

Dynamic Programming and Business Cycles
 IDEA Program
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Problem Set #3
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Problem 1 (Recursive Stochastic Endowment Model). *Consider the recursive representation of the stochastic endowment economy we saw in class:*

$$\begin{aligned}
 V(z^i, a, \{b_z\}_{z=1}^N) &= \max_{c, a', \{b_{z'}\}_{z'=1}^N} U[c] + \beta \sum_{z'=1}^N \Gamma_{z^i z'} V(z', a', \{b_{z'}\}_{z'=1}^N) \\
 \text{s.t.} \\
 c + a' + \sum_{z'=1}^N q_{z^i z'} b_{z'} &= (1 + r(z))a + e_z + b_{z^i}
 \end{aligned}$$

(note that the notation indicates that the current state is z^i)

1. Write an equation for the volatility of GDP in this economy, in terms of model primitives.
2. Derive the Euler equations for savings (a') and Arrow securities ($b_{z'}$ for $z' = 1, 2, \dots, N$)
3. Find a condition (usually called the no-arbitrage condition) such that the agent will not need to use Arrow securities (hint: compare Euler equations)
4. Suppose we are currently in state $z = 1$. How much would an agent in this economy be willing to pay for a constant endowment $e_z = \varepsilon$ at each period/history from now on?
5. What is the value of being born into this economy?
6. Show that individuals save more in this economy than in a deterministic economy.
7. What is the value of removing all uncertainty from this world? Express this value in terms of the quantity of assets you would need to give to a newborn in the economy, to be indifferent between the stochastic and the deterministic case. Is this value unique?