Information Disclosure under Strategy-proof Voting Rules*

Salvador Barberà†
MOVE, Universitat Autònoma de Barcelona and Barcelona GSE

Antonio Nicolò‡
University of Padua and University of Manchester

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Abstract

We consider collective decision problems where some agents have private information about alternatives and others don't. Voting takes place under strategy-proof rules. Prior to voting, informed agents may or may not disclose their private information, thus eventually influencing the preferences of those initially uninformed. We provide general conditions on the voting rules guaranteeing that informed agents will always be induced to disclose what they know. In particular, we

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†salvador.barbera@uab.cat. Universitat Autònoma de Barcelona; Facultat d'Economia i Empresa; Edifici B; 08193 Bellaterra (BARCELONA); Spain.

‡antonio.nicolo@unipd.it. Department of Economics, University of Padua, via del Santo 33, 35124 Padova - Italy.

apply this general result to environments where agent's preferences are restricted to be single-peaked or separable, and characterize the strategy-proof rules that ensure information disclosure in these settings.

Keywords: strategy-proofness, information disclosure, voting rules, single-peaked preferences, committees.

JEL Codes: D70, D71, D82.

1 Introduction

Whenever the people are well-informed, they can be trusted with their own government. (*Thomas Jefferson, letter to Richard Price Jan. 8, 1789.*)

If you agree with Jefferson's view, you would like to live in a society that favors the flow of reliable information among its members. And a social designer may want to include among its objectives that of favoring the transmission from informed to uninformed agents of any available piece of reliable news, whatever these are and whatever their consequences on people's opinions and preferences.

The connection between the preferences of individuals and the information they hold is treated very differently by the two great lines of thought underlying the choice of voting institutions. In the Arrowian tradition, it is not discussed: agents' preferences, which may be the result of many factors, including what they know, are taken as given. The focus there is to aggregate heterogeneous preferences into criteria that allow for satisfactory social decisions. At the other end, Condorcet's jury theorem abstracts from the heterogeneity of agents' interests, assumes that they all have the same objective (discovering the truth), and focuses on the issue of aggregating the pieces of information that are available to each, in order again to achieve an acceptable social outcome. In this paper we discuss the interaction between preferences, information and social outcomes that arises once we accept that individuals need not share the same objectives (departing from Condorcet) and yet can modify their preferences if availed with new pieces of information (departing from Arrow). We adopt the point of view of a mechanism designer who wishes to favor information exchange and proposes a voting rule to arrive at a social outcome given the individual preferences that are finally declared by agents, once availed with the information that is voluntarily dispersed throughout society. Our design of a mechanism will thus involve allowing for a stage during which agents may share information, and a second one where they vote according to a given rule. Our purpose is to discuss the choice of voting rules to be used in the second stage of the decision process, and to identify those that favor information disclosure in the first stage. In particular, we concentrate on those rules that are strategy-proof, and thus guarantee the use of truthful information about agents' preferences at the voting stage. We prove a very general result showing that, in order for a strategy-proof voting rule to induce information disclosure, it must provide informed agents with a specific form of veto power, and then apply it to a variety of environments where attractive strategy-proof voting rules do exist. The ability of different rules to induce information disclosure becomes then a powerful additional requirement to select among them. In fact, if information is held by more than half of the population, anonymous strategy-proof rules can be selected among those that satisfy our requirements, with simple majority playing a major role. When the set of informed voters is small, then in order to induce disclosure one has to resort to strategy-proof rules that skew the power of voters in favor of those who are informed, although uninformed voters can still influence the social outcome. In most of the paper, we consider that some agents will always be informed, the rest will never be, and the designer will know a priori which agents are informed. But we also briefly discuss the case where the identity of informed agents is unknown to the designer.

Unlike most of the papers that we shall survey below, we work in a deterministic setting that does not require the use of Bayesian games. This is the result of several modeling decisions that allow us to isolate the choice of voting rules as our unique objective, and to avoid other related issues. Here are some basic assumptions we make. First, we concentrate on hard information. This is a natural requirement, given our premise that knowing more is always desirable, and it avoids issues regarding whom to trust, and to what extent, that would arise if facts were debatable. Second, we restrict attention to mechanisms that use strategy-proof rules in the second stage. This eliminates the need to model agents' guesses about the strategic voting behavior of others. And we analyze situations where, once a state of nature is realized, the agents involved have full information. There is still room in our model for informed agents to form expectations about the reaction of uninformed ones when they receive information, and when they do not, but as the reader will see they are modeled in a deterministic manner.

The issue of information disclosure has received the attention of many authors in many different economic settings, with applications to general and also to specific contexts (see for instance, Butters (1977) on advertising, Grossman (1981), Milgrom (1981), and Milgrom & Roberts (1986) for more general settings, or for more recent literature on mechanism design with evidence (Bull and Watson (2004, 2007) and Kartik and Tercieux (2012)). A leading application where these issues become highly prominent is the study of voting rules. Since the literature at large is too vast to cover all relevant references beyond those that we have already mentioned, in what follows we concentrate our comments to those papers dealing with issues of information disclosure in voting, and try to emphasize the differences between our approach and that of preceding works. A natural starting point is to enrich the context of Condorcet's jury theorem, where agents with common preferences must pool their private information through voting, by allowing some form of communication prior to the decision stage. Austen-Smith and Feddersen (2005, 2006) modeled communication among agents as cheap talk, building on previous works that already emphasized the fact that agents would not always behave straightforwardly, as assumed by Condorcet (see Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998)). In all these papers, social decisions involve the choice between only two alternatives and concentrate on simple rules that are the natural ones to consider in that setting. Our paper considers much wider families of rules, since we are interested in choices involving possibly many alternatives. Jackson and Tan (2012) build on the idea that experts (who can be voters or not) may observe private signals about the relative values of alternatives and decide whether or not to reveal them. Unlike in the previous papers, but like in ours, they assume that information is hard: agents cannot lie, but only conceal information. However, their analysis is again restricted to the case of binary decisions. Bhattarharya (2013) studies the robustness of the Condorcet jury theorem to relaxations of the assumption that all voters share the same goal, thus allowing the preferences of different agents not to be affected in the same manner, even if they all get the same information. This heterogeneity of responses to information is part of our model. But again, his paper concentrates on the case with two alternatives. Another general model of communication and disclosure is Hagenbach, Koesler and Pérez-Richet (2014). These authors consider Bayesian games augmented by a pre-play communication phase in which players can make announcements. When applied to voting games, they characterize conditions under which a fully revealing equilibrium will prevail, under, again, the assumption that the choice involves two alternatives only. Other papers consider specific voting situations that are in some ways similar and in other different than ours. Guo (2013) studies individual incentives to disclose hard information in the context of three-person committee voting. Esö, Hansen and White (2014) consider vote trading as a way to influence the outcome of corporate decisions where there is an exogenously determined "correct" way to vote, and voters may or may not be informed about the consequences for them of following that prescription. Ali and Bohrenz (2014) analyze a committee's decision to end its search for information when its members observe private signals about the current alternatives and may have heterogeneous search costs. Goertz and Maniquet (2013) investigate whether plurality rule aggregates information efficiently in large elections with multiple alternatives, when voters have common, state-dependent preferences and imprecise information about the state of nature. Gräuner and Kiel (2004) analyze collective decision problems where agent's preferences are correlated but not identical, each agent holds private information about her own bliss point, and this information also affects the bliss points of others. Rosar (2012) studies collective decisions when agents have one-dimensional interdependent preferences, a utilitarian planner must choose the reports through which they can communicate, and decide whether the median or the average report should be implemented. Each of the works we reference shares some feature with ours: they study mechanisms from the point of view of information disclosure, they involve voting by committees, they allow for some agents to be differentially informed. But each one is also different than ours in at least one way: they do not concentrate on strategy-proof rules, need not share the notion that information is hard, are restricted to study very specific rules, or introduce forms of uncertainty that require the use of tools that we don't need. And, finally, unlike in most works à la Condorcet, and in most of the models we just surveyed, we do not restrict our analysis to the two-alternative case, but are able to study rules to choose among many alternatives.

The paper proceeds as follows. In Section 2 we present the model and our general question. In Section 3 we start by analyzing the simplest setting when informed agents may either fully disclose their private information or to hide it and therefore their action space at the disclosure stage is binary. We provide a sufficient condition for information disclosure and we discuss under which conditions it is necessary. Then we look at one application of this binary setting, when agents are restricted to have single-peaked preferences. In Section 4 we generalize our approach, allowing for partial information disclosure, and show that the condition provided in Section 3 still holds in this more general case. This section also contains an application within this setting where partial disclosure is possible, to the case where alternatives are multidimensional and agents' preferences are separable. Section 5 discusses which voting rules provide more incentives to disclose information when the mechanism designer does not observe who are the informed voters. All proofs are in the Appendix.

2 The model and our general question

A finite set $N = \{1, ..., n\}$ of agents must choose one alternative from a set X, that contains at least two alternatives, which may be finite or infinite. Some of the agents have hard information about the alternatives, which they privately own. This information, or parts of it, may be relevant to shape the preferences of other, initially uninformed agents, should these get to know it. We consider two-stage collective decision processes. In the first one, the disclosure stage, informed agents simultaneously decide whether to disclose what they know about the alternatives, or not to. In the second, the decision stage, all agents vote to determine what alternative will be chosen.

In order to identify the precise role of voting rules over disclosure decisions, we make a number of modeling decisions that we now spell out in order.

We assume that the set of agents that are informed is fixed and known in advance by the designer and by the voters.[1]

Let $I \subset N$ denote the set of informed agents and U the set of uninformed agents, with $I \cup U = N$. Without loss of generality let $U = \{1, ..., l\}$ and $I = \{l + 1, ..., n\}$. At the first stage of the process, informed agents may or may not disclose to initially uninformed ones the information they have. We assume, to begin with, that each of the informed agents can only choose one of two actions: either to disclose to all agents all they know, action $a_i = 1$, or not to disclose any piece of information, action $a_i = 0$.[2] Let A be the set of vectors of actions in $\{0, 1\}^{n-l}$ that may be played by the informed agents in the disclosure stage. Let $a^0 = (0, ..., 0)$ denote the vector of actions where

[1] This is the case for instance for Congressional committees in the US, that are designed to gather and evaluate information, Krishna and Morgan (2001) study which legislative rules, among open, closed, and modified ones, provide a committee more incentives to acquire and transmit relevant information.

[2] This assumption of binarity of actions will be relaxed later on, in Section 4.

no agent has disclosed any information.

Informed and uninformed agents have preferences over alternatives, in the form of complete, transitive, reflexive binary relations. As customary, we denote by $R_i \in \mathcal{R}$ the preferences of agent i and by P_i the associated strict preference relation, where $\mathcal{R}$ is the set of all possible preferences on X. We shall consider situations where the admissible preferences of agents are restricted to belong to some subset $\mathcal{D} \subset \mathcal{R}$, called a domain. Admissible preference profiles will be $n-tuples$ $(R_1, ..., R_n) \in \mathcal{D}^n$. We restrict our attention to preference domains $\mathcal{D}$ where each $R_i \in \mathcal{D}$ has a unique best alternative $B(R_i)$.

The preferences of all informed agents I are based on a commonly shared full knowledge of the nature of alternatives, and stay the same whether or not any information is disclosed: but even if all informed agents share the same information, their preferences may differ. We denote by $R_I \in \mathcal{D}^{n-l}$ the subprofile of preferences held by informed agents. As for agents who are initially uninformed, we assume that their preferences at decision time depend on the disclosure actions taken by the informed agents in the first stage of the process. We model the impact of such actions through a reaction function $g : A \to \mathcal{R}^l$, associating to each action profile $a \in A$, a preference profile $R_U \in \mathcal{R}^l$. We focus on the case of public disclosure of hard facts, meaning that it suffices that one agent discloses what she knows for that to be learnt and believed by everyone. In this initial case of binary messages we can write $g : \{a^0, A\backslash a^0\} \to \mathcal{D}^l$ to stress that the function g associates the same outcome to any vector other than a^0, and at most two different subprofiles of preferences are obtained for initially uninformed voters. We call every $a \neq a^0$, in which at least one informed agent i has played $a_i = 1$, a fully informative action profile and we denote $g(a) \equiv R_U^{g;full}$. Notice that we have required that, for a given family $\mathcal{D}$ of admissible preferences and set A of action profiles of the informed agents, the image of g be some subset of $\mathcal{D}^l$. This is a restriction that we stress by saying that g is a $\mathcal{D}-admissible$ reaction function.

Informed agents have complete information: each informed agent observes the preferences of other informed agents as also the preferences of initially uninformed agents, both in case information is disclosed and in case is not, and therefore she knows the reaction function g that describes the effect of information disclosure on uninformed agents' preferences. However, we keep the standard assumption in implementation theory that agents and the designer act at different points in time and are endowed with different information. The designer must propose one voting rule without knowing which of the many circumstances that he may envisage as possible will eventually arise. Specifically, he just knows the domain D of possible preferences for informed agents, and the family $\mathcal{H}(\mathcal{D})$ of functions according to which uninformed agents may react when information is disclosed, and when it is not. The triple $(I, \mathcal{D}, \mathcal{H}(D))$ specifying what the designer knows, is called the information structure of the problem. Sometimes, we'll make the additional assumption that the set $\mathcal{H}(\mathcal{D})$ contains enough reaction functions to span the full preference domain $\mathcal{D}$. This is the assumption of completeness expressed in our next definition.

Definition 1 *A family $\mathcal{H}(\mathcal{D})$ of $\mathcal{D} -$ admissible reaction functions is* complete *if for every pair $R, R' \in \mathcal{D}^l$ there exists $g \in \mathcal{H}(\mathcal{D})$ such that $g(a^0) = R$ and $g(a) = R'$ for all $a \neq a^0$.*

We denote by $\mathcal{H}^C(\mathcal{D})$ a complete family of $\mathcal{D} - admissible$ reaction functions. Notice that our reaction functions determine the preference changes associated to every disclosure decision in a deterministic manner. Hence, we eliminate any uncertainty from the second stage of our game, and can simply leave open the choice of the deterministic rule to be used in aggregating the declared preferences.

Definition 2 *A voting rule (on $\mathcal{D}^n$) is a function $f : \mathcal{D}^n \to X$, mapping each preference profile reported by agents to one alternative.*

As already stated in the introduction, we'll concentrate on the choice of voting rules that satisfy the normatively attractive condition of strategy-proofness, in addition to inducing information disclosure.

Definition 3 *Given a voting rule $f : \mathcal{R}^n \to X$, we say that agent $i \in N$ can manipulate at profile $R \in \mathcal{R}^n$ via R'_i, if $f(R_{-i}, R'_i))P_i f(R_{-i}, R_i)$. A voting rule $f : \mathcal{R}^n \to X$ is strategy-proof if no agent can manipulate f at any profile $R \in R$.*

We shall also assume that agents operating under these rules are partially honest, in the following sense (Dutta and Sen 2011): if truthfully reporting their preferences is a dominant strategy, then agents declare their true preferences.

Our interest in rules satisfying these properties is normative, but admittedly also pragmatic. Indeed, they allow us to eliminate strategic behavior from the second stage of the mechanism, and thus concentrate all attention on the only strategic consideration left, that of information disclosure, which is our focus of interest.

To complete a list of conditions that voting rules may or may not satisfy, we add here some further definitions that will be useful to qualify our findings.

Definition 4 *A voting rule $f : \mathcal{R}^n \to X$ satisfies voter sovereignty if for each $x \in X$ there exists $R \in \mathcal{R}^n$ such that $f(R) = x$.*

Voter sovereignty is necessary for but milder than efficiency, the other relevant property that we may want sometimes to impose on a voting rule.

Definition 5 *An alternative $x \in X$ is Pareto undominated if there is no other alternative $y \neq x$ such that yR_ix for all $i \in N$ and yP_jx for some $j \in N$. A voting rule $f : \mathcal{R}^n \to X$ is Pareto efficient (or simply efficient) if it selects a Pareto undominated alternative at any profile $R \in \mathcal{R}^n$.*

Finally, we define anonymity. A voting rule is anonymous if the names of agents are immaterial in deriving social choices.

Definition 6 *A voting rule $f : \mathcal{R}^n \to X$ is anonymous if for all $R, \hat{R} \in \mathcal{R}^n$, $f(R) = f(\hat{R})$ whenever R is a permutation of $\hat{R}$.*

Having spelled out our assumptions regarding the voting rule, we can now go back to model the strategic decisions regarding information disclosure as a one stage simultaneous move game $\Gamma \equiv (I, R_I, A, f(R_I, g(\cdot)))$. The consequences of each action profile a on voter's preferences gets translated into a single chosen alternative, $x = f(R_I, g(a))$, the one chosen by the strategy-proof voting rule f when the preferences of uninformed agents are $g(a)$ and informed agents' preferences are R_I. Observe that, even if the only active players of the game are the informed voters, the uninformed still play an important role in defining the overall problem of the designer, since the function g reflects their preferences' reaction to information and the function f also depends on their preferences.

We are now ready for our main definitions. Fix a set of informed agents I, a set of action profiles A, and a reaction function g.

Definition 7 *A strategy-proof voting rule* $f : \mathcal{D}^n \to X$ *is information manipulable at* $(R_I, g(a)) \in R$ *if there exists an informed agent* $i \in I$ *and an action* $a'_i \neq a_i$ *such that*

$$f(R_I, g(a'_i, a_{-i}))P_i f(R_I, g(a)).$$

Definition 8 *Under information structure* $(I, \mathcal{D}, \mathcal{H}(\mathcal{D}))$*, a strategy-proof voting rule* f *ensures information disclosure, if for all* $R_I \in \mathcal{D}^{n-l}$*, and all* $g \in \mathcal{H}(\mathcal{D})$*, (i) there exists a fully informative action profile* a *such that* f *is not information manipulable at* $(R_I, g(a))$*; and (ii) for all* $a \in A$*,* $f(R_I, g(a)) \neq f(R_I, R_U^{g,full})$ *implies that* f *is information manipulable at* $(R_I, g(a))$*.*

Notice that the action profile a is a Nash equilibrium of the simultaneous move game Γ if and only if f is not information manipulable at $(R_I, g(a))$. Hence, a strategy-proof voting rule f ensures information disclosure if and only if for all $g \in \mathcal{H}(A, \mathcal{D})$ and for all $R_I \in \mathcal{D}^{n-l}$ any Nash equilibrium outcome $f(R_I, g(a))$ is the outcome of the voting rule at the preference profile $(R_I, R_U^{g,full})$ in which all agents are informed.

3 A first set of results and applications

In this section we propose a property of voting rules that is sufficient to ensure information disclosure and then we show it to also be necessary when the set of admissible reaction functions is complete.

Definition 9 *A voting rule* $f : \mathcal{D}^n \to X$ *attributes coalitional veto power to a set* $M \subseteq N$*, if the following two conditions hold*
(i) for all $R \in \mathcal{D}^n$*, if* $B(R_i) = x$ *for all* $i \in M$*, then* $f(R) = x$*;*
(ii) for all pairs $R, R' \in \mathcal{D}^n$ *with* $R_i = R'_i$ *for all* $i \in M$*, either* $f(R) = f(R')$ *or there is a pair* $i, j \in M$ *such that* $f(R')P_i f(R)$ *and* $f(R)P_j f(R')$*,*

Notice that a voting rule attributes coalitional veto power to a set $M \equiv \{i\}$ if and only if f is dictatorial and agent i is the dictator.[3]

[3] A voting rule f is defined dictatorial if, there exists $i \in N$ such that for all $R \in \mathcal{D}^n$, $f(R)$ is the preferred alternative by agent i in the range of f.

Proposition 1 *Under any information structure* $(I, \mathcal{D}, \mathcal{H}(D))$, *a strategy-proof voting rule* $f : \mathcal{D}^n \rightarrow X$ *that satisfies voter sovereignty ensures information disclosure if it attributes coalitional veto power to the set of informed voters* I.

Proof. See the Appendix. ■

Coalitional veto power is sufficient to ensure information disclosure for any preference domain $\mathcal{D}$ and any arbitrary family of reaction functions $\mathcal{H}(D)$. If, in addition, the family $\mathcal{H}(D)$ is complete, then it is also necessary.

Proposition 2 *Under an information structure* $(I, \mathcal{D}, \mathcal{H}^C(D))$, *a strategy-proof voting rule* $f : \mathcal{D}^n \rightarrow X$ *that satisfies voter sovereignty ensures information disclosure only if it attributes coalitional veto power to the set of informed voters* I.

Proof. See the Appendix. ■

The previous results have a nice interpretation. A trivial way to solve the tension between information disclosure and preference aggregation is to make the former irrelevant by attributing all decision power to some or all of the informed voters.[4] Our characterization results establish how far we can go in the direction of allowing uninformed voters to have some influence on the social outcome while keeping the incentives for the informed voters to disclose what they know. As we shall see later there will be cases where the power can be equally distributed between informed and uninformed voters, and others where those who are informed will have to be given a more prominent role.

A first application

As an interesting application of the previous result, we now characterize the family of strategy-proof voting rules that ensure information disclosure in the domain of single-peaked preferences. To do that, we need a few definitions. Consider a set X of ordered alternatives, which may be identified with an interval in the real line, or with a finite integer interval $[a, b]$. For each $i \in N$, R_i is single-peaked over X if there exists a unique $B(R_i) \in X$ (agent i's peak), and xP_iy for all $x, y \in X$ such that $y < x \leq B(R_i)$ or

[4] It is well known that when the set of admissible preferences is the universal domain only dictatorial voting rules are strategy-proof and efficient. A simple consequence is that under an information structure $(I, \mathcal{P}, \mathcal{H}^C(P))$, an efficient and strategy-proof voting rule $f : \mathcal{P}^n \rightarrow X$ ensures information disclosure if and only if it is a dictatorial rule and the dictator is an informed agent.

$B(R_i) \geq x > y$. Let $\hat{\mathcal{D}}$ denote the set of all single-peaked preferences over X. In the single-peaked preferences domain the class of strategy-proof voting rules coincides with the class of generalized median voter rules (Moulin 1980).

Before providing formal definitions, let us describe informally how generalized median voter rules work. Let us first consider the case when we must choose among two alternatives only, identified by 0 and 1. Ask agents what is their best alternative. Then, a given rule would be to choose 1 unless there is "enough" support for 0, in which case this lower value will be selected. What do we mean by "enough support"? We could establish the list of coalitions that will get 0 if all their members prefer it to 1; and it is natural to require that, if a coalition can enforce 0, then its supersets are also able to. Such a family of "winning" coalitions will fully describe the rule. We can now extend this same idea to cases where we must select an alternative among a set X of ordered alternatives. Let each voter declare her best value. Now, we can start by asking whether value x should be chosen. If "enough" people have voted for values at least as high as x, but not "enough" of them have voted for values below x, then x is chosen. To determine what we mean by "enough", we associate a list of coalitions $C \in 2^N$ to each possible alternative x, and consider that support by any of these coalitions is "enough". In order to guarantee that the rules so described do satisfy strategy-proofness and voter sovereignty, we require from them that 1) if a coalition is "strong enough" to support an outcome, its supersets are too; 2) if a coalition is "strong enough" to support the choice of a given value, it is also "strong enough" to support any higher value; and 3) any coalition is "strong enough" to guarantee that the choice will not exceed the maximum value in X, if that exists.

In particular we can generate anonymous generalized median voter rules by requiring that if a coalition is strong enough for a given alternative, all other coalitions of the same size are also strong enough.

We now provide the formal definitions.

Definition 10 *A left coalition system on $X = [a, b]$ is a correspondence C assigning to every $x \in X$ a non-empty collection of non-empty coalitions $C(x)$, satisfying the following requirements:*
1. if $c \in C(x)$ and $c \subset c'$, then $c' \in C(x)$;
2. if $x' > x$ and $c \in C(x)$ then $c \in C(x')$; and
3. $C(b) = 2^N$.

Definition 11 *A left coalition system C on X is anonymous if for all $x \in X$ and for all $c, c' \in C$ with $|c'| = |c|$, $c \in C(x) \Leftrightarrow c' \in C(x)$.*

Definition 12 *Given a left coalition system C on X, its associated generalized median voter rule $f : \hat{\mathcal{D}}^n \to X$ is defined so that, for all profiles R,*

$$f(R) = x \text{ iff } \{i|B(R_i) \leq x\} \in C(x)$$

and

$$\{i|B(R_i) \leq y\} \notin C(y) \text{ for all } y < x.$$

Proposition 3 *(Moulin 1980) A voting rule on profiles of single-peaked preferences over a set X is strategy-proof and satisfies voters' sovereignty if and only if it is a generalized median voter rule.*

We first present an example of a generalized median voter rule that, as we shall see, ensures information disclosure. Let $B_I^{\min}$ ($B_I^{\max}$) denote the minimum (maximum) peak among informed agents' peaks.

Example 1 *Suppose n is odd, $|I| < \frac{n+1}{2}$ and $X = [0, 1]$. Let $x_{med}(R)$ denote the median alternative when agents' preference profile is R. For all $R \in \hat{\mathcal{D}}$, the voting rule selects*

$$f(R) = \begin{cases} x_{med}(R) \text{ if } B_I^{\min}(R) \leq x_{med} \leq B_I^{\min}(R) \\ B_I^{\max}(R) \text{ if } x_{med} > B_I^{\max}(R) \\ B_I^{\min}(R) \text{ if } x_{med} < B_I^{\min}(R). \end{cases}$$

This rule is defined by the following left coalition system: for all $x \in X$, i) $I \in C(x)$, and ii) for all $c \neq I$, $c \in C(x)$ only if there exists $i \in I \cap c$ and $|c| \geq \frac{n+1}{2}$.

Informally, the rule chooses the median of the best alternatives of all voters as long as it lays between the min and the max of the peaks of agents in I. Otherwise, it chooses one of the these two peaks, the one closer to the median.

To see that this rule ensures information disclosure, notice that when all informed agents have the same best alternative, then this is chosen by the rule, as required by (i) in the Definition 9. Also observe that for any pair of profiles under which the rule selects different outcomes, they will both fall between the maximum and the minimum peak of the informed agents, hence by single-peakdness there will exist two informed agents who have opposite views regarding these two outcomes, as required by (ii) in the Definition 9.

Our second example describes a generalized median voter rule that fails to ensure information disclosure.

Example 2 *Let $N = \{1, 2, 3\}$ and $I = \{1\}$. Three agents must decide whether to locate a public facility in one of three alternative locations $X = \{y, w, z\}$ with $y < w < z$ and only agent 1 is an informed agent. Consider a family $\mathcal{H}(\hat{D})$ of reaction functions such that for all $g \in \mathcal{H}(\hat{D})$ and for all $a_1 \in \{a_1^0, a_1^1\}$, either $g(a_1) = R_U$ or $g(a_1) = R'_U$, with wR_iyR_iz, and $zR'_iwR'_iy$ for both $i \in \{2, 3\}$. Consider the voting rule f that selects the median among the reported preferred locations by the agents.*[5] *This function does not ensure information disclosure, because there exists a preference profile R such that $B(R_1) = w$ and a reaction function $g \in \mathcal{H}(\hat{D})$ such that $g(a_1^0) = (R_2, R_3)$ and $g(a_1^1) = (R'_2, R'_3)$. In the induced simultaneous move game Γ, a_1^0 is a NE and $f(R_1, g(a_1^0)) = w \neq f(R_1, R_2^{g,full}, R_3^{g,full})) = z$.*

The median rule in the above example fails to satisfy coalitional veto power, because it does not select at every profile the best alternative of the informed agent.

The following result characterizes the rules that ensure full disclosure under our informational restrictions, when preferences of agents are single-peaked, and have the previous examples as special cases.

Proposition 4 *Under information structure $(I, \hat{D}, \mathcal{H}^C(\hat{\mathcal{D}}))$, a voting rule $f : \hat{\mathcal{D}}^n \to X$ is strategy-proof, satisfies voter sovereignty and ensures information disclosure if and only if it is a generalized median voter scheme with an associated coalition system such that for each alternative x, (i) there exists $c \in C(x)$ such that $c \subseteq I$; and (ii) for all $c \in C(x)$, $c \cap I \neq \varnothing$.*

Remark 1 *A generalized median voter rule that satisfies the above requirements at every profile $R \in \hat{\mathcal{D}}^n$ selects an alternative which is in between the minimum and maximum peak of the informed voters.*

Proof. see the Appendix. ■

[5] This rule is associated to the following left coalition system: for all $x \in X$, $c \in C(x)$ if and only if $|c| \geq 2$.

The class of voting rules characterized in Proposition 4 is large. It will contain anonymous rules only when at least half of the voters are informed. One such rule will be the one that selects the median peak of the agents.

However, when the number of informed agents is smaller, the special treatment that coalition veto power reserves to informed agents, is incompatible with anonymity. In fact if $|I| < \frac{n+1}{2}$ condition (i) of Proposition 4 requires that for each $x \in X$ there exists $c \in C_x$ such that $c \subseteq I$, therefore $|c| \leq \frac{n+1}{2}$. Anonymity imposes that all coalitions c' with $|c'| = |c|$ belong to C_x and therefore condition (ii) of Proposition 4 is not compatible with the property of anonymity and condition (i).

4 Extending the set of messages

In the preceding sections we have limited attention to the binary information case, where informed agents either reveal all or none of the facts they know. In this section we show that the results described in Propositions 1 and 2 can be immediately generalized to a setting where informed agents may also disclose partial information. Let Σ be a family of objects to be interpreted as elementary pieces of hard information. The hard information available to informed agents in a given environment, will be given by $S \in 2^{\Sigma}$.[6] Available actions for each informed agent $i \in I$, will be subsets $a_i \in 2^S$. We will denote by $A(S)$ the family of $(n-l)$ tuples of messages $a = a_1, ...a_{n-l} \in \left(2^S\right)^{n-l}$ that may be used at the disclosure stage.[7] For each a, $\bigcup\limits_{i \in I} a_i \equiv T^a$ is the *total amount of information* disclosed by the informed agents. We assume that the reaction functions of uninformed agents only depend on the total amount of information disclosed and not on the agent who provides each piece of it.

Assumption A1. *For all $a, a' \in A(S)$ such that $T^a = T^{a'}$, $g(a) = g(a')$.*

Let $a^0 \equiv \bigcup\limits_{i \in I} a_i^0 = \varnothing$ and let a^1 stand for any action profile such that $\bigcup\limits_{i \in I} a_i \equiv S$. Notice that a fully informative action profile a^1 can be obtained when at least a single agent fully discloses the information, or when the

[6] The assumption that information is hard implies that the set S only contains pieces of information that are not contradictory with each other.

[7] To simplify notation we only specify the set S of available information when it is necessary.

combination of the pieces of partial information disclosed by more than one agent is fully informative.

Definition 13 *We say that a profile of actions $a \in A$ is fully informative if $T^a = S$, uninformative if $T^a = \varnothing$, and (partially) informative otherwise.*

Notice that Proposition 1 is general and not restricted to the binary case, even if we presented it when discussing that simple case. The proof we provide in the Appendix is general. Also Proposition 2 still holds, but we need to modify the definition of a *complete family* of reaction functions, that was given for the binary case, to encompass the general case.[8]

Definition 14 *A family $\mathcal{H}(\mathcal{D})$ of $\mathcal{D}-$admissible reaction functions is* complete *if for every triple $R, R', R'' \in \mathcal{D}^l$ there exists $g \in \mathcal{H}(\mathcal{D})$ such that $g(a^0) = R$, $g(a^1) = R''$ and $g(a) = R'$ for all $a \neq a^0, a^1$.*

A second application

We offer now an application of our general result to a context in which the dichotomy between no information and full information is clearly too coarse and it seems natural to allow informed agents to partial disclose private information. This application refers to a voting context introduced by Barberà Sonnenschein and Zhou (1993). Under appropriate separability restrictions on preferences, nice classes of strategy-proof rules exist for that context, and our problem is well defined there. Yet, as we shall see, some additional qualifications regarding the information structure will be added. Therefore our analysis also points at the need to study each environment with special reference to its singularities.

We consider the problem faced by the members of a club who must select which new candidates to admit. Let N denote the set of voting members, and X the set of candidates with $|X| = k \geq 3$. We assume that there are no constraints on the number of candidates that may be elected. Hence, alternatives are subsets of candidates, denoted by T. Let $\mathcal{P}$ denote the set of all strict preferences (asymmetric orderings) on 2^X, and P_i (belonging to $\mathcal{P}$) stands for voter i's preferences. Given a preference relation P_i and a class of subsets of X, Θ, $\arg\max(P_i, \Theta)$ denotes the best element in Θ according to

[8] Notice that both definitions coincide when we restrict to the binary case, $A_i = \{a^0, a^1\}$ for all $i \in I$. Hence, we keep the same name for the newly defined, extended notion.

P_i. In particular, let $B(P_i) = \arg\max(P_i, 2^X)$.[9] A voting rule in this context is a function $f : \mathcal{P}^n \to 2^X$. The following definition will help us describe a family of voting rules in terms of the power they attribute to coalitions.

Definition 15 *A committee (or a monotonic simple game) is a pair $C = (N, \mathcal{W})$, where $N = \{1, ..., n\}$ is the set of voters, $\mathcal{W}$ is a nonempty set of nonempty coalitions of N, such that $M \in \mathcal{W}$ and $M' \supseteq M$ implies $M' \in \mathcal{W}$.*

Coalitions in $\mathcal{W}$ are called winning. $M \in \mathcal{W}$ is a *minimal winning coalition* if and only if $M' \subsetneq M$ implies $M' \notin \mathcal{W}$.

Definition 16 *A voting rule $f : \mathcal{P}^n \to 2^X$ is based on voting by committees, if for each candidate x, there exists a committee $C_x = (N, \mathcal{W}_x)$ such that: for all preference profiles $P \in \mathcal{P}^n$, $x \in f(P)$ if and only if $\{i | x \in B(P_i)\} \in \mathcal{W}_x$.*

One nice feature of methods based on voting by committees is that they only depend on the subset of X that is ranked higher by each voter. Barberà, Sonnenschein and Zhou (1991) show that voting by committees satisfies desirable properties when agents have separable preferences, defined as follows. Let $G(P_i) = \{x \in X | \{x\} P_i \varnothing\}$ denote the set of good candidates for agent i, and $G^c(P_i)$ its complement. Separability means that once agents partition the set of candidates in the two sets of desirable (good) candidates and undesirable ones, their preference relations satisfy the following condition.

Definition 17 *A preference relation P_i is separable if for all $T \in X$ and all $x \notin T$, $T \cup \{x\} P_i T$ if and only if $x \in G(P_i)$. The family of all separable preferences is denoted by $\tilde{\mathcal{P}}$.*

Clearly $G(P_i) = B(P_i)$ under separability. We already defined non-manipulability and the same definition applies in this setting.

Proposition 5 (Barberà et al. (1991)) *A voting rule $f : \tilde{\mathcal{P}}^n \to 2^X$ is strategy-proof on $\tilde{\mathcal{P}}^n$ and satisfies voter sovereignty if and only if it is based on voting by committees.*

[9] Since sets can be identified with their characteristic functions, the model can be reinterpreted as one where agents choose among any kind of alternatives identified by $0-1$ vectors of characteristics. Geometrically, the alternatives can be seen as the vertices of a k-dimensional hypercube.

Notice that, in order to accommodate the specific characteristics of our problem, and to obtain a characterization based on the incentives to disclose, we must be careful to avoid some potential conflicts between properties that one could find interesting to impose here. In particular, here is an example of a strategy-proof voting rule that may not induce disclosure, in the absence of additional qualifications.

Example 3 *Let $N = \{1, 2, 3\}$, $I = \{1, 2\}$ and $X = \{y, w, z\}$. Let f be based on voting by quota two: that is, an alternative is selected if and only if at least two agents report that it is good. Suppose that voter 1's preferences $P_1 \in \tilde{\mathcal{P}}$ are such that $G(P_1) = \{w, z\}$, voter 2's preferences $P_2 \in \tilde{\mathcal{P}}$ are such that $G(P_2) = \{y, z\}$, and for both $i \in \{1, 2\}$ $\{z\} P_i \{y, w, z\}$. Consider the following $\tilde{P}-$admissible reaction function g such that $g(a^0) = P_3''$ with $G(P_3'') = \{z\}$ and $g(a) = P_3'$ with $G(P_3') = \{y, w, z\}$ for all $a \neq a^0$. If agent 3 remains uninformed she only likes alternative z, otherwise, if any of the two informed voters $i = 1, 2$ partially or fully discloses information, then she likes all three alternatives. It is easy to check that $f(P_{-3}, g(a^0)) = \{z\}$ and $f(P_{-3}, g(a)) = \{y, w, z\}$ for all $a \neq a^0$, and therefore informed agents have incentives to keep silent and there is a Nash equilibrium a^* of the game such that $a^* = a^0$.*

What happens in this example is that the impact of information on the preferences of the initially uninformed agent jointly affects the valuation of several candidates, and thus one cannot find meaning for the idea of partially informing about candidates one by one, nor to the further property that such pieces of information should not interfere with one another. This is in contrast with our notion of separable preferences, which allows each one of them to be considered good or bad independently of agent's opinion about other candidates. And yet, the reaction function g we have used complies with the requirements we have assumed till now.

To find a formulation that fits the world of separable preferences, we identify the messages of an informed agent i with the set of candidates about whom he will reveal the hard fact he knows.

Formally for each $i \in N$, let $A_i = 2^X$, meaning that each informed agent is allowed to disclose information about any set $M \subseteq X$ of candidates.[10] Let $D(a) \equiv \bigcup_{i \in N} a_i$ denote the set of candidates about which information has been

[10] We implicitly assume that choices in A_i by different agent will be restricted to reveal the same information, if they inform about the same candidate.

disclosed when the action profile a has been played at the disclosure game. Therefore $D(a^0) = \varnothing$ and $D(a^1) = X$. And we focus attention on reaction functions that satisfy the following property.

Definition 18 *A reaction function $g : A \to \mathcal{R}^l$ is separable if (i) it is a $\tilde{\mathcal{P}} -$ admissible reaction function; and (ii) for each $a \in A$, and for each $x \in X$, such that $x \in D(a)$, $x \in G(g(a))$ if and only if $x \in G(g(a^1))$ and for each $y \notin D(a)$, $y \in G(g(a))$ if and only if $y \in G(g(a^0))$.*

Let $\mathcal{H}_{sep}(\tilde{\mathcal{P}})$ denote a family of separable reaction functions. Separability of the reaction functions requires that the consequences on preferences of initially uninformed agents over the information disclosed about a candidate does not affect their preferences over the candidates about whom information has not been disclosed, nor they are affected by what additional information may or may not be provided about other candidates. Therefore if a candidate x is reputed good by an initially informed agent $j \in U$ at $g(a^1)$, i.e. when information about all candidates has been disclosed, then it is also reputed a good candidate at any profile $g(a)$ with $x \in D(a)$. And similarly if a candidate is reputed good at $g(a^0)$, that is when no information has been disclosed, then it is reputed good at any profile such that no information has been disclosed.

It is easy to realize that a family $\mathcal{H}_{sep}(\tilde{\mathcal{P}})$ of separable reaction functions cannot be complete(i.e., satisfy Definition 14), because separability imposes some constraints on how partial information affects uninformed voters' preferences. But we can define a milder condition, satisfied by a family of separable reaction functions, that is equivalent to that of a complete family in the binary case.

Definition 19 *A family $\mathcal{H}(\tilde{\mathcal{P}})$ of separable reaction functions is* rich *if for every pair $R, R' \in \tilde{\mathcal{P}}^l$ there exists $g \in \mathcal{H}(\tilde{\mathcal{P}}))$ such that $g(a^0) = R$ and $g(a^1) = R'$.*

We denote by $\mathcal{H}^R_{sep}(\tilde{\mathcal{P}})$ a rich family of separable reaction functions.

The following theorem characterizes the set of strategy-proof rules that satisfy voter sovereignty and ensure information disclosure for a rich family of separable reaction functions.

Proposition 6 *Under an information structure $(I, \tilde{P}, \mathcal{H}^R_{sep}(\tilde{\mathcal{P}}))$, a voting rule $f : \tilde{\mathcal{P}}^n \to X$ is strategy-proof, satisfies voter sovereignty and ensures* *information disclosure, if and only if it is voting by committees and for each $x \in X$, a) there exists $T_x \subseteq I$ such that $T_x \in \mathcal{W}_x$; and b) for each $M \in \mathcal{W}_x$, $M \cap I \neq \varnothing$.*

Proof. See the Appendix. ■

Notice that the rule in Example 4 satisfies the condition of Proposition 6 since for all $x \in X$, i) $I \in \mathcal{W}_x$; ii) $M \in \mathcal{W}_x$ implies that there exists $i \in I \cap M$. However, the reaction function g is not separable, Notice also that for all $i \in I$, $\{z\}\, P_i \,\{y, w, z\}$, and $f(P_{-3}, P_3') = \{z\} \neq \{y, w, z\} = f(P_{-3}, P_3'')$ therefore condition (ii) in Definition 9 and consequently Proposition 1 is violated.

5 Societies with dispersed information

Up to now we have assumed that the mechanism designer knows who are the initially informed agents. In this section we want to extend our analysis to provide some hints for the case in which the designer cannot observe who are the set of informed voters.

To simplify matters, we consider again the simple binary case where informed agents can either disclose all they know ($a_i = 1$), or no information at all ($a_i = 0$).

Here, nature draws the set of voters $I \subset N$ that will be informed, out of a family $\mathcal{W}$ of nonempty coalitions of N, interpreted as the sets of agents who may eventually become informed. For each set I of informed voters we denote A^I the set of profiles of action that informed agents can play. The designer knows $\mathcal{W}$, but must choose the mechanism before nature's choice, and seek to promote information disclosure. In that context, we'll first show that it is impossible to achieve this objective at large, along with that of strategy-proofness. Then, we shall argue that simple majority stands out as the strategy-proof method that provides the maximal chances to induce disclosure when preferences are single-peaked or separable.

Definition 20 *Under information structure $(\mathcal{W}, \mathcal{D}^n, \mathcal{H}(\mathcal{D}))$, a strategy-proof voting rule f ensures **strong information disclosure** with respect to $\mathcal{W}$, if for all $M \in \mathcal{W}$, for all $R_M \in \mathcal{D}^{|M|}$, and all $g : A^M \to \mathcal{R}^{n-|M|} \in \mathcal{H}(\mathcal{D})$, (i) there exists a fully informative action profile a such that f is not information manipulable at $(R_I, g(a))$; and (ii) for all $a \in A$, $f(R_I, g(a)) \neq f(R_I, R_U^{g,full})$ implies that f is information manipulable at $(R_I, g(a))$*

The following proposition follows as an immediate corollary of Proposition 2.

Corollary 1 *Under information structure* $(2^N, \mathcal{D}^n, \mathcal{H}^C(\mathcal{D}))$ *no strategy-proof voting rule that satisfies voter sovereignty ensures* ***strong information disclosure.***

This impossibility result also holds if there are only two alternatives to be chosen. Suppose for simplicity that $X = \{a, b\}$. By strategy-proofness and voter sovereignty if $B(P_i) = q$ for all $i \in N$, then $f(P) = q$ for both $q \in \{a, b\}$. So both a and b are in the range of f. Now consider a preference profile P' such that for some arbitrary $M \in 2^N$ with $n > |M| > 1$, $B(P_i') = a$ for all $i \in M$, and $B(P_j) = b$ for all $j \in N \backslash M$. By Proposition 2 when $I = M$ then $f(P') = a$ while when $I = N \backslash M$, $f(P') = b$.

Given the previous impossibility result, we propose a criterion to evaluate whether a voting rule provides better incentives to disclose information than another one, when the designer cannot observe who are the informed agents.

Definition 21 *Under the information structure* $(\mathcal{W}, \mathcal{D}^n, \mathcal{H}(\mathcal{D}))$*, the strategy-proof voting rule* f *ensures* ***better information disclosure*** *than the voting rule* f'*, if (i) for any* $M \in \mathcal{W}$ *such that* f' *ensures information disclosure when* M *is the set of informed voters, then* f *also ensures it and (ii) there is some* $T \in \mathcal{W}$ *such that* f *ensures information disclosure when* T *is the set of informed voters, while* f' *does not.*

The previous definition provides a partial ordering over voting rules with respect to the incentives they provide to disclose information.

Although in many cases two voting rules cannot be compared in terms of information disclosure, this partial ordering turns out to be powerful enough to univocally identify a voting rule which provides the maximal incentives to disclose in several relevant settings. In particular this is true when we restrict attention to anonymous strategy-proof voting rules. This is quite a natural restriction when each agent has the same ex-ante chances to become informed, and the designer has no reason to provide coalitional veto power to a specific set of agents.

The next proposition shows the very special role of the simple median when preferences are single-peaked.

Proposition 7 *Under information structure* $(2^N, \hat{\mathcal{D}}, \mathcal{H}^C(\hat{\mathcal{D}}))$, *the median voter rule ensures better information disclosure than any other anonymous, efficient, and strategy-proof voting rule.*

Proof. See the Appendix. ■

A similar result can be proved for the case of separable preferences, when we restrict attention to the same setting as in Section 4. Barberà, Sonnenschein and Zhou (1991) prove that a strategy-proof voting rule is anonymous and satisfies voter sovereignty, if and only if for each $x \in X$ there is a quota q^x such that a candidate is selected if and only if there are q^x voters for whom x is a good candidate. A voting by quota rule f^q is a rule with the same quota q for each alternative.

Proposition 8 *Under the information structure* $(2^N, \tilde{P}, \mathcal{H}^R_{sep}(\tilde{\mathcal{P}}))$, *the voting by quota rule* f^q *with* $q = \frac{n+1}{2}$ *ensures better information disclosure than any other anonymous, strategy-proof voting rule that satisfies voter sovereignty.*

Proof. See the Appendix. ■

6 Conclusions

We have identified conditions under which a mechanism designer could induce information disclosure through an appropriate choice of voting rules which, in addition, may be required to be strategy-proof. There will be environments where the only solutions to our problem of design are trivial and undesirable, leading to the award of all power to a single agent or to an oligarchy. But in many other cases of interest one can find ways to induce information disclosure while giving a say to all voters, whether they are informed at the outset or not. And even if it is true in very broad terms that informed voters must be given some additional power than uninformed ones, this difference may vanish as the relative size of the informed increases. This suggests that any policies that increase the information level of the population will favor the task of designers who seek transparency. Moreover, these designers will find the simple majority rule to be increasingly effective for their disclosure purposes, as the proportion of a priori informed citizens gets close to one half.

7 Appendix

Proof of Proposition 1. We directly provide a proof for the more general case presented in Section 4, that also holds in the special case in which each informed voter has two available actions, to fully disclose all the information or to not disclose any pieces of it. Let $S \in 2^{\Sigma}$ be the information available to informed agents and $A(S)$ the set of $(n-l)$ tuples of messages sent by the set of informed voters I. Assume that assumption A1 holds. Suppose f attributes coalitional veto power to the set I. For any $R_I \in \mathcal{D}^{n-l}$ and $g \in \mathcal{H}(\mathcal{D})$, consider the game $\Gamma \equiv (I, R_I, A, f(R_I, g(a)))$. When only one agent is informed, $|I| = \{i\}$, then for all $a \in A$, $f(R_I, g(a)) = f(R_I, R_U^{g,full}) = B(R_i)$, and information disclosure is guaranteed. If $|I| \geq 2$, let $\bar{a}$ be the action profile such that $\bar{a}_i = S$ for all $i \in N$. It follows that $f(R_I, g(\bar{a})) = f(R_I, R_U^{g,full})$ and f is not information manipulable at $(R_I, g(\bar{a}))$, then full disclosure is a Nash equilibrium of the game. To prove that there are no other Nash equilibria involving partial or no disclosure, let a be a (purported) Nash equilibrium with $T^a \neq S$, and suppose $f(R_I, g(a)) \neq f(R_I, R_U^{g,full})$. This inequality cannot hold if all informed agents have the same best alternative, by condition (i) in the Definition 9. Therefore, by condition (ii) in Definition 9, there should be an agent $i \in I$ for whom $f(R_I, R_U^{g,full}) P_i f(R_I, g(a))$. Then, agent i would profitably deviate by announcing $a'_i = S$, and since $g(a_{-i}, a'_i) = R_U^{g,full}$, f is information manipulable at $(R_I, g(a))$.

Proof of Proposition 2. We directly provide a proof for the more general case presented in Section 4, using Definition 14, which collapses to Definition 1 for the binary case. If f does not attribute coalitional veto power to the set I, then either condition (i) or condition (ii),in 9 or both, are violated. Suppose first that condition (i) is violated. Then, $\exists \hat{R} \in \mathcal{D}$ such that for all $i \in I$, $B(\hat{R}_i) = x$ but $f(\hat{R}) \neq x$. Let $\bar{R}_U \in \mathcal{D}^l$ be such that for all $i \in U$, $B(\bar{R}_i) = x$. Since the family of reaction functions is complete, there exists $g \in \mathcal{H}^C(\mathcal{D})$ such that $g(a) = \hat{R}_U \equiv R_U^{g,full}$ for all $a \neq a^0$, and $g(a^0) = \bar{R}_U$. By voter sovereignty and strategy-proofness $f(\bar{R}) = x$. Since for all $i \in I$, $B(\hat{R}_i) = x$ strategy-proofness implies that $f(\hat{R}_I, \bar{R}_U) = x$. Because of our choice of the reaction function g, then a^0 is a Nash equilibrium of the game $\Gamma \equiv (I, \hat{R}_I, A, f(\hat{R}_I, g(\cdot)))$ since $f(\hat{R}_I, g(a^0)) = f(\hat{R}_I, \bar{R}_U) = x = B(\hat{R}_i)$ for all $i \in I$. Since $x \neq f(\hat{R}_I, R_U^{g,full})$, f does not ensure information disclosure. Suppose now that (ii) is violated because there exists a pair $R, R' \in \mathcal{D}$ with $R_i = R'_i$ for all $i \in I$, such that $f(R) P_i f(R')$ for all $i \in I$. Consider a $\mathcal{D}-admissible$ reaction function $\bar{g} \in \mathcal{H}^C(\mathcal{D})$ such that $\bar{g}(a) = R'_U$

for all $a \neq a^0$, and $\bar{g}(a^0) = R_U$. Notice that by definition $R_U^{\bar{g},full} = R'_U$. Then a^0 is a Nash equilibrium of the game $\Gamma \equiv (I, R_I, A, f(R_I, \bar{g}(a)))$ and $f(R_I, \bar{g}(a^0)) \neq f(R_I, R_U^{\bar{g},full})$.

Proof of Proposition 4 Sufficiency. Consider any generalized median voter rule with a left coalition system C such that for all $x \in X$, $i)$ there exists $c_x \in C(x)$ such that $c_x \subseteq I$, and $ii)$ $c \in C(x)$ only if there exists $i \in I \cap c$. Consider any $R \in \hat{\mathcal{D}}^n$, such that for all $i, j \in I$, $B(R_i) = B(R_j) = z$. By (i) there exists $c_z \subset I$, hence $f(R) \leq z$. By (ii) for all $x < z$, and for all $c \in C(x)$, there is $i \in I \cap c$, and therefore $f(R) \neq x$. Hence $f(R) = z$ and condition (i) in 9 is satisfied. Consider any $R \in \hat{\mathcal{D}}^n$ such that there exist $i, j \in I$ with $B(R_i) \neq B(R_j)$. Let $h \in I$ be such that for all $j \in I$, $B(R_h) \leq B(R_j)$ and let $l \in I$ be such that for all $j \in I$, $B(R_l) \geq B(R_j)$. By (ii) $f(R) \geq B(R_h)$ and by (i) $f(R) \leq B(R_l)$. Consider now any $R' \in \hat{\mathcal{D}}^n$ such that for all $j \in I$, $R_j = R'_j$. For the same arguments as above $B(R_i) \leq f(R') \leq B(R_l)$. It follows that if $f(R') \neq f(R)$, then there exist $i, j \in I$ such that $f(R')P_i f(R)$ and $f(R)P_j f(R')$.
Necessity. Consider any rule f that is strategy-proof, satisfies voter sovereignty and ensures information disclosure when I is the set of informed agents. By strategy-proofness it has to be a generalized median voter rule. By Proposition 2 the voting rule must satisfy coalitional veto power relative to I. Let C be its associated left coalition system and $X = [a, b]$. Suppose first that there exists $x < b$ such that for each coalition $c \in C(x)$, a member of c is an uninformed agent. Consider $R \in \hat{\mathcal{D}}^n$ such that for all $i \in I$, $B(R_i) = x$ and for all $j \notin I$, $B(R_j) = b$; by definition of 12 $f(R) > x$,and coalitional veto power relative to I is violated at profile R. Suppose now that there exists $x < b$ and $c \in C(x)$ such that $I \cap c = \varnothing$. Consider $\bar{R} \in \hat{\mathcal{D}}^n$ such that for all $i \notin I$, $B(\bar{R}_i) = x$ and for all $j \in I$, $B(\bar{R}_j) = b$. By definition of 12 $f(R) = x$ and therefore coalitional veto power relative to I is violated at profile $\bar{R}$.

Proof of Proposition 6 A voting rule $f : \tilde{\mathcal{P}}^n \to X$ is strategy-proof and satisfies voter sovereignty, if and only if it is based on voting by committees, as proved by Barberà et al. (1991). We prove that conditions (a) and (b) above are necessary and sufficient for ensuring information disclosure.
Necessity. Suppose by contradiction that (a) is violated. There exists $x \in X$ and $M \in \mathcal{W}_x$ such that $M \cap I = \varnothing$. Consider $P \in \tilde{\mathcal{P}}^n$ such that for all $i \in I$, $G^C(P_i) = \{x\}$ and for all $j \in U$, $x \in G(P_j)$. Since $M \cap I = \varnothing$ then $M \subseteq U$ and $x \in f(P)$. The voting rule f does not satisfy coalitional veto power relative to I, contradicting Proposition 2. Suppose by contradiction

that (b) is violated: there exists $x \in X$ such that for all $M \in \mathcal{W}_x$ $M \cap U \neq \varnothing$. Consider $P \in \tilde{\mathcal{P}}^n$ such that for all $i \in I$, $G(P_i) = \{x\}$ and for all $j \in U$, $x \in G^C(P_j)$. Then $x \notin f(P)$ and therefore f does not satisfy coalitional veto power relative to I, contradicting Proposition 2.
Sufficiency. Suppose that f is a voting rule based on committees that satisfies conditions a) and b). Consider any $P \in \tilde{\mathcal{P}}^n$ such that for all $i, j \in I$, $P_i = P_j$. Since (b) holds then there is no $x \in G^C(P_i)$ for all $i \in I$ such that $x \in f(P)$. Moreover since a) holds there is not any $x \in G(P_i)$ for all $i \in I$, such that $x \notin f(P)$. Hence $f(P) = G(P_i)$ for every $i \in I$. Therefore condition (i) of definition 9 holds. Suppose, by contradiction, that there exist P, P' such that $P_i = P'_i$ for each $i \in I$ and $f(P) \succ_i f(P')$ for all $i \in I$. By assumption $\cap_{i \in I} G(P_i) = \cap_{i \in I} G(P'_i) \equiv \bar{G}_I$ and $\cap_{i \in I} G^C(P_i) = \cap_{i \in I} G^C(P'_i) \equiv \bar{G}_I^C$.By condition (a) $\bar{G}_I^C \cap f(P) = \emptyset$ and $\bar{G}_I^C \cap f(P') = \emptyset$. By condition (b) $\bar{G}_I \subseteq f(P)$ and $\bar{G}_I \subseteq f(P')$. Therefore if $f(P) \succ_i f(P')$ for all $i \in I$, then there exist $x, y \in X$ and $j, l \in I$ such that $x \in G(P_l)$, $x \in G^C(P_j)$, $y \in G^C(P_l)$, $y \in G(P_j)$. That is, there are two alternatives x and y such that informed voter l likes x and dislikes y, while informed voter j likes y and dislikes x, and either
Case 1: $x, y \in f(P)$, $x, y \notin f(P')$ or
Case 2 $x, y \notin f(P)$, $x, y \in f(P')$.
Consider case 1 (case 2 is analogous). Since $x \in f(P)$, then there is $M \in \mathcal{W}_x$ such that for all $i \in M$, $x \in G(P_i)$. Since $x \notin f(P')$ there is no $\hat{M} \in \mathcal{W}_x$ such that for all $i \in \hat{M}$, $x \in G(P'_i)$. Moreover, since $x \notin f(P')$ but $x \in f(P)$, there exists $h \in M \cap U$ such that $x \in G^C(P'_h)$ and $x \in G(P_h)$. Therefore, P_h and P'_h differ. For each $i \in U$, let $P_i^{|x}$ denote voter $i's$ preferences restricted to alternative x (by separability they are well defined: either $x \in G(P_i)$ or $x \in G^C(P_i)$). Consider now the profile P_U^{full}. Suppose first that $x \in f(P_I, P_U^{full}) = f(P'_I, P_U^{full})$. We show that $f(P')$ cannot be a Nash equilibrium outcome of the game $(I, P'_I, 2^X, f\,(P'_I, g(\cdot)))$. Let a be the purported NE of this game with $g(a) = P'_U$. Since g is separable by assumption, then $x \notin D(a)$. Hence, voter $l \in I$ can deviate and play $\hat{a}_l$ such that $D(a_{-l}, \hat{a}_l) = D(a) \cup \{x\}$: voter l can disclose information about x. Hence $g(a_{-l}, \hat{a}_l) = \bar{P}_U$ such that for all $i \in U$, $\bar{P}_i^{|x} = P_i^{|x}$. It follows that $f(P'_I, \bar{P}_U) = f(P') \cup \{x\}$ and $f(P'_I, \bar{P}_U) P'_l f(P')$ contradicting that $f(P')$ is a NE outcome of the game. Suppose now that $x \notin f(P_I, P_U^{full}) = f(P'_I, P_U^{full})$.Using a symmetric argument, it follows that $f(P)$ cannot be a Nash equilibrium outcome of $(I, P_I, 2^X, f\,(P_I, g(\cdot)))$ because voter $j \in I$ can profitably deviate and disclose information about x.

Proof of Proposition 7 By Proposition 4 we know that no anonymous strategy-proof voting rule that satisfies voter sovereignty, ensures information disclosure when $|I| < \frac{n+1}{2}$, and that the median voter rule ensures information disclosure when $|I| \geq \frac{n+1}{2}$. We prove that every anonymous strategy-proof voting rule that satisfies voter sovereignty, different than the simple median does not ensure information when $|I| \geq \frac{n+1}{2}$. Consider an arbitrary set of voters M with $|M| = \frac{n+1}{2}$. Consider any anonymous, strategy-proof voting rule f that that satisfies voter sovereignty such that for some $z \in X$ $c \in C(z)$ if and only if $|c| \geq k$ with $k \neq \frac{n+1}{2}$. Suppose first $k < \frac{n+1}{2}$. Consider a pair of preference profiles R^0, R^1 with $R^1_M = R^0_M = R_M$, $B(R_i) = z$ for all $i \in M$, $B(R^0_j) = z$, $B(R^1_j) = y$ for all $j \notin M$ with $y < z$, and a reaction function $g \in \mathcal{H}^C(\hat{\mathcal{D}})$ such that $g(a^0) = R^0_{-M}$ and $g(a) = R^1_{-M}$ for all $a \neq a^0$. Consider the game $\Gamma = (M, R_M, A, f(R_M, g(\cdot)))$. Then, there exists a Nash equilibrium of Γ such that $a^* = a^0$ and $f(R_M, g(a^0)) = z \neq f(R_M, R^{g,full}_{-M}) = y$. The proof for the case $k > \frac{n+1}{2}$ is analogous: consider a pair of preference profiles $\bar{R}^0, \bar{R}^1$ with $\bar{R}^1_M = \bar{R}^0_M = \bar{R}_M$, $B(\bar{R}_i) = y$ for all $i \in M$, $B(\bar{R}^0_j) = y$, $B(\bar{R}^1_j) = z$ for all $j \notin M$ and a reaction function $g \in \mathcal{H}^C(\hat{\mathcal{D}})$ such that $g(a^0) = \bar{R}^0_{-M}$ and $g(a^1) = \bar{R}^1_{-M}$.

Proof of Proposition 8 To prove this proposition we first show that an anonymous voting by committees f ensures information disclosure at information structure $(I, \tilde{P}, \mathcal{H}^R_{sep}(\tilde{\mathcal{P}}))$ if and only if for each alternative $x \in X$, the quota q^x satisfies the following two conditions: $|I| \geq q^x$ and $n - |I| < q^x$. Sufficiency. Suppose that for all $x \in X$, $|I| \geq q^x$ and $n - |I| < q^x$. Therefore, for every alternative $x \in X$, every winning coalition contains at least one expert and there exists a subset of experts who are a winning coalition. Then by Proposition 6 the anonymous voting by committees rule ensures information disclosure under information structure $(I, \tilde{P}, \mathcal{H}^R_{sep}(\tilde{\mathcal{P}}))$.

Necessity. Suppose that there exists $x \in X$ such that $|I| < q^x$. Consider a preference profile $\bar{P}$ such that $G(\bar{P}_i) = X$ for all $i \in N$. For each $T \in 2^X \setminus \{\emptyset\}$, let $\bar{P}^T$ denote a preference profile such that $G(\bar{P}^T_i) = X$ for all $i \in I$, and $G(\bar{P}^T_j) = X \setminus T$ for all $j \in U$. Let $g \in \mathcal{H}^R_{sep}(\tilde{\mathcal{P}})$ such that $g(a^0) = \bar{P}_U$ and for all $a \neq a^0$, $g(a) = \bar{P}^{D(a)}$. It immediately follows that $(I, \bar{P}_I, A, f(\bar{P}_I, g(\cdot))$ has a NE $a^* = a^0$ and $f(\bar{P}_I, g(a^0)) \neq f(\bar{P}_I, g(a^1))$ because $f(\bar{P}_I, g(a^0)) = X$ by voter sovereignty and strategy-proofness, while $x \notin f(\bar{P}_I, g(a^1))$. Suppose now that there exists $x \in X$ such that $n - |I| \geq q^x$. The same argument can be replicated considering a preference profile $\hat{P}$ such that $G^C(\hat{P}_i) = X$ for all

$i \in N$. For each $T \in 2^X \backslash \{\emptyset\}$, let $\hat{P}^T$ denote a preference profile such that $G^C(\hat{P}_i^T) = X$ for all $i \in I$, and $G(\hat{P}_j^T) = T$ for all $j \in U$. Let $g \in \mathcal{H}_{sep}^R(\tilde{\mathcal{P}})$ such that $g(a^0) = \hat{P}_U$ and for all $a \neq a^0$, $g(a) = \hat{P}^{D(a)}$.

Suppose there are less than $\frac{n+1}{2}$ informed voters. Then the above conditions cannot be satisfied. Suppose there are exactly $\frac{n+1}{2}$ informed voters. The conditions are satisfied if and only if for all $x \in X$, $q^x = \frac{n+1}{2}$. Hence the only anonymous voting by committees rule which ensures information disclosure when $|I| = \frac{n+1}{2}$ is a voting by quota rule with $q = \frac{n+1}{2}$. This rule satisfies the two conditions when the cardinality of the set of informed voters is larger than $\frac{n+1}{2}$. This concludes the proof.

References

[1] S. Nageeb Ali and J. Aislinn Bohrenz, (2014). “Information Aggregation in Search Committees”, working paper.

[2] David Austen-Smith and Jeffrey. S. Banks, (1996). “Information Aggregation, Rationality, and the Condorcet Jury Theorem”, *American Political Science Review* 90, 34–45.

[3] David Austen-Smith, and Timothy J. Feddersen, (2005). “Deliberation and Voting Rules.” In Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks, eds. David Austen-Smith and John Duggan. Heidelberg: Springer.

[4] D. Austen-Smith and Timothy J. Feddersen, (2006) “Deliberation, Preference Uncertainty, and Voting Rules", *American Political Science Review*, 100 (2), 211-217.

[5] Salvador Barberà, Faruk Gul, and Ennio Stacchetti, (1993). “Generalized Median Voter Schemes and Committees”, *Journal of Economic Theory* 61, 262-289.

[6] Salvador Barberà, Hugo Sonnenschein and Lin Zhou, (1991). "Voting by committees", Econometrica, 59, 3, 595-609.

[7] Sourav Bhattacharya, (2013). "Preference Monotonicity and Information Aggregation in Elections", *Econometrica*, 81, 3, 1229–1247.

[8] Jesse Bull and Joel Watson, (2004). "Evidence Disclosure and Verifiability", *Journal of Economic Theory*, 118, 1-31.

[9] Jesse Bull and Joel Watson, (2007). "Hard Evidence and Mechanism Design", *Games and Economic Behavior*, 58: 75-93.

[10] Gerard R. Butters, (1977). "Equilibrium Distributions of Sales and Advertising Prices", *Review of Economic Studies*, 44, 465-491.

[11] Bashkar Dutta and Arunava Sen, (2011). "Nash Implementation with Partially Honest Individuals," Games and Economic Behavior, 74, 154–169.

[12] Peter Eso, Stephen Hansen and Lucy White, (2014). "A Theory of Vote-Trading and Information Aggregation", CEPR working paper.

[13] Sanford J. Grossman, (1981). "The informational role of warranties and private disclosure about product quality", *Journal of Law and Economics*. 24, 461-83.

[14] Hans Peter Gräuner and Alexandra Kiel, (2004). "Collective decisions with interdependent valuations", *European Economic Review*, 48, 1147–1168.

[15] Yingni Guo, (2013). "Information Transmission and Voting", working paper Yale University.

[16] Jeanne HagenBach, Frédéric Koesller, and Eduardo Perez-Richet, (2014). "Certifiable Pre-Play Communication: Full Disclosure", *Econometrica*, 82, 3, 1093-1131.

[17] Matthew O. Jackson and Xu Tan, (2012). "Deliberation, Disclosure of Information, and Voting ", *Journal of Economic Theory,*

[18] Navin Kartik and Olivier Tercieux, (2012). "Implementation with Evidence", *Theoretical Economics*, 7, 323-355.

[19] Vijay Krishna and John Morgan, (2001). "Asymmetric Information and Legislative Rules: Some Amendments". American Political Science Review, 95, 2, 435-452.

[20] Steven Matthews and Andrew Postlewaite, (1985). "Quality testing and disclosure". RAND Journal of Economics. 16, 328-340.

[21] Paul Milgrom, (1981). “Good news and bad news: Representation theorem and applications”, *The Bell Journal of Economics*, 12, 380-91.

[22] Paul Milgrom and John Roberts, (1986). “Relying on the information of interested parties”, *Rand Journal of Economics*, 17, 18-32.

[23] Hervé Moulin, (1980). “On Strategyproofness and Single-peakedness”, *Public Choice* 35(4), 437-455.

[24] Frank Rosar (2012). “Continuous decisions by a committee: median versus average mechanisms”, University of Bonn, working paper.