_{n\geq 0} E(U(\Pi(x,n)) = \int_0^D U(\Pi_-(x,n))dF(x,n) + (1 - F(D,n))U(\Pi(D,n)) \qquad \#$$

where E denotes the expectation operator.

The optimality condition reduces to:

$$E(U)_n \equiv -cEU'(\Pi(x,n)) - p\int_0^D U'(\Pi_-(x,n))F_n(x,n)dx = 0$$

Where $F_n(.,.)$ denote the first derivative of the cumulative distribution function with respect to n. Note that by definition $F_n(x,n) < 0 \ \forall \ x \in [0,D]$.

The first term is the marginal cost in terms of utility of increasing n. The second term is the marginal benefit in terms of utility from the decrease in the loss probability. This equation characterizes the optimal number of reserve goods.

When the firm is risk neutral $(U'' = 0)$, equation (ref: P2) yields the following condition $-c - p\int_0^D F_n(x,n)dx = 0$. The marginal cost of producting an additional good must equal the decrease in the loss probability.

Assuming that an interior solution exists footnote , we study how the optimal number of reserve goods is affected by changes in various parameters. We find that, under constant absolute risk aversion (CARA), an increase in the production cost induces the firm to reduce the demand for reserve goods: there exists a shift between the additional production and risk shifting. The demand for reserve goods is unchanged when the initial wealth or the level of demand are modified.

To complete the comparative static findings for the case of DARA or IARA, an additional assumption is necessary. Let $G(x,n)$ denote the cumulative distribution function of the profit $\Pi(x,n)$. footnote It is assumed that there exists $t \in [0,D]$ such that $G_n(k,n) \leq 0 \ \forall k \in [0,t]$ and $G_n(k,n) \geq 0 \ \forall k \in [t,D]$. This condition, known as the Single Crossing Condition (SC1) footnote , supposes that an increase in the level of production modifies the distribution of the profit such that there is a shift of probability mass from the highest to the lowest values of profit.

Under this assumption, we find that, under DARA, an increase in the production cost reduces the demand for reserve goods: the marginal cost for risk-shifting increases with c and a wealth effect reinforces this substitution effect. Under IARA, the two effects are opposite and the final result is therefore ambigous. If the initial wealth or the level of demand increase, the production of reserve goods increases (decreases) as the absolute risk aversion decreases (increases). Clearly, the demand for reserve goods is a normal good (inferior good) for DARA (IARA).

The reserve goods strategy and insurance

We assume now that in addition to the use of the self protection strategy, the firm can insure the risk. We suppose that insurers are risk-neutral. Against a premium P, the market offers him an indemnity payment $I(x)$ contingent on the realizations of the loss and the number of reserve goods footnote . It is assumed that the cost of providing the insurance policy is proportional to the actuarial value of the policy, i.e. it involves a fixed percentage loading. The premium must at least cover the expected value of the indemnity payment.

$$P \geq (1+k)\int_0^D I(x)dF(x,n)$$

where k is the loading factor. Insurance is unfair in the sense that $k > 0$. The insurance industry is assumed to be perfectly competitive so that (ref: prime) holds with equality.

The profit of the firm is then given by

$$\Pi(x,n) = \begin{cases} \Pi_-(x,n) = w + px + I(x) - P - c(D+n) & if\ x \leq D \\ \Pi_+(x,n) = w + pD - P - c(D+n) & if\ x > D \end{cases}$$

The optimal insurance contract and the optimal number of reserve goods must solve :

$$\max_{I(x),n,P} \int_0^D U(\Pi_-(x,n))dF(x,n) + (1-F(D,n))\Pi(D,n)$$

mbox $I(x) \geq 0$ #

$$P = (1+k)\int_0^D I(x)dF(x,n) \quad \#$$

The maximization problem specifies that the insured will maximize the expected utility $E(U)$ of final wealth by choosing an optimal indemnity schedule $I(x)$, an optimal premium P, the optimal number of reserve goods n, subject to the non negativity constraint on $I(x)$.

The Lagrangean for this problem can be written as :

$$L(I(x), n, P, \lambda) = \int_0^D U(\Pi_-(x,n))dF(x,n) + (1 - F(D,n))\Pi(D,n) \qquad \#$$
$$+ \ \lambda[P(1+k)^{-1} - \int_0^D I(x)dF(x,n)]$$

The Kuhn and Tucker conditions reduce to :

$$U'(\Pi_-(x,n)) - \lambda \leq 0 \ \forall \ x \leq D \qquad \#$$
$$< 0 \ \Rightarrow \ I(x) = 0$$

$$\lambda(1+k)^{-1} = EU'$$

$$P = (1+k)\int_0^D I(x)dF(x,n) \qquad \#$$

$$\int_0^D U(\Pi_-(x,n)))F_{xn}(x,n)dx - F_n(D,n)U(\Pi(D,n)) \qquad \#$$
$$- cEU' - \lambda\int_0^D I(x)F_{xn}(x,n)dx = 0$$

Condition (ref: C1) implies that the indemnity payment involves a total insurance of losses below footnote a deductible $\bar{x} > 0$, i.e. the indemnity function can be written $I(x) = max\{0, p(\bar{x} - x)\}$. As Raviv (1974) has shown the deductible clause in the insurance payment exists due to two sources : the nonnegativity constraint on the transfer from the insurer to the insured and the variable insurance cost.

The firm is then endowed with the following profit function.

$$\Pi(x,n) = \begin{cases} \Pi_-(\bar{x},n) = w + p\bar{x} - P - c(D+n) & if \ x \leq \bar{x} \\ \Pi_-(x,n) = w + px - P - c(D+n) & if \ \bar{x} < x \leq D \\ \Pi_+(D,n) = w + pD - P - c(D+n) & if \ x > D \end{cases}$$

Assuming that the loading cost k is zero, we obtain that the firm chooses a total insurance of losses, i.e $\bar{x} = D$. The firm obtains the same final wealth in all the states of nature, is indifferent toward risk and therefore interested only in maximizing his expected final wealth.

It is important to note that the indemnity is independent of the quantity of reserve goods produced whereas the premium is a decreasing function of the reserve goods quantity. The random variable for the firm is the quantity of non defective goods. The firm does not want to insure each good produced but a given quantity of goods. The premium only is affected by the excess production since it affects the successful goods distribution.

Comparative static analysis

In all this section it is assumed that an interior solution to the program exists.
Since our interest focuses on the interaction between the two decisions we first study how one of the two decision variables reacts to an exogeneous increase of the other decision variable. prop
Proof : See appendix.

These first results of comparative static analysis indicate that market insurance and reserve goods are complements in the sense that the availability of one instrument can increase the demand for the second. This implies that the optimal number of reserve goods is greater in the model with insurance than in the model without insurance. A non insured firm is more interested in reducing the maximal loss, which leads to less self protection. This also suggests that if the insurance market makes premium responsive to the number of reserve goods produced, it can expect an increase in the insurance demand.

prop

Proof : See appendix.

Under CARA, in equilibrium, insurance and reserve goods are complements. If we reasonably suppose that an increase in the production cost reduces the demand for reserve goods, Proposition 2 implies that insurance demand decreases. It also implies that an increase in the insurance cost reduces both the demand for insurance and the demand for reserve goods.

Conclusion

This paper analyses a particular strategy : the strategy of producing goods in excess of demand or reserve goods strategy. It was shown that the optimal insurance contract involves a deductible and the optimality condition for a positive level of reserve goods was given. The comparative static analysis indicates that under constant absolute risk aversion and the single-crossing condition reserve goods and insurance are complement.

This analysis clearly rationalizes the behaviour of space industry firms which jointly insure the risk of financial loss on the insurance market and produce one or several reserve satellites. If we consider risk neutral firms, we can observe that the incentive to produce reserve goods still exists contrary to the incentive to insure the risk as the variable insurance cost is positive. The complementarity of the two strategies, as previously noted, has an important implication for all these firms. It effectively encourages insurers, when the number of reserve goods produced can be observable, to make the premium responsive to these reserve goods.

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Appendix

Proof of Proposition 1

Substituting for $I(x) = max\{0, p(\bar{x} - x)\}$ in (ref: C1)-(ref: C4), and integrating (ref: C4) by parts yields :

$$U^{'}(\Pi_{-}(\bar{x}, n)) = (1 + k)[U^{'}(\Pi_{-}(\bar{x}, n))F(\bar{x}, n) + \int_{\bar{x}}^{D} U^{'}(\Pi_{-}(x, n))dF(x, n) ++(1 - F(D, n))U'(\Pi(D, n))] \quad \#$$

$$P = (1 + k)\int_{0}^{\bar{x}}(p\bar{x} - px)dF(x, n)$$

$$- cU^{'}(\Pi_{-}(\bar{x}, n))F(\bar{x}, n) \quad \#$$
$$- c\int_{\bar{x}}^{D} U^{'}(\Pi_{-}(x, n))dF(x, n)$$
$$- c(1 - F(D, n))U'(\Pi(D, n))$$
$$- pU^{'}(\Pi_{-}(\bar{x}, n)) \int_{0}^{\bar{x}} F_{n}(\bar{x}, n)dx$$
$$- p\int_{\bar{x}}^{D} U^{'}(\Pi_{-}(x, n))F_{n}(x, n)dx = 0$$

Differentiating equation (ref: e5) with respect to n and $\bar{x}$, where the latter is treated as an exogeneously determined variable, gives :

$$0 = [pU^{''}(\Pi_{-}(\bar{x}, n))[cF(\bar{x}, n) + p\int_{0}^{\bar{x}} F_{n}(x, n)dx] \quad \#$$
$$- p(1 + k)F(\bar{x}, n)[cE(U^{''}) + pU^{''}(\Pi_{-}(\bar{x}, n)) \int_{0}^{\bar{x}} F_{n}(\bar{x}, n)dx$$
$$+ p\int_{\bar{x}}^{D} U^{''}(\Pi_{-}(x, n))F_{n}(x, n)dx]]d\bar{x}$$
$$+ [-c^{2}E(U^{''}) - cpU^{''}(\Pi_{-}(\bar{x}, n)) \int_{0}^{\bar{x}} F_{n}(\bar{x}, n)dx$$
$$- 2cp\int_{\bar{x}}^{D} U^{''}(\Pi_{-}(x, n))F_{n}(x, n)dx$$
$$+ p\int_{0}^{D} U^{'}(\Pi_{-}(x, n))F_{nn}(x, n)dx - [cE(U^{''})$$
$$+ pU^{''}(\Pi_{-}(\bar{x}, n)) \int_{0}^{\bar{x}} F_{n}(\bar{x}, n)dx$$
$$+ p\int_{\bar{x}}^{D} U^{''}(\Pi_{-}(x, n))F_{n}(x, n)dx](1 + k)p\int_{0}^{\bar{x}} F_{n}(x, n)dx]dn$$

Observe that $E(U'') - (1+k)^{-1}U''(\Pi_-(\bar{x},n)) \equiv V$

$V = \int_{\bar{x}}^{D} U'(\Pi_-(x,n))(A(\Pi_-(\bar{x},n)) - A(\Pi_-(x,n)))dF(x,n) + (1-F(D,n))(A(\Pi_-(\bar{x},n)) - A(\Pi(D,n)))U$

$A(w)$ being the measure of absolute risk aversion, under DARA (IARA) $V > (<) 0$ and V is equal to zero if the firm exhibits constant absolute risk aversion.

Denote $\delta \equiv (1+k)p\int_0^{\bar{x}} F_n(x,n)dx + c$

$\delta = -(1+k)p\int_{\bar{x}}^{D} U'(\tilde{w})F_n(x,n)dx / U'(\Pi_-(\bar{x},n)) > 0.$

Denote $Z \equiv cE(U'') + pU''(\Pi_-(\bar{x},n))\int_0^{\bar{x}} F_n(\bar{x},n)dx + p\int_{\bar{x}}^{D} U''(\tilde{w})F_n(x,n)dx$

Note that under CARA, Z must be equal to zero for (ref: e5) to hold with equality. If the single crossing condition is satisfied, $Z \leq (\geq) 0$ as the absolute risk aversion decreases (increases).

Using the fact that under CARA $V = Z = 0$, equation (ref: a6) becomes :

$$[pU''(\Pi_-(\bar{x},n))[cF(\bar{x},n) + p\int_0^{\bar{x}} F_n(x,n)$$

$$+ [-cp\int_{\bar{x}}^{D} U''(\Pi_-(x,n))F_n(x,n)dx + p\int_0^{D} U'(\Pi_-(x,n))F_{nn}(x,n)dx]dn = 0$$

$-cp\int_{\bar{x}}^{D} U''(\tilde{w})F_n(x,n)dx + p\int_0^{D} U'(\Pi_-(x,n))F_{nn}(x,n)dx$ is positive by the assumption that an interior solution exists and $pU''(\Pi_-(\bar{x},n))[cF(\bar{x},n) + p\int_0^{\bar{x}} F_n(x,n)dx]$ is negative if the single-crossing condition is satisfied.

We therefore obtain that an exogeneous increase of the level of the deductible increases the optimal number of reserve goods, displaystyle $\frac{dn}{d\bar{x}} > 0$.

Differentiating equation (ref: e2) with respect to $\bar{x}$ and n, where the latter is treated as an exogeneously determined variable, gives :

$$\begin{aligned} 0 = &-[(1+k)p[U''(\Pi_-(\bar{x},n))\beta^2 + F(\bar{x},n)[\int_{\bar{x}}^{D} U''(\Pi_-(x,n))dF(x,n) \\ &+ (1-F(D,n))U''(\Pi(D,n))]]]d\bar{x} \\ &+ [((1+k)^{-1}U''(\Pi_-(\bar{x},n)) - E(U''))((1+k)p\int_0^{\bar{x}} F_n(x,n)dx + c) \\ &- p\int_{\bar{x}}^{D} U''(\Pi_-(x,n))F_n(x,n)dx]dn \end{aligned}$$

where

$Y \equiv U''(\Pi_-(\bar{x},n))\beta^2 + F(\bar{x},n)[\int_{\bar{x}}^{D} U''(\Pi_-(x,n))dF(x,n) + (1-F(D,n))U''(\Pi(D,n))] < 0$

and $\beta \equiv (1+k)^{-1} - F(\bar{x},n)$

$\beta = E(U')/U'(\Pi_-(\bar{x},n)) - F(\bar{x},n)$ from (ref: e2)

$\beta = [\int_{\bar{x}}^{D} U'(\Pi_-(x,n))dF(x,n) + (1-F(D,n))U''(\Pi(D,n))]/U'(\Pi_-(\bar{x},n)) > 0$

Under CARA equation (ref: a5) becomes :

$$-[(1+k)pY]d\bar{x} - [p\int_{\bar{x}}^{D} U''(\Pi_-(x,n))F_n(x,n)dx]dn = 0.$$

We therefore obtain that an exogeneous increase of the number of reserve goods increases the optimal level of the deductible, displaystyle $\frac{d\bar{x}}{dn} > 0$.

Proof of Proposition 2

When the firm exhibits CARA, the effects of a change in the production cost on the optimal deductible and the optimal number of reserve goods are given by the two following equations :

$$-[(1+k)pY]d\bar{x} - [p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx]dn + 0dc = 0$$

$$0 = [pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]]d\bar{x} + E(U')dc \qquad \#$$
$$+ [-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]dn$$

and therefore :

$$d\bar{x}/dc = E(U')[p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx] \qquad \#$$
$$[[-p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx][pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]$$
$$+ [(1+k)pY][-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]]^{-1}$$

$$dn/dc = -E(U')[(1+k)pY] \qquad \#$$
$$[[-p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx][pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]$$
$$+ [(1+k)pY][-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]]^{-1}$$

See that the denominator of the two equations is the same, then observe that the sign of the two numerators is positive is sufficient to show that $d\bar{x}/dc$ and dn/dc are of the same sign.

If we consider a change in the insurance cost k :

$$-[(1+k)pY]d\bar{x} - [p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx]dn + E(U')dk = 0$$

$$0 = [pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]]d\bar{x} + 0dk \qquad \#$$
$$+ [-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]dn$$

and then

$$d\bar{x}/dk = E(U')[-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx \qquad \#$$

$$+ p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]$$

$$[[-p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx][pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]$$

$$+ [(1+k)pY][-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]]^{-1}$$

$$dn/dk = -E(U')[pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx] \qquad \#$$

$$[[-p\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx][pU''(\Pi_{-}(\bar{x},n))[cF(\bar{x},n) + p\int_{0}^{\bar{x}} F_n(x,n)dx]$$

$$+ [(1+k)pY][-cp\int_{\bar{x}}^{D} U''(\Pi_{-}(x,n))F_n(x,n)dx + p\int_{0}^{D} U'(\Pi_{-}(x,n))F_{nn}(x,n)dx]]^{-1}$$

The denominator of equations (ref: a21) and (ref: a22) is equal to the denominator of (ref: a17) and (ref: a18). Observe that the numerator of (ref: a21) and (ref: a22) are positive ends the proof.

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