First Term Macroeconomics
Barcelona GSE
MSc in Macroeconomic Policy and Financial Markets
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Problem Set
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Problem 1 (2-period, endowment OLG model)**.** *Consider the classic set up for the 2-period overlapping generations model we have seen in class. The lifetime utility of an agent is given by*

$$U(c_{t1}, c_{t2}) = u(c_{t1}) + \beta u(c_{t2})$$

with the per-period utility $u(\cdot)$ *being increasing, concave and differentiable. Each generation receives endowments of* $\{e_1, e_2\}$ *when young and old, respectively and there is no population growth.*

1. *Set up the individual problem in sequential/recursive form*
2. *Let* $e_2 = e_1(1+g)$ *and* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ *with* $g \in (0,1)$ *and* $\sigma > 0$. *Find the optimality condition for the individual decision problem and derive the condition for the Pareto optimality of the competitive equilibrium. How do* g *and* σ *relate in this last condition? Explain intuitively*
3. *Redo the last question, but when there is population growth, i.e.,* $N_t = (1+n)N_{t-1}$ *where* N_t *is the size of generation* t

Problem 2 (Organizational Equilibrium)**.** *Recall the setting proposed by Prescott and Ríos-Rull (2005) that we saw in class.*

1. *Recalculate the sequence of transfers needed to support organizational equilibrium (by satisfying the No Restarting Condition) when lifetime utility is of the constant relative risk aversion (CRRA) kind, i.e.*

$$u(c_{t1}, c_{t2}) = \frac{c_{t1}^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t2}^{1-\sigma}}{1-\sigma}$$

   and endowments are $e_1 = 3$ *and* $e_2 = 1$ *for all generations. Do the above when* $\sigma = 2$ *and when* $\sigma = 10$. *Which sequence converges faster? (note, you can use the computer to solve this for many other cases, i.e.,* β *close to zero and* β *close to 1)*
2. *Show that when* $e2 > e1$ *there are no sequences of transfers that can be supported as an organizational equilibrium*

Problem 3 (Stochastic Endowments)**.** *Write the standard 2-period endowment OLG model in recursive form, as an unconstrained maximization problem where individuals just chose* s_t *(their savings when young) given endowments* e_1 *and* e_2. *Now, consider a different scenario where* e_2 *can be either* e_2^H *or* e_2^L*, each with probability* 0.5

1. *Write the expected utility problem for the second scenario. How does the aggregate market clearing condition look like?*

2. *Write the optimal condition for savings in both scenarios (when e_2 is known for sure and when individuals have to make decisions given their expectation of e_2)*

3. *Find the optimal savings function (for both cases) when lifetime utility is CRRA*

4. *Under which scenario are savings higher?*

5. *Consider the condition for a competitive equilibrium to be Pareto optimal. Can you think of a similar condition for the stochastic endowment case?*

Problem 4 (OLG with outside Money). *Set up the OLG model when there is a fixed amount M of fiat money in the economy, lifetime utility is CRRA and the value of money in period t is given by q_t*

1. *Find an equilibrium condition for the sequence of monetary values, i.e. something of the form*
$$q_{t+1} = f(q_t)$$

2. *Find the steady state value of money and the steady state value of the intergenerational transfer, in terms of e_1, e_2, σ and β*

Problem 5 (Pay as you go Social Security). *Once again, consider the 2-period endowment OLG model in recursive form. Now, suppose there is a government that taxes the young in the amount τ and gives the proceeds to the current old generation in the form of benefits b.*

1. *Compute the maximum welfare improving tax rate if lifetime utility is CRRA*

2. *Now assume that the tax τ is not a "per head" tax but a proportional tax to the amount of first period endowment e_1 (the proceeds are still redistributed lump sum to the old). Get a condition for this PAYG system to be welfare improving and compute the maximum welfare improving tax rate with CRRA utility*

Problem 6 (OLG and production). *Once again, consider the 2-period endowment OLG model in recursive form, but now with an aggregate production function $Y_t = F(K_t, N_t)$ that exhibits constant returns to scale*

1. *Show that in equilibrium, the optimal savings decision is a function only of the amount of capital per capita and the growth rate of the population in the economy*

2. *Derive a condition for the evolution of capital per capita of the form*
$$k_{t+1} = \widetilde{s}(k_t, k_{t+1})$$
(hint: start from the definition of aggregate investment in the economy)

3. *Show that if* k_0 *is a steady state in the level of capital per capita, it is locally stable if*

$$\left.\frac{\partial \widetilde{s}}{\partial k}\right|_{k=k_0} < 1$$

4. *Check the above statement for the particular case of logarithmic lifetime utility*