

Life-Cycle Consumption: Can Single Agent Models Get it Right?

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June 2011

Abstract

In the quantitative macro literature, single agent models are heavily used to explain 'per-adult equivalent' household data, a common example being household consumption deflated by some form of equivalence scale. In this paper, we show theoretically and quantitatively that per-adult equivalent predictions from a model that explicitly features household size effects are under some configurations, but not all, very different from those of an otherwise comparable single agent model. The key driving force of these differences is the interaction between the amount of economies of scale arising from living in a multi-person household and how the utility of each household member from consumption is valued. Since demographics in our simple setup affects both household and per-adult equivalent consumption (a statistic usually treated as if it was free from demographic effects), models without demographics might attribute too much importance to other mechanisms to replicate life-cycle consumption facts.

Keywords: Consumption, Life-Cycle Models, Households

JEL classification: D12, D91, E21, J10

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1 Introduction

To understand life-cycle consumption, single agent models are frequently used given their tractability. In the quantitative macroeconomic literature, a standard approach entails extracting per-adult equivalent consumption facts from household survey data and use them as targets to be replicated by single agent models, which are also calibrated using per-adult equivalent household or individual worker’s income. Some recent papers in this vein include [Heathcote, Storesletten, and Violante \(2008\)](#), [Blundell, Low, and Preston \(2008\)](#), [Low and Pistaferri \(2010\)](#), [Fernández-Villaverde and Krueger \(2010\)](#) and [Guisar and Smith \(2010\)](#).

However, this approach faces the inherent challenge that consumption decisions might depend on household size and composition through non trivial channels. [Cubeddu and Ríos-Rull \(2003\)](#) for example, demonstrate that changes in marital status over the life-cycle affect aggregate savings in the same order of magnitude as idiosyncratic income uncertainty. Although this approach has the benefit of considering explicitly multi-person households, its drawback is that the model structure becomes very complicated and computationally intensive to solve. The same criticism can be made of models where demographic transitions occur endogenously, as in [Aiyagari, Greenwood, and Guer \(2000\)](#) and [Mazzocco, Ruiz, and Yamaguchi \(2007\)](#). In this paper, we abstract from these difficulties and propose a simple framework in order to understand the sources and magnitudes of bias when single agent models are used to make predictions for aggregate household consumption or consumption related measures (i.e. welfare). Specifically, we are interested in predictions from the standard incomplete markets model, which has become a workhorse in modern macroeconomic analysis.

We follow [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and [Gourinchas and Parker \(2002\)](#) and perform our analysis by extending the standard incomplete markets model to allow for deterministic changes in household size and composition during the life-cycle and let these changes affect optimal decisions on consumption and savings in a unitary model approach (what we label the *Demographics* model). But unlike these two papers, which use a general ‘demographic’ taste shifter in the utility function, we propose a formulation where economies of scales inside the household are considered explicitly through equivalence scales¹ and households might have a different preference

¹Equivalence scales are functions of household size and composition and typically used to deflate total household information (like consumption and income) by a number less than the actual household size. This approach has gained

for consumption depending on household size. This setup is useful since it can accommodate the case of single households (the *Single Agent* model) as a special case. Although the *Single Agent* model provides predictions for a single/bachelor consumption only and the *Demographics* model predicts total household consumption, a common practice in the literature is to transform household into individual (or per-adult equivalent), consumption and vice versa through equivalence scales, making predictions directly comparable.

Using a simple two period model of household consumption, we show theoretically that single agent models produce in general different predictions of per-adult equivalent consumption profiles than in the *Demographics* case: agents in *Single Agent* models ignore the fact that the relative price of consumption across periods in which family size is changing might be affected by economies of scale and direct preferences over household size.

We also perform a quantitative exercise to assess how different these predictions between the two approaches are in a standard model of life-cycle consumption with income uncertainty. We find that they crucially depend on the interaction between the degree of economies of scale in the household and how the utility of per-adult equivalent consumption of each household member is valued. This comprises the full range from negligible to substantial differences in key model moments as e.g. the peak age of per-adult equivalent consumption and the implied 'hump' in life-cycle consumption.

The key mechanism that drives our results is already used in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) to highlight the importance of demographics for the life-cycle profile of household consumption. Our key contribution is to show the importance of demographics for the life-cycle profile of per-adult equivalent consumption, a statistic usually treated as if it was free from demographic effects. Our formulation of the utility function permits a direct comparison to the recent empirical practice of obtaining per-adult equivalent consumption by dividing household consumption by an equivalence scale. Since demographics in our simple setup affects both household and per-adult equivalent consumption models, without demographics might attribute too much importance to other mechanisms to replicate life-cycle consumption facts.

importance in the macro literature: in the 2010 special issue of the Review of Economic Dynamics, equivalence scales are used to obtain consistent per-adult equivalent cross sectional facts for a wide range of countries (See [Krueger, Perri, Pistaferri, and Violante \(2010\)](#) for a general description and [Heathcote, Storesletten, and Violante \(2010\)](#) for the US economy). In general, equivalence scales are widely used to study income and consumption inequality, see e.g. [Cutler and Katz \(1992\)](#), [Krueger and Perri \(2006\)](#) and [Meyer and Sullivan \(2010\)](#). [Fernández-Villaverde and Krueger \(2007\)](#) discuss the properties of different types of equivalence scales and their effect on life-cycle consumption profiles.

The structure of the paper is as follows: in Section 2 we discuss our proposed preferences for the household and present theoretical predictions in a stylized two period framework. In Section 3 we discuss the model we use to quantify these theoretical predictions. In Section 4 we show the quantitative features of the model and the calibration strategy while Section 5 shows our main quantitative results. Section 6 presents our identification exercise, while in the last section, we conclude.

2 Demographics in a Life-Cycle Model

In this section we compare the optimal consumption allocations in the presence of household size changes over time from two models. The first model abstracts from these household size changes inside the model, but is 'calibrated' in a fashion that controls for these effects outside the model. The second model incorporates the household size changes directly in the problem of the household. The following paragraphs will be very explicit about the respective approaches and argue why and how a fair comparison between the two different models can be made. To keep the theoretical analysis as simple as possible we focus on a two period framework where the household experiences an exogenous but deterministic change in household size. In particular, we assume that the household size is one in the first period ($N_1 = 1$), e.g. a young person living alone, and larger than one in the second period ($N_2 > 1$), e.g. because a child is born. For the theoretical analysis all we need is a change in household size between the two periods whereas the quantitative analysis will feature differences in household composition, i.e. a distinction between the number of adults and children in the household. The household receives an income stream y_1 and y_2 , and can borrow (up to the natural borrowing constraint) and save at an interest r which without loss of generality is set to zero. Similarly, the discount factor is set to one, i.e. from the perspective of period one the utility in period two is not discounted.

A standard approach in macroeconomics (obviously with more than two periods) is to assume that households consist only of a single member, a single agent or bachelor household. The work by [Attanasio and Weber \(1993\)](#) and [Attanasio and Browning \(1995\)](#) shows how important demographics are for understanding the patterns of household consumption over the life-cycle. In order to keep on working with the bachelor household model, a response to these findings is to clean

household consumption data for household size and household composition, i.e. demographic, effects. One popular approach is to divide these data by an equivalence scale which transforms total household consumption into a per-adult equivalent consumption, against which the predictions of the bachelor household are then compared, see e.g. [Krueger and Perri \(2006\)](#), [Blundell, Low, and Preston \(2008\)](#), [Heathcote, Storesletten, and Violante \(2008\)](#) or [Low and Pistaferri \(2010\)](#).²

The three mechanisms through which household size affects the intra-temporal rate of transformation between expenditures and consumption services, and that are captured partially through equivalence scales, are family/public goods, economies of scale, and complementarities, see e.g. [Lazear and Michael \(1980\)](#). As a concrete example, consider the widely used OECD equivalence scale which is given by

$$\phi_{OECD} = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch}, \quad (1)$$

with N_{ad} being the number of adults and N_{ch} the number children in the household. According to Equation (1) it takes 1.7\$ of consumption expenditures to generate the same level of welfare out of consumption for a two adult household that 1\$ achieves for a single member household. [Fernández-Villaverde and Krueger \(2007\)](#) list a summary of representative equivalence scales, including the OECD scale, which are all normalized to one for single person households and are increasing in household size by less than one. In our concrete setup this implies that the equivalence scale ϕ_t equals one in the first period and is larger than one in the second period, i.e. $\phi_1 = 1$ and $\phi_2 > 1$.

To ensure consistency between the model and the data, income fed into the model is cleaned as well for household size and household composition effects. One popular approach is to divide household income with an equivalence scale as done by [Krueger and Perri \(2006\)](#) or [Blundell, Low, and Preston \(2008\)](#). If the same equivalence scale is used as for consumption, this strategy could be interpreted as dividing the per-period household budget constraint by the equivalence scale. We label this approach as the *Single Agent* model and the corresponding optimization problem is thus

$$\max_{c_{1,S}, c_{2,S}} U = u(c_{1,S}) + u(c_{2,S}) \quad (2)$$

²There are obviously other methods to create per-adult equivalent information from household data. One alternative method is to estimate household size/composition effects directly from micro data using least squares regressions, see e.g. [Aguiar and Hurst \(2009\)](#). Although studying heterogeneity in household size/composition (which is a pre-requisite to understand the regression methodology) is beyond the scope of our paper, this approach generates adjustments that can be trivially converted to an ad-hoc equivalence scales.

subject to

$$c_{1,S} + c_{2,S} = \frac{y_1}{\phi_1} + \frac{y_2}{\phi_2} \equiv Y_S. \quad (3)$$

Another alternative to this approach is to use only the household head's income as done by [Heathcote, Storesletten, and Violante \(2008\)](#) or [Low and Pistaferri \(2010\)](#). In the context of this simple two period model ϕ_2 could in this case be interpreted as the share of the household head's income from household income.

Not only do the consumption data come in household format but also the household consumption choices are made taking into account household size. We model this by letting household size N_t affect household utility, which is represented by the following optimization problem:

$$\max_{c_{1,D}, c_{2,D}} U = u(c_{1,D}, N_1) + u(c_{2,D}, N_2) \quad (4)$$

subject to

$$c_{1,D} + c_{2,D} = y_1 + y_2 \equiv Y_D. \quad (5)$$

For the utility function we employ the following specification:

$$u(c_t, N_t) = \delta(N_t)u\left(\frac{c_t}{\phi(N_t)}\right). \quad (6)$$

We label this setup as the *Demographics* model. Household utility $u(c_t, N_t)$ is the product of the utility from per-adult equivalent consumption $u\left(\frac{c_t}{\phi(N_t)}\right)$, and a utility weight δ as a function of household size N_t , reflecting in how far the households values the per-adult equivalent consumption enjoyed by each household member. If $\delta_t = 1$, the household would only care about per-capita utility in each period whereas for $\delta_t = N_t$ the sum of the per-capita utility over all household members would be relevant. For the moment, we are completely agnostic about how household size exactly affects δ_t . Furthermore, there is no need to assume that the same equivalence scale is used for obtaining a per-adult equivalence consumption and per-adult equivalent income, as e.g. mentioned above if household head's is fed into the model. We nevertheless use the same equivalence scale for both variables because it is the standard procedure in studies that investigate jointly income and consumption inequality, see e.g. [Cutler and Katz \(1992\)](#), [Krueger and Perri \(2006\)](#), [Meyer and Sullivan \(2010\)](#), and the 2010 special issue of the Review of Economic Dynamics. We will however

comment on how this affects our theoretical results.

Comparing the two setups, it becomes evident what we meant in the introduction, with the first, the *Single Agent* model abstracting from household (changes) inside the model, but being ‘calibrated’ in a fashion that controls for these effects outside the model. Household effects enter only via the budget constraint. In the second, the *Demographics* model household size (changes) affect utility directly but not via the budget constraint.

[Attanasio, Banks, Meghir, and Weber \(1999\)](#) were the first to let demographics affect household utility in a quantitative life-cycle model via a more general taste shifter $[\exp(\xi_1 N_{ad} + \xi_2 N_{ch})u(c)]$. [Gourinchas and Parker \(2002\)](#) follow their approach whereas [Fuchs-Schündeln \(2008\)](#) uses the same structure as in Equation (6) and sets $\delta_t = \phi_t$. Similarly, [Cubeddu and Ríos-Rull \(2003\)](#), and [Hong and Ríos-Rull \(2007\)](#) set $\delta = \min\{N_{ad}, 2\}$ considering only the household’s head’s and, if present, the spouse’s utility from per-adult equivalent consumption, ignoring any further dependents in the household.³ The latter three papers do not provide any further justification for their choice of the utility weight δ .

We think of this framework as the mildest departure from the *Single Agent* model which considers something akin to household size effects. The only additional twist in the *Demographics* model is that household size and composition affect the (marginal) utility of consumption. As a consequence, changes in household demographics over time impact the intertemporal allocation of consumption. However, and as in the *Single Agent* model, the optimal consumption saving choices are undertaken by a single decision maker who equates marginal utilities over time. This mechanism is already used in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) to highlight the importance of demographics for the life-cycle profile of household consumption. Our key contribution is to show the importance of demographics for the life-cycle profile of per-adult equivalent consumption. Our formulation of the utility function permits a direct comparison to the recent empirical practice of obtaining per-adult equivalent consumption via the division of household consumption by an equivalence scale. Note that the *Single Agent* model directly predicts an per-adult equivalent

³In [Cubeddu and Ríos-Rull \(2003\)](#), and [Hong and Ríos-Rull \(2007\)](#) couples solve a joint maximization problem with exogenously fixed utility weights (λ) for the household head’s (h) and the spouse’s (s) utility and choose next period’s asset holdings a' to maximize the following objective function: $\sum_{j=h,s} \lambda^j \left[u\left(\frac{c}{\phi(N_t)}\right) + \beta EV^j(a') \right]$. For their assumption of $\lambda^h = \lambda^s = 0.5$, the optimization problem boils down to $0.5 \left[2u\left(\frac{c}{\phi(N_t)}\right) + \beta \sum_{j=h,s} EV^j(a') \right]$. However, they consider stochastic transitions between marital states, making the comparison to a unitary model difficult.

consumption because the household receives a per-adult equivalent income. In the *Demographics* model, household consumption is predicted which then has to be deflated by the equivalence scale in order to be comparable, i.e. $\frac{c_{1,D}}{\phi_1} = \frac{c_{1,D}}{1}$ and $\frac{c_{2,D}}{\phi_2}$.

The following sections discuss how the *Single Agent* model performs relative to our benchmark, the *Demographics* model.

2.1 Consumption Profiles

Result 1. *The per-adult equivalent consumption profile in the Demographics model and Single Agent model coincide only if $\phi_2 = \delta_2$.*

This result can be immediately read of from the two Euler equations which for the *Demographics* model is given by

$$u'(c_{1,D}) = \frac{\delta_2}{\phi_2} u' \left(\frac{c_{2,D}}{\phi_2} \right), \quad (7)$$

and for the *Single Agent* model by

$$u'(c_{1,S}) = u'(c_{2,S}). \quad (8)$$

In both first-order conditions only per-adult equivalent consumption appears. For the *Single Agent* model the consumption levels $c_{1,S}$ and $c_{2,S}$ in fact reflect per-adult equivalent consumption because income as an input to the optimization problem has already been deflated by the equivalence scale. For the *Demographics* model it is obvious in the second period as the household receives the (marginal) utility from per-adult equivalent consumption $\frac{c_{2,D}}{\phi_2}$ which is however also true in the first period because household size is one in period one ($\phi_1 = 1$).

Equation (8) predicts a flat per-adult equivalent consumption profile for the *Single Agent* model. The per-adult equivalent consumption profile in the *Demographics* model is however only flat if $\phi_2 = \delta_2$ but upward sloping if $\delta_2 > \phi_2$, i.e. $c_{1,D} = \frac{c_{1,D}}{\phi_1} < \frac{c_{2,D}}{\phi_2}$, while the opposite is true for $\delta_2 < \phi_2$. The intuition behind this result can be best explained when decomposing the benefit of consuming one additional unit of consumption in the second period in the *Demographics* model which

1. is associated with the marginal utility of per-adult equivalent consumption in period one $\left[u' \left(\frac{c_{2,D}}{\phi_2} \right) \right]$
2. accrues to all household members reflected through the multiplication by the weighting factor

$[\delta_2]$

3. has to be divided by the equivalence scale $[\phi_2]$ because each household member does not get the full unit to consume but only the fraction $\frac{1}{\phi_2}$.

As an example, consider the case of the weighting factor being equal to household size, i.e. $\delta_2 = N_2$. The larger household size in period two provides an incentive to allocate more consumption to period two because the household enjoys a larger utility from consuming as each unit of per-adult equivalent consumption is weighted by δ_2 . However, in period two every unit of consumption has to be shared with more people which is reflected through the division with the equivalence scale ϕ_2 . This in turn reduces the incentive to allocate more consumption to period two. Since for all empirically estimated equivalence scales (see [Fernández-Villaverde and Krueger \(2007\)](#)), $\phi_2 < N_2$, the case of $\delta_2 = N_2$ implies that per-adult equivalent consumption in period two exceeds per-adult equivalent consumption in period one. Relative to period one, the absolute loss in consumption in period two because of the sharing across household members is outweighed by the fact that each household member enjoys the extra per-adult equivalent consumption. Interestingly, such a configuration provides an additional explanation for the hump observed in per-adult equivalent consumption documented in [Fernández-Villaverde and Krueger \(2007\)](#). The ratio $\frac{\delta_2}{\phi_2}$ can be interpreted as changing the effective discount factor in the Euler equation or alternatively as changing the relative price of per-adult equivalent consumption between two periods whenever there is a change in household size. This channel is ignored in the *Single Agent* model.

Note that this result is completely independent from the chosen procedure to construct the per-adult equivalent income measure feed in the *Single Agent* model.

2.2 Consumption Levels

Result 2. *Life-time per-adult equivalent consumption in the Demographics model coincides with life-time per-adult equivalent consumption in the Single Agent model, only if in the Demographics model period two household consumption $c_{2,D}$ and period two household income y_2 coincide.*

Life-time per-adult equivalent consumption from the *Demographics* model can be written as

$$C_D = c_{1,D} + \frac{c_{2,D}}{\phi_2} = \underbrace{y_1 + y_2 - c_{2,D}}_{c_{1,D}} + \frac{c_{2,D}}{\phi_2} = y_1 + y_2 - \left(1 - \frac{1}{\phi_2}\right) c_{2,D} \quad (9)$$

while in the *Single Agent* model life-time per-adult equivalent consumption equals life-time per-adult equivalent income Y_S (see also Equation (3)):

$$C_S = c_{1,s} + c_{2,s} = y_1 + \frac{y_2}{\phi_2} = Y_S \quad (10)$$

Hence, the difference in life-time per-adult equivalent consumption between the *Demographics* and the *Single Agent* model is given by

$$C_D - C_S = C_D - Y_S = (y_2 - c_{2,D}) \left(1 - \frac{1}{\phi_2}\right). \quad (11)$$

which proves Result 2. Whenever $y_2 > c_{2,D}$, i.e. the household in the *Demographics* model is a borrower, the life-time per-adult equivalent consumption under the *Demographics* model is larger than under the *Single Agent* model. The opposite is true for $y_2 < c_{2,D}$, i.e. when the household in the *Demographics* model is a saver.

The intuition for this result can be explained best with a concrete example. Assume that the household income is zero in the first period ($y_1 = 0$), and positive in the second period ($y_2 > 0$). In this case life-time per-adult equivalent income in the *Single Agent* model is $\frac{y_2}{\phi_2}$ which by the budget constraint equals life-time per-adult equivalent consumption. In the *Demographics* model in turn, the household has the income y_2 available for consumption. For any utility function satisfying the Inada condition period one consumption will be positive such that $c_{2,D} < y_2$. Given that household size is one in period one, in the calculation of life-time per-adult equivalent consumption in the *Demographics* model only period two consumption is deflated by the equivalence scale. Since $c_{2,D} < y_2$, “less” in absolute terms is lost through the deflation by the equivalence scale in the calculation of life-time per-adult equivalent consumption in the *Demographics* model compared to the *Single Agent* model.⁴

Essentially, Result 2 is the implication of a pure accounting exercise. The key driving force behind is that households can shift consumption between periods whereas income is predetermined, at least in any model with exogenous labor supply. If income and consumption allocations are not fully synchronized, then transforming household income to a per-adult equivalent drives a wedge between per-adult equivalent consumption in the *Demographics* model and per-adult equivalent

⁴More formally, for $y_1 = 0$ and $y_2 > 0$, $c_{2,D} < y_2$ implies that $C_D = y_2 - \frac{\phi_2 - 1}{\phi_2} c_{2,D} > y_2 - \frac{\phi_2 - 1}{\phi_2} y_2 = \frac{y_2}{\phi_2} = Y_S = C_S$.

income, and, as a direct consequence also, between per-adult equivalent consumption in the *Demographics* and *Single Agent* model. This wedge is however decreasing in the economies of scale, reflected in a smaller ϕ_2 , as the equalization simply matters less, see also Equation (11). If income would be divided by another equivalence scale than consumption, Result 2 may also hold if $c_{2,D} \neq y_2$ but only under very strict conditions.⁵

These differences in life-time per-adult equivalent consumption are also important in the presence of income heterogeneity. First, in the *Single Agent* model the timing of income matters as it determines life-time per-adult equivalent income. Even for the same life-time household income $y_1^A + y_2^A = y_1^B + y_2^B$ but a different timing $\frac{y_1^A}{y_2^A} \neq \frac{y_1^B}{y_2^B}$ life-time per-adult equivalent incomes differ in the *Single Agent* but not in the *Demographics* model. This implies an artificial inequality in life-time per-adult equivalent consumption in the *Single Agent* model that is not present in the *Demographics* model. Second, it is straightforward to show that for heterogeneity in life-time household income $y_1^A + y_2^A \neq y_1^B + y_2^B$ but the same timing of income $\frac{y_1^A}{y_2^A} = \frac{y_1^B}{y_2^B}$, the implied inequality in life-time per-adult equivalent consumption between the *Single Agent* and *Demographics* model is proportional to the differences in life-time per-adult equivalent consumption in the two models, i.e. $\frac{\text{Var}(C_S)}{\text{Var}(C_D)} = \left(\frac{C_S^A}{C_D^A}\right)^2$.

Note that the derivation and implications of Result 2 are completely independent of the relationship between δ_2 and ϕ_2 which do however determine $c_{2,D}$ and thus, for a given y_1 and y_2 , the relationship between the two per-adult equivalent consumption levels.

2.3 CRRA Preferences

In quantitative life-cycle models, CRRA preferences are the prevailing choice for the utility function. We now briefly discuss the role of the parameter of relative risk aversion in the *Demographics* model. For our simple setup given by Equations (4) and (5) we obtain closed form solutions for

⁵ Assume that period two income is adjusted for household size by a factor $\kappa_2 \neq \phi_2$, i.e. $\frac{y_2}{\kappa_2}$ with $\kappa_2 \geq 1$. κ_2 may be simply another equivalence scale or another procedure to transform household income into per-adult equivalent income. In this case Equation (11) would be given by $C_D - Y_S = y_2 \left(1 - \frac{1}{\kappa_2}\right) - c_{2,D} \left(1 - \frac{1}{\phi_2}\right)$ which is only equal to zero if $\kappa_2 = \frac{y_2}{y_2 - c_{2,D} \left(1 - \frac{1}{\phi_2}\right)} > 1$, as long as $\phi_2 > 1$ which is the case for all empirically estimated equivalence scales, see Fernández-Villaverde and Krueger (2007).

the optimal per-adult equivalent consumption allocation

$$c_{1,D} = \frac{1}{1 + \left(\frac{\delta_2}{\phi_2}\right)^{\frac{1}{\alpha}} \phi_2} \quad \text{and} \quad \frac{c_{2,D}}{\phi_2} = \frac{\left(\frac{\delta_2}{\phi_2}\right)^{\frac{1}{\alpha}}}{1 + \left(\frac{\delta_2}{\phi_2}\right)^{\frac{1}{\alpha}} \phi_2}. \quad (12)$$

Since CRRA preferences are just a special case of the general utility function discussed before, we can see here again that it is only the ratio $\frac{\delta_2}{\phi_2}$ which determines the profile of the per-adult equivalent consumption. Larger values of α imply a flatter profile, as the lower intertemporal elasticity of substitution decreases the willingness to have differences in per-adult equivalent consumption between the two periods.

While the profile of equivalized consumption is entirely determined by $\left(\frac{\delta_2}{\phi_2}\right)^{\frac{1}{\alpha}}$, the profile of household consumption, i.e. $c_{2,D} - c_{1,D}$, is in turn determined by $\left(\frac{\delta_2}{\phi_2}\right)^{\frac{1}{\alpha}} \phi_2$. Any choice of α exceeding the threshold value of $\bar{\alpha} = 1 - \frac{\ln \delta_2}{\ln \phi_2}$ features therefore an increasing household consumption profile.

2.4 Summary

Two mechanisms introduce a bias into per-adult equivalent consumption predicted by the *Single Agent* model relative to the *Demographics* model. First, the *Single Agent* model ignores the change in the effective discount factor or relative price of per-adult equivalent consumption induced by changes in household size in the presence of economies of scale. With CRRA preferences this channel loses importance as the coefficient of relative risk aversion increases or equivalently as the intertemporal elasticity of substitution decreases. Second, as an outcome of a pure accounting exercise, the *Single Agent* model is generally associated with a different life-time per-adult equivalent consumption than in the *Demographics* model, which in the presence of income heterogeneity, feeds into inequality measures.

3 Quantitative Model

By constructing a quantitative model, our aim is to test and evaluate the implications of our theoretical analysis with a simple, stripped-down version of a standard incomplete markets life-cycle model, which can then be compared to actual US data.

Households start their economic life in period t_0 with zero assets. During their working life until period t_w they receive a stochastic income in every period. There is no labor supply choice. From period $t_w + 1$ onwards households are retired and have to live from their accumulated savings during working life and social security benefits. Life ends with certainty at age T and households do not leave bequests and cannot die with debt. Households have access to a risk-free bond a which pays the interest rate r and can borrow at the same interest rate up to the natural borrowing constraint, i.e. an age specific level $a_{min,t}$ of debt that they can repay for sure.

In the *Demographics* model household size changes over the life-cycle deterministically as in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and [Gourinchas and Parker \(2002\)](#) and is homogenous across all households. The maximization problem is given by

$$\max_{\{a_{t+1}\}_{t=t_0}^{T-1}} E_0 \sum_{t=t_0}^T \beta^{t-t_0} N_t u \left(\frac{c_t}{\phi_t} \right) \quad \text{subject to} \quad (13)$$

$$c_t + a_{t+1} \leq (1+r)a_t + (1-\tau)y_t \quad \forall t \leq t_w \quad (14)$$

$$c_t + a_{t+1} \leq (1+r)a_t + p(\bar{y}) \quad \forall t_w < t \leq T \quad (15)$$

$$a_{t+1} \geq a_{min,t} \quad (16)$$

where ϕ is a function of household size and its composition ($N_{ad,t}$ and $N_{ch,t}$). Income y_t is stochastic during working life, i.e. as long as $t \leq t_w$, and given by the following process:

$$\ln y_t = \varrho_t + \epsilon_t, \quad (17)$$

where ϱ_t is an age-dependent, exogenous experience profile and

$$\epsilon_t = \rho\epsilon_{t-1} + \varepsilon_t \quad \text{with } \varepsilon_t \sim N(0, \sigma^2). \quad (18)$$

During retirement, i.e. for $t_w < t \leq T$, households receive fixed social security contributions that may depend on the realization of income over the working life with $\bar{y} = \frac{1}{t_w - t_0 + 1} \sum_{t_0}^{t_w} y_t$.

The Euler equation to this problem is given by

$$\frac{N_t}{\phi_t} u' \left(\frac{c_t}{\phi_t} \right) = \beta(1+r) \frac{N_{t+1}}{\phi_{t+1}} E_t \left[u' \left(\frac{c_{t+1}}{\phi_{t+1}} \right) \right]. \quad (19)$$

The structure of the *Single Agent* problem is very similar. Demographics do not affect the utility function while income y_t is deflated by household size through equivalence scales ϕ_t :

$$\max_{\{a_{t+1}\}_{t=0}^{T-1}} E_0 \sum_{t=0}^{T-1} \beta^{t-t_0} u(c_t) \quad \text{subject to} \quad (20)$$

$$c_t + a_{t+1} \leq (1+r)a_t + \frac{y_t}{\phi_t} \quad (21)$$

$$a_{t+1} \geq a_{\min,t}, \quad (22)$$

with y_t following the same process as for the *Demographics* model.

The Euler equation to this problem is given by

$$u'(c_t) = \beta(1+r) E_t [u'(c_{t+1})]. \quad (23)$$

As in the theoretical analysis and in line with the studies that investigate jointly income and consumption inequality, see e.g. [Cutler and Katz \(1992\)](#), [Krueger and Perri \(2006\)](#), [Meyer and Sullivan \(2010\)](#), and the 2010 special issue of the Review of Economic Dynamics, we use the same equivalence scale for computing per-adult equivalent income and per-adult equivalent consumption.

4 Quantitative Features of the Model

A model period is one year. Agents start life at age 25, retire when 65 and live with certainty until age 75. To maintain simplicity, they receive a common pension, independent of life-cycle wages. We set the interest rate at 2.25%, the average of the 1 year t-bill minus the inflation rate between 1984 and 2003 (IMF statistics) and the CRRA coefficient α to 1.57, the same value used in [Attanasio, Banks, Meghir, and Weber \(1999\)](#).

In the next section, we use as a benchmark the square root scale $\phi_t^{SQR} = \sqrt{N_{ad} + N_{ch}}$, and compare our results with the OECD and the Nelson scales. These choices follow closely the discussion of equivalence scales in [Fernández-Villaverde and Krueger \(2007\)](#). The OECD scale has

the lowest economies of scale while the opposite is true for the Nelson scale. The square root scale is almost identical to the 'Mean' scale in [Fernández-Villaverde and Krueger \(2007\)](#) which is their preferred choice.⁶

As for utility weights, we remain agnostic and compare three cases: (i) $\delta_t = 1$ represents the case when households do not value household size; (ii) $\delta_t = N_t = N_{ad,t} + N_{ch,t}$ is the opposite, since households always enjoy having more members; and (iii) an intermediate case when $\delta_t = \phi_t$, i.e., we let the utility weight take the same value as the equivalence scale.

4.1 Income

We use data from the Current Population Survey, from 1984 to 2003, in particular the March supplements for years 1985 to 2004, given that questions about income are retrospective. We use total wage income (deflated by CPI-U, leaving amounts in 2000 US dollars), and apply the tax formula of [Gouveia and Strauss \(1994\)](#) to get after-tax income.⁷

We construct total household income $W_{i\tau}$ for household i observed in year τ , as the sum of individual incomes in the household for all households with at least one full time/full year worker. The latter is defined as someone who worked more than 40 hours per week and more than 40 weeks per year and earned more than \$2 per hour. Then, we estimate the following regression:

$$\log\left(\frac{W_{i\tau}}{\phi_{i\tau}}\right) = D_{i\tau}^{age} \varrho^{age} + X_{i\tau}\gamma + \epsilon_{i\tau} \quad (24)$$

where $\phi_{i\tau}$ is an equivalence scale, $D_{i\tau}^{age}$ represents a set of age dummies of the head of household, ϱ^{age} and γ are estimated coefficients and ϵ are estimation errors. Note that for the *Demographics* model we use household income for the estimation, i.e. $\phi_{i\tau} = 1 \forall i, \tau$. We also control for cohort effects and time effects by introducing birth year and year dummies in $X_{i\tau}$.⁸

⁶For explicit formulations of the different equivalence scales used in empirical consumption literature, see Table 1 in [Fernández-Villaverde and Krueger \(2007\)](#).

⁷Wage income in the CPS is pre tax income.

⁸Since year dummies are perfectly collinear with age and birth cohort dummies, we follow [Fernández-Villaverde and Krueger \(2007\)](#) and [Aguar and Hurst \(2009\)](#) and include normalized year dummies instead, such that for each year τ

$$\sum_{\tau} \gamma_{\tau} = 0 \quad \text{and} \quad \sum_{\tau} \tau \gamma_{\tau} = 0$$

where $\{\gamma_{\tau}\}$ are the coefficients associated to these normalized year dummies. This procedure was initially proposed by [Deaton and Paxson \(1994\)](#). To compare life-cycle profiles across different cohorts/time periods, we normalize the estimated coefficients associated to age dummies by adding the effect of a particular cohort/time. More specifically,

Figure 1: Experience Profile for Households

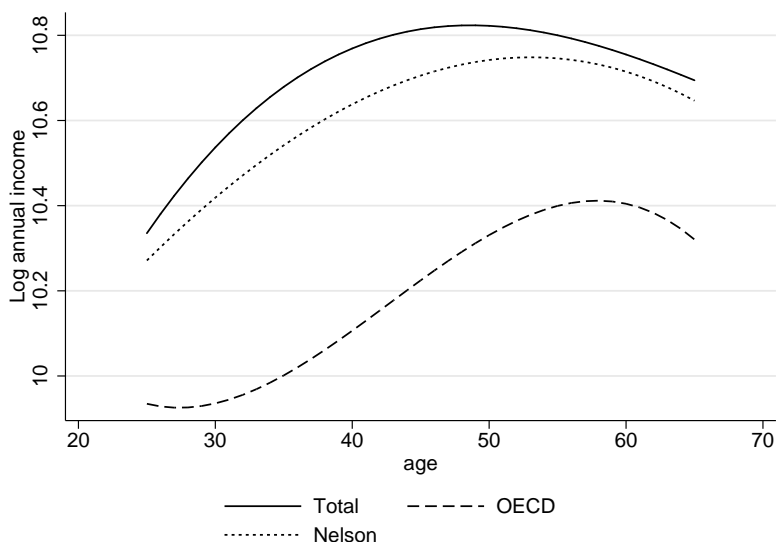


Table 1: Calibrated Parameters

ρ	σ	σ_0
0.9906	0.0189	0.1575

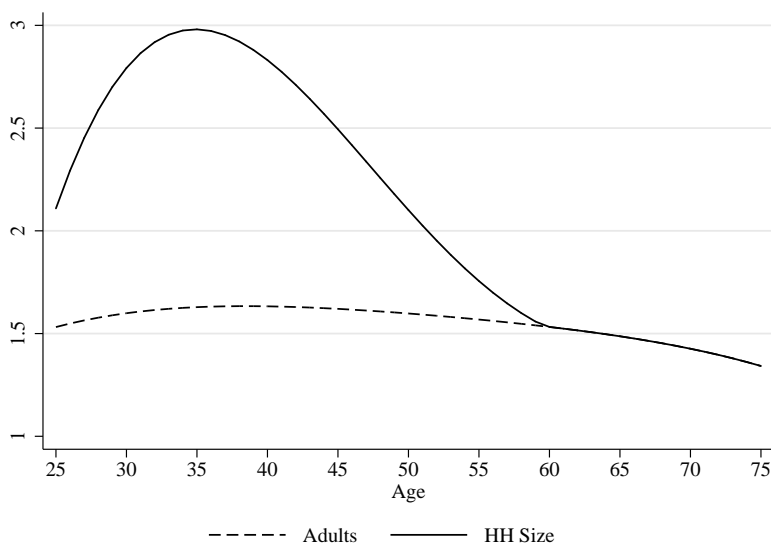
From this estimation, we are interested in the regression coefficients associated with age dummies of the household head (experience profiles in the model). In our exercise below, we use smoothed profiles, which we show in Figure 1 for different choices of equivalence scales.

From the estimation residuals, we calibrate the income process in (18). Our calibration procedure is standard and follows Storesletten, Telmer, and Yaron (2004): we pick values of ρ and σ in order to minimize the square difference between the profile of observed cross-sectional variances of income and the simulated one (given the chosen parameters). We also pick values of σ_0 , the standard deviation for the unconditional distribution of the first income shock ε_0 in order to match the cross sectional variance of income for our first age group (25 years old). We present these values in Table 1. We discretize this calibrated process using the Rouwenhorst method, using 20 points for the shock space. This methodology is specially suited for our case, given the high persistence of the process, see the discussion in Kopecky and Suen (2010).

To maintain full comparability with our simple theoretical model, we perform an *ex-post equiv-*

we picked the cohort corresponding to the median age at the last observed year.

Figure 2: Profiles for Household Size and Composition



Note: Both figures are constructed using data from the CPS, 1983-2003.

alization procedure for the income process in the *Single Agent* model: we use the same calibrated income profiles and shocks in both the *Demographics* and *Single Agent* models (the calibration in Figure 1 and Table 1) and then feed the per-adult equivalent experience profiles to the *Single Agent* model. Besides making the quantitative model more comparable to the theoretical model, this approach maintains the same shock structure across considered equivalence scales, making the comparison of biases more direct, since no extra 'noise' is being introduced by different volatility parameters. This would be the case for an alternative approach, or an *ex-ante equalization*: estimating Equation (24) with a particular equivalence scale $\phi_{i\tau}$, resulting in different age profiles and calibrated income shocks for the *Single Agent* model. Since then the income would have already been turned into per-adult equivalents *ex-ante*, there would be no need to do so in budget constraint (21).⁹

We assume that each household receives the same social security benefits with a replacement rate of 0.45 of the average cross-sectional net income. This replacement rate corresponds to what a worker earning average labor income in each year during her working life would be entitled to,

⁹In our computations below, we do not find major differences between the *ex-post* or *ex-ante equalization* strategies, so we show results only for the former. The results of these exercises are available on request.

see [Mitchell and Phillips \(2006\)](#).

4.2 Family Structure

We use the March supplements of the CPS for years 1984 to 2003.¹⁰ For each household, we count the number of adults (individuals age 17+) and the number of children: individuals age 16 or less who are identified as being the “child” of an adult in the household. We compute two separate profiles: one for number of adults and one for number of children. As above, we run dummy regressions to extract life-cycle profiles, where the considered age is that of the head (irrespective of gender) and control for cohort and year effects. After extracting these life-cycle profiles, we smooth them using a cubic polynomial in age, and restrict the number of children to zero after age 60. The results of this procedure are in [Figure 2](#).

As in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and [Gourinchas and Parker \(2002\)](#), the number for adults and children in the household over the life-cycle are not integers. The transformation into adult equivalents for the Square Root and OECD scale is trivial; for the Nelson scale, we use

$$\phi(N_{ad}, N_{ch}) = 1 + 0.06(N_{ad} - 1) + 0.1\min\{1, N_{ch}\} + 0.07\max\{0, N_{ch} - 1\}.$$

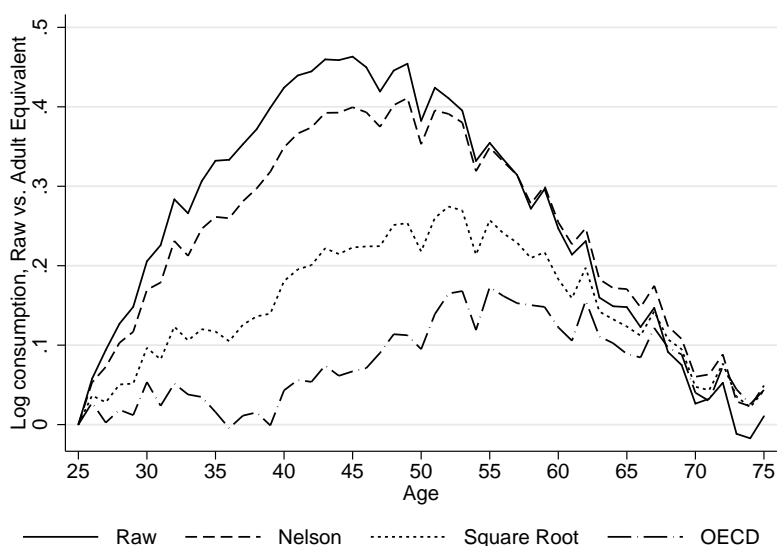
A similar adjustment would need to be done with all other equivalence scales that distinguish between the order of additional children.

4.3 Discount Factor

Usually the discount factor β is calibrated in order to match the economy-wide prevailing wealth or capital to output ratio. We deviate from this strategy in order to highlight the models’ mechanisms. In particular, we calibrate β such that per adult equivalent consumption is the same at age 25 and at age 75, a fact that can be observed for US data (see next section, [Figure 3](#)). This target is attractive because it is essentially the same across the different equivalence scales and leaves other moments, like e.g. the peak age of consumption, as moments to evaluate the models’ predictions.

¹⁰Since we are only interested in the average household size and composition by age rather than the evolution over the life-cycle for an individual, we use the CPS instead of the PSID because of the substantially larger sample size.

Figure 3: Per-Adult Equivalent Consumption Relative to Age 25



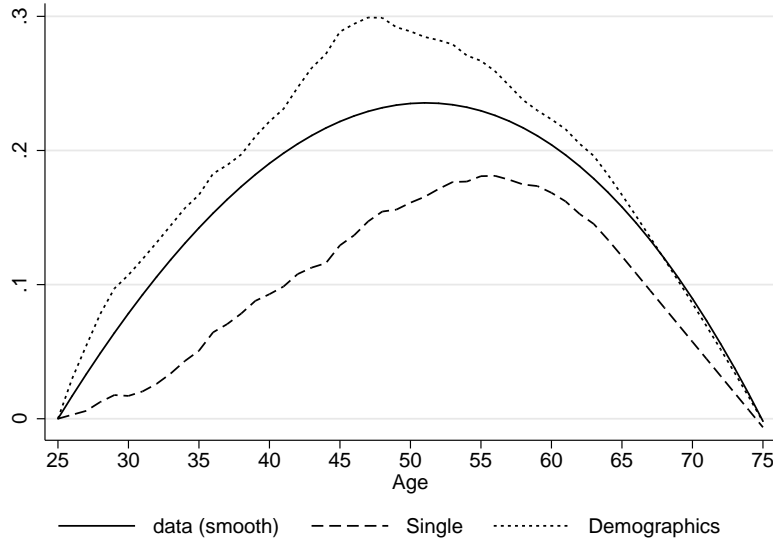
5 Results

In this section we compare the implications from each model (*Single Agent* and *Demographics*) against evidence on household consumption from the survey of Consumer Expenditures (CEX) for the years 1984 to 2003.¹¹ We use the definition of nondurables in [Aguiar and Hurst \(2009\)](#), which consists of household expenditures not including housing services.

From the CEX we extract life-cycle profiles of per adult equivalent household consumption in a similar way as we do for income profiles: we estimate a regression with age dummies controlling for both cohort and time effects. Figure 3 shows the coefficients associated to the age dummies in the regression with the log of nondurable consumption as a dependent variable, when we control for different equivalence scales. As in [Fernández-Villaverde and Krueger \(2007\)](#) and [Aguiar and Hurst \(2009\)](#), we normalize the consumption profile with respect to that of age 25. The figure shows the well known 'hump' shape of household consumption and the fact that the level of consumption at age 25 is almost the same as the one at age 75 (we use this fact to calibrate β in the model, as explained above). As seen in the figure, deflating total household consumption by the Nelson scale produces a per adult equivalent profile that follows closely total household consumption, due to the

¹¹As in [Fernández-Villaverde and Krueger \(2007\)](#), we ignore years 1982 and 1983 due to methodological differences in the survey.

Figure 4: Per-Adult Equivalent Consumption, Model vs. Data, Square Root scale



Note: Calibration of β (discount factor) implies a value of 0.959 for both models.

fact that this equivalence scale does not increase significantly when household size changes, given its implied economies of scale. The opposite is true for the OECD scale, which produces the flattest per adult equivalent consumption profile. Finally, the square root scale produces an intermediate profile.

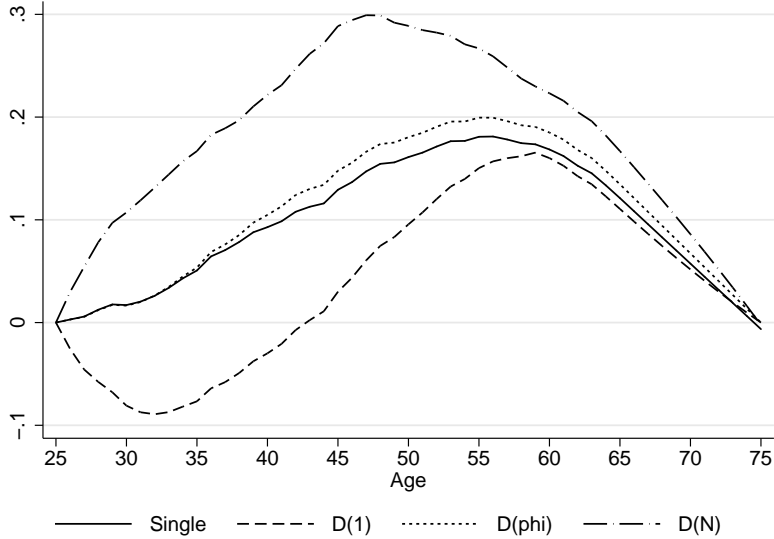
5.1 Per-Adult Equivalent Consumption

For expositional purposes, in the next two figures we focus our attention to the square root equivalence scale ($\phi_t = \sqrt{N_t}$), which is very close to the mean value of the different equivalence scales reported in [Fernández-Villaverde and Krueger \(2007\)](#) and their preferred choice.

For each model, we simulate fifty thousand life-cycles and compute profiles for mean household consumption in per adult equivalent terms. This is shown in [Figure 4](#) for the case when the utility weight in the *Demographics* model is set equal to N , while [Figure 5](#) presents the profiles for the *Demographics* model under different utility weights $\delta_t = \{1, \phi_t, N_t\}$.

In [Figure 4](#) we compare the predictions for the *Single Agent* and *Demographics* models and also show the smoothed profile from the CEX data. Our analysis confirms the result we derived in the theoretical section: whether the model incorporates economies of scale inside the problem of

Figure 5: Per-Adult Equivalent Consumption, Different δ_t s



Note: D stands for the *Demographics* model, and the figure in parenthesis, represents the utility weight (δ_t) considered: 1, equivalence scale (ϕ) and household size N . Table 4 in the Appendix lists the calibrated β s for each model.

the agent (as in the *Demographics* model) or uses equivalence scales only as an accounting device (the *Single Agent* case) has strong quantitative implications for the size of the predicted 'hump' in per-adult equivalent consumption, even after controlling for household size via equivalence scales. In this particular case (square root scale and $\delta_t = N_t$), we see that both models are relatively close to the data, but that the *Demographics* model predicts a more pronounced midlife hump and earlier peaks than the *Single Agent* model. The calibrated value of β together with the choice of the interest rate ($r = 2.25$) implies that in the *Single Agent* model, the household would like to have a decreasing consumption profile over the life-cycle. The presence of income uncertainty however makes the household accumulate precautionary savings (recall that we assume natural borrowing constraints), which shifts the peak age of consumption to age 55 and induces a 20% increase in consumption at that age relative to age 25. The earlier peak age in the *Demographics* model of per-adult equivalent consumption (age 47) and the higher consumption relative to age 25 at this age (30%) stems, on the other hand, directly from 'demographic' effects. With $\delta_t = N_t$, the household prefers to have a higher per-adult equivalent consumption when household size is large.

Table 2: Peak Age and Ratio of Per-Adult Equivalent Cons. for Different Equivalence Scales

	OECD		Square Root		Nelson	
	Peak Age	Ratio (%)	Peak Age	Ratio (%)	Peak Age	Ratio (%)
Data	59	11.8	51	23.5	46	39.5
Demographics Model						
$\delta_t = 1$	59	14.3	59	16.5	56	16.0
$\delta_t = \phi$	55	23.2	55	19.9	53	15.4
$\delta_t = N$	48	30.0	47	29.9	42	32.2
Single Agent Model	56	19.7	56	18.1	53	14.8

Note: *Peak Age* refers to the age where per-adult equivalent consumption is the highest, while *Ratio* reflects by how much % per-adult equivalent consumption at *Peak Age* exceeds per-adult equivalent consumption at age 25. The figures for the data refer to the smoothed profiles. Table 4 lists the calibrated β s for each model, and Figures 7 and 8 the complete per-adult equivalent consumption profiles relative to age 25 for the OECD and Nelson scale, respectively.

Still, per-adult equivalent consumption peaks much later than household size (around age 35, see Figure 2) due to income uncertainty.

By inspecting Figure 5, we see that the value of the utility weight is very important for determining the size and shape of per adult equivalent consumption profiles over the life-cycle. As described in the theoretical section, the case when the utility weight is equal to the equivalence scale ($\delta_t = \phi_t$) we get similar results as with the *Single Agent* model. When $\delta_t = 1$, per adult equivalent consumption actually decreases in the early part of the life-cycle, tracking the increase in household size in the opposite direction. In this case, consuming when household size is large is only costly because it has to be shared among a lot of members which is however not valued by the household.

In Table 2, we present all combinations of the three border cases for utility weights, $\delta_t = \{1, \phi_t, N_t\}$, and equivalence scales across *Demographics* and *Single Agent* models. The columns are arranged from lowest to highest implied economies of scale in the household (OECD, Square Root and the Nelson scale, respectively). For both US data and each model, we compute the age at which per-adult equivalent consumption is the highest (*Peak Age*) and the percentage increase in per-adult equivalent consumption relative to age 25 (*Ratio (%)*). Table 4 in the Appendix lists the calibrated β s for each model. All of them are such that in the absence of uncertainty the

household in the *Single Agent* model or in the *Demographics* model, if household size would be constant between two periods, would prefer a decreasing consumption profile. Note that we do not view these results primarily as a data fitting exercise but rather as a means to show how the different models work and in how far they can contribute to explain the differences in the data across the different equivalence scales.

Reflecting the information from Figure 3, we see that for the CEX data, the peak age is reduced when total household data is deflated by scales of decreasing value (increasing implied economies of scale in the household), while the difference between log consumption at that age relative to age 25 log consumption increases.

The different equivalence scales affect the *Single Agent* model only through their effect on the budget constraint. Since the natural borrowing constraint is imposed, this effect is minimal on the peak age of consumption when moving across different equivalence scales (55 vs. 53 for the OECD and Nelson scale) which is also true for the peak ratio (19.7% vs. 14.8% for the OECD and Nelson scale).

Further, consider the performance of the different *Demographics* models. The first interesting comparison to the *Single Agent* model can be drawn for $\delta_t = \phi_t$. This was the case where our theoretical model predicted the same profile for both models. In line with this result, the peak ages are nearly the same and per-adult equivalent consumption at the peak age is only slightly higher relative to age 25 for the *Demographics* model for all equivalence scales. Second, deviating from $\delta_t = \phi_t$ implies substantial deviations between the *Demographics* and *Single Agent* model, again along both dimensions shown in Table 2. Higher economies of scale decrease the peak age and increase the corresponding consumption ratio as it less expensive in terms of per-adult equivalent consumption to consume when household size is larger. Similarly, as δ_t depends more on household size, it becomes more attractive to allocate consumption to periods where household size is larger as the valuation of each household member's utility from per-adult equivalent consumption matters more. Hence, higher economies of scale or a larger weight of the utility of per-adult equivalent consumption for each household member essentially increase the incentive to allocate more consumption to periods where household size is large. Obviously, the opposite is true when δ_t is low/close to 1.

The OECD scale exhibits the lowest economies of scale and per-adult equivalent consumption

in the data peaks at the latest age with the lowest consumption ratio. The *Demographics* model with $\delta_t = 1$ has the best prediction for the two moments because in this setup, households want to allocate consumption to periods where household size is small, albeit the fact that for a fixed household size they would prefer a decreasing consumption in the absence of uncertainty. For the square root scale, the peak age and ratio decrease substantially relative to the data which the *Demographics* model with $\delta_t = 1$ is no longer able to replicate. The household must value the utility of per-adult equivalent consumption from additional members in order to generate that fact and the empirical results are 'bracketed' by the case of $\delta_t = \phi_t$ and $\delta_t = N_t$. The *Demographics* model with $\delta_t = N_t$ is the only that can replicate the substantial decrease in the peak age and ratio that we observe in the data for the Nelson scale. As already mentioned, the *Single Agent* model is always close to the *Demographics* model with $\delta_t = \phi_t$ and therefore has a hard time to replicate the changes in the data across the different equivalence scales.

The most important message from this exercise is that to control for household size and composition in a consistent way, not only equivalence scales matter, but rather the interplay between those and the chosen model of the household. The *Demographics* model we consider here (with its different variants) is a simple stand in for different configurations of preferences or household structures. The fact that the predictions from the *Demographics* model are close to those of the *Single Agent* model for some cases (i.e., OECD scale and a utility weight between 1 and ϕ) and widely different for others (Nelson scales) is a clear example of this.

5.2 Life-Time Per-Adult Equivalent Consumption

As an implication of a pure accounting exercise, Result 2 stated that life-time per-adult equivalent consumption in general differs between the two models. Table 3 shows by how many percentage points the mean of life-time per-adult equivalent consumption in the *Demographics* model exceeds the one in the *Single Agent* model. The differences range from 0.15% ($\delta_t = N_t$, Nelson scale) to 1.42% ($\delta_t = 1$, OECD scale). First, for a given equivalence scale the differences in the mean are decreasing in the dependence of δ_t on household size. For $\delta_t = 1$ ($\delta_t = N_t$), household consumption and income in the *Demographics* are the least (most) synchronized, as e.g. measured by the average absolute difference between the two variables. Second, for a given δ_t the differences in the mean are decreasing in the economies of scale as both household income and household consumption are

Table 3: Life-Time Per-Adult Equivalent Cons. *Demographics* Relative to *Single Agent* Model

δ	OECD		Square Root		Nelson	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
$\mathbf{1}$	1.42%	3.40%	1.29%	2.88%	0.54%	1.14%
ϕ	0.78%	2.58%	0.76%	2.31%	0.42%	1.02%
\mathbf{N}	0.50%	2.15%	0.36%	1.71%	0.15%	0.74%

divided by a lower number. These observations are exactly in line with Result 2, see Equation (11).

Another implication of Result 2 is that the differences in means might feed through into inequality in life-time per-adult equivalent consumption. Table 3 also displays by how many percentage points the standard deviation of life-time per-adult equivalent consumption in the *Demographics* exceeds the one in the *Single Agent* model. Higher differences in means are as well associated with higher differences in the standard deviations, ranging from 0.74%. ($\delta_t = N_t$, Nelson scale) to 3.40% ($\delta_t = 1$, OECD scale).¹²

6 Discussion

We have discussed at length different cases for utility weights and equivalence scales and the sources of differences between what *Demographics* and *Single Agent* models predict. However, the question still remains with regard to which values we should consider for empirical work. For example, as we stated in Section 2 Cubeddu and Ríos-Rull (2003) and Hong and Ríos-Rull (2007) set the utility weights to $\min\{N_{ad}, 2\}$, while Fuchs-Schündeln (2008) sets it equal to the equivalence scale. In both papers no further justification for this choice is provided.

By comparing the preferences in Attanasio, Banks, Meghir, and Weber (1999) and the ones used in this paper, we can back out an analytical expression for the utility weight:

$$\delta(N_{ad}, N_{ch}) = \exp(\zeta_1[N_{ad} - 1] + \zeta_2 N_{ch}) \phi(N_{ad}, N_{ch})^{1-\alpha} \quad (25)$$

¹²Note that this is still true for a scale free measure of inequality, as e.g. the coefficient of variation which divides the standard deviation by the mean. The profiles of the standard deviation of log per-adult equivalent consumption over the life-cycle are very similar between the *Demographics* and *Single Agent* model.

where ζ_1 , ζ_2 and α are estimated directly from CEX data. Note that it is not possible to uniquely pin down individual household member weights but only the sum of the weights given by δ . In Figure 6, we show the calculated δ_t/ϕ_t ratios, for different household sizes and various equivalence scales. In addition, we plot the ratios implied by the two extreme cases we considered in the quantitative analysis, $\delta_t = 1$ and $\delta_t = N_t$. Finally, we plot the ratio implied by a utility weight of $\delta_t = \phi_t$, represented by a flat line equal to 1, as a reference.

In [Attanasio, Banks, Meghir, and Weber \(1999\)](#), the preference structure is estimated and fed into a life-cycle model of household consumption. These simulations provide a good fit of the life-cycle profile of household consumption as shown in the CEX. Given these estimates, we can ask which value of δ is consistent with the evidence, *given* a fixed equivalence scale, or in other words, do the empirical estimates select any particular version of our *Demographics* model?

For the OECD scale, the previous section showed that the *Demographics* model was the closest to the data for something in-between $\delta = 1$ and $\delta = \phi$. Figure 6 implies for this scale a value $\frac{\delta}{\phi}$ close to one, or alternatively a value of $\delta \approx \phi$. In contrast, for the Nelson scale, the *Demographics* model was closest to the data for $\delta = N$ which again is also consistent with the information in Figure 6, where the implied ratio $\frac{\delta}{\phi} \approx \frac{N}{\phi}$.

We take this as supportive of our previous statement: the choice of the equivalence scale cannot be made independent of the choice of the model.

7 Conclusions

In this paper we suggest a simple framework to understand the sources of bias when single agent models are used to make predictions for aggregate consumption or consumption related variables, such as aggregate welfare.

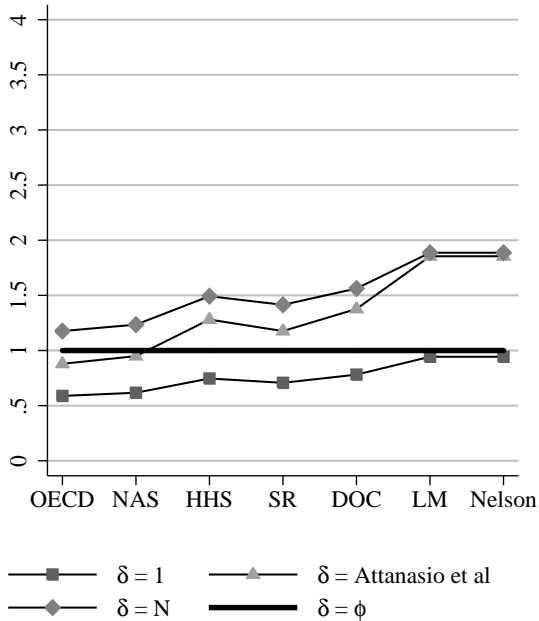
Our proposed *Demographics* model acknowledges that the interaction of economies of scale in household consumption (measured by equivalence scales, which are widely used in the quantitative literature) and the degree to which household value the utility of consumption of each household member have an effect on the household's effective discount factor or alternatively the perceived relative prices of consumption over the life-cycle when household size and composition change. This is in stark contrast to the common practice of simulating a single agent model, where these induced

price effects are ignored. However, depending on the configuration of the *Demographics* model the implied bias might be negligible or substantial.

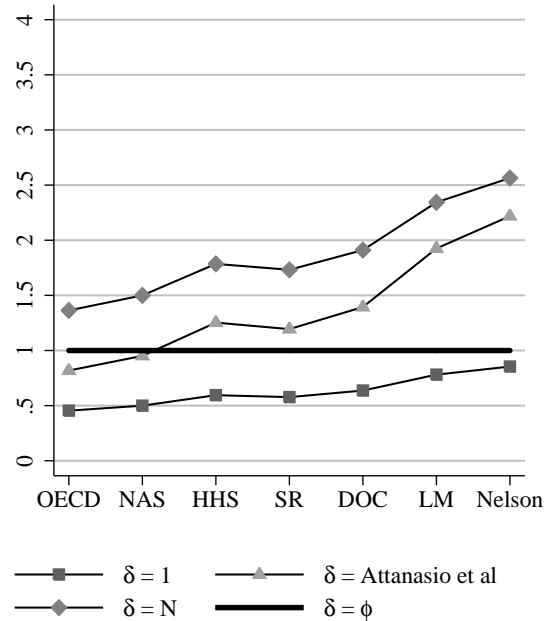
Another contribution is that we show the importance of demographics for the life-cycle profile of per-adult equivalent consumption, a statistic usually treated as if it was free from demographic effects. Since demographics in our simple setup affects both household and per-adult equivalent consumption, models without demographics might attribute too much importance to other mechanisms to replicate life-cycle consumption facts.

Figure 6: Ratio $\frac{\delta}{\phi}$ for Various Utility Weights and Equivalence scales

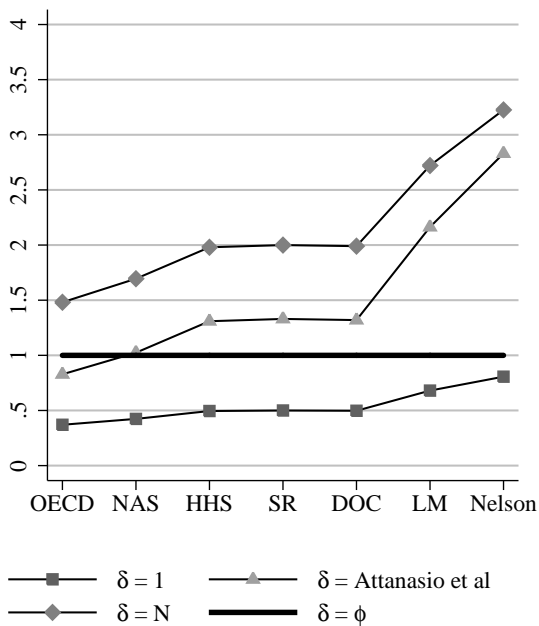
(a) No children



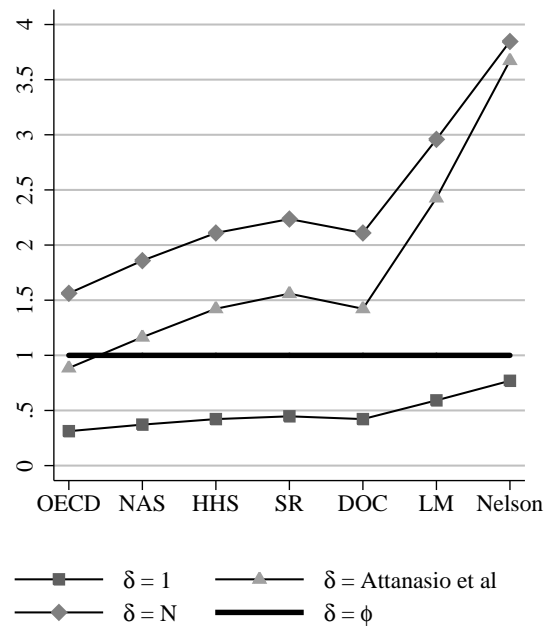
(b) One child



(c) Two children



(d) Three children



Note: Calculations based on households with two adults and the respective number of children.

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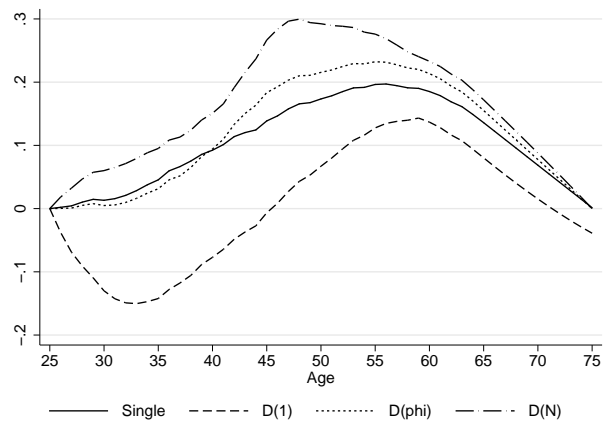
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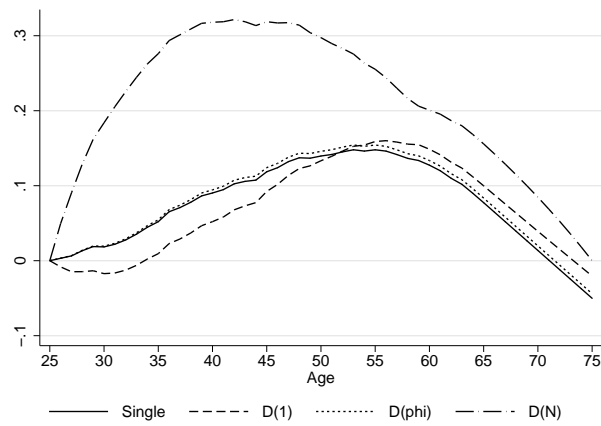
Appendix

Figure 7: Adult Equivalent Consumption Relative to Age 25, OECD scale



Note: D stands for the *Demographics* model, and the figure in parenthesis, represents the utility weight (δ_t) considered: 1, equivalence scale (ϕ) and household size N . Table 4 in the Appendix lists the calibrated β s for each model.

Figure 8: Adult Equivalent Consumption Relative to Age 25, Nelson scale



Note: D stands for the *Demographics* model, and the figure in parenthesis, represents the utility weight (δ_t) considered: 1, equivalence scale (ϕ) and household size N . Table 4 in the Appendix lists the calibrated β s for each model.

Table 4: Calibrated β s for Different Equivalence Scales (ϕ) and Utility Weights (δ)

	OECD	Square Root	Nelson
<i>Demographics Model</i>			
$\delta_t = 1$	0.950	0.955	0.959
$\delta_t = \phi_t$	0.954	0.957	0.959
$\delta_t = N_t$	0.955	0.959	0.966
<i>Single Agent Model</i>	0.957	0.959	0.959