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On the advantages and disadvantages of being the first mover under rules of k names

Salvador Barberà* and Danilo Coelho†

Rules of k names are methods that allow two potentially conflicting parties to share the power to appoint officers. One of the parties (the proposer) selects k candidates from a larger pool, and then the other party (the chooser) selects the winner from this restricted list. We investigate conditions under which the two parties could agree ex ante on the distributions of roles, one of them preferring to be the chooser and the other preferring to be the proposer. We show that this may not always be possible, and discuss what are the relevant characteristics of the environments where agreement can be reached.

Key words voting rule, constitutional design, strong Nash equilibrium, rule of k names

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1 Introduction

In many social instances, two parties with potentially conflicting interests must come to agreement regarding a decision that affects both of them. A paradigmatic example is the cutting of a cake that can be divided in infinite ways. But a lot of social decisions involve the choice of indivisibles: in particular, the social decision may involve choosing one individual from a set of potential candidates. Different methods have been proposed in this setting with indivisibilities allowing each of the parties to have some partial say on the final outcome and to achieve some sort of compromise between them.

One family of such methods, which are reminiscent of the divide and choose procedure, are those that we have called in previous work rules of k names (Barberà and Coelho 2010, 2017), where the aim is to choose one out of $\mathbf{c}$ candidates ($1 \leq k \leq \mathbf{c}$). They work as follows: one of the parties is allowed to select k candidates out of the $\mathbf{c}$ available, and then the other party chooses one winner from those selected by the opponent. Rules of that kind have been used for centuries, and are still very much resorted to in many countries to adopt different types of decisions. Already at the beginning of the sixth century BC, the clergy and the chief of the citizens of some eastern European countries were entitled to choose three names from which the archbishop could select the bishop. The use of rules of k names was widespread within the Roman Church from the early Middle Ages, when

* MOVE, UAB and Barcelona GSE , Barcelona, Spain. Email: salvador.barbera@uab.cat

† IPEA, Rio de Janeiro, Brazil.

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secular rulers tried to control the appointment of bishops, while the clergy would rather reserve for itself the decision on its leaders. And they are still used to share power between Rome and local congregations. Nowadays, rules of *k* names, in different variants, are resorted to in many countries, including Argentina, Brazil, Canada, Chile, Haiti, Mexico, Spain, Turkey, and the USA, to decide how to appoint a wide range of important officers. Positions in judiciary bodies are filled through rules of *k* names in many Latin American countries. And a closely related procedure is the commission-selection political appointment method of selecting judges to the supreme courts of several US states, sometimes referred to as merit selection or the Missouri Plan. For a final example, in some German universities, in order to fill each vacant professorship, a jury must propose a list of three candidates, from whom the academic authorities, or the local government, end up selecting one.[1]

In many cases, the proposers or choosers are themselves not a single individual, but a legislative body or an ad hoc committee. More important for our present concern is the fact that the roles of proposer and chooser are not always attributed according to the same criteria. In most cases where the decision is shared between a collective body and a single officer it is the larger body that submits the *k* names to a single decision-maker.

This is the case in Chile, for example, where the members of the Supreme Court are appointed by the President of the Republic from a list of five candidates proposed by the sitting members of the Supreme Court.[2] Yet, in other cases it is the single agent who is in charge of proposing the names of candidates from whom the larger body will select. This happens, for example, in Mexico and Colombia. In Mexico, the president submits a list of candidates to the Supreme Court of Justice to the Senate, and it is this body that makes the final choice. In Colombia, the Attorney General is elected by the Supreme Court from a list with three names presented by the President of the Republic. Even in cases like the ones we will examine here, where two single individuals (such as the prince and the Pope) must share the power to appoint, the division of tasks is again possible in both directions, and not always easy to justify. To our knowledge, the criteria that may underlie the assignment to the parties of these two distinct roles, as choosers and proposers, have not been discussed in the literature. This will be the main subject of our paper.

To simplify matters, from now on we concentrate on the case where the proposer and the chooser are single individuals. Several important decisions end up shaping the specific rule within this general class that will be used in each specific situation. One of them is the choice of *k*. Another is the assignment of roles, since in general agents will not be indifferent between acting as a chooser and as a proposer. These two decisions interact. An agent's preference for *k* will depend on her position as proposer or chooser. And this agent's position on which role she would like to play will in turn depend on what value of *k* is to be used. Once the roles of agents, the value of *k*, and the preferences of agents are fixed, a strategic game arises that will result in the choice of a final alternative. Hence, our inquiry is a very standard one in the analysis of sequential games: is it better to be the leader or the follower, to play first or last? However standard, though, the question is relevant in our case because it allows us to address important issues in mechanism design.

One of these issues is whether the exact form of the mechanism must be imposed, or else may result from the choice, by consensus among the interested parties, from rules within a given

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1 Who plays the role of the chooser varies across states. Actually, the list must be ordered, but the order is not binding, and there are actually many cases where it is not respected. Hence, the analogy is very close.

2 In Brazil and Argentina, the nomination of the members of federal courts is almost identical to this procedure, but instead of five the list must have three candidates.

class.[3] We look at this question from an ex ante point of view, assuming that agents may have a say about what rule will be used from then on, knowing that the circumstances under which it will be applied will not always be the same. We model this variability of situations as if each agent had some expectations regarding their distribution, and some knowledge about her levels of risk aversion and that of her opponent. Of course, these distributional and informational assumptions matter, but our modeling decisions may easily be adapted to alternative scenarios. What is important is that the analysis allows us to address the following question. Can the mechanism be chosen by consensus between the parties, or will it have to be imposed because they will face an irreducible conflict? And when consensus is possible, what parameters should be adjusted to attain it? This is particularly relevant in cases where these parties are powerful and may not accept the rule unless it gives them a sense of fairness. Consider, for example, the historical situation where the ruler of a medieval city-state and the chapter of its cathedral ended up using of a rule of k names to choose the bishop. This was obviously a compromise solution, but the nature of the compromise might have been different depending on circumstances. In some cases, both parties may have agreed on who should play the role of chooser and that of proposer. In others, the stronger side of the deal might have chosen its preferred role even if the other party also coveted it. One can expect the consensual solution to be more durable in time than the imposed one. And, from another point of view, analyzing who played the more favorable role under a historically used rule may provide information about the correlation of forces between the parties involved.

Because of that, we discuss the advantages and the disadvantages that agents will find in playing one role or another, as a function of k, and how this may affect the process whereby rules are chosen. In particular, we shall focus on the circumstances under which it is possible to assign to both agents the roles that they would prefer to play. When an appropriate choice of k makes this agreement possible, the choice of roles may be left to the contending parties and this reinforces the value of the procedure as a method to find consensus. We shall see that this possibility depends on the information that the parties have at the time of accepting, or not, to operate under such rules, and on their attitudes toward risk. We will prove that such agreement may not always possible, and will also characterize the range of values for k that allow for agreement when possible. In the next two sections we propose our formal model, the basic informational scenario under consideration, and state our main qualitative results for this scenario. We then conclude by proposing how to use of our results, qualifying their coverage, suggesting alternative scenarios and further research topics.

2 The model

Let $\mathbf{C}$ be a finite set of candidates with cardinality c. Denote by $\mathbf{C_k}$ the family of subsets of $\mathbf{C}$ with cardinality k. Two agents, 1 and 2, are entitled to participate in the choice of one of the candidates.

We assume that each agent is endowed with preferences on the lotteries over candidates, and that the preferences of agent i on these lotteries are representable through a Bernoulli utility function u_i. We denote by $\mathbf{U}$ the set of all Bernoulli utility functions assigning different values to different candidates. Given any $u_i \in \mathbf{U}$, we can identify the first ranked candidate (the best), or, in general, the jth ranked one, according to u_i. We will denote by u_{ij} the utility of the jth ranked candidate

[3] A similar question can be asked with respect to the choice of rules of a different kind than those in the class we consider here. See, for example, Barberà and Jackson (2004), Coelho (2005), or Attanasi *et al*. (2017).

according to u_i, since we assume that all these values are different from each other, and thus $u_{i1} > u_{i2} > \ldots > u_{i\mathbf{c}}$. Preferences profile are pairs of utilities (u_1, u_2) in $\mathbf{U} \times \mathbf{U}$.

We now define rules of k names for the two-agent case.

Definition 1 *For any* $\mathbf{C}$ *and any* $k \leq \mathbf{c}$*, the rule of k names selects one candidate as follows: one of the agents (the proposer) selects k candidates from the* $\mathbf{c}$ *available ones, and then the other agent (the chooser) picks one winner from those selected by the proposer.*

Notice that agents operating under a rule of k names are involved in a simple strategic game, in which the proposer's strategies are the sets in $\mathbf{C}_k$, and the chooser's strategies are choice functions over the sets in $\mathbf{C}_k$. The solution to these games, and hence the chosen candidates for each given profile, will depend on the information available to agents at the time of voting, and on the solution concepts to be used. In what follows, we shall think of the choice as the result of a sequential process, represented by an extensive form, and use subgame perfection as our equilibrium notion.

It is then easy to characterize the backward induction equilibrium outcomes of the complete information game associated with the use of a rule of k names to choose from a set $\mathbf{C}$.

Proposition 1 *(Barberà and Coelho 2010) Consider a preference profile on a set of candidates* $\mathbf{C}$*, and the game induced by the rule of k names (for* $k \leq \mathbf{c}$*). The backward induction equilibrium outcome (the winning candidate) is the proposer's best candidate out of the chooser's* $(\mathbf{c} - k + 1)$ *top candidates.*

Here is the intuition for this result. Allowing the proposer to select k candidates is giving her the power to veto the remaining $\mathbf{c} - k$. This means that even in the worse of cases, where both agents have reverse preferences, the proposer cannot lead the chooser to pick anything worse than her $(\mathbf{c} - k + 1)$th ranked alternative. Thus, the proposer must select his best candidate from the chooser's $\mathbf{c} - k + 1$ top alternatives and submit a list where this candidate is the chooser's best candidate in the list.[4]

Defining rules of k names, the games they induce and finding their pure strategy solution does not require any use of the preferences of agents over lotteries. For these sole purposes we could have simply defined the ordinal preferences of agents over candidates. However, we shall now start assessing the performance of those rules under different forms of uncertainty, and from now on the assumption that agents can compare lotteries will become necessary.

As stated in the introduction, our goal is to explore to what extent the choice of roles under rules of k names could be the result of a voluntary agreement by the parties involved. And for this we must start by acknowledging that once these rules are set, parties must in principle abide by them, and refer our analysis to the moment prior to their establishment.[5] Knowing that the rule will be applied under varying circumstances, involving different degrees of conflict among the parties, what could the calculations of rational agents be before accepting one of them?

We model the uncertainty regarding the different situations under which the rule may be applied by assuming that the relevant profiles will be the result of random draws from independent and

[4] Notice that the equilibrium in the case of reverse preferences has the proposer submitting her k most preferred candidates from which the chooser selects the candidate that has rank k in the proposer's preference ordering.

[5] Recent work by de Clippel *et al*. (2014) and by Brams *et al*. (2017) also proposes and analyzes similar methods to ours, but not from this ex-ante perspective.

uniform distributions over the class of preferences. And we initially assume that at the time when a decision has to be taken, both parties will know the profile.[6]

We can start by calculating the probability that a rule of k names attaches to the choice of an agent's jth ranked alternative, given our distributional assumptions, as a function of k and of the agent's role as proposer or chooser.

For each value of $k \in \{1, \ldots, \mathbf{c}\}$, denote by $\text{Prob}(u_i(x) = u_{ij}|k, \textit{proposer})$ and $\text{Prob}(u_i(x) = u_{ij}|k, \textit{chooser})$ the probabilities that agent i receives utility u_{ij} conditional on i's role (proposer or chooser) under the rule of k names. Let us compute these probabilities for our present scenario. For every agent $i \in \{1, 2\}$:

$$\text{Prob}(u_i(x) = u_{ij}|k, \textit{proposer}) = \begin{cases} \dfrac{\dbinom{\mathbf{c}-j}{\mathbf{c}-k}}{\dbinom{\mathbf{c}}{\mathbf{c}-k+1}} & \text{for any } j \in \{1, \ldots, k\} \\ 0 & \text{for any } j \in \{k+1, \ldots, \mathbf{c}\}; \end{cases} \tag{1}$$

$$\text{Prob}(u_i(x) = u_{ij}|k, \textit{chooser}) = \begin{cases} \frac{1}{\mathbf{c}-k+1} & \text{for any } j \in \{1, \ldots, \mathbf{c}-k+1\} \\ 0 & \text{for any } j \in \{\mathbf{c}-k+2, \ldots, \mathbf{c}\}. \end{cases} \tag{2}$$

Notice that all of the chooser's $\mathbf{c} - k + 1$ top candidates have the same probability, $1/(\mathbf{c} - k + 1)$, of being the equilibrium outcome. By contrast, notice that candidates who are among the proposer's k top candidates have larger probabilities of being chosen, the better they are ranked. The only exception arises when $k = \mathbf{c}$. Then, from the proposer's perspective, all candidates have the same probability of being the equilibrium outcome, since the chooser's favorite candidate may be any one, and this will be the equilibrium outcome.

Having presented these probabilities, let us now see their consequences, when combined with the agent's Bernoulli utilities, on the desirability for an agent to act as a proposer or as a chooser.

Denote by $E(u_i(x)|k, \textit{proposer})$ and $E(u_i(x)|k, \textit{chooser})$ agent i's expected utility in the role of the proposer and the chooser, respectively. Then

$$E(u_i(x)|k, \textit{proposer}) \equiv \sum_{j=1}^{k} \frac{u_{ij}\dbinom{\mathbf{c}-j}{\mathbf{c}-k}}{\dbinom{\mathbf{c}}{\mathbf{c}-k+1}} \qquad \text{for any } k \in \{1, \ldots, \mathbf{c}\}, \tag{3}$$

$$E(u_i(x)|k, \textit{chooser}) \equiv \sum_{j=1}^{c-k+1} \frac{u_{ij}}{\mathbf{c}-k+1} \qquad \text{for any } k \in \{1, \ldots, \mathbf{c}\}. \tag{4}$$

[6] Notice that, as long as we use backward induction equilibrium as our solution concept, whether or not the chooser knows the proposer's preferences is irrelevant. But this may not be the case in other possible situations.

3 Proposer or chooser: Which would you like to be?

Before carrying out the general calculations regarding the advantages or disadvantages of playing either of the two roles under rules of *k* names, let us consider an example showing that the possibility of agreement on how to distribute these roles depends on the value of *k* and the degree of risk aversion of the parties. The example also shows that, in some cases, no agreement is possible. Our calculations after that will extend these remarks to consider general cases.

Example 1 Consider $\mathbf{c} = 6, k = 4$. Suppose that agents 1 and 2 have the following Bernoulli utility function, described also in Figure 1: $(u_{11} = 6.8, u_{12} = 6.7, u_{13} = 6.3, u_{14} = 4.6, u_{15} = 2.6, u_{16} = 0.1)$ and $(u_{21} = 6, u_{22} = 5, u_{23} = 4, u_{24} = 3, u_{25} = 2, u_{26} = 1)$. Using formulas (3) and (4), agent 1's expected utility in each role is:

$$E(u_1(x)|k = 4, \mathit{proposer}) = \frac{6.8\binom{6-1}{6-4}}{\binom{6}{6-4+1}} + \frac{6.7\binom{6-2}{6-4}}{\binom{6}{6-4+1}} + \frac{6.3\binom{6-3}{6-4}}{\binom{6}{6-4+1}} + \frac{4.6\binom{6-4}{6-4}}{\binom{6}{6-4+1}} = 6.59,$$

$$E(u_1(x)|k = 4, \mathit{chooser}) = \frac{6.8}{6-4+1} + \frac{6.7}{6-4+1} + \frac{6.3}{6-4+1} = 6.60,$$

$$E(u_2(x)|k = 4, \mathit{proposer}) = \frac{6\binom{6-1}{6-4}}{\binom{6}{6-4+1}} + \frac{5\binom{6-2}{6-4}}{\binom{6}{6-4+1}} + \frac{4\binom{6-3}{6-4}}{\binom{6}{6-4+1}} + \frac{3\binom{6-4}{6-4}}{\binom{6}{6-4+1}} = 5.25,$$

$$E(u_2(x)|k = 4, \mathit{chooser}) = \frac{6}{6-4+1} + \frac{5}{6-4+1} + \frac{4}{6-4+1} = 5.00.$$

Thus, since $E(u_1(x)|k = 4, \mathit{chooser}) > E(u_1(x)|k = 4, \mathit{proposer})$ and $E(u_2(x)|k = 4, \mathit{proposer}) > E(u_1(x)|k = 4, \mathit{chooser})$, agent 1 would prefer to be the chooser, while agent 2 would prefer to be the proposer.

Table 1 shows that if $\mathbf{c}$ were equal to 5 instead of 6, there would be no value of *k* such that agents 1 and 2 would agree on who should play first.

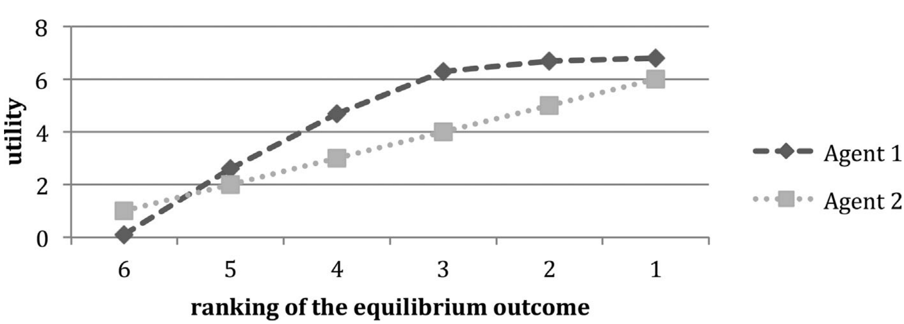

Figure 1 Agents' utility functions over the equilibrium outcome.

Table 1 Agents' utility functions over the equilibrium outcome

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$E(u_1(x)\|k, \textit{proposer})$	6.80	6.78	6.72	6.45	5.40
$E(u_1(x)\|k, \textit{chooser})$	5.40	6.10	6.60	6.75	6.80
$E(u_2(x)\|k, \textit{proposer})$	6.00	5.80	5.50	5.00	4.00
$E(u_2(x)\|k, \textit{chooser})$	4.00	4.50	5.00	5.50	6.00

In Example 1 we used specific values of the Bernoulli function. But some of our conclusions can be arrived at without reference to any specific functional form. Proposition 2 below gives the values of k for which we do not need to have any information about the functional form of the utility function of a generic agent i in order to know the position (proposer or chooser), under a given value of k, that gives the highest expected payoff for that agent.

Before presenting Proposition 2, first notice that there is a $k^* \in \{1, \dots, \mathbf{c}\}$ such that $E(u_i(x)|k, \textit{proposer}) > E(u_i(x)|k, \textit{chooser})$ for every $k < k^*$ and $E(u_i(x)|k, \textit{chooser}) > E(u_i(x)|k, \textit{proposer})$ for every $k > k^*$. This is because $E(u_i(x)|k, \textit{proposer})$ is strictly decreasing and $E(u_i(x)|k, \textit{chooser})$ is strictly increasing with respect to k. Moreover, $E(u_i(x)|k = 1, \textit{proposer}) = E(u_i(x)|k = \mathbf{c}, \textit{chooser}) = u_{i1}$ and, given the assumption of independently uniformly distributed preferences, we have that $E(u_i(x)|k = \mathbf{c}, \textit{proposer}) = E(u_i(x)|k = 1, \textit{chooser}) = (u_{i1} + \dots + u_{ic})/\mathbf{c}$. Of course, the value of k^* may change when we change the form of agents' utility functions.

Proposition 2 *For any $i \in \{1, 2\}$ and any utility function $u_i(\cdot)$ on $\mathbf{C}$:*

$$E(u_i(x)|k, \textit{proposer}) > E(u_i(x)|k, \textit{chooser}) \quad \textit{for every } k \leq \frac{\mathbf{c}+1}{2},$$

$$E(u_i(x)|k, \textit{chooser}) > E(u_i(x)|k, \textit{proposer}) \quad \textit{for every } k \geq \mathbf{c} - \sqrt{\mathbf{c}} + 1.$$

PROOF: Notice that a direct corollary of our equilibrium characterization is that the equilibrium outcome is among the proposer's k top candidates. Thus, the equilibrium outcome is the proposer's best candidate in the intersection between the proposer's k top candidates and the chooser's $\mathbf{c} - k + 1$ top candidates. Thus, when $k \leq \mathbf{c} - k + 1$, being the proposer gives a higher expected utility than being the chooser. Now let us analyze the second item. Notice from expression (1) that the probability of a candidate being the equilibrium outcome increases, the higher he is ranked in the proposer's preferences. Notice also that the probability of the proposers' best candidate is equal to $\frac{\mathbf{c}-k+1}{\mathbf{c}}$, while the probability of the chooser's best candidate is equal to $\frac{1}{\mathbf{c}-k+1}$. Thus, when $k = \mathbf{c} - \sqrt{\mathbf{c}} + 1$, these two probabilities are equal, and when $k > \mathbf{c} - \sqrt{\mathbf{c}} + 1 > \mathbf{c} - k + 1$, any of the chooser's $(\mathbf{c} - k + 1)$ top candidates has a higher probability of being the equilibrium outcome than any of the proposer's k top candidates. Therefore, when $k > \mathbf{c} - \sqrt{\mathbf{c}} + 1$ it is always better to be the chooser. □

The next two propositions show that, if we can specify the form of the utility functions, this enlarges the interval of values k for which we know for sure which position would be best for each agent. The next proposition considers linear utility functions.

Definition 2 *Given any number of candidates $\mathbf{c} \geq 3$, we say that $u_i(\cdot)$ is linear over the set of candidates if and only if $u_{ij-1} - u_{ij} = u_{ij+1} - u_{ij+2}$ for every $j \in \{2, \dots, \mathbf{c} - 2\}$.*

Proposition 3 *For any $i \in \{1, 2\}$ and any linear utility function $u_i(\cdot)$ on* $\mathbf{C}$*:*

$$E(u_i(x)|k, proposer) > E(u_i(x)|k, chooser) \quad \text{for every } k < \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2},$$
$$E(u_i(x)|k, proposer) = E(u_i(x)|k, chooser) \quad \text{if } k = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2} \text{ is an integer},$$
$$E(u_i(x)|k, chooser) > E(u_i(x)|k, proposer) \quad \text{for every } k > \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}.$$

PROOF: First notice that if agent i has a linear utility function then $u_{ij} = \alpha - j\beta$, where $\alpha = u_{i1} + \beta$ and $\beta = u_{i1} - u_{i2}$, for every $j \in \{2, \dots, \mathbf{c}\}$. Thus, by the probability distributions described in expressions (1) and (2), we have that

$$E(u_i(x)|k, proposer) = \alpha - \beta \sum_{j=1}^{k} \frac{-j \binom{\mathbf{c} - j}{\mathbf{c} - k}}{\binom{\mathbf{c}}{\mathbf{c} - k + 1}} \text{ and } E(u_i(x)|k^*, chooser) = \alpha - \beta \sum_{j=1}^{\mathbf{c}-k+1} \frac{-j}{\mathbf{c} - k + 1}.$$

We learned from Barberà and Coelho (2017) that we can simplify these two formulas as follows:

$$E(u_i(x)|k, proposer) = \alpha - \beta \frac{\mathbf{c} + 1}{\mathbf{c} - k + 2} \quad \text{and} \quad E(u_i(x)|k, chooser) = \alpha - \beta \frac{\mathbf{c} - k + 2}{2}.$$

Thus, it is easy to see that $E(u_i(x)|k, proposer)$ is strictly decreasing with k and that $E(u_i(x)|k, chooser)$ is strictly increasing with k. Take any $\alpha \geq 0$ and $\beta > 0$; we only need to show that if $E(u_i(x)|k, proposer) = E(u_i(x)|k, chooser)$ then $k = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. Suppose that there is $k^* \in \{1, \dots, \mathbf{c}\}$ such that $E(u_i(x)|k^*, proposer) = E(u_i(x)|k^*, chooser)$. Notice that the product between $(E(u_i(x)|k, proposer) - \alpha)$ and $(E(u_i(x)|k, chooser) - \alpha)$ is equal to $\beta^2 \frac{\mathbf{c}+1}{2}$ for every $k \in \{1, \dots, \mathbf{c}\}$. Thus, $E(u_i(x)|k^*, chooser) = \alpha + \beta\sqrt{\frac{\mathbf{c}+1}{2}}$. This implies that $E(u_i(x)|k^*, chooser) = \alpha - \beta\frac{\mathbf{c}-k^*+2}{2} = \alpha + \beta\sqrt{\frac{\mathbf{c}+1}{2}}$ and, rearranging the terms, we have that $k^* = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. □

Notice that the result above determines for each possible value of k what is the position that gives the highest expected payoff. When the utility is strictly concave our results are not so sharp because for some values of k strictly concavity is not sufficient in order to determine which is the best position.

Definition 3 *Given any number of candidates $\mathbf{c} \geq 3$, we say that agent i's utility function is strictly concave over the set of candidates if and only if $u_{ij+1} - u_{ij+2} > u_{ij-1} - u_{ij}$ for every $j \in \{2, \dots, \mathbf{c} - 2\}$.*

Proposition 4 *For any $i \in \{1, 2\}$ and any strictly concave utility function $u_i(\cdot)$ on* $\mathbf{C}$*:*

$$E(u_i(x)|k, proposer) > E(u_i(x)|k, chooser) \quad \text{for every } k \leq \frac{\mathbf{c} + 1}{2},$$
$$E(u_i(x)|k, chooser) > E(u_i(x)|k, proposer) \quad \text{for every } k \geq \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}.$$

PROOF: The first inequality of Proposition 4 has already been show in Proposition 2. Let us prove the second inequality. Let $r_{ch} \equiv -(1 + \#\{y \in \mathbf{C}| y \succ_{chooser} x\})$ $(r_p \equiv -(1 + \#\{y \in C| y \succ_{proposer} x\})$

be the negative ranking of equilibrium outcome according to the chooser's preference (proposer's preferences). First notice that Proposition 3 implies that the negative ranking of the equilibrium outcome under the role of chooser is higher than under the role of proposer for every $k > \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. Thus, it will be enough to show that the statement holds even in the case where $k = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. Suppose that $\mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$ is an integer number and that agent i's utility function is strictly concave. Let $k^* = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. Notice that Proposition 3 implies that the expected ranking of the equilibrium outcome according to the chooser's and proposer's preferences are equal if and only if $k^* = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. From (1) and (2), it is easy to see that the probability distribution of r_{ch} is a mean-preserving spread of the probability distribution of r_p under $k^* = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$ given that $k^* > \mathbf{c} - k^* + 1$. This implies, given that $u_i(x)$ is strictly concave, that $E(u_i(x)|k^*, chooser) > E(u_i(x)|k^*, proposer)$. Therefore $E(u_i(x)|k, chooser) > E(u_i(x)|k, proposer)$ for any $k \geq \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$. □

We can now define the agreement zone as the set of values of k under which one agent would rather be the proposer and the other would prefer to be the chooser.

Definition 4 *Consider any pair of utility functions $u_1(\cdot)$ and $u_2(\cdot)$. We say that $k \in \{1, \ldots, \mathbf{c}\}$ belongs to the agreement zone if and only if the following conditions hold:*

$$\text{if } E(u_1(x)|k, chooser) \geq E(u_1(x)|k, proposer) \text{ then } E(u_2(x)|k, proposer) \geq E(u_2(x)|k, chooser);$$

$$\text{if } E(u_1(x)|k, proposer) \geq E(u_1(x)|k, chooser) \text{ then } E(u_2(x)|k, chooser) \geq E(u_2(x)|k, proposer).$$

Proposition 5 *Consider any pair of agents with utility functions $u_1(\cdot)$ and $u_2(\cdot)$.*

1. *The agreement zone can be empty or can contain more than one k. If k belongs to the agreement zone then $\frac{\mathbf{c}+1}{2} < k \leq \mathbf{c} - \sqrt{\mathbf{c}} + 1$.*
2. *Suppose that $u_1(\cdot)$ and $u_2(\cdot)$ are strictly concave. If k belongs to the agreement zone then $\frac{\mathbf{c}+1}{2} < k < \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$.*
3. *Moreover, if $u_1(\cdot)$ and $u_2(\cdot)$ exhibit constant absolute risk aversion then, under any k belonging to the agreement zone, the agent who is more risk averse weakly prefers to be the chooser and the other weakly prefers to be the proposer.*

Proof: Statements (1) and (2) are direct corollaries of Propositions 2–4. To prove (3), suppose that agent 1 is more risk averse than agent 2. Let $k^* \in \{1, \ldots, \mathbf{c}\}$ be any of the ks that belong to the agreement zone. We only need to prove that agent 1 weakly prefers to be the chooser. First notice that given expressions (1) and (2), the role of the chooser provides a lottery described by a homogeneous distribution over the chooser's $\mathbf{c} - k + 1$ top candidates. The role of proposer provides a lottery over the k proposer's top candidates such that $Prob(u_i(x) = u_{ij-1}|k, proposer) > Prob(u_i(x) = u_{ij}|k, proposer)$ for every $j \in \{2, \ldots, k\}$. Notice that there is no second-order stochastic domination between these two lotteries, otherwise the agreement would not be possible. However, k^* is larger than $\mathbf{c} - k^* + 1$, given that $k^* > (\mathbf{c} + 1)/2$ by Proposition 2. Thus, the advantage of being the chooser is that the candidates ranked worst than $(\mathbf{c} - k^* + 1)$th cannot be selected. And the advantage of being the proposer is that the proposer's best candidate has a higher probability of being selected. Suppose, by contradiction, that agent 2 prefers to be the chooser and agent 1 prefers to be the proposer. If it is the case, it means that agent 2 prefers the lottery with less probability mass in the extreme, while agent 1 prefers the reverse. This is a contradiction since agent 1 is the more risk averse. □

The next result is a direct consequence of statement (1) of Proposition 5 and the fact that $\mathbf{c} - \sqrt{\mathbf{c}} + 1 < \frac{\mathbf{c}+1}{2} + 1$ for every integer $\mathbf{c} \leq 5$.

Corollary 1 *If the number of candidates is smaller than* 6 *then the agreement zone is always empty.*

4 Concluding remarks

We have modeled a scenario in which two parties agree that a rule of k names will be used to let them both have a say on the choice of candidates, but where the roles of proposer and chooser are still to be assigned. We have identified situations where agents could agree on the choice of roles, because each prefers the role that the other dislikes. We have also argued that in many cases this consensual agreement over roles may not be possible, and the parties, or the mechanism designer, will have to resort to criteria other than consensus to settle the issue. We have discussed the main factors that determine whether consensus may or may not be reached: the value of k and the attitudes of the players toward risk.

Our analysis has been carried out in a context where the two parties understand that specific decisions will have to be taken for a wide variety of situations, involving different degrees of conflict between them. We have presented specific calculations for the case where all possible preference profiles are equally likely to arise. This is a standard way to model ignorance, but of course it could be modified, by changing our distributional assumptions, to study cases where some degrees of conflict are more likely to arise than others. In addition, we have assumed that, even if ignorant of the specific problem to be solved in each situation, both agents believe that at the time of voting they will be perfectly informed about their own interests and those of the opposite party. This assumption allows us to identify the payoffs of players as those of perfect equilibrium outcomes in an extensive game of complete information. And again, we are aware that other informational scenarios could be studied. A natural one would assume that, even if aware of their own interests, the parties are uncertain about those of their opponent, even at the time of voting. In that case a different game would arise, requiring alternative solution concepts to be used.

In spite of its limitations, we think that our analysis is important because, after periods of relative acceptance of given rules, one may observe that some agents become increasingly uncomfortable with them. That may be due, *inter alia*, to changes in behavioral parameters that call for adjustment. This paper does not solve these potential conflicts, but it allows us to point to possible sources and eventual solutions for societies that have chosen to operate under rules of k names.

References

Attanasi, G., L. Corazzini, and F. Passarelli (2017), "Voting as a lottery," *Journal of Public Economics* **146**, 129–37.

Barberà, S., D. Coelho (2017), "Balancing the power to appoint officers," *Games and Economic Behavior* **101**, 189–203.

Barberà, S., and D. Coelho (2010), "On the rule of k names," *Games and Economic Behavior* **70**, 44–61.

Barberà, S., and M. Jackson (2004), "Choosing how to choose: Self-stable majority rules and constitutions," *Quarterly Journal of Economics* **119**, 1011–48.

Brams, S. J., D. M. Kilgour, and C. Klamler (2017), "Maximin envy-free division of indivisible items," *Group Decision and Negotiation* **26**, 115–31.

Coelho, D. (2005), "Maximin choice of voting rules for committees," *Economics of Governance* **6**, 159–75.

de Clippel, G., F. Eliaz, and B. Knight (2014), "On the selection of arbitrators," *American Economic Review* **104**, 3434–58.