JOURNAL OF ECONOMIC THEORY 46, 34–44 (1988)

Maximin, Leximin, and the Protective Criterion: Characterizations and Comparisons*

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Received January 24, 1986; revised August 26, 1987

In this paper we characterize maximin, leximin, and the protective criterion. In particular, we provide a new proof for a classical characterization of maximin, we provide a new characterization of leximin, and we define and characterize a new variant of maximin: the protective criterion. The decision criteria are considered and compared in both a game agains nature framework and a social welfare framework. *Journal of Economic Literature* Classification Numbers: 024, 025, 026.

INTRODUCTION

The object of this paper is to define, to compare, and to characterize three variants of the maximin criterion. More specifically, we (1) provide a new proof for a classical characterization of maximin, (2) characterize leximin by a new set of axioms, and (3) define and characterize a new variant of maximin, to be called the protective criterion.

* This research was conducted while Barberà was visiting the Graduate School of Business at Stanford University. He wishes to acknowledge their hospitality as well as financial support from the Comisión Conjunta Hispano–Norteamericana para la Cooperation Cultural y Educativa. Jackson acknowledges the support of a National Science Foundation Graduate Fellowship and the Graduate School of Business at Stanford University. We thank Bhaskar Dutta, John Roberts, Hugo Sonnenschein, and Alain Trannoy for helpful discussions and comments. The insightful comments of two referees and the associate editor of *Journal of Economic Theory* led to a reevaluation of our results and their interpretations.

0022-0531/88 $3.00

We consider situations where different actions must be compared to each other in terms of their consequences, which are described by vectors of real numbers. Ranking actions is then formally equivalent to ranking vectors, and a decision criterion can be seen as a particular binary relation on the set of real vectors.[1] We shall evaluate decision criteria, and the axioms characterizing them, in the light of two alternative interpretations. The first interpretation comes from decision theory and is classically known as a game against nature (see [9] and [10]). The actions taken by a decision maker have consequences which may differ depending on what state of the world prevails; the ith component a_i of a vector a is interpreted as the utility for the agent if he takes action a and state i occurs. The second interpretation comes from the welfare economists' rendition of Rawls' ideas [12], [13]. An agent must evaluate different social arrangements from behind a "veil of ignorance" regarding what social position he will be assigned to. Then, the ith component a_i of a vector a represents the utility[2] that he will receive if social arrangement a prevails and he is assigned to the ith position in society.[3] (See also [5], [7], [8], [14], and [15]).

Actions (or social arrangements) are thus vectors of numbers. It is best to think of these numbers as Von Neumann–Morgenstern utilities, characterizing the agent's preferences with respect to actions (or social arrangements) if the probabilities of different states (or different positions in society) were known to him. Our problem is to rank actions (or social arrangements) when the agent does not know these probabilities. Given the complete parallel between our two interpretations, in the rest of the paper we refer to the decision theory framework.

The basic principle of maximin is to rank actions according to the utility levels which they guarantee to the agents involved. Guaranteed levels correspond to the smallest components of the vector describing the action. Within the present intepretation, only vectors of the same dimension are compared to each other.[4] A shortcoming of maximin is that it ranks

[1] Decision criteria could just select sets of actions without reference to any ranking of these. We concentrate here on those criteria where choices of actions derive from some ranking.

[2] As one referee pointed out, Rawls uses vectors of "primary goods," not of utilities, to describe the consequences of different social arrangements. What we describe here is thus the economists' version of Rawls.

[3] A third interpretation would also be possible, again from welfare economics in the tradition of Arrow. Here, the consequences of an action do not depend on chance, each action has a different impact on the welfare of the n members of society, and the ith component a_i of action a represents the utility that individual i will receive if action i is taken. However, several of the axioms we use in this paper would not be generally acceptable under such a broad interpretation, and we will no longer refer to it.

[4] This assumes that we want to compare different courses of action that apply to one society of fixed size, or to a decision situation with well-defined contingent states of the world. For a different perspective, see Pattanaik and Peleg [11].

as indifferent vectors (of the same dimension) with the same guaranteed utility levels, making no use of additional available information. The lexicographical version of maximin (leximin) engages in many more comparisons, based on two pieces of additional information. The first is that when minimum guaranteed levels agree, leximin takes into account the number of components of each action where the minimum is realized. If this does not provide a strict comparison, leximin uses the additional information of the next levels of utility that actions guarantee when the worst possible contingencies do not occur (in the welfare interpretation, they may give different utilities to those individuals which are just above the strictly worse off). This leads to lexicographic comparisons (see [4], [6], and [11]). The protective criterion we propose here is intermediate (see [2] and [3]). It also relies on lexicographic comparions at different utility levels, but it avoids comparisons based solely on frequency (without reference to state), since we feel that these are not in the spirit of maximin.

The paper is organized as follows. In Section 1 we define maximin, leximin, and the protective criterion, we propose different axioms on decision criteria, and we characterize the above by combinations of these axioms. Section 2 discusses the plausibility of the criteria.

1. Notations, Definitions, and the Characterization Results

Let $\mathbb{R}$ denote the set of real numbers and let $\mathscr{R}$ denote the set of finite-dimensional vectors of real numbers. The following notation is adopted. For $x \in \mathscr{R}$, dx is the dimension of x, x_i is the ith component of x, and x_{-i} is the $(dx-1)$-dimensional vector obtained from x by deleting its ith component; when convenient, we write $x = (x_i, x_{-i})$. I denotes the set of positive integers. For $a \in \mathbb{R}$, $x \in \mathscr{R}$, let $J(a, x) = \{i \in I \mid x_i \leqslant a\}$, and let $|J(a, x)|$ denote the cardinality of $J(a, x)$.

For $\mathscr{T} \subseteq \mathscr{R} \times \mathscr{R}$, a binary decision criterion on $\mathscr{T}$ is a subset $\succ$ of $\mathscr{T}$. If $(x, y) \in \succ$, we write $x \succ y$. The maximin, leximin, and protective criteria are binary decision criteria on $\mathscr{R} \times \mathscr{R}$. They are defined as follows:

The Maximin Criterion $\succ_{Mm}$.

$$x \succ_{Mm} y \leftrightarrow [dx = dy \;\&\; \exists a \in \mathbb{R} \ni (J(a, x) = \varnothing \;\&\; J(a, y) \neq \varnothing)].$$

The Leximin Criterion $\succ_{Lm}$.

$$x \succ_{Lm} y \leftrightarrow [dx = dy \;\&\; \exists a \ni (|J(a, x)| < |J(a, y)| \;\&\; (\forall b < a)\; |J(b, x)| = |J(b, y)|)].$$

The Protective Criterion $\succ_P$.

$$x \succ_P y \leftrightarrow [dx = dy \;\&\; \exists a \ni (J(a, x) \subset J(a, y) \;\&\; (\forall b < a)\, J(b, x) = J(b, y))],$$

where $\subset$ denotes proper inclusion.

The Axioms.

(0) $x \succ y \rightarrow dx = dy$.

This axiom states that only actions of the same dimension are to be compared. It is consistent with our interpretation that actions to be compared only differ by their consequences for the same possible states of the world.

(1) $\succ$ is irreflexive, transitive, and asymmetric.

This axiom leads to the interpretation that $x \succ y$ means "x is better than y," and requires that "x is better than y" and "y is better than z" imply "x is better than z." When (1) holds, we define the indifference relation $\sim$ by

$$x \sim y \leftrightarrow [dx = dy, \text{ not } x \succ y \text{ and not } y \succ x]$$

and the relation $\succcurlyeq$ by

$$x \succcurlyeq y \leftrightarrow [x \succ y \text{ or } x \sim y].$$

Thus, when x and y have the same dimension, but none is better than the other, we declare them indifferent, and $x \succcurlyeq y$ means that "x is better than or indifferent to y." The relation $\succcurlyeq$ is not complete because vectors of different sizes are not compared.

(2) (Symmetry) Let $dx = dy = n$, and let σ be a permutation of $\{1, ..., n\}$.

Then, $x \succ y \leftrightarrow x' \succ y'$, where $(\forall i \in (1 \cdots n\})\; x'_i = x_{\sigma(i)}$ and $y'_i = y_{\sigma(i)}$. This axiom states that the labeling of states of the world should not matter in the ranking of actions.

(3) (Domination) $x \geqslant y \rightarrow x \succ y$.

If the consequences of action x are always at least as good as those of action y, and sometimes better, then x is preferred to y.

(4) (Independence of Duplicated States).

Let x, y be such that, for some $i, j \in I$, $x_i = x_j$ and $y_i = y_j$. Then

$$x \succ y \leftrightarrow x_{-i} \succ y_{-i} \leftrightarrow x_{-j} \succ y_{-j}.$$

This axiom captures the notion that the agent is ignorant of the probability that a given state will occur. If the consequences of action x in states i and j are identical, and the consequences of action y in the same two states are also identical, the axiom declares the distinction between these two states to be irrelevant.[5] (See [1] and [8].)

(5) (Convexity) Whenever $dx = dy = dz$

(a) $[x \succ z \;\&\; y \succ z] \Rightarrow \frac{1}{2}(x+y) \succ z$

(b) $[x \succcurlyeq z \;\&\; y \succcurlyeq z] \Rightarrow \frac{1}{2}(x+y) \succcurlyeq z.$

Convexity embodies a notion of risk-aversion: averaging actions, thus increasing guaranteed payoffs, leads to new actions that are better than the original.

(6) (Independence of Identical Consequences; The Sure-Thing Principle)

If $x_i = y_i$, $[x \succ y \leftrightarrow x_{-i} \succ y_{-i}]$.

This axiom requires that, when comparing two actions, only the states of the world for which these actions have different consequences should count. Notice that the sure-thing principle convenitionally relates preferences on lotteries, whereas we do not have probabilities. Moreover, our axiom relates preferences between vectors of a given dimension with preferences between vectors of a different dimension. However, we think that the present "Independence of Identical Consequences" is in the same spirit as the conventional sure-thing principle: if two actions have some common consequences, it is not these consequences, but rather those which are different from one action to the other, which determine the ranking between the two.

(7) (Continuity) Let x^k, y^k be sequences of vectors.

$$[(\forall k)\, x^k \succ y^k \;\&\; x^k \to x \;\&\; y^k \to y] \to x \succcurlyeq y.$$

(8) (Shuffling) Let $dx = dy = n$ and let σ, ρ be permutations of $\{1, ..., n\}$. Then, $x \succ y \leftrightarrow x' \succ y'$, where $(\forall i \in \{1 \cdots n\})$ $x'_i = x_{\sigma(i)}$ and $y'_i = y_{\rho(i)}$.

Let $\mathcal{S} = \{(x, y) \in \mathcal{R} \times \mathcal{R} \mid \min_i\{x_i\} \neq \min_j\{y_j\}\}$. $\mathcal{S}$ is the set of pairs of vectors whose minima are not identical. It is easy to check that our three decision criteria coincide on this set: any two vectors of the same dimen-

[5] As pointed out be a referee, for Axioms (4) and (6) to make sense, it is useful to have the Symmetry Axiom (2) hold. Otherwise, ambiguities could arise when relabeling the states after deleting or adding components to vectors.

sion, the one with the largest minimum is preferred. The following lemma characterizes these criteria on $\mathscr{S}$.

LEMMA 1. *The maximin, leximin, and protective criteria agree on* $\mathscr{S}$. *They are characterized (on* $\mathscr{S}$*) by Axioms* (0)–(5) *(on* $\mathscr{S}$*).*

Proof. We first prove that, for $a, b, c, d \in \mathbb{R}$,

(i) $[b > a, c > a, d > a] \rightarrow [(c, d) \succ (a, b)]$.

Suppose not. Then, $(a, b) \succcurlyeq (c, d)$. Let $\varepsilon \in (0, \infty)$ be such that $b > a + \varepsilon$, $c > a + \varepsilon$, and $d > a + \varepsilon$. By (3), $(c, d) \succ (a + \varepsilon, a + \varepsilon)$, and then by (1), $(a, b) \succcurlyeq (a + \varepsilon, a + \varepsilon)$. By (4), $(a, a, b) \succcurlyeq (a + \varepsilon, a + \varepsilon, a + \varepsilon)$. By (2), $(a, b, a) \succcurlyeq (a + \varepsilon, a + \varepsilon, a + \varepsilon)$. By (5), $(a, (a + b)/2, (a + b)/2) \succcurlyeq (a + \varepsilon, a + \varepsilon, a + \varepsilon)$. By (4), $(a, (a + b)/2) \succcurlyeq (a + \varepsilon, a + \varepsilon)$. Repeating this argument n times we find

$$\left(a, \frac{(2^n - 1)\, a + b}{2^n}\right) \succcurlyeq (a + \varepsilon, a + \varepsilon).$$

Note that, as $n \rightarrow \infty$,

$$\frac{(2^n - 1)\, a + b}{2^n} \rightarrow a.$$

Since $a + \varepsilon > a$ we can find N large enough so that $((2^N - 1)\, a + b)/2^N < a + \varepsilon$. For this N,

$$\left(a, \frac{(2^N - 1)\, a + b}{2^N}\right) \succcurlyeq (a + \varepsilon, a + \varepsilon),$$

which contradicts Domination (3). This contradiction establishes (i).

Now, consider a pair of actions in $\mathscr{S}$, x and y, with $dx = dy$. Let $a = \min_i\{x_i\}$ and $b = \min_i\{y_i\}$. Assume without loss of generality that $b > a$, Let $d = (2b + a)/3$ and $e = (b + 2a)/3$. Then, by Domination (3), $y \succ (d, d, ..., d)$. Let $c = \max\{x_i\}$. By Symmetry (2), we can assume without loss of generality that $a = x_1$. Then, by Domination (3), $(e, c, ..., c) \succ x$. Now, by (i), $(d, d) \succ (e, c)$. Then, by (4), $(d, d, ..., d) \succ (e, c, ..., c)$. Thus, by transitivity (1), $y \succ x$.

We have thus shown that, if $dx = dy$, and $\min_i\{y_i\} > \min_i\{x_i\}$, then $y \succ x$. This provides an ordering of all pairs $(x, y) \in \mathscr{S}$ such that $dx = dy$, coinciding with maximin, leximin, and the protective criterion on $\mathscr{S}$. ∎

Even though our criteria agree on $\mathscr{S}$, they may differ on the way they rank vectors of the same dimension having identical minima. The following theorems characterize each of the criteria separately. (See Table I for a listing of the relation between each of the axioms and the decision criteria, both on $\mathscr{S}$ and $\mathscr{R} \times \mathscr{R}$.)

THEOREM 1. *Maximin is characterized by Continuity* (7) *and* (0) *on* $\mathscr{R} \times \mathscr{R}$ *and Axioms* (1)–(5) *on* $\mathscr{S}$.

Remarks. Notice that Axioms (0)–(5) are only required to hold on $\mathscr{S}$. All but domination are also satisfied on $\mathscr{R} \times \mathscr{R}$. For example, maximin does not rank (0, 1) above (0, 0) when it is very compelling to do so. However, maximin still respects a weaker version of dominance, namely that $x_i > y_i$ for all i implies that $x \succ y$. This weaker property, along with the rest of the axioms, is the one used in Milnor's [10] characterization. In spite of the close similarity between the two axiom systems, the following proof is quite different from his.

Proof. Let x and y be such that $dx = dy$ and $\min_i\{x_i\} = \min_i\{y_i\}$. Let $\{\varepsilon^k\}$ be a sequence of strictly positive real numbers, tending to zero.

Let

$$x_+^k = (x_1 + \varepsilon^k, x_2 + \varepsilon^k, ..., x_n + \varepsilon^k)$$

and

$$x_-^k = (x_1 - \varepsilon^k, x_2 - \varepsilon^k, ..., x_n - \varepsilon^k).$$

For all k, the pairs (x_+^k, y) and (x_-^k, y) belong to $\mathscr{S}$. Thus, by Lemma 1, $x_+^k \succ y$ and $y \succ x_-^k$. By Continuity, $x \succcurlyeq y$ and $y \succcurlyeq x$. Thus, by (1), neither $x \succ y$ nor $y \succ x$. It is easily verified that maximin satisfies the stated properties. ∎

TABLE I

	Maximin		Leximin		Protective criterion	
Axiom	$\mathscr{S}$	$\mathscr{R} \times \mathscr{R}$	$\mathscr{S}$	$\mathscr{R} \times \mathscr{R}$	$\mathscr{S}$	$\mathscr{R} \times \mathscr{R}$
(0) Dimension	+[a]	++	+	++	+	++
(1) Trans., Assym.	++	+	++	+	+	++
(2) Symmetry	++	+	++	+	+	++
(3) Domination	++	−	++	+	+	++
(4) Duplicated State	++	+	++	−	+	++
(5) Convexity	++	+	++	+	+	++
(6) Sure-Thing	+	−	+	++	+	++
(7) Continuity	+	++	−	−	−	−
(8) Shuffling	+	+	+	++	+	−

[a] (+) satisfies; (−) does not satisfy; (++) is characterized by.

THEOREM 2. *Leximin is characterized by the Sure-Thing Principle* (6), *Shuffling* (8) *and* (0) *on* $\mathscr{R} \times \mathscr{R}$, *and Axioms* (1)–(5) *on* $\mathscr{S}$.

Proof. It is easily checked that the leximin criterion satisfies the required properties. Now, let $\succ$ be any relation on $\mathscr{R} \times \mathscr{R}$ satisfying the axioms. No pair of vectors is in the relation unless they are of the same dimension, by Axiom (0). Consider x and y such that $dx = dy = n$. By Axiom (8) (Shuffling), we can assume without loss of generality that $x_i \leqslant x_{i+1}$ and $y_i \leqslant y_{i+1}$, for $i \in \{1 \cdots n\}$. Eliminate all components i, where $x_i = y_i$, and let x', y' denote the resulting vectors. By the Sure-Thing Principle (6), $x \succ y \leftrightarrow x' \succ y'$. By construction, $(x', y') \in \mathscr{S}$. Since Shuffling (8) implies Symmetry (2), Lemma 1 applies to x', y', an thus $x' \succ y' \leftrightarrow \min_i\{x'_i\} > \min_i\{y'_i\}$. But,

$$\min_i \{x'_i\} > \min_i \{y'_i\} \leftrightarrow [\exists a \ni (|J(a, x)| < |J(a, y)|$$
$$\& \; (\forall b < a) \; |J(b, x)| = |J(b, y)|)].$$

Thus, $\succ$ is the leximin criterion. ∎

THEOREM 3. *Axioms* (0)–(6) *on* $\mathscr{R} \times \mathscr{R}$ *characterize the protective criterion.*

Remark. Although Axiom (1), which requires the transitivity of the strict relation $\succ$, is satisfied by the protective criterion, the relation $\succcurlyeq_P$ implies by the protective criterion is not fully transitive. For example, $(0, 1) \succcurlyeq_P (1, 0)$ and $(1, 0) \succcurlyeq_P (0, 2)$, however, $(0, 2) \succ_P (0, 1)$.

Proof. It is easily checked that the protective criterion satisfies Axioms (0)–(4) and (6).

We now verify that the protective criterion satisfies Axiom (5), convexity. Suppose $x \succ_P z$ and $y \succ_P z$. Eliminate all states where all three actions agree. Denote the resulting vectors x', y', and z'. By Axiom (6), $x' \succ_P z'$ and $y' \succ_P z'$. By the definition of the protective criterion we know that for any i where z' does not achieve its minimum, $x'_i > \min_j\{z'_j\}$ and $y'_i > \min_j\{z'_j\}$, which implies $\frac{1}{2}(x'_i + y'_i) > \min_j\{z'_j\}$. For any i where z' achieves its minimum, we know that $x'_i \geqslant z'_i$ and $y'_i \geqslant z'_i$ and since we have eliminated all columns where the three actions agree, then either $x'_i > z'_i$ or $y'_i > z'_i$. Thus $\frac{1}{2}(x'_i + y'_i) > z'_i$. So $\frac{1}{2}(x'_i + y'_i) > \min_j\{z'_j\}$ for all i and therefore $\frac{1}{2}(x' + y') \succ_P z'$. By Axiom (6), $\frac{1}{2}(x + y) \succ_P z$. A similar argument holds for $\succcurlyeq_P$.

Now, let $\succ$ be any relation on $\mathscr{R} \times \mathscr{R}$ satisfying the axioms, and we must show that it is $\succ_P$. No pair of vectors is in the relation unless they are of the same dimension, by Axiom (0). We first prove some facts about two-dimensional vectors.

(ii) Let $a, b \in \mathbb{R}$, $a > b$. Then, neither $(a, b) \succ (b, a)$ nor $(b, a) \succ (a, b)$. This follows directly from Symmetry, Transitivity, and Irreflexivity.

(iii) Let $a, b, c \in \mathbb{R}$, $a > b$, $c > b$. Then, neither $(a, b) \succ (b, c)$ nor $(b, c) \succ (a, b)$.

If $a = c$, (iii) is (ii). Thus, we assume $a \neq c$, and let $c > a$ without loss of generality.

Suppose $(a, b) \succ (b, c)$. By Domination (3), $(c, b) \succ (a, b)$; by Transitivity (1), $(c, b) \succ (b, c)$. This contradicts (ii). Suppose $(b, c) \succ (a, b)$. By the Sure-Thing Principle (6), $(b, b, c) \succ (b, a, b)$. By Symmetry (2), $(c, b, b) \succ (b, a, b)$. By convexity, $((b+c)/2, b, (b+c)/2) \succ (b, a, b)$. By Axiom (4), $(b, (b+c)/2) \succ (a, b)$. Repeating the argument iteratively we find that $(b, ((2^n - 1)\, b + c)/2^n) \succ (a, b)$, for all n. For N large enough, $((2^N - 1)\, b + c)/2^N < a$, and thus, by Domination (3), $(b, a) \succ (b, ((2^N - 1)\, b + c)/2^N)$. By Transitivity (1), $(b, a) \succ (a, b)$, contradicting (ii). This completes the proof of (iii).

We now extend fact (iii) to cover comparisons of vectors of any size.

(iv) Let x, y be such that $dx = dy$, $\min_i\{x_i\} = \min_i\{y_i\} = m$, and $\exists i, j$ such that $x_i = m$, $y_i > m$, $y_j = m$. Then, neither $x \succ y$ nor $y \succ x$.

By Symmetry, we can assume $i = 1$, $j = 2$. Suppose $x \succ y$. Let $M = \max_i\{x_i\}$. By Domination (3), $(m, M, ..., M) \succ x$ or $(m, M, ..., M) = x$. Likewise, $y \succ (y_1, m, ..., m)$ or $y = (y_1, m, ..., m)$. By Transitivity (1), $(m, M, ..., M) \succ (y_1, m, ..., m)$, and this would imply $(m, M) \succ (y_1, m)$, by (4), contradicting (iii). Therefore, $x \succ y$ cannot hold. A similar argument would prove that $y \succ x$ is also impossible.

Now consider an two vectors x and y, with $dx = dy$. Eliminate all components i where $x_i = y_i$, and let x', y' denote the resulting vectors. By the Sure-Thing Principle, $x \succ y \leftrightarrow x' \succ y'$. If $\min_i\{x'_i\} = \min_i(y'_i\}$, (iv) applies and neither $x' \succ y'$ nor $y' \succ x'$. Thus, in this case, neither $x \succ y$ nor $y \succ x$. If $\min_i\{x'_i\} \neq \min_i\{y'_i\}$, $x' \succ y'$ or $y' \succ x'$ depending on which of the minima is greater, and thus either $x \succ y$ or $y \succ x$. Since $\min_i\{x'_i\} \neq \min_i\{y'_i\}$ iff $\exists a \ni [J(a, x) \subset J(a, y)\ \&\ (\forall b < a)\ J(b, x) = J(b, y)]$, $\succ$ coincides with the protective criterion. ∎

2. A Comparison of the Criteria

Before comparing the three variants of maximin discussed here, let us notice their common features and discuss their merits. Lemma 1 brings out these common features. Whenever we compare vectors with different minimum components, the criteria are characterized by Axioms (0)–(5).

We find these axioms to be very compelling. Axioms (3)–(5) deserve special comment. Axiom (3) is clearly attractive in both interpretations, and in the welfare context it is just a Paretian criterion. Independence of Duplicated States (Axiom (4)) is extremely appropriate in a context of uncertainty. If the agent is completely ignorant about the probabilities that the different states of nature will occur or that he will be assigned to a certain position in a given society, the axiom conveys this notion of ignorance. For, if the number of states having identical consequences for each action did matter, implicit considerations about the frequency of occurrence of these states would be influencing the ranking. Convexity represents a restriction that we may or may not want to impose on a decision criterion, but it has a definite meaning as a condition of risk aversion.

We turn now to the differences among the three criteria. If continuity is desired, then the classical version of maximin emerges. Several of the axioms that hold on $\mathscr{S}$ are also respected on all of $\mathscr{R} \times \mathscr{R}$, but not Domination (3). Thus, for example, (0, 1) is not ranked better than (0, 0) by maximin, and yet it seems intuitively obvious that it should be, under any reasonable interpretation. Maximin violates domination on this larger domain because it fails to satisfy the Sure-Thing Principle (6); this is probably the most important of the axioms involved in the axiomatization of expected utility that is not satisfied by classical maximin, and it is one that both leximin and the protective criterion will meet.

Leximin has been the better studied extension of maximin, expecially as an attractive social welfare functional in the spirit of Rawls' notion of justice. The axiomatization that we provide here is a simple one, and it differs from the ones we have found in the literature. Once again, several of the axioms that are only required on $\mathscr{S}$ would also carry on to $\mathscr{R} \times \mathscr{R}$, but not all. For example, $(3, 1) \succ_L (1, 2)$, yet $(1, 2, 2) \succ_L (3, 1, 1)$. Thus, leximin violates Independence of Duplicated States on $\mathscr{R} \times \mathscr{R}$. This violation is quite disturbing since this axiom expresses the idea of ignorance about the probability with which states of the world or social positions will occur, and violating it amounts to admitting considerations about frequencies of states.

The protective criterion is proposed here as one way of extending maximin while respecting at the same time two important axioms that the preceding criteria do not jointly satisfy. The protective criterion satisfies the Sure-Thing Principle, and at the same time still meets Independence of Duplicated States. Thus, it captures the notion of absolute ignorance about the probabilities that each state of the world would occur, while respecting domination and the rest of the requirements. It extends maximin, but not as far as leximin. It is, we feel, the "right" extension of maximin in the decision-theory framework and in the economists' version of Rawls' idea of justice.

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