On the rule of k names ☆

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A R T I C L E   I N F O

A B S T R A C T

The rule of k names can be described as follows: given a set of candidates for office, a committee chooses k members from this set by voting, and makes a list with their names. Then a single individual from outside the committee selects one of the listed names for the office. Different variants of this method have been used since the distant past and are still used today in many countries and for different types of choices. After documenting this widespread use by means of actual examples, we provide a game theoretical analysis. We concentrate on the plausible outcomes induced by the rule of k names when the agents involved act strategically. Our analysis shows how the parameter k, the screening rule and the nature of candidacies act as a means to balance the power of the committee with that of the chooser.

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1. Introduction

In the beginning of the sixth century, the clergy and the chief of the citizens of some Eastern European countries chose three names from whom the archbishop selected the bishop (The Catholic Encyclopedia, 1917). Nowadays, several institutions around the world use variants of this system to fill public offices. For example, this system is known as the "rule of three names" in the United States, "regla de la terna" in Spain and "lista tríplice" in Brazil. Sometimes the list consists of more than three names. Since this does not complicate our analysis, we shall from now on refer to the "rule of k names."

The rule of k names can be formally described as follows: given a set of candidates for office, a committee chooses k members from this set by voting, and makes a list with their names. Then a single individual from outside the committee selects one of the listed names for the office.

We emphasize that an important part of a rule of k names is the procedure used by the committee in order to screen out those k candidates that will be presented to the final chooser. A diversity of screening rules are actually used to select

the k names. Because of this diversity, the "rule of k names" is in fact a family of different rules. Let us review some variants of the rule that have been used in the past and are used in the present.

In the Catholic Church, the bishops are appointed under the rule of three names. According the Code Canon Law #377, the Pope may accept one of the candidates in the list proposed by the Apostolic Nuncio (papal ambassador), or consult further. The Apostolic Nuncio makes the list after consultation with the members of the ecclesiastical province. In many countries the list is decided by means of voting. In Ireland, each canon of the cathedral and parish priest can cast a vote for three candidates to bishop.[1] In England, the canons vote three times, and select each time the most voted candidate. In both cases, the list is made with the most voted candidates (The Catholic Encyclopedia, 1917 and The Code of Canon Law, 1999).

In the nomination of the rectors in Brazilian federal public universities, the university councils are permitted, since 1996, to consult their university communities. The law requires that during this consultation, each voter shall cast a vote for only one candidate, and that the three most voted candidates will form the list.[2] The President of the Republic shall select one of the listed names (see Decreto n⁰ 1916 (Brasil, Governo Federal, 1996)).

The committee that takes the final decision is in most of the cases a single individual and this is what we shall study here. But we could easily embed our definition of the rule of k names, with one committee and one chooser, into a larger class of procedures where both the screening and the choice are made by more than one agent. Here are two examples in which the first committee is smaller than the second committee. One is in the Article 96, paragraph 5°of the Mexican Constitution that states that the President of the Republic shall propose three names to the Senate, which shall appoint one of them to become member of the Supreme Court of Justice (Mexico, 2008). Another, is the Brazilian law of corporate finance, approved in 2001, that states that the preferred stockholders who hold at least 10% of the company capital stock shall choose one name among the three names listed by the controller of the company to become their representative on the company's board of administration (see Lei n⁰ 10303 (Brasil, Governo Federal, 2001)).[3]

There are many variations of the rule of k names, involving more than two committees. In Chile, according to Article 75 of the Chilean Constitution, the members of the Superior Court of Justice are designated by the President of the Republic among those in the list with five names proposed by the Superior Court of Justice,[4] and must get the approval of two thirds of the Senate. If the Senate does not approve the proposal of the President, then the Superior Court must substitute the rejected name in the list, and the procedure is repeated until a presidential nominee is finally approved by the Senate (Chile, 2005). Another example also comes from Brazil. According to the Brazilian Constitution, one-third of the members of the Superior Court of Justice shall be chosen in equal parts among lawyers and members of the Public Prosecution nominated in a list of six names by the entities representing their respective classes. Upon receiving the nominations, the court shall organize a list of three names and send it to the President of the Republic, who selects one of the listed names for appointment (Brazil, 2005).

Since two or more candidates may have the same number of votes during the preparation of the list, a tie-breaking criterion is often used. In some institutions ties are broken randomly and in others by some deterministic rule. For example, in the nomination of the minister of the Superior Court of Justice in Chile, ties are broken randomly, while in Brazil, the age and tenure in the public service of the candidates are used to break ties.

Let us mention that there are versions of the rule of k names in which one of the parties is supposed to be impartial regarding the alternatives. We have two examples and both come from the US. In the first example, the neutral party is the one who makes the list; in the second example, it is the one that makes the final choice.

For the first example, consider the following variant of the rule of three names, which the US Federal Government uses to recruit and select new employees. The US civil service law requires federal examining officers to assign each job applicant a numerical score, based on assessment tools, performance tests or by evaluations of her training and experience. Then a manager hiring people into the civil service must select one from among the top three candidates available (U.S. Merit Systems Protection Board, 1995). Notice that this may be viewed as a variant of the rule of three names, but one where the list is not obtained by voting. Rather, the list must appear as being obtained by applying a neutral scoring test.

Let us turn to the second example. Under the 1993 Labor Reform, California's Labor Code 4065 states that the workers' compensation judge (WCJ) is constrained, in determining a permanent disability rating, to choose among the offers of one of the two parties. This procedure to settle disputes is referred to in the literature as the final-offer arbitration (FOA) (Neuhauser and Swezey, 1999). It is also known as "baseball arbitration" since US major baseball leagues used it to determine wages in disputed contracts. This system is often presented as an improvement over the conventional arbitration procedure, where the arbitrator is not constrained to choose only among the parties' offers. The final-offer arbitrage model was first proposed by (Stevens, 1966), who argued that it would induce convergence among the offers of the two parties,

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[1] Most of Brazilian states adopts this procedure to make the list in the choice of Justice General Prosecutor. The Article 128, paragraph 3 of the Brazilian Constitution states that the governors of the states shall choose one name of those three names proposed by the members of the Justice Prosecution Service.

[2] In additional, the sum of the weighted votes of teaching staff need to be a minimum of 70% of the total. In Brazil, before 1996, the list was made with six names proposed by the university council without consulting the university community (see Decreto-Lei n⁰ 5540 (Brasil, Governo Federal, 1968)).

[3] After 2006, the law states that there will be not such restriction.

[4] Each member of the Court cast a vote for three candidates, the list is made with the five most voted candidates.

and presented this conjectured property as an advantage over the conventional arbitration scheme. However, the theoretical literature does not support this conjecture. Farber (1980), Chatterjee (1981), Crawford (1981), Whitman (1986) and Brams and Merrill (1983) show that the offers still diverge under FOA Brams and Merrill (1986). Whatever its formal properties, FOA can be interpreted as a rule of 2 names, with the added qualification that the chooser is not guided by self interest, and is assumed to choose in the name of fairness.

Surprisingly, despite of the extensive economics and political science literature on voting, we do not know any article that is specifically devoted to study the rule of k names when both parties are not neutral regarding the alternatives. This paper is an attempt to start filling this gap.

Many questions come to mind. Why are these rules used? What type of decisions are they well suited for? What could be the intentions and expectations of those who decided to set them up? Is there reason to believe that such expectations could be fulfilled? We do not address these questions directly. Our final remarks will hint at some possible answers. Others will have to wait for a thorough historical analysis of the circumstances under which each specific rule did emerge. What we attempt to do in this paper is to start the study of these rules by addressing some initial but non-trivial issues. For example, questions regarding how each rule manages to balance the tension between the interests of the proposers and those of the chooser. For example, if k was equal to one, this would amount to give all decisive power to the committee. At the other extreme, when k equals the number of alternatives, then no alternative is eliminated from the list, in which case the chooser decides everything.

In order to be precise about the type of balances implied by the rule of k names, we adopt a game theoretical approach. We study what outcomes one may expect from applying the rule of k names, when agents act strategically and cooperatively. Part of the discussion refers to the type of screening rules that are used to select the k names. We shall discuss two classes of such screening rules, to be called majoritarian and weakly majoritarian. Another point of the discussion refers to the best way to model the game that is induced by the use of the rule of k names. In fact, we work under two alternative assumptions regarding the ability of the chooser to commit. This leads us to study two closely related games in normal form. For each one of these games, we characterize the set of strong Nash equilibrium outcomes in the game. We are aware that there are other possible choices how to model and solve for the strategic interplay among agents within these rules. We hope that further work will continue to complete this initial analysis. Yet, our approach already allows us to reach a number of interesting results. The first one is that the choice of the screening procedure to select the k names is not as crucial as one could think. This is because rules of k names based on different majoritarian screening rules lead to the same sets of strong Nash equilibrium outcomes. This result is reminiscent of, although not directly related to, the recent findings in the literature on juries that establishes that, under some circumstances, the size of the majority required to convict is not as crucial as could be expected. However, we shall see that the comparison between the outcomes becomes trickier when one is based in a majoritarian screening rule and the other is not. We also obtain a number comparative static results. In particular, we determine the effects on the equilibria of changing k and of adding undesirable candidates. Each of these effects has implications on the agents' preferences over different variants of the rule of k names.

2. An introductory example: the case of one proposer and one chooser

The rule of k names is definitely a procedure to balance the power of the committee and that of the chooser, though making this statement more precise will take some effort. Before engaging in any complicated analysis, let us consider a simple and suggestive case involving only two agents: one of them, the proposer, selects a subset of k alternatives, from which the other agent has to choose one.[5] We hope that the example will provide the reader with some insights. However, we also warn about the fact that the general analysis will not follow exactly the same lines that we exploit here. This is because, with more than one proposer, we will have to model the interplay leading to the proposal of the k names. This will lead us to some modelling decisions that are described in Section 3.

For now, let us consider two agents, 1 and 2, facing four alternatives a, b, c and d. Assume that their preferences over alternatives are as follows: $a \succ_1 b \succ_1 c \succ_1 d$ and $c \succ_2 b \succ_2 a \succ_2 d$ where $x \succ_i y$ means that agent i prefers x to y. Assume that agent 1 can propose k alternatives, from which agent 2 makes a final choice. Clearly, $k = 1$ is the case where 1's choice is final, and 2 has no influence, whereas, $k = 4$ gives all decision power to 2. What about the intermediate cases where $k = 2$ or $k = 3$? Let us informally discuss what outcomes we might expect under different strategic assumptions.

In one scenario, assume that the agent 2 cannot commit to a choice before observing what set of k alternatives he is presented with. In that case, the only reasonable behavior for 2 is to choose his best alternative out of the proposed by 1, whose best proposal is then easily computed, given 2's choice rule. Clearly, we are describing what would be the backward induction equilibrium of the obvious extensive form game associated to the rule when 1 plays first, sets of k names are his strategies, and the strategies of 2 are just choice rules on sets of size k. The set of backward induction equilibrium outcomes for different values of k is displayed in the table below:

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[5] This situation is reminiscent of the classical problem of how to cut a cake, though here we are dealing with a finite set of possibilities and we are not introducing any prior normative notion regarding the outcome.

Set of backward induction equilibrium
outcomes.

k	
1	$\{a\}$
2	$\{a\}$
3	$\{b\}$
4	$\{c\}$

Notice that we can identify one of the strategies of the chooser with the choice rule generated by his preference relations on alternatives. Because of that, we can simply look at the simultaneous game where agent 2 is bound to use only this specific strategy. In the two person case, this leads us to a degenerate game, a decision problem for agent 1, whose choices will always coincide with backward induction equilibrium of the extensive form game described above. This remark will be extended and qualified in Section 3.

As a second scenario, suppose 2 can choose his strategy simultaneously with agent 1, so now 2 can commit to a given choice rule, even before the set of alternatives is presented to him. So, let us consider a normal form game where 2's strategy is a choice rule that dictates the winning alternative from every list which can be proposed by 1. The reader could think that, given our description of the rule of k names as the result of a well defined sequence, where the proposer goes first and the chooser goes last, there is no point in considering this case. However, we think that it is worth studying, for two different and complementary reasons. The first reason is because it allows to model possible threats from the chooser to the proposer, which may play a role in many real life situations. In fact, our model of the interaction between the committee and the chooser is a very simple one. We do not model some important facts that will arise in real contexts, like the fact that the relationship among these main actors is durable and complex, and that the choice of alternatives is only a part of it. For example, universities interact with the government in many ways other than the election of a rector, which only happens from time to time. Since introducing these unmodelled aspects would complicate our analysis very much, we simply admit that threats from the chooser may sometimes be credible. Paying attention to all possible Nash equilibria of the normal form game is a simple device to study the consequences of such threats.

The second reason why the simultaneous game is interesting is because, as soon as we introduce two players with actual possibilities to choose, we create conditions to study cooperation, in addition to conflict. And we shall now see that, although the Nash equilibria of the game can be inefficient, agents could cooperate to prevent inefficiencies, one of the several ideas that can be captured through the use of a different solution concept, that of Strong Nash.

In our second scenario, a strictly Pareto dominated alternative can be the outcome of a Nash equilibrium. An alternative is strictly Pareto dominated if some other alternatives is considered better for both agents 1 and 2. To understand this point, let $k = 3$ and suppose that agents 1 and 2 have the same preferences over alternatives, $a \succ_i b \succ_i c \succ_i d$ for $i = 1, 2$. Given this preference profile a is the only alternative that is not Pareto dominated. It turns out that there exists a strategy profile that can sustain b as a Nash equilibrium outcome. Agent 1 proposes a list with b, c and d and agent 2 declares a choice rule $C(\cdot)$ such that $C(a, b, c) = C(a, b, d) = C(b, d, c) = b$ and $C(a, c, d) = c$. Notice that under this strategy profile, b is the winning alternative and it is a Nash equilibrium since no agent can profitably deviate, given that the other keeps its strategy unchanged. However both 1 and 2 would be better off if agent 1 substituted d by a in the proposed list, and 2 changed the choice rule from C to C', so that $C'(B) = C(B)$ unless $B = \{a, b, d\}$, and $C'(B) = a$. However, we may find that this outcome is not reasonable, and one way to eliminate it is through the use of the strong Nash equilibrium concept. A strategy profile is said to be a pure strategy strong Nash equilibrium of a game, if no coalition of players (maybe singletons) can profitably deviate from this strategy profile, given that the strategies of other players remain unchanged. Notice that, under this equilibrium concept, any Pareto dominated alternative is ruled out as an equilibrium outcome.

Let us go back to the previous preference profile where $a \succ_1 b \succ_1 c \succ_1 d$ and $c \succ_2 b \succ_2 a \succ_2 d$. Now, if $k = 2$, agent 2 can threaten 1 by pledging to choose d if $\{a, d\}$ is proposed. Under this threat, 1's best response would be to propose the set $\{a, b\}$ to let b be chosen by 2. The outcome b is now the result of a strong Nash equilibrium play. However, a is still the outcome of another strong equilibrium where 2 does not threaten and 1 proposes a and d. The table below presents the set of pure strong Nash equilibrium outcomes for different values of k.

Set of pure strong Nash equilibrium
outcomes.

k	
1	$\{a\}$
2	$\{a, b\}$
3	$\{c, b\}$
4	$\{c\}$

Notice that the passage from $k = 2$ to $k = 3$ or $k = 4$ makes a difference. We can see that the case $k = 3$ still leaves room for 2 to get the preferred outcome c, while this is out of the question with $k = 1$ and $k = 2$.

This simple example suggests that one important concern of a careful study of rules of k names is precisely to asses the impact of the choice of k, from the point of view of the different parties involved. A second hint is that, in order to evaluate the likely consequences of establishing a rule of k names, we'll have to analyze the game that naturally arises, and that this

analysis will be quite different depending on whether or not we think that the chooser can commit and/or make credible threats. Because of that, in the sequel we analyze several games, and let the reader decide which one will suit each practical situation better.

The consequences of the choice of k cannot be analyzed independently of the cardinality of the set of alternatives. Even the alternatives that no one likes play a role in the functioning of the rule: since numbers count, having an undesirable alternative is not the same as not having that alternative at all. To illustrate this simple point, let us go back to the previous example. Clearly, alternative d could never be a strong Nash equilibrium outcome since it is the last option for both agents. However its presence as an alternative makes a difference. Had it not been there, the set of equilibrium outcomes for different values of k would have been as shown in the table below.

Set of pure strong Nash equilibrium outcomes.

k	
1	$\{a\}$
2	$\{b\}$
3	$\{c\}$

The reader can verify that the presence of d helps 1 and harms 2 for some k's, but never the reverse. For $k = 2$, the presence of d is crucial for agent 1 because, without d, agent 1 will not able to propose a list with two alternatives in which agent 2's best listed name is a. For $k = 3$, agent 2's favorite alternative c is elected in equilibrium if d is out. Yet, in the presence of d, b is chosen.

Since, for a given k, the number of alternatives affects the outcome, there is a lot of room to study how and why alternatives emerge. Adding undesirable alternatives to the contest, or introducing very similar alternatives (clones) to run are obvious forms to manipulate a rule of k names. We shall not pursue formally the related issue of strategic candidacies in the present paper.

Here is a last point regarding equilibria under the rule of k names, for the two games we have considered. We can express the equilibria as the result of a procedure where agent 2 takes the initiative of vetoing $k - 1$ alternatives and agent 1 chooses one out of those not vetoed for appointment. It turns out that, in terms of strong equilibrium outcomes, the rule of k names is equivalent to the rule of $k - 1$ vetoes where agent 2 is the one who has the right to veto.

The aim of the next section is to build on the preceding remarks, extend and qualify them, and provide an analysis of the case where the proposer is not a single person, but a committee.

3. General results: several proposers and one chooser

The case of one proposer and one chooser is interesting per se. Moreover it allows us to understand some interesting features of the rule of k names. However, our rules will involve, in general, not one but many proposers, whose interests may be at least partially divergent. The choice of the best set to propose will then no longer be a matter of one agent, but the result of a collective decision. Because of that, we can enlarge our analysis in directions that take into account the possibilities of cooperation among sets of proposers, or even between some proposers and the chooser.

In this paper, we define and analyze two games in normal form, and we study the strong Nash equilibria for each game. We first explain what these games are, what we attempt to model through them, and why they are different. After that, we comment on alternative modelling strategies that could also be interesting to pursue in later work.

First we define the Unconstrained Chooser Game, where each of the players is endowed with a variety of strategies, the outcome function is dictated by the use of the rule of k names, and where the objectives of the agents are given by their preferences over outcomes.

The idea here is to capture all the possibilities that arise from the interaction between the choosers and the proposer. In the preceding section we have already pointed out at the possibility that the chooser threatens the proposer, and also that they both cooperate. The possibilities of cooperation become richer as the set of proposers enlarges. Not only some proposers may coalesce, but also some may join forces with the chooser. And they can do it not only to further joint proposals, but also to avoid the advance of others. We find that the combination of simultaneity, as captured by a normal form game, and the mix of threats and cooperation, as captured by the notion of strong Nash equilibrium, provides a rich set of considerations regarding the possible interactions among these agents.

Of course, we could have considered modelling and analyzing the game in other ways. Since the description of the rule of k names suggests a sequential structure in the plays, we could have modelled it in extensive form, with the proposers advancing a set of candidates in a first stage, and the chooser choosing after that. This could be a way to combine sequentially with the implicit idea that the chooser cannot commit. Alternatively, if the chooser is availed with a commitment device, we can also think of an extensive form where the chooser pre-announces a strategy, and then the proposers offer the set accordingly, à la Stackelberg. For both extensive form games it would be natural to then use a notion of strong Nash subgame perfect equilibrium.

We do not follow this strategy fully, but rather propose a second game in normal form, that we call the Constrained Chooser Game, because we actually limit the possible behavior of the chooser to one single strategy, the one that leads this

player to always choose the best outcome out of those that are proposed. The Constrained Chooser Game captures the idea that the chooser might not be able to commit to any choice other than selecting the best, once presented with a list. Thus, we capture some of the flavor of subgame perfection in extensive form games. But the analysis is still based on studying the strong Nash equilibria of this new normal form game, and this allows for interesting comparisons with the preceding model (what we called, by contrast, the Unconstrained Chooser Game). As we shall see, the comparison is non-trivial, since there is no general systematic connection between the sets of strong Nash equilibrium outcomes for each of the games. On the one hand, the fact that the chooser can use more strategies would seem to go in favor a larger set of equilibria in the Unconstrained Chooser Game, by allowing equilibria where the chooser uses non-sincere strategies. But this richness of strategies also allows the chooser to join coalitions that disqualify certain outcomes as equilibria, and this force works in the direction of eliminating some equilibrium outcomes from the Unconstrained Chooser Game once we allow a richer set of strategies for the chooser. There is a systematic relation between the two sets, however, in the case where the number of proposers is odd (see Corollaries 1, 2 and 3).

We offer the analysis of both games because the ability of the chooser to commit may be different in different real-life cases. Regarding this, let us insist that the election of a candidate for office is just one part of the long lasting relationship between those institutions that this candidate will represent (as a University Rector, as a member of a Supreme Court, as a Bishop), and those that the chooser stands for (as President of a government or as the ruler of a Church). Since the election happens only from time to time, and often involves changing sets of players, we have not found appropriate to model the situation as a repeated game. And because the behavior of the players may have an impact on the overall connections between the institutions involved, we think that it makes sense to study situations where the chooser can precommit in varying degrees, even if he formally chooses last.

Much in the same way than we could have chosen to model the game differently, we could also have analyzed alternative types of equilibria. We are aware that Strong Nash is a very demanding concept, one that often leads to non-existence, and we comment on that at the end of Section 3.3.1. Our hope is that the results we can offer, in spite of this generic difficulty, are still rich and useful enough. And, we do hope for follow-ups of this work where other equilibrium concepts for these or for similar games, can be systematically studied.

In order to describe formally the rule of k names and the games induced by this rule, we need first to provide some notation and define screening rules (set valued functions), which are used by the committee members, the proposers, to select the k names.

3.1. Screening rules for k names

Let $\mathbf{A}$ be the finite set of candidates. We denote by $\mathbf{A}_k \equiv \{B \subseteq \mathbf{A} \mid \#B = k\}$ the set of all possible subsets of $\mathbf{A}$ with cardinality k where $\#B$ stands for the cardinality of B and $B \subseteq \mathbf{A}$ means that B is contained in $\mathbf{A}$. Denote by $\mathbf{N} \equiv \{1, \dots, n\}$ the finite set of committee members, the proposers, that selects a set B from $\mathbf{A}_k$ from which an individual that does not belong to $\mathbf{N}$, the chooser, selects a candidate for the office.

Let W be the set of all strict orders (transitive,[6] asymmetric,[7] irreflexive[8] and complete[9]) on $\mathbf{A}$. Each member $i \in \mathbf{N} \cup \{chooser\}$ has a strict preference $\succ_i \in W$. For any nonempty subset B of $\mathbf{A}$, $B \subseteq \mathbf{A} \setminus \emptyset$, we denote by $\alpha(B, \succ_i) \equiv \{x \in B \mid x \succ_i y$ for all $y \in B \setminus \{x\}\}$ the preferred candidate in B according to preference $\succ_i$.

Definition 1. Let $M^{\mathbf{N}} \equiv M_1 \times \cdots \times M_n$ with $M_i = M_j = M$ for all $i, j \in \mathbf{N}$ where M is the space of actions of a proposer in $\mathbf{N}$. Given $k \in \{1, 2, \dots, \#\mathbf{A}\}$, a *screening rule for k names* is a function $S_k : M^{\mathbf{N}} \to \mathbf{A}_k$ associating to each action profile $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ the k-element set $S_k(m_{\mathbf{N}})$.

In words, a screening rule for k names is a voting procedure that selects k alternatives from a given set, based on the actions of the proposers. These actions may consist of single votes, sequential votes, the submission of preference of rankings, the filling of ballots, etc.[10] For example, if the actions in $M^{\mathbf{N}}$ are casting single votes then $M \equiv \mathbf{A}$. If the actions in $M^{\mathbf{N}}$ are submissions of strict preference relation then $M \equiv W$.

We have found six different screening rules that are actually used by institutions around the world. We classify them into two different groups depending on some properties that are relevant to our analysis. Those in the first group are called majoritarian screening rules. The second group consists of rules that are not majoritarian, but they still satisfy a weaker condition.

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[6] Transitive: For all $x, y, z \in A$: $(x \succ y$ and $y \succ z)$ implies that $x \succ z$.

[7] Asymmetric: For all $x, y \in A$: $x \succ y$ implies that $\neg(y \succ x)$.

[8] Irreflexive: For all $x \in A$, $\neg(x \succ x)$.

[9] Complete: For all $x, y \in A$: $x \neq y$ implies that $(y \succ x$ or $x \succ y)$.

[10] Barberà and Coelho (2008) also study screening rules, but under a normative perspective. In other studies, procedures that select sets have been analyzed, but then they focus on the problem of selecting a committee of representatives of a fixed size. Fishburn (1981), Gehrlein (1985), Kaymak and Sanver (2003) discuss the Condorcet winner criterion for this type of rules. In Barberà et al. (1991), the sets that can be selected by the rule may be of variable size.

Definition 2. We say that a screening rule $S_k : M^\mathbf{N} \to \mathbf{A}_k$ is *majoritarian* if and only if for every set $B \in \mathbf{A}_k$ there exists $m \in M$ such that for every strict majority coalition $C \subseteq N$ and every profile of the complementary coalition $m_{\mathbf{N} \setminus C} \in M^{\mathbf{N} \setminus C}$ we have that $S_k(m_{\mathbf{N} \setminus C}, m_C) = B$ provided that $m_i = m$ for every $i \in C$.

In words, we say that a screening rule is majoritarian if and only if for every set with k candidates there exists an action such that every strict majority coalition of proposers can impose the choice of this set provided that all of its members choose this action.

Definition 3. We say that a screening rule $S_k : M^\mathbf{N} \to \mathbf{A}_k$ is *weakly majoritarian* if and only if for every candidate $x \in \mathbf{A}$ there exists $m \in M$ such that for every strict majority coalition $C \subseteq N$ and every profile of the complementary coalition $m_{\mathbf{N} \setminus C} \in M^{\mathbf{N} \setminus C}$ we have that $x \in S_k(m_{\mathbf{N} \setminus C}, m_C)$ provided that $m_i = m$ for every $i \in C$.

In words, we say that a screening rule is weakly majoritarian if and only if for every candidate there exists an action such that every strict majority coalition of proposers can impose the inclusion of this candidate among the k chosen candidates provided that all of its members choose this action.

Notice that by definition any majoritarian screening rule is weakly majoritarian.

We now present the six screening rules. We begin by the three that are majoritarian.

- Each proposer votes for three candidates and the list has the names of the three most voted candidates, with a tie break when needed. It is used in the election of Irish Bishops and that of Justice General Prosecutors in Brazilian states.
- The list is made with the names of the winning candidates in three successive rounds of plurality voting, with a tie break when needed. It is used in the election of English Bishops.
- Each proposer votes for three candidates from a set with six candidates, and if there are three candidates with more votes than half of the total number of voters, they will form the list. If there are positions left, the candidate with less votes is eliminated, so as to leave twice as many candidates as there are positions to be filled in the list. The process is repeated until three names are chosen. It may be that, at some stage (including the first one), all candidates have less than half of the total number of voters. Then the voters are asked to reconsider their vote and vote again. Notice that, if they persist in their initial vote, the rule leads to stalemate. Equivalently, we could say that the rule is not completely defined. However, in practice, agents tend to reassess their votes on the basis of strategic cooperative actions. It is used in the election of the members of the Brazilian Superior Court of Justice.

Notice that these three rules above guarantee the election of any set of k names, provided that a strict majority votes for them (in the same order).

The next three rules are only weakly majoritarian but not majoritarian.

- Each proposer votes for 3 candidates and the list has the names of the five most voted candidates, with a tie break when needed. It is used in the election of the members of the Superior Court of Justice in Chile.
- Each proposer votes for 2 candidates and the list has the names of the three most voted candidates, with a tie break when needed. It is used in the election of the members of the Court of Justice in Chile.
- Compute the plurality score of the candidates and include in the list the names of the three most voted candidates, with a tie break when needed. It is used in the election of rectors of public universities in Brazil.

Of course, many others screening rules are conceivable, with or without the properties described above. In what follows, our analysis is general, since we are not specific about the exact form of the screening rule. The results, however, will depend on some of the properties of the screening rule that is used.

We now propose and analyze two normal form games with complete information induced by the rule of k names.

3.2. A game theoretical analysis

In the first game, called the Unconstrained Chooser Game, we assume that the chooser can commit to a variety of strategies, each of which will indicate what alternative he would pick from each of the possible lists that the committee may submit. The strategy space of the proposers is the space of admissible messages associated with the screening rule used to select the k names. Based on these messages a list with k names is made and, given the chooser's strategy, one of them is nominated as the winning candidate. It is assumed that the players only care about the identity of the winning candidate.

Definition 4. Given $k \in \{1, 2, \ldots, \#\mathbf{A}\}$, a screening rule for k names $S_k : M^\mathbf{N} \to \mathbf{A}_k$ and a preference profile $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$, the *Unconstrained Chooser Game* can be described as follows: It is a simultaneous game with complete information where each player $i's \in \mathbf{N}$ strategy space is M, while the chooser's strategy space is $M_{chooser} \equiv \{f : \mathbf{A}_k \to \mathbf{A} \mid f(B) \in B$ for every $B \in \mathbf{A}_k\}$. Given $m_{\mathbf{N} \cup \{chooser\}} = (m_\mathbf{N} \equiv \{m_i\}_{i \in \mathbf{N}}, m_{chooser}) \in M^\mathbf{N} \times M_{chooser}$, $S_k(m_\mathbf{N})$ is the chosen list with k names and the winning candidate is $m_{chooser}(S_k(m_\mathbf{N}))$.

In the second game, called the Constrained Chooser Game, the chooser's strategy set is restricted to a single element. In that sense, we could say that he is not an active player. Specifically, we take it that the chooser will simply select that candidate that is best for him among those that he will be presented with. Thus, the chooser's preferences will condition the outcome function, and therefore will have an impact on the equilibrium play of the proposers. But we exclude the possibility that he announces a choice rule that is not in accordance to his preferences, which are known in each game.

Definition 5. Given $k \in \{1, 2, \ldots, \#\mathbf{A}\}$, a screening rule for k names $S_k : M^{\mathbf{N}} \to \mathbf{A}_k$ and a preference profile $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$, the *Constrained Chooser Game* can be described as follows: It is a simultaneous game with complete information where each player $i \in \mathbf{N}$ chooses a strategy $m_i \in M_i$. Given $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$, $S_k(m_{\mathbf{N}})$ is the chosen list with k names and the winning candidate is $\alpha(S_k(m_{\mathbf{N}}), \succ_{chooser})$.

3.3. Strong Nash equilibrium outcomes

We investigate the equilibrium outcomes for these two games, when agents act strategically and cooperatively. More specifically, we study their pure strong Nash equilibrium outcomes when the screening rule used to select the k names is majoritarian.

Definition 6. Given $k \in \{1, 2, \ldots, \#\mathbf{A}\}$, a screening rule for k names $S_k : M^{\mathbf{N}} \to \mathbf{A}_k$ and a preference profile $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$, a joint strategy $m_{\mathbf{N} \cup \{chooser\}} \equiv (m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}}, m_{chooser}) \in M^{\mathbf{N}} \times M_{chooser}$ is a *pure strong Nash equilibrium of the Unconstrained Chooser Game* if and only if, given any coalition $C \subseteq \mathbf{N} \cup \{chooser\}$, there is no $m'_{\mathbf{N} \cup \{chooser\}} = (m'_{\mathbf{N}}, m'_{chooser}) \in M^{\mathbf{N}} \times M_{chooser}$ with $m'_j = m_j$ for every $j \in \mathbf{N} \cup \{chooser\} \setminus C$ such that $m'_{\mathbf{N} \cup \{chooser\}}(S_k(m'_{\mathbf{N}})) \succ_i m_{\mathbf{N} \cup \{chooser\}}(S_k(m_{\mathbf{N}}))$ for each $i \in C$.

Definition 7. Given $k \in \{1, 2, \ldots, \#\mathbf{A}\}$, a screening rule for k names $S_k : M^{\mathbf{N}} \to \mathbf{A}_k$ and a preference profile $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$, a joint strategy $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ is a *pure strong Nash equilibrium of the Constrained Chooser Game* if and only if, given any coalition $C \subseteq N$, there is no $m'_{\mathbf{N}} \equiv \{m'_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ with $m'_j = m_j$ for every $j \in \mathbf{N} \setminus C$ such that $\alpha(m'_{\mathbf{N}}, \succ_{chooser}) \succ_i \alpha(m_{\mathbf{N}}, \succ_{chooser})$ for each $i \in C$.

3.3.1. The case of an odd number of proposers

In this subsection, we assume that the number of proposers is odd and that all agents have strict preference over the set of candidates. Individual indifferences are ruled out. These two assumptions are convenient because they eliminate the need of specifying a tie-breaking criterion, if the screening rule is majoritarian. As we shall see, this simplifies the analysis and provides crispier results. Assumptions about the number of voters are usually considered artificial, and this is right when this number is exogenously determined (as in the case where voters are all members of a society). In our case, the size of the committee is a construct of the designer, and it is fixed by the voting rule. In that context, the assumption is not artificial. In fact a good designer may want to make sure that ties do not arise by adjusting the size of the prescribed committee.

We provide a full characterization of the set of strong Nash equilibrium outcomes of the games when the screening rule is majoritarian. Our main result holds for each one of the games: all majoritarian screening rules generate the same set of strong Nash equilibrium outcomes.

Before introducing the characterization results, we need to provide three definitions. The first one is the standard definition of a Condorcet winner. A candidate $x \in \mathbf{B} \subseteq \mathbf{A}$ is the *Condorcet winner* over $\mathbf{B}$ if $\#\{i \in \mathbf{N} \mid x \succ_i y\} > \#\{i \in \mathbf{N} \mid y \succ_i x\}$ for any $y \in \mathbf{B} \setminus \{x\}$. In words, a candidate is the Condorcet Winner over a subset of $\mathbf{A}$ if and only if it belongs to this subset and it defeats any other candidate in this subset in pairwise majority contests among proposers. It is important to note that the chooser's preferences over candidates are not taken into account in this definition. Notice also that there is at most one Condorcet winner over any set, and that such an alternative may not exist. In particular, if there is only one proposer, then the Condorcet winner over a set coincides with the proposer's preferred candidate in this set.

In the two following definitions, the preferences of the chooser will matter. A candidate is *dominated* if and only if there exists another candidate that is considered better by the chooser and by a strict majority of the proposers. A candidate is a *chooser's ℓ-top candidate* if and only if he is among the ℓ best ranked candidates according to the chooser's preference. These two definitions are important because only those candidates that are undominated and $(\#\mathbf{A} - k + 1)$-top candidates for the chooser can be strong equilibrium outcomes.

As we shall see later, even when the chooser is a player, the equilibrium conditions still require him to choose his truly preferred candidate among the k listed names. This remark is important because if a candidate is not a chooser's $(\#\mathbf{A} - k + 1)$-top candidate then it cannot be the best listed name for the chooser in any list with k names. This is why only those candidates that are $(\#\mathbf{A} - k + 1)$-top for the chooser can be strong equilibrium outcomes. Such outcomes need to be undominated, because any coalition formed by the chooser and by the strict majority of proposers is a winning coalition, i.e. are able to induce the election of any candidate.

Propositions 1 and 2 below characterize the pure strong Nash equilibrium outcomes of the Unconstrained Chooser Game and the Constrained Chooser Game, respectively.[11]

Proposition 1. *For any majoritarian screening rule, a candidate is a strong Nash equilibrium outcome of the Unconstrained Chooser Game if and only if*

1. *It is an undominated and chooser's ($\#\mathbf{A} - k + 1$)-top candidate, and*
2. *It is the Condorcet winner over some set of candidates with cardinality larger or equal than $\#\mathbf{A} - k + 1$.*

Proposition 2. *For any majoritarian screening rule, a candidate is a strong Nash equilibrium outcome of the Constrained Chooser Game if and only if it is the Condorcet winner over the chooser's ($\#\mathbf{A} - k + 1$)-top candidates.*

Notice that the Condorcet winner over the chooser's ($\#\mathbf{A} - k + 1$)-top candidates is an undominated candidate.

The proofs of all propositions appear in Appendix A.

Hence, given a set of candidates, a preference profile over this set and a value for the parameter k, we can easily identify the set of strong Nash equilibrium outcomes of the games with the help of Propositions 1 and 2. The following example illustrate it.

Example 1. Let $\mathbf{A} = \{a, b, c, d\}$, $\mathbf{N} = \{1, 2, 3\}$ and a majoritarian screening rule. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
a	a	b	d
b	b	c	b
c	c	a	a
d	d	d	c

Following Propositions 1 and 2, the first step in describing the equilibrium outcomes for each $k \in \{1, 2, 3, 4\}$, is to identify the set of undominated candidates. The second step is to find, for each undominated candidate, the largest set in which it is the Condorcet winner. The third and final step is to know the set of the chooser's ($\#\mathbf{A} - k + 1$)-top candidates.

Inspecting the preference profile above and recalling that $\#\mathbf{A} = 4$, we have that:

1. The set of undominated candidates is $\{a, b, d\}$.
2. Candidate a is the Condorcet winner over $\{a, b, c, d\}$, candidate b is the Condorcet winner over $\{b, c, d\}$ and candidate d is the Condorcet winner over $\{d\}$.
3. The chooser's ($\#\mathbf{A} - k + 1$)-top candidates are: for $k = 1$, $\{a, b, c, d\}$, for $k = 2$, $\{a, b, d\}$, for $k = 3$, $\{b, d\}$ and for $k = 4$, $\{d\}$. Combining the informations in steps 1–3 above and Propositions 1 and 2 we have the following: For the Constrained Chooser Game, when $k = 1$ or $k = 2$, candidate a is the strong equilibrium outcome. The outcome is b when $k = 3$, and it is d when $k = 4$. For the Unconstrained Chooser Game, the only change is that when $k = 2$, $\{a, b\}$ is the set of strong equilibrium outcomes.

Propositions 1 and 2 imply three corollaries. The first two refer to the existence and the number of strong equilibrium outcomes.

Corollary 1. *For any majoritarian screening rule, the set of strong Nash equilibrium outcomes of the Unconstrained Chooser Game may be empty, and its cardinality cannot be higher than the minimum between k and $\#\mathbf{A} - k + 1$.*

Corollary 1 follows from Proposition 1, the uniqueness of a Condorcet winner and because there are at most k candidates that can be a Condorcet winner over some set with cardinality $\#\mathbf{A} - k + 1$ and, by definition, there are exactly $\#\mathbf{A} - k + 1$ candidates that are chooser's ($\#\mathbf{A} - k + 1$)-top candidates. Since Condorcet winner may not exist, a strong equilibrium of the Unconstrained Chooser Game may not exist either.

Corollary 2. *For any majoritarian screening rule, the set of strong Nash equilibrium outcomes of the Constrained Chooser Game is either a singleton or empty.*

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[11] We follow closely the approach of Sertel and Sanver (2004). They consider a standard voting game where a committee elects a candidate for office, without any external interference. They show that the set of strong Nash equilibrium outcomes of their voting game is the set of generalized Condorcet winners.

Corollary 2 follows from Proposition 2 and from the fact that a Condorcet winner, if it exists, is unique.

Corollary 3 states the connection between the equilibrium outcomes of the two games we have studied.

Corollary 3. *For any majoritarian screening rule, if the strong Nash equilibrium outcome of the Constrained Chooser Game exists then it is the chooser's worst strong Nash equilibrium outcome of the Unconstrained Chooser Game.*

The example below shows that our characterization results are not valid when the screening rule is only weakly majoritarian.

Example 2. Let $\mathbf{A} = \{a, b, c, d, e, f\}$ and let $\mathbf{N} = \{1, 2, 3\}$. Assume that each proposer casts a vote for one candidate and the list is formed with the names of the three most voted candidates (a tie-breaking criterion is used when needed). So, this screening rule is only weakly majoritarian. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
a	a	f	f
b	b	e	e
c	c	d	a
d	d	c	b
e	e	b	c
f	f	a	d

Notice that, in both games, if the screening rule was majoritarian then candidate a would be the unique strong equilibrium outcome. However, the screening rule considered here is weakly majoritarian but not majoritarian. As can be verified, proposer 3 is able to force the inclusion of candidate f, his preferred candidate, in the chosen list independently of what the other proposers do. Notice that f is also the chooser's favorite candidate. Therefore candidate f is the unique strong Nash equilibrium outcome of both games.

Having presented the characterization results, we should admit that, like in many other cases, our analysis of the strategic behavior of agents under the rule of k names is marred by the fact that strong equilibria may fail to exist. Since this is pervasive, we shall not be apologetic about it, rather, we will offer two comments on the existence issue that go in opposite directions. First, Example 3 will show how easy it is to obtain nonexistence in some cases. But we will counterbalance this remark by showing (Corollary 4) that non-emptiness can be guaranteed whenever the profile of preferences of committee members is single peaked and the screening rule is majoritarian. Since single peakedness is a common and natural assumption in our context, we can argue that our characterizations, and the comparative statics results that they imply, are perfectly grounded for at least a large class of situations, in addition to being true in general.

Example 3. Let $\mathbf{A} = \{a, b, c, d, e\}$, $\mathbf{N} = \{1, 2, 3\}$ and a majoritarian screening rule. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
e	e	e	a
b	d	a	c
d	c	c	b
c	b	b	d
a	a	d	e

After a quick inspection of the preference profile above we have that: The set of undominated candidates is $\{a, c, e\}$, candidates a, c and e are the Condorcet winners over $\{a\}$, $\{a, b, c\}$ and $\{a, b, c, d, e\}$ respectively.

Applying Propositions 1 and 2, we have that the two games, for each k, share the same set of strong Nash equilibrium outcomes.

Set of strong Nash equilibrium outcomes.

k	
1	$\{e\}$
2	$\emptyset$
3	$\{c\}$
4	$\{c\}$
5	$\{a\}$

Notice that when $k = 2$, the set of strong Nash equilibrium outcomes is empty. The table above is interesting because we also can examine the effects of changing the parameter k on the set of strong equilibrium outcomes. For a moment, consider only those rules for which an equilibrium exists. Those are the rules with $k = 1$, $k = 3$, $k = 4$ and $k = 5$.

According to the agents' preferences over candidates, the chooser prefers $k = 5$ to $k = 3$ and $k = 4$ and $k = 3$ and $k = 4$ to $k = 1$. All the proposers agree that the best scenario is when $k = 1$. However, proposers 1 and 2 prefer $k = 3$ and $k = 4$ to $k = 5$ while 3 prefers $k = 5$ to $k = 3$ and $k = 4$. So proposer 3 does not have monotonic preferences over k's. It is easy to find a preference profile where one of the proposers always prefers a higher k. For instance, this may happen when one of the proposers shares with the chooser the same preferences over the candidates.

Now, the comforting news.

Definition 8. A preference profile satisfies *single peakedness* if and only if the elements of **A** can be linearly ordered as $x_1 > x_2 > \cdots > x_{\#A}$ such that for every $i \in \mathbf{N}$ and $a, b \in \mathbf{A}$ we have that if $b > a > \alpha(\mathbf{A}, \succ_i)$ or $\alpha(\mathbf{A}, \succ_i) > a > b$ then $a \succ_i b$, where $\alpha(\mathbf{A}, \succ_i)$ is i's preferred candidate in **A**.

Corollary 4. *For both games the following statement holds*:

For any majoritarian screening rule, if the proposers have single peaked preferences then the set of strong Nash equilibrium outcomes is not empty.

Corollary 4 follows by Propositions 1 and 2 and by the well-known result in social choice theory, proved by Black (1958), that whenever the preference profile is single peaked, a Condorcet winner candidate always exists, and that it is unique if there is an odd numbers of voters.

3.3.2. The case of an even number of proposers

We now qualify some of the statements in the previous subsection.

We first remark that if the number of proposers is even the set of strong Nash equilibrium outcomes may depend on how ties are broken. It can be seen in the following example.

Example 4. Let $\mathbf{A} = \{a, b, c, d\}$, and let $\mathbf{N} = \{1, 2, 3, 4\}$. Assume that each proposer casts votes for two candidates, and that the list is formed with the names of the two most voted candidates with a tie breaking rule in case of need. Notice that this screening rule is majoritarian. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Proposer 4	Chooser
a	a	b	b	b
d	d	d	d	a
c	c	c	c	d
b	b	a	a	c

The set of strong equilibrium outcomes of this game will depend on how ties are broken under this screening rule:

1) Suppose that the tie breaking criterion is as follows: $\{a, b\} \succ \{a, c\} \succ \{a, d\} \succ \{b, c\} \succ \{b, d\} \succ \{c, d\}$. Notice that proposers 3 and 4 are able to include in the chosen list their favorite candidate, which is b. Since candidate b is also the chooser's favorite candidate, there is no doubt that candidate b is the unique strong equilibrium outcome of the Constrained Chooser Game.
2) Suppose that the tie breaking criterion is as follows: $\{a, c\} \succ \{a, d\} \succ \{a, b\} \succ \{c, d\} \succ \{c, b\} \succ \{d, b\}$. Now proposers 1 and 2 are able to guarantee the choice of the list formed by candidates a and c. If this list is chosen, candidate a, which is proposers 1 and 2's preferred candidate will be elected, since he is the best listed name according to the chooser's preferences. Thus it is clear that candidate a is the unique strong equilibrium outcome of the Constrained Chooser Game.

Second, we notice that the relationship between the set of strong Nash equilibrium outcomes for our two games is no longer a simple one. In particular, the set of equilibrium outcomes for the Unconstrained Chooser Game may not contain the set of those that arise in the Constrained Chooser Game. This is due to the fact that some of the coalitional arrangements that would eliminate certain outcomes in the unconstrained games will not be available in the constrained one, because the chooser can only use one strategy.

This remark is important because it proves that these two games are indeed different from each other in important respects.

Example 5. Let $\mathbf{A} = \{a, b, c, d\}$, and let $\mathbf{N} = \{1, 2, 3, 4\}$. Consider the following majoritarian screening rule for two names: The chosen list of two names is $\{b, d\}$ unless the strict majority of the proposers agrees with another list. Notice that this screening rule is majoritarian. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Proposer 4	Chooser
a	a	b	b	d
b	b	a	a	b
c	c	c	c	a
d	d	d	d	c

The set of strong Nash equilibrium outcomes of the Constrained Chooser Game is $\{a, b\}$. Notice that all the proposers announcing that they support $\{a, c\}$ is a strong Nash equilibrium outcome of the Constrained Chooser Game and it leads the victory of candidate a. All the proposers supporting $\{b, c\}$ is a strong equilibrium as well. In this equilibrium, candidate b is the winner.

However in the context of the Unconstrained Chooser Game, candidate b is the unique strong Nash equilibrium outcome. There exists no strategy profile that can sustain candidate a as a strong equilibrium outcome under this game because the coalition formed by the chooser and by proposers 3 and 4 can always find a profitable deviation.

3.4. Comparative statics with an odd number of proposers

Our purpose now is to examine the consequences of changing the parameter k, of adding undesirable candidates and of replacing a majoritarian screening rule by non-majoritarian screening rule. By knowing these consequences, we can infer the agent's preferences over different variants of the rule of k names. This can provide some insights into the questions raised in the introduction. Let us recall some of these questions: Why are these rules used? What could be the intentions and the expectations of those who decided to set them up? What is the type of strategic behavior that these rules induce on the different agents involved? Why choose three names in some cases, six in other occasions?

Since the following analysis is already quite involved, we return here again to the assumption that the number of proposers is odd. We have already discussed partially some of these issues for the one proposer case. Allowing several proposers complicates our analysis because the strong equilibrium may fail to exist. In the latter example, does the chooser prefer $k = 1$ to $k = 3$ or the reverse? This is a difficult question since for $k = 3$ there is no equilibrium. We could partially avoid this problem by assuming that the preference profile satisfies single-peakedness (see Corollary 4). Unfortunately, this assumption would not avoid the non existence of equilibrium when screening rules are not majoritarian. Thus it cannot help us to compare the performance of majoritarian screening rules with others that are only weakly majoritarian. Another drawback is the possibility of multiple equilibrium outcomes in the Unconstrained Chooser Game.

In what follows, and with this warning, we will try to make our best in tackling with those added difficulties. We assume that the agents have preferences over sets of strong Nash equilibrium outcomes that satisfy two mild requirements: Let $\mathbf{P}$ denote a generic individual strict preference relation on $2^A \equiv \{B \subseteq \mathbf{A} \mid B \neq \emptyset\}$. Consider any $X, Y \in 2^A$ and $X \neq Y$. (1) If $X \subset Y$ then $X \mathbf{P}_i Y$ if $x \succ_i y$ for all $x \in X$ and for all $y \in Y \setminus X$. (2) If $X \nsubseteq Y$ then $X \mathbf{P}_i Y$ if $x \succ_i y$ for all $x \in X \setminus Y$ and for all $y \in Y$. Notice that if an agent i prefers x to y then condition (1) implies that $\{x\}$ is preferred to $\{x, y\}$ and condition (2) implies that $\{x\}$ and $\{x, y\}$ are preferred to $\{y\}$. These are very natural conditions since the elements of a set of strong Nash equilibrium outcomes are mutually exclusive alternatives.[12]

Consider any of the two games. Let us denote by $\mathbf{SET}(\mathbf{S}'; \mathbf{A}'; k')$ the set of strong Nash equilibrium outcomes of this game when $k = k'$, the set of candidates is $\mathbf{A}'$ and the screening rule is $\mathbf{S}'$. We say that the agent i prefers the triple $(\mathbf{S}''; \mathbf{A}''; k'')$ to $(\mathbf{S}'; \mathbf{A}'; k')$ if and only if $\mathbf{SET}(\mathbf{S}''; \mathbf{A}''; k'') \mathbf{P}_i \mathbf{SET}(\mathbf{S}'; \mathbf{A}'; k')$.

In the context of the Constrained Chooser Game, we know by Corollary 2 that the set of equilibrium outcomes is singleton or empty. So, the condition above says that agent i prefers the triple $(\mathbf{S}''; \mathbf{A}''; k'')$ to $(\mathbf{S}'; \mathbf{A}'; k')$ if and only if the strong equilibrium outcome under $(\mathbf{S}''; \mathbf{A}''; k'')$ is preferred to the respective outcome under $(\mathbf{S}'; \mathbf{A}'; k')$ according to agent i's preferences over candidates.

The next proposition states that, in the context of the Constrained Chooser Game, if the chooser is asked to choose between a rule of k' names and of k'' names, and both rules use majoritarian screening rules then the chooser prefers the rule with the highest k.

Proposition 3. *For the Constrained Chooser Game the following statement holds*:

If $\emptyset \neq \mathbf{SET}(S''; \mathbf{A}, k'') \neq \mathbf{SET}(\mathbf{S}'; \mathbf{A}, k') \neq \emptyset$, $k'' > k'$, and both $\mathbf{S}'$, and $\mathbf{S}''$ are majoritarian then the chooser prefers the triple $(\mathbf{S}''; \mathbf{A}, k'')$ to $(\mathbf{S}'; \mathbf{A}, k')$.

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[12] Examples of preferences over sets can be found in Barberà et al. (2004).

Surprisingly, this proposition is not valid in the context of the Unconstrained Chooser Game. It can be seen in the following example.

Example 6. Let $\mathbf{A} = \{a, b, c, d\}$, and let $\mathbf{N} = \{1, 2, 3\}$. The preferences of the chooser and the committee members are as follows:

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
d	c	b	b
a	a	c	a
b	b	d	c
c	d	a	d

Given the preference profile above, the set of undominated candidates is $\{a, b\}$, candidates a and b are the Condorcet winners over $\{a, b\}$, $\{b, c, d\}$, respectively.

Applying Proposition 1, we have the following equilibrium outcomes:

Set of strong Nash equilibrium outcomes.

k	Unconstrained Chooser Game
1	$\emptyset$
2	$\{b\}$
3	$\{a, b\}$
4	$\{b\}$

Examining the table above, we can see that the chooser prefers $k = 2$ to $k = 3$, while the majority of the proposers, 1 and 3, prefers $k = 3$ to $k = 2$. The intuition of this result is the following: the passage from $k = 2$ to $k = 3$ makes it more difficult for the proposers to coordinate their strategies. Notice that, under $k = 2$, the set of strong Nash equilibrium outcomes of the Constrained Chooser Game is empty while, under $k = 3$, candidate a is the equilibrium outcome of this game. It means that under $k = 2$ the proposers are not able to coordinate themselves in order to form a majority winner coalition without the chooser.

Proposition 4. *For the Unconstrained Chooser Game the following statement holds*:

If $\emptyset \neq \mathbf{SET}(\mathbf{S}''; \mathbf{A}, k'') \neq \mathbf{SET}(\mathbf{S}'; \mathbf{A}, k') \neq \emptyset$, $k'' > k'$, both $\mathbf{S}'$ and $\mathbf{S}''$ are majoritarian and the proposers have single peaked preferences then the chooser prefers the triple $(\mathbf{S}''; \mathbf{A}, k'')$ to $(\mathbf{S}'; \mathbf{A}, k')$.

Now, let us analyze the role of a candidacy that, at first glance, one could imagine that has no influence in the game. We say that a candidate is an *undesirable candidate* in $\mathbf{A}$ if the chooser and all proposers dislike him more than any other candidate in $\mathbf{A}$.

The next results show that the withdrawal of an undesirable candidate has an effect similar to that of passing from k to $k + 1$.

Proposition 5. *For both games the following statement holds*:

If candidate u is an undesirable candidate of $\mathbf{A}$ and both $\mathbf{S}'$ and $\mathbf{S}''$ are majoritarian then $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \subseteq \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k + 1)$.

Corollary 5. *For the Constrained Chooser Game the following statement holds*:

If candidate u is an undesirable candidate of $\mathbf{A}$, $\mathbf{S}'$ is majoritarian and the proposers have single peaked preferences then $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) = \mathbf{SET}(\mathbf{S}'; \mathbf{A}; k + 1) \neq \emptyset$.

The next proposition states that the chooser cannot be worse off if an undesirable candidate decides to withdraw from the contest.

Proposition 6. *For both games the following statement holds*:

If $\emptyset \neq \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k) \neq \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \neq \emptyset$, candidate u is an undesirable candidate of $\mathbf{A}$ and both $\mathbf{S}'$ and $\mathbf{S}''$ are majoritarian, then the chooser prefers the triple $(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$ to $(\mathbf{S}''; \mathbf{A}; k)$.

How about the chooser's preferences over screening rules? This is a natural question since half of the screening rules that we have documented are not majoritarian. We now comment on the advantages and the disadvantages for the chooser

of operating under either type of screening rule. Going back to Example 2 above, the reader may see that under both games the chooser can be better off when a majoritarian screening rule is substituted by another that is only weakly majoritarian. Examples 7 and 8, below, show that the opposite can also happen.

Example 7. Let $A = \{a, b, c, d, e\}$, and let $N = \{1, 2, 3\}$. Consider the following screening rule: Each proposer casts a vote for list A or list B. List A is formed by candidates a, b and e and list B is formed by candidates b, c and d. The screened list is the most voted list. Notice that this screening rule is weakly majoritarian but not majoritarian.

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
b	b	e	e
c	c	a	a
a	a	b	b
d	d	c	c
e	e	d	d

For $k = 3$, under this screening rule, candidate b is the unique strong Nash equilibrium outcome of the Unconstrained Chooser Game. However under any majoritarian screening rule, $\{a, b\}$ is the set of strong Nash equilibrium outcomes of the Unconstrained Chooser Game. Therefore, the chooser is better off under any majoritarian screening rule. Notice that the reverse can be said to the majority of the proposers.

Example 8. Let $A = \{a, b, c, d\}$, and let $N = \{1, 2, 3\}$. Each proposer votes for one candidate and the list has the names of the two most voted candidates, with a tie break when needed. The tie breaking criterion is as follows: $c \succ d \succ b \succ a$. Notice that this screening rule is weakly majoritarian but not majoritarian.

Preference profile.

Proposer 1	Proposer 2	Proposer 3	Chooser
b	a	a	b
c	c	c	a
d	d	d	c
a	b	b	d

Notice that candidate a cannot be a strong equilibrium outcome of the Constrained Chooser Game, because as long as proposer 1 votes for b, proposers 2 and 3 cannot get a to be the outcome, even if they can force a to be in the list. Short of that, proposers 2 and 3 coordinate their actions so that one of them votes for c and the other for d. If 1 persists in voting for b, this creates a tie between the three candidates that is solved in favor of c and d, out of which the chooser selects c. If 1 votes for c instead, the same outcome ensues. And all other actions by any combination for agents would lead some of them to outcomes that would be worse than c for some of them. Hence, c is the unique strong Nash equilibrium of the Constrained Chooser Game under our proposed rule. Yet, under any majoritarian screening rule, 2 and 3 could ensure that a, c, or a, d became the set proposed to the chooser, and candidate a would be the unique strong Nash equilibrium outcome. This shows that for here the chooser is better off under any majoritarian screening rule than under this weakly majoritarian one.

3.5. The rule of q vetoes

As we said in Section 1, in Mexico, the President of the Republic shall propose three names to the Senate, which shall appoint one of them to become member of the Supreme Court of Justice. Since vetoing q names is equivalent to selecting $\#A - q$ names, we can say that the Mexican president vetoes $\#A - 3$ candidates, and then the Senate chooses one of the remaining candidates for appointment. This system is thus a member of the family of the "rules of q vetoes" that can be described as follows: given a set of candidates for office, a single individual vetoes q members from this set. Then a committee selects one candidate by plurality voting, among those not vetoed, for appointment.

Proposition 7. *When the number of proposers is odd, any rule of k names based on majoritarian screening is equivalent to a rule with $k - 1$ vetoes, in the sense they have the same sets of strong equilibrium outcomes.*

A interesting implication of the proposition above is that the balance of power between the Mexican President and the Senate would not be changed, if this nomination system was substituted by the rule of $\#A - 2$ names where the Senate is the committee of proposers and the screening rule is majoritarian.

3.6. Concluding remarks

As shown by our analysis the rule of k names is a method to balance the power of the two parties involved in decisions: the committee and the final chooser. We have provided examples of several institutions around the world that use the rule of k names to take decisions. We described six different screening rules that are actually used. Two of them are majoritarian and the others are only weakly majoritarian.

As part of our attempt to understand the widespread use of these rules, we have engaged in a game theoretical analysis of two games induced by them. We have shown that the choice of the screening procedure to select the k names is not too crucial when agents act strategically and cooperatively. This is because rules of k names based on different majoritarian screening rules lead to the same sets of strong equilibrium outcomes. We characterized the set of strong equilibrium outcomes of these games under any majoritarian screening rules. As we already pointed out, this result is reminiscent of results in the Condorcet jury theorem literature showing that some institutional details are crucial in determining the voters' behavior, while others are not. It is not unreasonable to conjecture that the choice between strongly and weakly majoritarian screening rules, at the moment of their establishment, might have resulted from the relative strength of the interested parties, i.e. of the chooser and the proposers, in each particular historical case. Checking this conjecture, which we leave open here, would require an accurate analysis of historical data.

For both games, we determined the effects on the equilibria of increasing k, and of adding undesirable candidates. We also discussed the possible consequences for the chooser and for the voters of using majoritarian screening rules, versus those of using weakly majoritarian ones. Knowing these effects, we were able to derive endogenously the agents' preferences over different variants of the rule of k names.

For both games, the chooser weakly prefers high k's as well as adding to the contest an undesirable candidate, i.e. a candidate that nobody likes, goes against the chooser's interests.

Without the assumption of single peaked preferences, many interesting voting paradoxes emerge. For instance, we give an example where chooser strictly prefers $k = 2$ to $k = 3$ while a majority of proposers strictly prefers $k = 3$ to $k = 2$.

We have also shown the equivalence of rule of $k - 1$ vetoes with the rule of k names in terms of strong Nash equilibrium outcomes of the Unconstrained Chooser Game. In other words, we proved that the set of strong equilibrium outcomes would not change if instead of taking the final decision, the chooser vetoes $k - 1$ candidates and then let the committee select by plurality one of the remaining candidates for appointment.

We interpret our present work as a first step for understanding the implications of using such methods, and hope to generate interest in its further study. In particular, we have emphasized the mix of cooperative and non-cooperative considerations which are embodied in the concept of strong Nash equilibrium. Further work may concentrate on the study of the rules under different formalizations and for alternative solution concepts, and eventually introduce other important issues we have omitted. For example, the role of communication under this type of rules, or the optimal choice of the set of proposers and choosers.

Appendix A

Proof of Proposition 1. Suppose that candidate x is the outcome of a strong equilibrium of the Unconstrained Chooser Game. In any strong Nash equilibrium where x is the outcome, the screened set is such that x is the best candidate in this set according to the chooser's preferences. Otherwise the chooser would have incentives to choose another name in this set. So x is a chooser's ($\#\mathbf{A} - k + 1$)-top candidate. Since the screening rule is majoritarian there exists no other set with k names where all the candidates in this set are considered better than x by a strict majority of proposers. Otherwise this coalition would have incentives to impose the choice of this set. This is only true when candidate x is the Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A} - k + 1$. For this same reason there exists no candidate that is considered better than x by a strict majority of proposers and the chooser. This implies that candidate x is a undominated candidate. Therefore the first part of the proposition is proved.

To complete the proof we need to show that if a candidate is (1) undominated and chooser's ($\#\mathbf{A} - k + 1$)-top candidate, and (2) Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A} - k + 1$ then there exists a strategy profile that sustains him as a strong Nash equilibrium outcome. Let us call this candidate x. Take any set with k candidates contained in $\mathbf{A}$ such that x is the chooser's best candidate in this set. Notice that this set exists, since candidate x is a chooser's ($\#\mathbf{A} - k + 1$)-top candidate. Let B be such a set. Since the screening rule is majoritarian there exists an action such that every majority coalition of proposers can impose the choice of B provided that all of its members choose this action. Let m be such an action. Suppose the strategy profile where all proposers choose the action m. Let the chooser declare a choice rule such that if candidate x is in the screened set, x is the winning candidate. Otherwise, the winner is a candidate in the screened set that is considered worse than x by a strict majority of proposers (notice that this choice rule exists since x is the Condorcet winner over a set with cardinality $\#\mathbf{A} - k + 1$). Under this strategy profile, candidate x will be elected since the screening rule is majoritarian. Notice that the chooser's strategy eliminates any incentive of any majority coalition of proposers to deviate. Notice also that candidate x is the chooser's best candidate in the screened set, so that the chooser has no incentive to unilaterally deviate either. No coalition formed by a majority of proposers and the chooser has incentives to deviate either, since x is a undominated candidate. Therefore this strategy profile is a strong Nash equilibrium of the Unconstrained Chooser Game. Therefore, the proof of the proposition is established. □

Proof of Proposition 2. Suppose that candidate x is the outcome of a strong equilibrium of the Constrained Chooser Game. In any strong Nash equilibrium where x is the outcome, the screened set is such that x is the best candidate in this set according to the chooser's preferences. So x is a chooser's $(\#\mathbf{A}-k+1)$-top candidate. Since the screening rule is majoritarian there exists no other chooser's $(\#\mathbf{A}-k+1)$-top candidate that is considered better that x by a strict majority of proposers. Otherwise, this coalition could impose the choice of a set where this candidate would be the preferred candidate according to the chooser's preference. Therefore, candidate x is the Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates, and the first part of the proposition is proved.

To complete the proof we need to show that if a candidate is the Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates then there exists a strategy profile that sustains him as a strong Nash equilibrium outcome. Let x be such a candidate. Take any set with k candidates contained in $\mathbf{A}$ such that x is the chooser's best candidate in this set. Notice that this set exists, since candidate x is a chooser's $(\#\mathbf{A}-k+1)$-top candidate. Let B be such a set. Since the screening rule is majoritarian there exists an action such that every majority coalition of proposers can impose the choice of B provided that all of its members choose this action. Let m be such an action. Consider the strategy profile, where all proposers choose action m. Then, candidate x will be elected since the screening rule is majoritarian. By this same reason, any coalition with less than half of the proposers cannot change the outcome. Notice also that any majoritarian coalition does not have any incentive to deviate, since there is no candidate among the chooser's $(\#\mathbf{A}-k+1)$-top candidate that is considered better than x by all proposers in the coalition (recall that only the chooser's $(\#\mathbf{A}-k+1)$-top candidates can be the chooser' best name among the candidates of a set with cardinality k). Otherwise, x would not be a Condorcet winner over the set of the chooser's $(\#\mathbf{A}-k+1)$-top candidates. Therefore, this strategy profile is a strong Nash equilibrium of the Constrained Chooser Game. $\square$

Proof of Corollary 3. Let x be the strong equilibrium outcome of the Constrained Chooser Game and z be a strong equilibrium outcome of the Unconstrained Chooser Game. By Proposition 2, x is the Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates. This information and Proposition 1 imply that x is also a strong equilibrium outcome of the Unconstrained Chooser Game. Again by Proposition 1, z is (1) undominated and chooser's $(\#\mathbf{A}-k+1)$-top candidate, and (2) the Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A}-k+1$. Let us prove that the chooser does not prefer x to z. Suppose by contradiction that the chooser prefers x to z. This implies that the strict majority of proposers prefers z to x. Otherwise, z would not be undominated. Since z is chooser's $(\#\mathbf{A}-k+1)$-top candidate, it implies that x is not the Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates. This is a contradiction. $\square$

Proof of Proposition 3. Suppose $k'' > k'$, both $\mathbf{S}'$ and $\mathbf{S}''$ are majoritarian screening rules and $\emptyset \neq \mathbf{SET}(\mathbf{S}'';\mathbf{A};k'') \neq \mathbf{SET}(\mathbf{S}';\mathbf{A};k') \neq \emptyset$. Let $x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A};k'')$ and $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A};k')$ such that $x \neq y$. It will suffice to show that $x \succ_{chooser} y$. Suppose by contradiction that the chooser prefers y to x. By Proposition 2, we have that x is the Condorcet winner over the set of the chooser's $(\#\mathbf{A}-k''+1)$-top candidates. And y is the Condorcet winner over the of chooser $(\#\mathbf{A}-k'+1)$-top candidates. Since x is one of the chooser's $(\#\mathbf{A}-k''+1)$-top candidate, the chooser prefers y to x and $k'' > k'$, we also have that y is a chooser's $(\#\mathbf{A}-k''+1)$-top candidate. But then this contradicts the fact that x is the Condorcet winner over the set of a chooser's $(\#\mathbf{A}-k''+1)$-top candidates. $\square$

Proof of Proposition 4. Suppose $k'' > k'$, both $\mathbf{S}'$ and $\mathbf{S}''$ are majoritarian screening rules and $\mathbf{SET}(\mathbf{S}'';\mathbf{A};k'') \neq \mathbf{SET}(\mathbf{S}';\mathbf{A};k')$. In order to prove this proposition we need to prove the chooser prefers $\mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$ to $\mathbf{SET}(\mathbf{S}';\mathbf{A},k')$. It will suffice to show that:

(1) if $\mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \subset \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ then there are no x and y such that $y \succ_{chooser} x$, $x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$ and $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k') \setminus \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$.
(2) If $\mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \not\subseteq \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ then there are no x and y such that $y \succ_{chooser} x, x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \setminus \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ and $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$.

Let us prove item (1). Suppose by contradiction that $\mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \subset \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ and there are $x, y \in A$ such that , $y \succ_{chooser} x$, $x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$ and $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k') \setminus \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$. Notice that the fact $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k') \setminus \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$ implies that y is not a chooser's $(\#\mathbf{A}-k''+1)$-top candidate. It follows because y is a Condorcet winner over a set of candidates with cardinality equal to $\#\mathbf{A}-k''+1$ since $k'' > k'$ and Proposition 1. Hence, since x is a chooser's $(\#\mathbf{A}-k''+1)$-top candidate. By Proposition 1, it implies that $x \succ_{chooser} y$. It is a contradiction.

Now let us prove item (2). Suppose by contradiction that $\mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \not\subseteq \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ and there are $x, y \in A$ such that $y \succ_{chooser} x, x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \setminus \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$ and $y \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$. By Proposition 1, since $y \succ_{chooser} x$ and $x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'')$ then y is a chooser's $(\#\mathbf{A}-k''+1)$-top candidate and x is undominated. It implies that the majority of the voters prefers x to y, otherwise x would not be undominated. Notice, also by Proposition 1, that y is a Condorcet winner over some set of candidates with cardinality larger or equal than $\#\mathbf{A}-k'+1$. It implies that x is also a Condorcet winner over some set of candidates with cardinality larger or equal than $\#\mathbf{A}-k'+1$. It follows by single peakedness and the fact that the majority of the voters prefers x to y. Since x is also chooser's $(\#\mathbf{A}-k'+1)$-top candidate and undominated, it implies that $x \in \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$. It is a contradiction since $x \in \mathbf{SET}(\mathbf{S}'';\mathbf{A},k'') \setminus \mathbf{SET}(\mathbf{S}';\mathbf{A},k')$. $\square$

Proof of Proposition 5. Consider first the Unconstrained Chooser Game. Let u be an undesirable candidate of $\mathbf{A}$. It is easy to see that the set of the chooser's $(\#\mathbf{A} \setminus \{u\} - k + 1)$-top candidates is equal to the set of the chooser's $(\#\mathbf{A} - (k + 1) + 1)$-top candidates. Moreover, the set of undominated candidates does not change when the set of the candidates is $\mathbf{A} \setminus \{u\}$ or $\mathbf{A}$. We also have that the set of candidates that are Condorcet winners over some set with cardinality $\#\mathbf{A} \setminus \{u\} - k + 1$ is contained in the set of candidates that are Condorcet winners over some set with cardinality $\#\mathbf{A} - (k + 1) + 1$. By Proposition 1, these informations imply that $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \subseteq \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k + 1)$.

For the Constrained Chooser Game, the proof requires a similar argument. For this reason it is omitted. $\quad\square$

Proof of Proposition 6. Let u be the undesirable candidate of the set $\mathbf{A}$ and $\emptyset \neq \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k) \neq \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \neq \emptyset$.

First let us prove that the statement holds for the Unconstrained Chooser Game. Notice that the set of undominated candidates does not change whenever the set of candidates is $\mathbf{A} \setminus \{u\}$ or $\mathbf{A}$. We also have that the set of candidates that are Condorcet winners over some set with cardinality $\#\mathbf{A} \setminus \{u\} - k + 1$ is equal to the set of candidates that are Condorcet winners over some set with cardinality $\#\mathbf{A} - k + 1$. Moreover, any chooser's $(\#\mathbf{A} \setminus \{u\} - k + 1)$-top candidate is also chooser's $(\#\mathbf{A} - k + 1)$-top candidate. Therefore, by Proposition 1, we have that $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \subseteq \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k)$. Thus it suffices to show that we have $x \succ_i y$ for all $x \in \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$ and $y \in \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k) \setminus \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$. Suppose by contradiction that there is $y \in \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k) \setminus \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$ and $x \in \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$ such that the chooser prefers y to x. Notice that this information implies that y is a chooser's $(\#\mathbf{A} \setminus \{u\} - k + 1)$-top candidate. But then, we have y is undominated, chooser's $(\#\mathbf{A} \setminus \{u\} - k + 1)$-top candidate and the Condorcet winner of a set with cardinality $\#\mathbf{A} \setminus \{u\} - k + 1$. So, by Proposition 1, we have that $y \in \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$. This is a contradiction, since we had assumed that $y \in \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k) \setminus \mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$.

Now let us prove that the statement for the Constrained Chooser Game. By Proposition 5, we have that $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) = \mathbf{SET}(\mathbf{S}''; \mathbf{A}; k + 1)$ since $\mathbf{SET}(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k) \neq \emptyset$. Hence, by Proposition 3, we have that the chooser prefers the triple $(\mathbf{S}'; \mathbf{A} \setminus \{u\}; k)$ **to** $(\mathbf{S}''; \mathbf{A}; k)$. $\quad\square$

Proof of Proposition 7. By Proposition 1, it suffices to show that a candidate is a strong equilibrium outcome under the rule of $k - 1$ vetoes if and only if it is (1) undominated and chooser's $(\#\mathbf{A} - k + 1)$-top candidate, and (2) Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A} - k + 1$.

Suppose that candidate x is the outcome of a strong equilibrium outcome under rule of $k - 1$ vetoes. In any strong Nash equilibrium where x is the outcome, x is the Condorcet winner over the set of not vetoed candidates. Otherwise, there would be a coalition formed by a majority of proposers that would have incentives in voting for another not vetoed candidate and this candidate would be elected. So x is the Condorcet winner over a set with cardinality $\#\mathbf{A} - k + 1$, since this is the cardinality of the set of available candidates after the veto made by the chooser. Notice that there exists no subset with cardinality $\#\mathbf{A} - k + 1$ where all candidates are considered better than x by the chooser. Otherwise, the chooser would have an incentive to veto all but those in this subset. This is only true when x is a chooser's $(\#\mathbf{A} - k + 1)$-top candidate. Since the proposers use plurality, there exists no other candidate that is considered better than x by a majority of proposers and by the chooser. Otherwise, this coalition would be able to elect this candidate. This is only true when x is an undominated candidate. Thus, we have proved that if a candidate is a strong equilibrium outcome under rule of $k - 1$ vetoes then it is (1) undominated and chooser's $(\#\mathbf{A} - k + 1)$-top candidate, and (2) Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A} - k + 1$. So we have completed the first part of the proof.

To finish the proof we need to show that if a candidate is (1) undominated and chooser's $(\#\mathbf{A} - k + 1)$-top candidate, and (2) Condorcet winner over some set of candidates with cardinality higher or equal than $\#\mathbf{A} - k + 1$ then there exists a strategy profile that sustains him as a strong Nash equilibrium outcome. Let x be such a candidate. Consider the following strategy profile: The chooser vetoes all candidates but the set with cardinality $\#\mathbf{A} - k + 1$ in which candidate x is the Condorcet winner (if there are more than one set with this characteristic choose one of them). All proposers unanimously cast a vote for candidate x if candidate x is a not vetoed; otherwise they vote for candidate considered worse than x according to the chooser's preference (this strategy is feasible since x is a chooser's $(\#\mathbf{A} - k + 1)$-top candidate). Notice that under this strategy profile, candidate x will be elected. Notice that the proposers' threat eliminates any chooser's incentive in unilaterally deviating. Notice that candidate x is the Condorcet winner over the set of not vetoed candidates. So the majority of proposers do not have incentive to deviate. Notice also that a coalition formed by a majority of the proposers and the chooser does not have any incentive to deviate either, since x is an undominated candidate. Thus, this strategy profile is a strong equilibrium under rule of $k - 1$ vetoes. Therefore, the proof of the proposition is established. $\quad\square$

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