Fertility Risk in the Life-Cycle

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Abstract

I present a life-cycle model of consumption-savings and fertility decisions in an environment with uninsurable income shocks and imperfect fertility control. I quantify the importance of fertility risk (due to imperfect birth control usage) in determining fertility outcomes in the life-cycle. My model presents a unified framework in which opportunity costs of child rearing as well as technological restrictions (in the form of contraception effectiveness) play significant roles to understand lifetime fertility.

Keywords: Stochastic fertility, Life-cycle model, Heterogeneous agents model, Birth control

JEL Classification: D31,D91,E21,J13,J31

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1 Introduction

Around half the pregnancies in the United States are unwanted and half of those end up in an abortion (See for example Henshaw (1998)). In this paper I argue that the imprecision with which individuals carry out their fertility plans (what can be labeled *fertility risk*) needs to be acknowledged and included in dynamic models of fertility choice. Since the arrival of children into a household represent important costs in terms of resources and time for a significant period in the life-cycle, this fertility risk needs to be understood to make meaningful welfare and public policy prescriptions.

In this paper I develop a model to understand and quantify the driving forces behind fertility decisions across educational groups. My model is described by a life-cycle, consumption-savings problem with uninsurable earnings risks, heterogeneous demographics (stochastic marriage/divorce) and exogenous educational types.[1] I introduce fertility decisions into the model, with the property that the effectiveness of birth control and reproductive technologies is subject to idiosyncratic shocks (*fertility risk*). I calibrate the model using data from the 1995 National Survey of Family Growth (NSFG), the Panel Study of Income Dynamics (PSID) and the Current Population Survey (CPS) using a simulated method of moments approach, where I choose education-specific fertility and abortion life-cycle rates as data moments to be matched by the model.

In the model, I assume that children induce a shift in the utility of public consumption inside the household (through equivalence scales) and a time cost for mothers: I model children as durable consumption goods that require time for maintenance (child rearing time). In previous literature this assumption is key to obtain a negative relationship between fertility and market skills, since the opportunity cost of a child is higher for females with higher income.[2]

However, in the data I find that life-cycle rates of unwanted and aborted pregnancies

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[1] This structure is similar to models in Hong and Ríos-Rull (2007) and Hong (2008).

[2] See for example Mincer (1963), Becker (1965), Galor and Weil (1996) and Greenwood, Seshadri, and Vandenbroucke (2005). See also Jones, Schoonbroodt, and Tertilt (2008) for a discussion in dynastic models, and Erosa, Fuster, and Restuccia (2010) for a life-cycle context.

across educational groups are remarkably similar. This motivates the introduction of differential fertility risks in the model: given the same contraception technology, more educated individuals are more effective using it. The need for this additional assumption comes from the fact that if the model matched fertility, unwanted and aborted pregnancies for individuals with high income (high education) say, it wouldn't be able to match jointly those same facts for low income individuals, given *the same level of fertility risk*, since the opportunity cost of children is lower for them in terms of forgone wages.

My model is closely related to Conesa (2000) and Sommer (2009): they both study life-cycle fertility decisions in environments with uninsurable labor income risk. I extend this framework by considering a richer demographic structure and the presence of fertility risk. On the other hand, the main mechanism in this paper is related to the findings in Rosenzweig and Schultz (1989) who show that more educated individuals are more efficient using different birth control methods.[3]

My approach borrows insights from the empirical microeconomic literature that studies life-cycle fertility[4] using structural and dynamic models of fertility choice. From that literature, my paper relates the most to Wolpin (1984) and Hotz and Miller (1993) who acknowledge the importance of the stochastic nature of fertility. Wolpin analyzes how child mortality risk shapes fertility choices using Malaysian data; Hotz and Miller estimate birth control method choices by females in a life-cycle framework. However, my approach differs in terms of assumptions regarding capital markets and preference heterogeneity: I assume imperfect capital markets in the sense that agents can save but not borrow against their future earnings while the above mentioned literature assumes away the existence of capital markets; also, I impose the same utility function for all agents, downplaying the role of unobserved heterogeneity in preferences to account for the data.

The structure of the paper is as follows: In the next section I describe my data sources and the main stylized facts I want to explain. In section 3, I describe my quantitative model.

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[3] I also present casual evidence on this, based from survey questions in the National Longitudinal Survey of Youth (NLSY1979). See the next section.

[4] See Hotz, Klerman, and Willis (1997) for a survey

Sections 4 and 5 describe the functional forms used in the model and the specific calibration method to obtain model parameters. I show the parameter values for the model and some quantitative experiments in section 6. The final section concludes.

2 Motivating Facts

I use information from the National Survey of Family Growth (NSFG) to put forward a set of facts on U.S. fertility. The NSFG is compiled by the National Center for Health Statistics (NCHS) and gathers information on family life, fertility, use of birth control and other health related questions. I use the survey for the year 1995, which comprises around ten thousand women between the ages of 15 and 44.

For every survey participant, the NSFG collects retrospective information on usage of birth control methods, on a monthly basis for up to 5 years. Participants also answer questions on wantedness and timing of births and pregnancy outcomes for all pregnancies conceived during that 5 year period.[5] The survey also contains information on educational attainment, marital status and other background information.

I present age specific fertility rates in figure 1 and age specific abortion rates in figure 2.

Both graphs present information on pregnancies occurring between 1994 and 1995, for each education-age group: High School (those without any post-secondary education) and College (those with at least some post-secondary education). Age specific fertility rates are defined by the ratio between the number of pregnancies in the specific education-age group and the total number of women in that group. Unwanted and abortion rates are the fraction of total pregnancies for each age-education group that were claimed as such by survey participants of the NSFG. In both figures, I show smoothed statistics.[6]

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[5] In terms of accuracy of this data, Fu, Darroch, Henshaw, and Kolb (1998) show that the introduction of computer assisted interviews in the NSFG for the year 1995 was accompanied by a reduction of underreporting of abortions and unplanned pregnancies. Nevertheless, their study shows (by comparing implied abortion rates from the NSFG to data from abortion providers in the U.S.) that non reported abortion cases are still present and are higher for lower income groups.

[6] I present 3 age averages, so that the statistics for age "i" comprise the information of females who were between $i-1$ and $i+1$ years old inclusively.

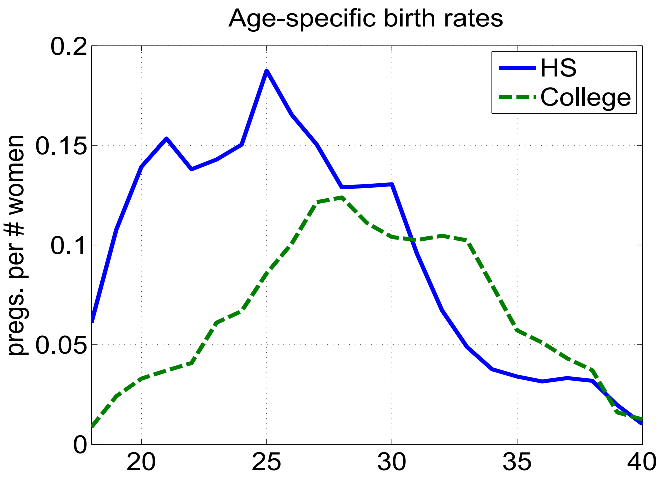

Figure 1: Life-cycle profiles of fertility (NSFG 1995)

Figure 1 presents two well known facts about fertility: first, a negative relationship between fertility and education of mothers (since more educated individuals have higher wages, this fact can be restated as a negative income-fertility relationship) and second, a delay in the timing of fertility for more educated individuals.

The observation that there is a negative relationship between income and fertility goes back to Becker (1960), while Jones and Tertilt (2008) study Census data and find that this negative relationship is robust across time and different definitions of income. The differential timing of births is documented and studied by Caucutt, Gunner, and Knowles (2002), who argue that returns to experience as well as marriage markets play an important role in explaining delay in childbirth.

On the other hand, figure 2 documents the prevalence and timing of errors in fertility plans across educational groups. Panel (a) of that figure presents the life cycle profile of unwanted pregnancies while panel (b) shows aborted pregnancies. Both figures are in terms of percentages over of pregnancies, excluding miscarriages.

For the high school group, the rate of unwanted pregnancies is higher (around 40%) than for the college group (around 36%).[7] In terms of timing, these profiles are very close for both

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[7] These numbers are slightly lower than those in Henshaw (1998), due to different sample selection criteria: Henshaw (1998) considers all pregnancies concluding in 1994, for women between 15 and 44 years of age. I

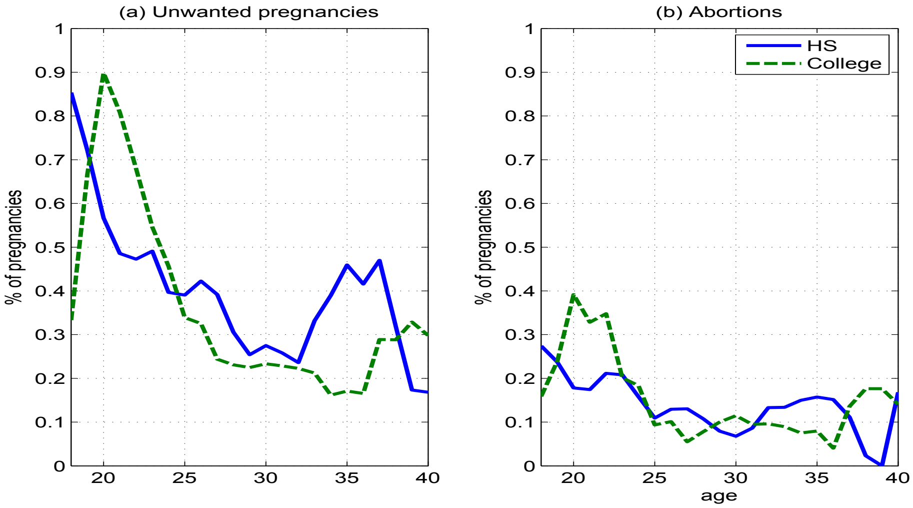

Figure 2: Fertility facts (NSFG 1995)

educational groups: they show a clear decreasing pattern, with some increase in unwanted rates at later periods. In terms of abortion rates, figure 2 shows that they are higher for the college group (averages of 13.8% for high school vs 15.2% for college) while both profiles are decreasing in age.

Considering only the information on births presented above (figure 1) in conjunction to the fact that more educated individuals have on average higher wages (this is depicted in figure 3 below, which shows annual earnings of *childless* women by marital status), it would be reasonable to think about a fertility theory in which the opportunity cost of children in terms of time (or disruption of a career) plays a first order role in explaining differential fertility outcomes. This is discussed by Jones, Schoonbroodt, and Tertilt (2008) in dynastic models, and Erosa, Fuster, and Restuccia (2010) in a life cycle context.

However, if we consider the profiles of unwanted and aborted pregnancies in the life cycle[8] the opportunity cost of time alone cannot explain all the facts in a unified way, since these rates behave remarkably similarly across education groups, despite differences in potential

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consider pregnancies ending between 1994 and 1995 and restrict my sample to 18 to 40 year old females.

[8] I take this information at face value. Rosenzweig and Schultz (1993) estimate that unwanted pregnancies in survey data might be over predicting the real figure due to ex-post rationalization of mothers.

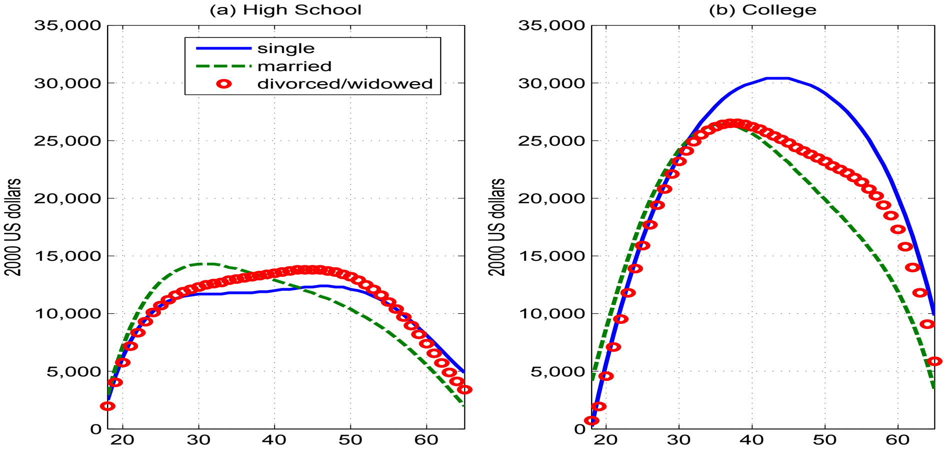

Figure 3: Annual Female Earnings in 2000 US$ (CPS: 1990-1995)

labor market rewards and thus, differences in the lifetime opportunity cost of children.

The quantitative theory I propose in the next section, assumes that the overall effectiveness of birth control usage is different across educational groups: given the same technology to prevent a pregnancy (pills, condoms, etc.), the college group is better at implementing it. For some intuition of the importance of this mechanism, consider earnings (figure 3) and fertility outcomes (figure 2). Any quantitative theory which relied only on the opportunity cost theory only, would have a chance to match the outcomes (births, unwanted and aborted pregnancies) for the college group but only births for the high school group: wages for this group are lower, hence their opportunity cost of bearing children is lower. However, the opportunity cost also determines wantedness of pregnancies: a model where the technology of contraception is *constant across educational groups* would predict a much lower rate in unwanted and aborted pregnancies for high school females, given the same preferences for children.

In table 1, I present some evidence on my proposed mechanism. In 1984, the following question was asked to women in the National Longitudinal Survey of Youth: *"when during the female monthly menstrual cycle is pregnancy most likely to occur?"*. In the table I present the fraction of females 25 to 27 years old (oldest cohorts in 1984) who correctly answered the question[9] by educational attainment.

Table 1: Fertility Knowledge, NLSY1979 Survey (1984 wave)

	HS	College
% Doesn't Know	54.4	29.9
% Knows	45.6	70.1

Table 1 shows that for the high school group, more than half (around 54%) didn't know the answer, while for the college group, a high percentage did know it (around 70%). This knowledge might play a complementary role in the effectiveness of birth control usage (for example, avoiding sexual activity during critical days) and in resulting differential rates of unwanted pregnancies given the heterogeneous awareness of this basic (and costless) piece of information across educational groups.

3 A Quantitative Model

The model environment is an economy populated by agents of different gender (males and females) and education level (high and low). I abstract from the education decisions in order to simplify the analysis. This can obviously introduce bias if individuals are self selected into those groups (e.g., dropping out of college due to an unplanned pregnancy); however, most births and abortions seem to take place after the age of 20 (see figure 1), hence this doesn't seem to be a significant issue. Agents live finite lives and face three types of exogenous and idiosyncratic shocks: to their life (survival shocks), to their household type (marital transition shocks) and to their earnings (shocks to the value of their market rewards). All agents derive utility from consumption and from the presence of children in the household. Agents supply labor inelastically to the market before retirement and every period they decide how much to consume and save for the future; they cannot borrow.

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[9] Answers were tabulated by week. Here I aggregate into those who answered correctly (2 weeks after the start of cycle) and those who pointed to other times during the cycle.

During the first part of their life-cycle, female agents are fertile (can conceive children) and decide on contraceptive efforts period by period. This effort influences imperfectly the probability of conception. Unwanted pregnancies can be aborted; both contraceptive effort and abortions come at a utility cost. After a birth, female agents must spend a fraction of their time at home rearing their children and this time cannot be substituted away (i.e., there is no child care). Males are not affected by this time requirement.

State space. Let z be the state space that defines an agent in this economy. From this point onwards in the discussion, I focus on the female's point of view:

$$z = \{i, a, k, e, e^*, m, \epsilon, \epsilon^*, i^*\} \tag{1}$$

asterisks represent values for spouses (when applicable). Age is indexed by $i = \{i_0, ..., I\}$, a is the amount of real assets in the household, $k = \{1, 2, ..., K\}$ represents the number of children living in the household, $e \in \{\underline{e}, \overline{e}\}$ represents the education type of the agent (low, high), $m = \{1, 2, 3\}$ is the type of household (1 = single, 2 = married, 3 = widowed/divorced[10]), and ϵ is the value of the multiplicative shock to labor earnings. For ease of exposition, in some sections of the paper I use the following partition of the sate space $\widetilde{z} = \{e, e^*, m, \epsilon, \epsilon^*, i^*\}$ so that $z = \{i, a, k\} \times \widetilde{z}$.

The Life-cycle proper. All agents start life at age i_0 (first year of adulthood) being one of two educational types: low ($\underline{e}$) or high ($\overline{e}$). This type doesn't change and can be considered as a decision taken before the events in the model. Agents can also start life as married or single and with or without children.

The maximum lifespan for all agents is of I years. Survival from age i to $i+1$ is subject to state dependent mortality risk, i.e., the probability of surviving an additional year depends on the gender and the educational type of the agent. I denote this probability as $\delta_{i,e}$ and $\delta^*_{i^*,e^*}$ for females and males respectively.

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[10] Features of widowed vs. divorced households are unified in a single state, since their distinctions in the data are not significant

With regard to labor markets, agents work until they reach age i_r. The retirement age is common for males and females. Female agents also make fertility decisions from i_0 to i_f, the last fertile age. This cut-off for the fertile period is common and known to all female agents.

Fertility and children. During their fertile years, females choose effort to determine the probability of a pregnancy. I denote this effort as $x \in \mathbb{R}$, which translates into a probability $\pi(x|i, m, e) \in (0, 1)$ of *no* conception (or status quo). This stochastic production function of no pregnancies depends on the age of the female agent (to capture biological constraints on women's reproductive systems), her marital status (since conception opportunities might differ if a mate is present or not) and her education. The exertion of this effort comes at a utility cost $C(x)$.

With complementary probability $(1 - \pi)$, a pregnancy occurs. The pregnancy is deemed as "unplanned/unwanted" if a positive amount of contraceptive effort was exerted. Agents have the opportunity of getting an abortion at a utility cost κ. If the pregnancy is intended the agent keeps the child and the household increases its size by one.[11]

I make the assumption that children are attached to females. I don't keep track of the age nor the sex of children in the household due to the computational burden of doing so. Instead, households face a constant hazard rate for the permanence of children in the household. I denote this hazard by s_k, which means that on average, children spend $1/s_k$ periods attached to their mothers.[12] Finally, no children can stay in the household after retirement of the mother.

Marital states. The transition through different marital status is stochastic and exogenous. The probability of going from m to m' (conditional on both spouses being alive, in case of agents being married) is given by $\Gamma_{i,e}(m'|m)$. I assume that mortality shocks hit the household before marital transition shocks.[13]

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[11] There is no child mortality risk nor multiple births. Also, note that the effort space is the real line, so trying to get a pregnancy can be identified with exerting a negative level of effort $x < 0$ which minimizes the probability of status quo (no pregnancy).

[12] This hazard rate is independent for each child in the household (if $k > 1$).

[13] This timing assumption simplifies the calculation of expectations over future states.

Markets. Agents sell their time to a spot market for labor, receiving a fixed price of w. They can also save positive amounts of resources, i.e., they can rent assets at the market rate r.

Labor endowments. Agents are endowed with state dependent efficiency profiles, $\varepsilon_{i,m,e}$ for females and $\varepsilon^*_{i^*,m^*,e^*}$ for males. They also face idiosyncratic and persistent multiplicative income shocks (ϵ and ϵ^*). The processes generating these shocks are also state dependent. Hence, for males of age i^*, marital status m^* and education level e^*, labor income is given by

$$w\epsilon^*\varepsilon^*_{i^*,m^*,e^*}$$

Note that w is the market rental rate for efficiency units of labor. On the other hand, if children are present in the household, females need to devote some time taking care of them. These time requirements are reflected in $b(m,k) \in (0,1)$, so that labor income of females/mothers is given by

$$w\epsilon\varepsilon_{i,m,e}(1-b(m,k))$$

Since I don't keep track of ages of children in the household, $b(m,k)$ is not time dependent. This simplifying assumption is in contrast of evidence that children require more time and money as they grow old.[14]

Preferences. Agents in the economy derive utility from per period consumption and the number of kids in the household. Hence, children are treated as durable goods in terms of utility and their characteristics (such as age and sex) are not qualities that enter agents utility function. In this paper I restrict attention to preferences that are separable in consumption and number of children of the form

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[14] For example, see Hotz and Miller (1988) and Attanasio, Low, and Sánchez-Marcos (2008)

$$u(c|z) + \gamma g(k)$$

Preferences for consumption depend on the characteristics of the household (z), namely, the number of members living under the same roof. This is to capture economies of scale in consumption and the idea that marriage might create consumption habits.[15]

Since the focus of this paper is on females and fertility, utility of married households is taken to be that of the female member. This could be the result of using unitary theories of the household or theories that allow for intra-household bargaining and the female having all the bargaining power. This assumption is restrictive, but necessary to keep this a feasible exercise. Finally, agents in this economy don't have the ability/desire of leaving bequests upon death and don't receive utility from their children once they leave the household.

The Dynamic problem when fertile. There are three distinct stages in the life-cycle of a woman in this model: (1) work-fertile stage, (2) work - infertile stage and (3) Retirement. I divide stage (1) into two subperiods. In subperiod 1, women make fertility decisions and in subperiod 2, they choose consumption and savings for the future. Before transiting to subperiod 1 again, households face an updating in their stock of children (due to kids leaving their mothers). The following bellman equation represents the problem of agents during sub-period 2 (once they have made contraceptive effort choices):

$$\begin{aligned} V(i,a,k,\widetilde{z}) &= \max_{c,y} u(c|z) + \gamma g(k) + \delta_{i,e}\beta E\left[v_f(i+1,a',k',\widetilde{z}')|z\right] \qquad (2) \\ st &: \\ c+y &= (1+r)a + w\epsilon_i\varepsilon_{i,m,e}(1-b(m,k)) \qquad \text{if } m=\{1,3\} \text{ or } m=2, i^* \geq i_r \\ c+y &= (1+r)a + w\epsilon_i\varepsilon_{i,2,e}(1-b(2,k)) + w\epsilon^*_{i^*}\varepsilon^*_{i^*,2,e^*} \qquad \text{if } m=2, i^* < i_r \\ a' &= \Phi(y,z'|z) \end{aligned}$$

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[15] See Hong and Ríos-Rull (2007).

The budget constraint accounts for different states, since married agents receive extra income from their spouses' labor, but only if the spouse is not retired ($i^* < i_r$). The Φ operator translates the amount of savings into next period assets given marital transitions and future states.[16] To update the number of children present in the household, I apply a binomial distribution with parameter s_k, i.e.,

$$P(k' = k_0|k) = \left(\frac{k!}{k!(k-k_0)!}\right) s_k^{k-k_0}(1-s_k)^{k_0}$$

Given optimal policies in subperiod 2, females make contraceptive effort choices in subperiod 1. The problem faced by them is:

$$\begin{aligned} v_f(i,a,k,\widetilde{z}) \;&=\; \max_x \;\; \pi(x|i,m,e)V(i,a,k,\widetilde{z}) \\ &\quad + [1-\pi(x|i,m,e)] \max \left\{ \begin{array}{c} V(i,a,k+1,\widetilde{z}), \\ V(i,a,k,\widetilde{z}) - \kappa \end{array} \right\} \\ &\quad - \; C(x) \end{aligned} \tag{4}$$

The value function at this stage is a convex combination of the continuation values with and without a new pregnancy. In the case of pregnancy (which occurs with probability $(1-\pi(\cdot))$), agents have the chance of having an abortion at utility cost κ. Note that even though there are discrete outcomes following this stage (number of children in the

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[16] The particular form of Φ is given by:

$$\Phi(y, z'|z) = \left\{ \begin{array}{ccl} y & if & (m'=2|m=2) \\ y & if & (m'=1,3|m=1,3) \\ y & if & (m'=3|m=2) \text{ (widowhood)} \\ 0.5y & if & (m'=3|m=2) \text{ (divorce)} \\ y+a^* & if & (m'=2|m=1) \end{array} \right. \tag{3}$$

where (m', m) refers to a transition from m to m' next period. For example, when going from $m = 2$ (married) to $m = 3$ (through divorce), assets next period are split and divided equally among agents, hence $a' = 0.5y$. Note that when going from $m = 1$ (single) to $m = 2$ (married), assets next period are given by current savings plus what the prospective spouse brings to the household. This last variable (a^*) is a random variable that depends on the distribution of single agents of the opposite sex in the economy.

household), the effort function convexifies the problem maintaining smoothness of the value function, which proves useful for solving (3) using standard continuous methods.[17]

This setup allows the probability of no conception to be flexible enough so that overall fertility is not only due to failed birth control but also as the result of conscious efforts of females to start a family. Specifically, this means that the domain of π is the entire real line (contraceptive effort can be negative, in order to maximize the probability of conception) and the cost function is always positive and restricted to be symmetric around zero. This general specification allows me to capture biological constraint on human fertility, which play a role in determining the optimal timing of births later in life.

The dynamic problem after fertile years. Once agents are past the fertile stage (cannot produce more children), they keep choosing optimal paths for consumption and savings until death. This stage in the life-cycle can also be divided into two: before and after retirement. Before retirement ($i \leq i_r$), the problem of the agent is:

$$\begin{aligned} V(i,a,k,\widetilde{z}) &= \max_{c,y} u(c|z) + \gamma g(k) + \delta_{i,e}\beta E\left[V(i+1,a',k',\widetilde{z}')|z\right] \qquad (5) \\ st &: \\ c+y &= (1+r)a + w\epsilon_i\varepsilon_{i,m,e}(1-b(m,k)) \qquad \text{if } m=\{1,3\} \text{ or } m=2, i^* \geq i_r \\ c+y &= (1+r)a + w\epsilon_i\varepsilon_{i,2,e}(1-b(2,k)) + w\epsilon^*_{i^*}\varepsilon^*_{i^*,2,e^*} \qquad \text{if } m=2, i^* < i_r \\ a' &= \Phi(y,z'|z) \end{aligned}$$

After retirement, the problem reduces to

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[17] Details of the numerical solution procedure are in the Appendix.

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\begin{aligned}
V(i,a,0,\widetilde{z}) &= \max_{c,y} u(c|z) + \gamma g(k=0) + \delta_{i,e}\beta E\left[V(i+1,a',0,\widetilde{z}')|z\right] \qquad (6)\\
st &: \\
c+y &= (1+r)a \qquad \text{if } m=\{1,3\} \text{ or } m=2, i^* \geq i_r \\
c+y &= (1+r)a + w\epsilon^*_{i^*}\varepsilon^*_{i^*,2,e^*} \qquad \text{if } m=2, i^* < i_r \\
a' &= \Phi(y,z'|z)
\end{aligned}
$$

at this stage no children are present in the household ($k = 0 \; \forall i \geq i_r$) and the only resources available for non-married agents are past savings. On the other hand, if agents are married to working age individuals, they enjoy the extra labor income $w\epsilon^*_{i^*}\varepsilon^*_{i^*,2,e^*}$.

4 Taking the Model to the Data

The solution of this model is a set of policy functions $x^{opt}(z|\Theta), y^{opt}(z|\Theta)$ for contraceptive effort and savings respectively, given the current state z and other parameters, Θ (including prices). As it's usual, analytical expressions for the optimal policies are unfeasible, so I approximate them using numerical solutions to an empirical model with the following quantitative features.

Demographics and life-cycle. A model period is one year. All agents start life at age 18 and cannot live longer than 95 years. Retirement is at 65 and the last fertile age is 40. Age specific mortality rates are taken from the National Center for Health Statistics and adjusted for educational attainment, as in Hong (2008).

I divide educational or skill types into those with at most a high school diploma or GED, and those with some post secondary education (college, community college, vocational school, etc.). To calculate the proportion of these types, I use the Current Population Survey (CPS) between 1990 and 1995. The proportion of high school individuals is around 40%.

The majority of agents start life as single and childless, but I allow some of them to be married and have children. The proportion of never married 18 year old females in the CPS is around 93% and females with kids is around 9%. When performing simulations of the model, I distribute women according to these statistics to determine their initial state.

Since non-married females can always find a (new) partner in the model, I need information on who they'd marry. Also from the CPS, I compute the proportion of couples by age and educational attainment of the partners, the age distribution of male partners for married females and the relative asset position of both non-married males and non-married females.[18] Given this information, I construct education-specific grids with probabilities of marrying someone of characteristics given by $\{e^*, i^*, a^*\}$ (education, age and assets of prospective husbands). Since I'm not computing equilibrium, this procedure doesn't check for internal consistency of measures of agents as in Hong and Ríos-Rull (2007), where all these probabilities are endogenous objects.

Transitions between marital states come from the Panel Study of Income Dynamics (PSID) for the years 1990-1995. I follow all heads of household older than 18 years old (inclusive) and compute annual age and education specific transition probabilities between three states: single, married and divorced/widowed.Given variable specification in the PSID, married couples include cohabitating couples.

Preferences. I use an additively separable specification for instantaneous, per period utility: $u(c|z) + \gamma g(k)$. The marginal utility from consumption depends on the size of a household:

$$u(c|z) \equiv \widetilde{u}\left(\frac{c}{1 + \mathbf{1}_{\{m=2\}}\phi_m + \mathbf{1}_{\{k>0\}}k\phi_k}\right) \tag{7}$$

where $\mathbf{1}_{\{cond\}}$ is the indicator function that takes a value of one when "*cond*" is true and zero otherwise; ϕ_m and ϕ_k are equivalence scales which discount consumption in married households and households with children respectively. If $\phi_m, \phi_k < 1$, economies of scale in

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[18] My proxy for individual assets is the sum of interest, dividend and rent income as defined in the March supplements of the CPS.

consumption exist in the household: expenditures to maintain the level of per capita utility constant, grow proportionally less than the number of household members. The specific functional forms for $\widetilde{u}$ and g are given by

$$\widetilde{u}(\mathbf{c}) = \frac{\mathbf{c}^{1-\eta_c} - 1}{1-\eta_c} \qquad\qquad g(k) = \frac{(1+k)^{1-\eta_k} - 1}{1-\eta_k} \tag{8}$$

Fertility. I use the following function for π, the probability of NO conception, given effort x:

$$\pi(x|i,m,e) = \frac{\exp\{x\}}{\exp\{x\} + \varphi_{i,m,e}\exp\{-x\}} \tag{9}$$

π is a modified logistic function with φ as a shift parameter. Note that the higher $\varphi_{i,m,e}$, the higher the probability of a pregnancy when effort (x) is positive (females trying to avoid fertility), which means that I can parameterize higher difficulty in controlling fertility by increasing this parameter. I further parameterize $\varphi_{i,m,e}$ by

$$\varphi_{i,m,e} = \widetilde{\varphi}_{i,0} + \mathbf{1}_{\{m=1,3\}}\widetilde{\varphi}_{i,s} + \mathbf{1}_{\{m=2\}}\widetilde{\varphi}_{i,m} + \mathbf{1}_{\{e=\underline{e}\}}\overline{\varphi}_i$$

where $\widetilde{\varphi}_{i,0}$ is a common fertility risk parameter, $\widetilde{\varphi}_{i,s}$ is the shifter for non-married females, $\widetilde{\varphi}_{i,m}$ is the shifter for married ones and $\overline{\varphi}_i$ is a shifter for females in the low education group. These parameters are age specific, in order to account for biological restrictions on conceptions. I assume that these parameters decay linearly to zero from age 30 onwards, so to parameterize $\varphi_{i,m,e}$, I only need 4 parameters. On the other hand, I parameterize the utility cost of exerting contraceptive effort as

$$C(x) = \frac{x^2}{2}\xi$$

Earnings and Labor Supply. Endowments of labor efficiency profiles come from the CPS (years 1990-1995). I calculate annual labor earnings for the two educational groups (high school and college), by age and marital status. As in Hong and Ríos-Rull (2007) and Hong (2008), I use annual earnings since they capture differences in the intensive margin of earnings by sex and marital status better than hourly earnings. To account for inflation, I adjust nominal values by the GDP deflator for the year 2000. I restrict attention to childless females throughout the sample period. For males, I don't make that distinction, since the change in income due to the presence of own children in the household is not significant.

I attribute the time cost of child-rearing $b(m, k)$ to annual labor income differentials of females in fertile age (18 to 40) by number of children. This is not exactly accounting for hours worked by number of children in the household; it stands alternatively for different ways in which a child might change earnings ability of the mother (e.g., getting a job with more flexible schedule but lower pay, getting a job with lower pay but closer to home, not getting tenured at an academic job or not being made partner at a law firm, etc.) other than through hours worked per week. Since the presence of children is persistent in the household, these "time costs" reflect both contemporaneous as well as dynamic effects (loss of occupation specific human capital, for example). The computed values are in table 2.

Children	$b(m = \{1, 3\}, k)$	$b(m = 2, k)$
0	0%	0%
1	5.9%	26.5%
2	16.9%	37.5%
3	41.0%	52.6%
4	61.3%	63.3%
5+	81.2%	72.8%

Table 2: Time cost of Children (in terms of full time work), CPS 1990-1995.

As seen from the table, time cost of children (or time away from the best paid market alternative) is increasing in the number of children present in the household. The cost increases faster in the number of kids for married women than for single ones. Also, these numbers imply that the time/career cost of children are proportionally higher for educated and married females.

For the earnings shocks, I use an AR(1) specification

$$\epsilon'_e = \rho_e \epsilon_e + \mu'_e \tag{10}$$

where $\mu_e \sim N(0, \sigma_e)$. These shocks are gender and education specific. I take values of ρ_e, σ_e (for $e = \{\underline{e}, \overline{e}\}$) from Hong (2008), who uses the PSID between 1986-1992 to compute maximum likelihood estimates. As is common, I discretize both continuous processes using the method proposed by Tauchen (1986).

5 Calibration

Given the partial equilibrium nature of the exercise, I set several model parameters exogenously. First, the rental price of efficiency units of labor w is normalized to 1. I set the interest rate to equal the average of the 1-year Treasury Bill Rate (monthly auction averages).[19] I let the discount factor β to be $1/(1+r)$. For equivalence scales, I use $\phi_m = 0.7$ and $\phi_k = 0.5$ (i.e., the OECD values).

The rest of the model parameters are determined jointly, by minimizing the square difference between data and model moments. The procedure is standard in the literature: (i) select which data targets to use (ii) guess initial values for model parameters (iii) solve the model and calculate optimal policies (iv) simulate life-cycles for a large number of individuals and compute model equivalents to the data targets (v) calculate the error of the iteration (the sum of square values of the difference between every data and model moment) (vi) if the error is less than a pre-specified tolerance, exit; if not, update parameters according to some predefined rule and repeat from step (iii) until convergence. This is a simplified simulated method of moments estimation procedure, where the weighting matrix for moments is the identity matrix.

The list of moments is as follows:

- Age profile of pregnancy rates by education: 46 moments (23 ages × 2 education levels)

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[19] Series id TB1YA, on the St. Louis Fed Economic Data webpage.

- Age profile of unwanted pregnancy rates by education: 46 moments
- Age profile of abortion rates by education: 46 moments

In total, there are 138 moments to match. On the other hand, the model has 9 parameters to be determined jointly (which makes this an overidentified system):

- curvature in the utility of consumption η_c
- curvature in the utility of children η_k
- multiplicative parameter in utility of children γ
- utility cost of an abortion κ
- utility cost of contraceptive effort ξ
- contraceptive ability parameters: $\widetilde{\varphi}_{i,0}$, $\widetilde{\varphi}_{i,s}$, $\widetilde{\varphi}_{i,m}$
- contraceptive ability shifter for low skill/education group $\overline{\varphi}_i$

6 Results

The calibrated parameters are in table 3 and the model fit is shown in figure 4

η_c	η_k	γ	κ	ξ	$\widetilde{\varphi}_0$	$\widetilde{\varphi}_s$	$\widetilde{\varphi}_m$	$\overline{\varphi}_i$
1.41	1.96	1.04	22.89	3.84	0.08	0.04	0.10	0.37

Table 3: Model Parameters

Results from the joint calibration procedure show that single females are worse in using birth control than married ones since $\widetilde{\varphi}_s = 0.08 > \widetilde{\varphi}_m = 0.04 >$. On the other hand, the parameter associated to the contraceptive ability of the high school group ($\overline{\varphi}_i$) turns out to be significantly higher, showing the importance of this mechanism.

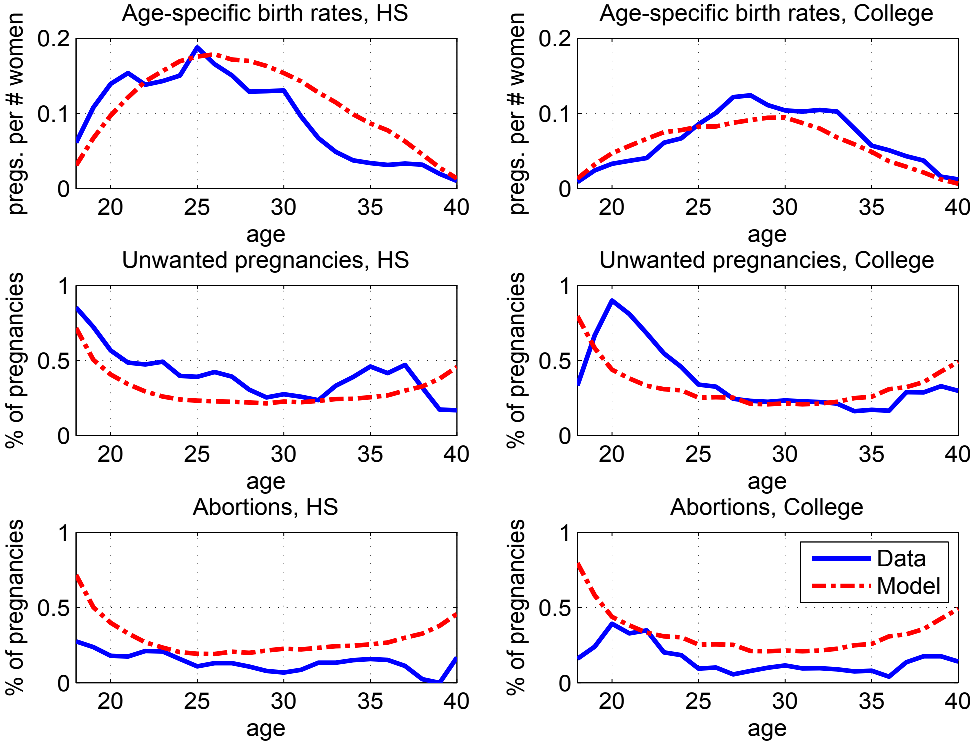

Figure 4: Goodness of fit of the Model

Figure 4 shows that the model matches the targeted models well. The model delivers the differential timing and level of overall births across educational groups (first row of figures), while matching relatively closely the timing profiles of unwanted (second row) and aborted pregnancies (third row).

Importance of Fertility Risk? I assess quantitatively the importance of the differential fertility risk process, by calibrating the model without the extra risk parameter for the high school group. In table 4, I present the parameters resulting from this exercise while figure 5 shows the simulations for that model.

	η_c	η_k	γ	κ	ξ	$\widetilde{\varphi}_0$	$\widetilde{\varphi}_s$	$\widetilde{\varphi}_m$	$\overline{\varphi}_i$
Baseline	1.41	1.96	1.04	22.89	3.84	0.08	0.04	0.10	0.37
No extra risk	1.74	2.58	0.84	22.50	3.18	0.19	0.15	$1.01E-06$	–

Table 4: Model Parameters

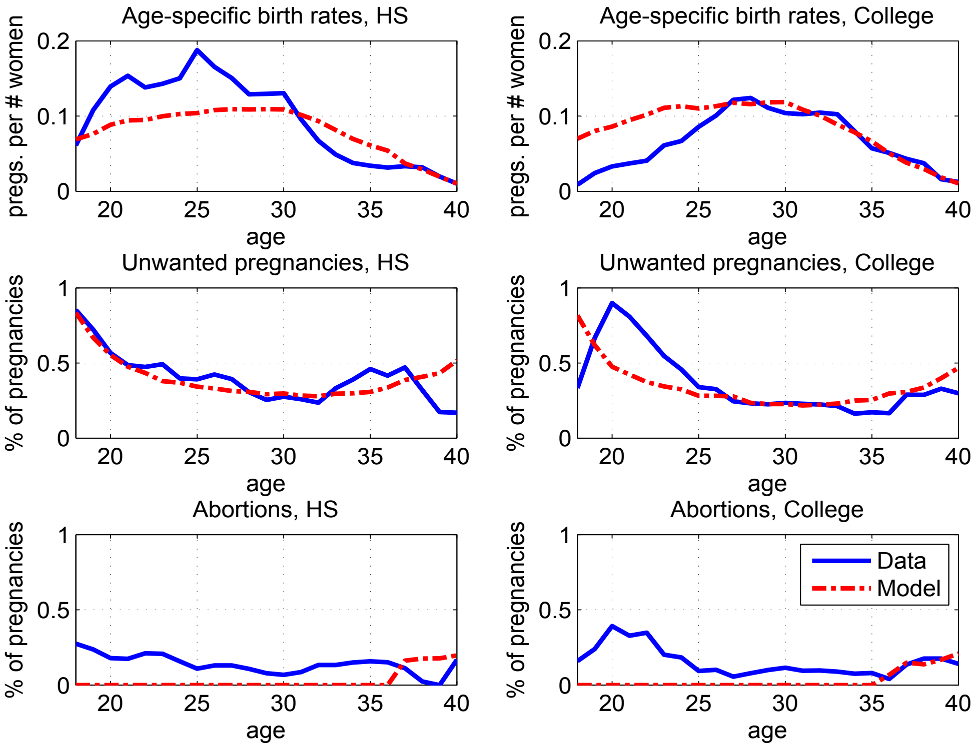

Figure 5: Model with no excess risk for High School group.

As seen from the table, the calibration procedure gives rise to different parameter values, where the most noticeable difference with the benchmark are in the fertility risk parameters, which have to readjust in order to match the selected data moments. However, as seen in figure 5, the model thus parameterized is unable to match the differential fertility rates by educational group nor the fact that abortion rate profiles are decreasing with age.

7 Conclusion

In this paper I study life-cycle fertility in the U.S., focusing on birth profile differences across educational groups (high school and college). To understand the facts on timing, number of births and abortions during the life-cycle, I develop a structural model where agents transit through different marital states, face idiosyncratic survival and earnings risk and capital markets are incomplete (individuals cannot borrow against their future earnings). In this setting, I include endogenous decisions about fertility, but where the outcome of those decisions cannot be controlled perfectly (i.e., fertility risk).

Using data from the National Survey of Family Growth, the Panel Study of Income Dynamics and the Current Population Survey, I calibrate the model using the simulated method of moments. From the exercise, I conclude that differential fertility risk (in the form of ability to control fertility plans) across education groups is the main determinant of differences in timing and levels of fertility. The main identification mechanism for the parameters in the model is the differential timing in the rates of unwanted pregnancies versus abortions for the different educational groups.

My results show that standard fertility theories, which rely solely on substitution effects to produce negative skill-fertility relationships, cannot account for life-cycle nor cross sectional facts. Furthermore, I show that fertility risk is an important source of life-cycle inequality and that life-cycle models built to ask questions about economic inequality in environments without complete insurance should incorporate endogenous and risky fertility decisions.

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8 Appendix (Not for publication)

8.1 Data

Figure 3 shows the profiles for labor endowments, computed from march supplements of the Current Population Survey (years 1990 to 1995). In the figure I show annual earnings for females, between ages 18 to 65, corrected for inflation using the GDP deflator for the year 2000. These profiles are smoothed using a 5th order polynomial.

To characterize the labor market, I also use gender and education specific idiosyncratic labor shocks. These shocks come from estimates from Hong (2008), who uses labor earnings data from the PSID to calculate the unobserved component of annual labor earnings. I use a standard discretization of the continuous AR(1) described in the paper. I choose to discretize the four processes (2 education groups and 2 genders) by a 3 state markov system. The standard in the literature is to use at least 5 states, but computational burden prevents me from using a more detailed shock structure. However, results in the paper don't rely in the dimensionality of these shocks.

Also from the CPS, I calculate the proportion of females (by education) married to college educated males (irrespective of presence of children in the household), in order to measure positive assortative matching in the marriage market. As seen in figure 6, marriage indeed shows the positive assortative matching property.

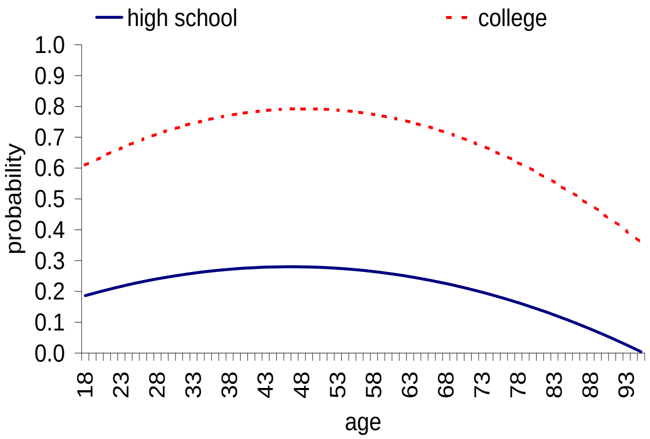


Figure 6: Probability of being married to college male, by education of female

I compute yearly survival probabilities by educational group using the information in Hong (2008). I interpolate his 5 year values and smooth the resulting series with a second order polynomial. The resulting probabilities for female individuals are in figure 7.

To calculate transitions through marital states, I use the Panel Study of Income Dynamics (PSID) for the years 1990 through 1995. I use heads of household and wives (as defined in the PSID) to compute the following probabilities, by education and age: probability of remaining single, the probability of remaining married and the probability of getting married condi-

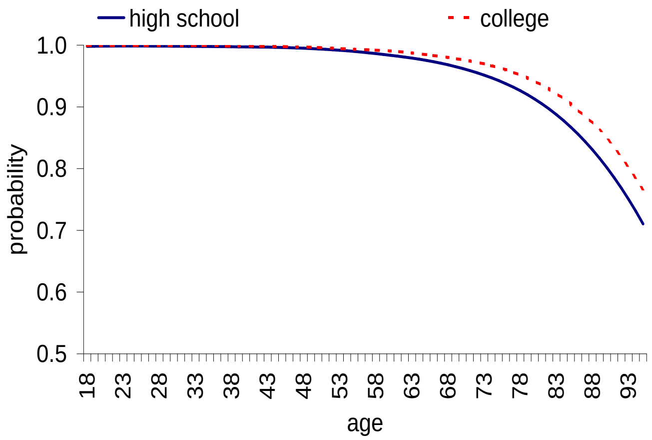


Figure 7: Survival probability by education

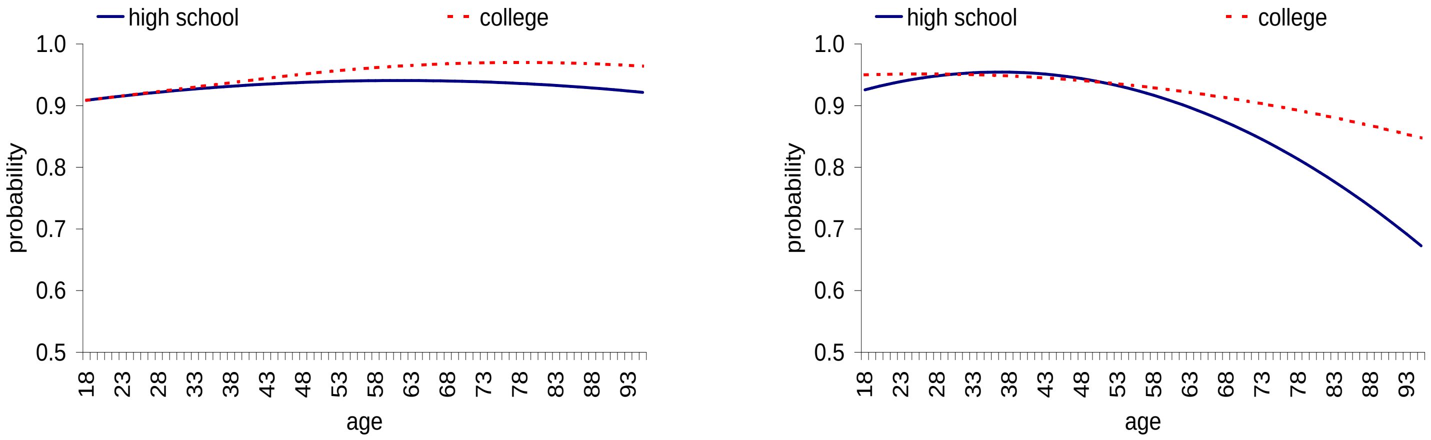
tional on being divorced/widowed. Given these three probabilities, I can span all transitions (e.g., some probabilities are zero by definition and others are just complements). I extrapolate these probabilities when necessary since the PSID doesn't have many observations for young/old heads of household. Given the short span of my chosen sample, individuals contribute at most 5 observations/years, making these probabilities a cross-section description of marital transitions during the mid 1990s in the U.S. Figure 8 shows these transitions.

I assume simple age and asset distribution of prospective male partners. For ages I consider only 3 possible alternatives: same age, one year older and two years older ($i^* = \{i, i+1, i+2\}$), each occurring with probabilities $P(i^* = i) = 0.4$, $P(i^* = i+1) = 0.41$ and $P(i^* = i+2) = 0.19$, which come from CPS data. Age of partners is important since they determine the extra income for the household in terms of partner's labor earnings and the probability of death (hence, transitioning to widowhood status). Since the profiles for both characteristics are smoothed, the tradeoff between accuracy and simplicity of the solution by assuming such a narrow age distribution is lessen.

For assets, I calculate from CPS data the average annual non-labor income (dividends, interests and rents) for both single males and females. Single males have on average 20% higher non-labor income than single females. Hence, I create a simple three point distribution for assets of prospective partners $a^* = \{1.1a, 1.2a, 1.3a\}$, centered around the fact that on average $a^*/a = 1.2$. This simple distribution is uniform (equal probabilities for each point). Changing this distribution doesn't alter any of the qualitative results from the exercise.

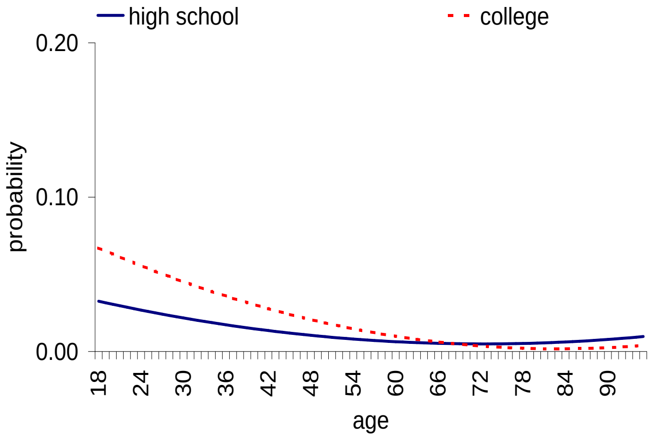
A note on Total Fertility Rates and cohort effects: throughout the paper, I assume no cohort effects in fertility rates. Although fertility has experienced significant changes during the 20th century, fertility rates for the cohorts considered in my analysis are quite stable. Figure 9 shows age specific fertility rates computed from the internet release of Vital Statistics of the United States for the year 1995 (tables 1-7).

The figure shows both total fertility rates for the cross section in 1995 and for cohorts



(a) single to single

(b) married to married



(c) div/wid to married

Figure 8: Transition probabilities for marital states

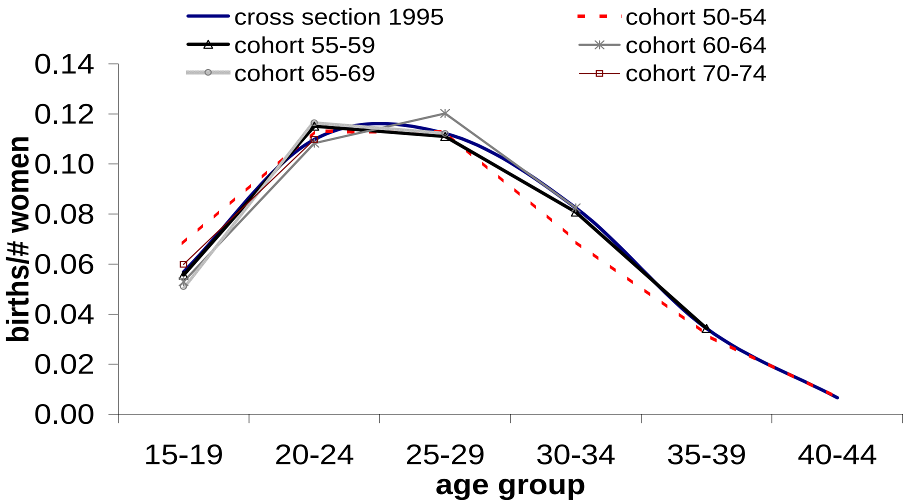


Figure 9: Age Specific Fertility Rates by birth cohort

(denoted by year of birth) across multiple survey years. The differences between the cross-sectional profile and actual cohort profiles is minimal. This comes from the fact that I am considering a small window in the life-cycle of cohorts that are close together (at most 20 years between births).

8.2 Computation and Calibration details

To solve the model, I use a Chebyshev regression (as described in Judd (1998) and Heer and Mauner (2004)) to approximate the optimal policies for savings and contraceptive effort and the value function along the asset space (the only continuous state variable in the model). My approximation is described by 7 collocation points and the use of a Chebyshev polynomial of degree 5. Increasing both the number of collocation points and/or the order of the polynomial doesn't improve the quality of the approximation significantly.

To accelerate the calibration algorithm, I use a Beowulf cluster with 20 processors. I parallelize at the parameter level, using the APPSPACK software available free on the web. See Gray and Kolda (2006) and Kolda (2005) for details.